(a) Initialization:

At $k=0$,

we have $N_0(S_t)=0$, $G_0(S_t)=0$, $V_0(S_t)=0$, $W_0(S_t)=0$

$$V_0(S_t)=W_0(S_t)=0$$

At $k=1$:

we have $N_1(S_t)=1$, $G_1=g_t^{(1)}$, $V_1=\frac{G_1}{N_1}=g_t^{(1)}$, $W_1=\frac{g_t^{(1)}}{N_1}=g_t^{(1)}=V_1$

So $V_k=W_k$ holds for $k=0, k=1$, $V_k=\frac{G_k}{N_k}$ holds for $k=1$

Maintanence:

Suppose at $k=i$, we have $V_i(S_t)=W_i(S_t)$, $V_i(S_t)=\frac{G_i(S_t)}{N_i(S_t)}$

At $k=i+1$:

$$N_{i+1}(S_t)=N_i(S_t)+1$$

$$G_{i+1}(S_t)=G_i(S_t)+g_t^{(i+1)}$$

$$V_{i+1}(S_t)=G_{i+1}(S_t)/N_{i+1}(S_t)$$

$$=(G_i+g_t^{(i+1)})/(N_i+1)$$

$$V_{i+1}N_i+V_{i+1}=G_i+g_t^{(i+1)}$$

$$V_{i+1}+V_{i+1}/N_i=V_i+g_t^{(i+1)}/N_i$$

$$V_{i+1}=\frac{N_i}{N_i+1}\left(V_i+\frac{1}{N_i}g_t^{(i+1)}\right)$$

$$=\frac{N_i}{N_i+1}V_i+\frac{1}{N_i+1}g_t^{(i+1)}$$

$$W_{i+1}(S_t)=W_i(S_t)+\frac{1}{N_{i+1}(S_t)}(g_t^{(i+1)}-W_i(S_t))$$

$$=V_i+\frac{1}{N_i+1}(g_t^{(i+1)}-V_i)$$

$$=\frac{N_i}{N_i+1}V_2+\frac{1}{N_i+1}g_t^{(i+1)}=V_{i+1}(S_t)$$

Since the assumption holds for $k=1$, by induction,

$W_k(S_t)=V_k(S_t)$ holds for $k\geq 1$

In all, $W_k(S_t)=V_k(S_t)$ holds for $k\geq 0$