Mathematisch-Naturwissenschaftliche Fakultät
Institut für Informatik, Anja Rey

Exercise Sheet 6

for the lecture on

Advanced Programming and Algorithms

Submission until **Monday, 4th December, 12:30 pm**.

Discussion in the exercise classes on 11th, 12th, and 15th December, 2023.

Problem 1 to hand in: *Data Structure*

Describe a data structure that can store m values together with an update history and undo function.

The following methods should be mananged by the data structure:

- `initialise(`m`)`: set up an empty instance that can manage m values (that are initially set to 0) in $\mathcal{O}(m)$,
- `update(`i`, `x`)`: change the ith value to x in $\mathcal{O}(1)$,
- `get(`i`)`: return the current ith value $\mathcal{O}(1)$,
- `undo()`: reset the latest update in $\mathcal{O}(1)$ (a redo is not needed),
- `reset()`: reset all values to 0 and delete the update history in $\mathcal{O}(1)$,
- `count_undo()`: count the number of available updates that can be undone in $\mathcal{O}(1)$.

Describe each of these methods (intuitively or in pseudocode) and explain why their running times are fulfilled.

Problem 2 as a programming exercise: *Compound Data Types and Numpy*

Work on the current jupyter notebooks `lecture_06_compound_types` and `lecture_06_numpy` in order to learn how to use collective types such as `list` and `set` as well as numpy arrays.

Problem 3 for discussion: *Set Problem*

Design an algorithm to solve the following problem: Given a set $U = \{1, \ldots, m\}$ and given sets $S_1, S_2, \ldots, S_n$ each of which is a subset of U, is there a pair of sets S_i, S_j that cover U completely, i.e., $S_i \cup S_j = U$?

Problem 4 for discussion: *Universal Hashing*

Let U be a set of possible keys, $S \subseteq U$ be an arbitrary set of currently used keys with $|S| = n$. Moreover, let H be a universal family of hash functions mapping from U to $T = \{0, \ldots, m-1\}$, $m \leq |U|$.

Show that, for $h \in H$ chosen uniformly, the total number of collisions is less than $\frac{n(n-1)}{m}$ with probability at least $\frac{1}{2}$.

Hint: Use an indicator random variable $X_{xy}(h)$ that is 1 if $h(x) = h(y)$, and 0 otherwise. Show that the expected number of collisions is $\frac{n(n-1)}{2m}$. Then use Markov's inequality.