Exercise 02

Saturday, November 4, 2023 3:48 PM

Task 3. Variance (2 points)

Show that the variance of a data set with n entries can be calculated using the following formula:

$$Var(X) = E(X^2) - E(X)^2 \tag{1}$$

$X \in \mathbb{R}^n \quad E(X), Var(X) \in \mathbb{R}$

$$\begin{aligned}
Var(X) &= \frac{1}{n}\sum_{i=1}^{n}(x_i - E(X))^2 \\
&= \frac{1}{n}\sum_{i=1}^{n}(x_i^2 - 2x_i E(x) + E(x)^2) \\
&= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - \frac{2}{n}\sum_{i=1}^{n}(x_i E(x)) + \frac{1}{n}\sum_{i=1}^{n}(E(X))^2 \\
&= E(X^2) - \frac{2}{n}E(X)\sum_{i=1}^{n}x_i + \frac{1}{n}E(X)^2\sum_{i=1}^{n}1 \\
&= E(X^2) - 2E(X)^2 + E(X)^2 = E(X^2) - E(X)^2 \quad \square
\end{aligned}$$