

## Exercise03

### Problem 1

a) Loop Invariant: At the start of iteration " $j$ " of the loop, the variable  $C$  contains the symmetric difference of sets  $A$  and  $B$  for the subarray  $A[0:j]$ . The symmetric difference represents the elements that are in  $A$  or  $B$  but not in both.

b) 1. Initialization: At the beginning,  $C$  is initialized with the elements of set  $B$ , making it a valid starting point for the symmetric difference. No elements in  $A$  have been processed yet, so it correctly represents the symmetric difference between  $A$  and  $B$  up to  $A[0:0]$ .

2. Maintenance: Assuming the loop invariant holds at iteration " $j$ ," we evaluate if the  $j^{th}$  element  $a_j$  belongs to  $C$ . If it does, it means  $a_j$  was in  $B$  as well, and we remove this element from  $C$ . This operation preserves the symmetric difference of sets  $A$  and  $B$ . If  $a_j$  does not belong to  $B$ , we add the element to  $C$ , which also maintains the symmetric difference. After this step, we move on to evaluate the next element,  $a_{j+1}$ , in  $A$ , ensuring that the loop invariant holds.

3. Termination: When all elements in  $A$  have been processed, the loop ends. At this point,  $C$  contains all the unique values from both  $A$  and  $B$ , correctly representing the symmetric difference between  $A$  and  $B$  as a whole.

c) Since the loop invariant holds at the beginning, during the loop body, and at the end, we can conclude that after the loop,  $C$  contains the symmetric difference between sets  $A$  and  $B$ , as required by the algorithm's goal.