Reinforcement Learning
Prof. Milica Gašić

Release: Thursday, 31 October 2024
Discussion: Wednesday, 6 November 2024

Exercise set #4

You do not have to hand in your solutions to the exercises and they will **not** be graded. However, there will be four short tests during the semester. You need to reach ≥50% of the total points in order to be admitted to the final exam (Klausur). The tests are held at the start of a lecture (room 2522.U1.74) at the following dates:

Test 1: Thursday, 31 October 2024, 10:30-10:45
Test 2: Thursday, 21 November 2024, 10:30-10:45
Test 3: Thursday, 5 December 2024, 10:30-10:45
Test 4: Thursday, 9 January 2025, 10:30-10:45

Please ask questions in the RocketChat
The exercises are discussed every Wednesday, 14:30-16:00 in room 2512.02.33.

1. **Monte Carlo prediction**

   (a) In Monte Carlo prediction the value function is estimated with the average return of collected episodes. Given an observed state s_t and return g_t, there are multiple ways to calculate the new value estimate:

   (i) $N_{k+1}(s_t) = N_k(s_t) + 1$
   $G_{k+1}(s_t) = G_k(s_t) + g_t$
   $V_{k+1}(s_t) = G_{k+1}(s_t)/N_{k+1}(s_t)$
   with $N_0(s_t) = 0, G_0(s_t) = 0, V_0(s_t) = 0$

   (ii) $N_{k+1}(s_t) = N_k(s_t) + 1$
   $W_{k+1}(s_t) = W_k(s_t) + \frac{1}{N_{k+1}(s_t)}(g_t - W_k(s_t))$
   with $N_0(s_t) = 0, W_0(s_t) = 0$

   Show that both approaches are equivalent, i.e., show by induction for all $k \geq 0$

   $$V_k(s_t) = W_k(s_t).$$

   Answer:
   Base case, $k = 0$:

   $$V_0(s_t) = 0 = W_0(s_t)$$

   Induction step, $k \to k + 1$:

Exercises by Stefan Harmeling, used with permission

(we already know that $V_k(s_t) = W_k(s_t)$)

$$\begin{aligned}
V_{k+1}(s_t) &= G_{k+1}(s_t)/N_{k+1}(s_t) \\
&= (G_k(s_t) + g_t)/N_{k+1}(s_t) \\
&= (G_k(s_t)/N_k(s_t) \cdot N_k(s_t) + g_t)/N_{k+1}(s_t) \\
&= (V_k(s_t)N_k(s_t) + g_t)/N_{k+1}(s_t) \\
&= (W_k(s_t)N_k(s_t) + g_t)/N_{k+1}(s_t) \\
&= (W_k(s_t)(N_{k+1}(s_t) - 1) + g_t)/N_{k+1}(s_t) \\
&= (W_k(s_t)N_{k+1}(s_t) - W_k(s_t) + g_t)/N_{k+1}(s_t) \\
&= W_k(s_t) + \frac{1}{N_{k+1}(s_t)}(g_t - W_k(s_t)) \\
&= W_{k+1}(s_t)
\end{aligned}$$

(b) Implement Monte Carlo prediction for tic-tac-toe with random moves. Follow the instructions in the Jupyter notebook `monte-carlo-prediction.ipynb`. What is the probability that the first player wins? Which initial action has the highest chance of winning?

2. **Monte Carlo control**

Implement Monte Carlo control and apply it to the Maze environment from the lecture. Follow the instructions in the Jupyter notebook `monte-carlo-control.ipynb`.