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Heinrich Heine Universität Düsseldorf
Lehrstuhl Finanz- und Versicherungsmathematik

Summer semester 2024

Markov Chains

Problem sheet 2

random walks

Problems to be discussed (in parts) during the exercise sections:

Problem 1
Let $(S_n)_{n\geq 0}$ be a simple random walk starting at 0 ($S_0 = 0$) and with $p = 0.4$ and $q = 1 - p = 0.6$. Compute the following probabilities:

- $\mathbb{P}(S_4 = 2, S_m \neq 3 \text{ for all } m = 1, 2, 3)$,
- $\mathbb{P}(S_2 = 2, S_5 = 1)$,
- $\mathbb{P}(S_2 = 2, S_4 = 3, S_5 = 1)$.

Problem 2
Use the reflection principle to find the number of paths for a simple random walk from $S_0 = 2$ to $S_{10} = 6$ that hit the line $y = 1$

Problem 3
Let $(S_n)_{n\geq 0}$ be a simple symmetric random walk. Compute $\mathbb{P}(S_n = y | S_m = x)$ for the two cases $n > m$ and $n < m$.

Problems to be handed in by:
Thursday, May 9th, 11:59 p.m., online via Ilias.

Problem 1 *(12 points)*
Let$(S_n)_{n\geq 0}$ be a simple random walk starting at 0 with $p = 0.4$ and $q = 1-p = 0.6$. Compute the following probabilities:

- $\mathbb{P}(S_2 = 0, S_4 = 0, S_5 = -1)$,
- $\mathbb{P}(\{S_4 = 4\} \cup \{S_4 = -2\})$,
- $\mathbb{P}(M_{17} \leq -5, S_7 = -5)$, where $M_{17} = \min_{0\leq i\leq 17} S_i$.

Problem 2 *(6 points)*

For a simple symmetric random walk $(S_n)_{n=0,1,2,\dots}$ starting in 0 ($S_0 = 0$), show that

$$\mathbb{P}(S_4 = 0) = \mathbb{P}(S_3 = 1).$$

Problem 3 *(6 points)*

Use the reflection principle to find the probability $\mathbb{P}(M_8 = 6)$, where $M_8 = \max_{0\leq i\leq 8} S_i$ and $(S_n)_{n\geq 0}$ is a simple symmetric random walk starting in 0 ($S_0 = 0$).

Problem 4 *(6 points)*

In an election candidate A receives 200 votes while candidate B only receives 100. Assume that the probability of getting a vote is identical (50% each) for A and B. What is the probability that A is always ahead throughout the count?