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Math for AI
Winter term 2023/24

Exercise sheet 1

Exercise 1 (10 points)

(a) Show that the sets

$$B := \left\{ \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\} \subseteq \mathbb{R}^2 \qquad B' := \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \right\} \subseteq \mathbb{R}^3$$

are bases of $\mathbb{R}^2$, resp. $\mathbb{R}^3$.

[Hint: It is enough to show that the sets are linearly independent, because of Observation 1.1.13 in the manuscript – two linearly independent vectors in $\mathbb{R}^2$ automatically form a basis, same for three vectors in $\mathbb{R}^3$.]

(b) Consider the linear map

$$f\colon \mathbb{R}^2 \to \mathbb{R}^3, \quad \begin{pmatrix} a \\ b \end{pmatrix} \mapsto \begin{pmatrix} a - b \\ b + 2a \\ 3b + 2a \end{pmatrix}.$$

Compute the matrices ${}_{S'}M(f)_S$, ${}_{S'}M(f)_B$ and ${}_{B'}M(f)_B$ where S, resp. S', denotes the standard basis of $\mathbb{R}^2$, resp. $\mathbb{R}^3$.

[Remark: If you get some slightly ugly fraction like $\frac{8}{3}$ as matrix entry, don't doubt yourself: That actually happens. **Do not write floating point numbers – write fractions!**]

Exercise 2 (10 points)

Find a polynomial $p(x)$ of degree at most three that satisfies $p(0) = 1$, $p(1) = 1$, $p(2) = 0$, $p(-1) = 1$. Is there more than one such polynomial?

[Hint: A polynomial looks like $p(x) = ax^3 + bx^2 + cx + d$, and you are asked to find the coefficients a, b, c, d. The above values of the polynomial give you four linear equations with the variables a, b, c, d. Again, some slightly ugly fractions may occur.]

Deadline: Friday, October 20, 10:00.

For the exercise sheet please form a team of two or three persons and hand in the solutions together. Please upload your solution in a single .pdf file named as in the following scheme:

`FirstName1stPerson-LastName1stPerson_FirstName2ndPerson-LastName2ndPerson_SheetNr.pdf`.

(notice that there should be a "minus" between the parts of the name of a single person and an "underscore" between different persons and before the sheet number)

Upload your solution to this link.