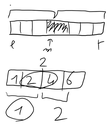
```
COST
1   m = r-l+1
1   DO_SOMETHING(A,l,r):
1   1  IF l==r THEN:
1   2  |  RETURN ∞
T(⌈m/2⌉) 3  m ← ⌊(r+l)/2⌋
T(⌊m/2⌋) 4  a ← DO_SOMETHING(A,l,m)
         5  b ← DO_SOMETHING(A,m+1,r)
1        6  c ← |A[m+1] - A[m]|
1        7  RETURN min{a,b,c}
```

A sorted array of integers $A[0..n-1]$ of distinct elements such that $A[0] < A[1] < \dots < A[n-1]$



b) $m = 2^k$

$$T(m) = \begin{cases} 2T\left(\frac{m}{2}\right) + O(1) & m > 1 \\ O(1) & m = 1 \end{cases}$$

M.T. $T(m) = \begin{cases} aT\left(\frac{m}{b}\right) + m^\beta & m > 1 \\ O(1) & m = 1 \end{cases}$

$\alpha > \beta \rightarrow T(n) \in O(m^\alpha)$
$\alpha = \beta \rightarrow T(n) \in O(m^\alpha \log m)$
$\beta < \alpha \rightarrow T(n) \in O(m^\beta)$

$\alpha = \log_b(a) = \log_2 2 = 1 > \beta = 0$

W. own Ex. $T(m) \in O(m)$

GUESS. $T(m) \in O(m)$

$\forall m \geq m_0, \exists c, b > 0$ s.t. $T(n) \leq cm - b$

$m = 1$

$T(1) = 1 \leq c + b \rightarrow c \geq 0 \rightarrow c = 10$

$\Leftrightarrow b \leq c + 1$

ASSUME $\forall k, 1 \leq k < m \quad T(k) \leq ck - b$

$T(m) = 2T\left(\frac{m}{2}\right) + 1$

$\overset{\text{IND. HP}}{\leq} 2\left(c\frac{m}{2} - b\right) + 1 \overset{!}{\leq} cm - b$

$-b + 1 \leq 0$

$b \geq 1$ $\quad b = 1$

$b = 1, c = 10$ $\quad T(m) \in O(m)$

c) CLAIM: DO_SOMETHING(A, l, r) returns the minimal difference between two consecutive entries and $0 \leq l \leq r \leq \text{lenght}(A) - 1$

$P(m)$ = this property when $m = r - l + 1$

d) We first use MERGESORT, QUICKSORT. They both run $O(m \log m)$. The overall time complexity increases to $O(m \log m)$

Ex. $m \rightarrow$ | 2 | 4 | 3 | 1 | $\rightarrow X = 3$

$i = 2$

return i if $\exists i$ s.t. $A[i] = X$

return -1 otherwise

runs in $O(\log m)$

```
BINARY_SEARCH(A, X, l, r):
  IF l == r:
    IF A[l] = X
      RETURN l
    ELSE
      RETURN -1
  m ← ⌊(r-l)/2⌋
  IF A[m] == X THEN:
    RETURN m
  ELSE IF A[m] < X THEN:
    RETURN BINARY_SEARCH(A, X, l, m-1)
  ELSE:
    RETURN BINARY_SEARCH(A, X, m+1, r)
```

$T(m) = \begin{cases} T\left(\frac{m}{2}\right) + O(1) & m \geq 1 \\ O(1) & m = 1 \end{cases}$

$a = 1 \quad \beta = 0$

$b = 2$

$\alpha = \log_2 1 = 0$

$\alpha = \beta \rightarrow T(n) \in O(\log m)$

	LEVEL	COST PER LEVEL
m	K	$1 = 1$
$\frac{m}{2}$	K-1	1
$\frac{m}{2^i}$	K-i	1
1	0	1

$T(m) = 1 + T\left(\frac{m}{2}\right)$

0 · 1 2 · K

$1 + 1 + \dots + 1$

$T_{\text{TOTAL COST}} = \sum_{i=0}^{k} \underbrace{1}_{\text{COST FOR EACH LEVEL}} = k + 1 = \log_2 m + 1$

$m = 2^k$

$k = \log_2 m$

$T(m) \in O(\log m)$

BASE CASE $m = 1$, $l = r = 1$

l1. → TRUE

l2. returns ∞

ASSUME $\forall K. \ 1 \leq k < m$

$P(K)$ HOLDS

IT. $P(m)$

l1. → FALSE, $m > 1 \Rightarrow r \neq l$

$m = r - l + 1$

$m = 1$ ↯

l4. $a \leftarrow$ DO_SOMETHING(A, l, m)

$m = \left\lfloor \frac{r+l}{2} \right\rfloor$

$m - l + 1 = \left\lfloor \frac{r+l}{2} \right\rfloor - l + 1 \leq \frac{r+l}{2} - l + 1 = \frac{r + l - 2l + 2}{2} = \frac{r - l + 2}{2} = \frac{m+1}{2} < m$

$m = r - l + 1$, $m > 1$

$\overset{\text{INDUCTIVE HP}}{a}$ IS THE MINIMAL DISTANCE IN $A[l, \dots, m]$

l5. $b \leftarrow$ DO_SOMETHING$(A, m+1, r)$

$r - (m+1) + 1 = r - \left(\left\lfloor \frac{r+l}{2} \right\rfloor + 1\right) + 1$

$= r - \left\lfloor \frac{r+l}{2} \right\rfloor - 1 \leq r - \frac{r+l}{2} + 1 - 1 \leq \frac{2r - r - l + 2}{2} = \frac{r - l + 2}{2} = \frac{m+1}{2} < m$

$m > 1$

b IS THE MINIMAL DIFFERENCE ... IN $A[m+1, \dots, r]$

l6. $c \leftarrow |A[m+1] - A[m]|$

l7. $\min\{a, b, c\}$ IS THE MINIMAL DISTANCE BETWEEN TWO NEIGHBORING ELEMENTS OF $A[l, \dots, r]$ ✓

$$T(m) = \begin{cases} 2T\left(\frac{m}{2}\right) + m & m > 1 \\ 1 & m = 1 \end{cases}$$

$m = 2^k$

k

COST PER LEVEL

	# CALLS	COST PER CALL (EXCLUDING RECURSIVE CALLS)
m	1	1
$\frac{m}{2}$	2	1
$\frac{m}{4}$	4	1
$\frac{m}{2^i}$	2^i	1
1	2^k	1

$T(m) = \sum_{i=0}^{k} \underbrace{2^i \cdot 1}_{i\text{th LEVEL COST}} = 2^0 + \sum_{i=1}^{k} 2^i = 1 + \frac{1 - 2^k}{1 - 2}$

$= 1 - 1 + 2^k$

$= 2^k = m = O(m)$

$$T(m) = \begin{cases} 1 & m = 1 \\ 2T(\sqrt{m}) + \log m & m > 1 \end{cases}$$

WITH M.T.

$$T(n) = \begin{cases} 1 & n = 1 \\ aT\left(\frac{n}{b}\right) + n^\beta & \end{cases}$$

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