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Math for AI
Winter term 2023/24

Exercise sheet 2

Exercise 3 (8 points, eigenvalues will be discussed on Tuesday)

Compute the eigenvalues of the following matrix (4 points):

$$A = \begin{pmatrix} 2 & 12 & 17 \\ 0 & 0 & 3 \\ 0 & 2 & -1 \end{pmatrix}$$

For each eigenvalue find a basis for its space of eigenvectors (4 points).

Exercise 4 (10 points)

Let A be an $n \times m$-matrix (i.e. n rows, m columns) of rank n.

(a) (3 points) Show that $x \mapsto A^T x$ is an injective map.

(b) (5 points) Show that $A \cdot A^T$ is an invertible $n \times n$-matrix.

(c) (2 points) Conclude from (b) that if B is an $n \times m$-matrix with m linearly independent columns, then $B^T \cdot B$ is an invertible $m \times m$-matrix.

[*Hint for (b):* Prove and use that for a vector v we have $v^T \cdot v = 0$ if and only if $v = 0$.]

Exercise 5 (12 points)

A pseudoinverse of a matrix A is a matrix A^+ such that all of the following equations hold:

(i) $AA^+A = A$

(ii) $A^+AA^+ = A^+$

(iii) $(AA^+)^T = AA^+$

(iv) $(A^+A)^T = A^+A$.

Let A be an $n \times m$-matrix with linearly independent columns. Show that $(A^TA)^{-1}A^T$ is a pseudoinverse of A (2 points for each property).

Show that for invertible matrices B one has $(B^T)^{-1} = (B^{-1})^T$ and use it on the way (2 points).

[Warning: By exercise 4 above, A^TA is indeed invertible – but A need not be invertible (not even quadratic), so you can in general not form A^{-1}.]

Exercise 6 (10 points)

Write a Python function that, given an invertible matrix A, returns a list of elementary matrices whose product is A.

See the Notebook in the download folder for hints and more details!

Deadline: Friday, October 27, 10:00.
Upload your solution to this link.