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New Design for Inferential Control System *

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Abstract: An H_∞ optimal controller is considered for the systems with infrequently sampled output. Based on the proposed I/O model, unmeasurable primary output can be inferred and the controller can be designed directly, so that inferential estimator and controller can be linked together and calculation is considerably simplified. By the polynomial equation method, H_∞ optimal control law is derived form LQG control law. The robustness of the inferential control system is explicitly expressed by sensitivity. This controller is also suitable for nonmininum-phase and unstable plants.

Key words: I/O model of inferential control system; H_∞ optimal control; robust control; polynomial equation method

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新的鲁棒推理控制系统设计方法

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摘要:针对被控输出采样间隔大的系统,设计 H_∞最优推理控制器.推理估计器与控制器的设计都基于推理控制系统 I/O 模型,简化了设计步骤和计算量.多项式方程方法亦使 H_∞控制器直接利用 LQG 控制器的结果,并用灵敏度函数定量描述推理控制系统的鲁棒性.该设计还适用于非最小相位与不稳定系统.

关键词:推理控制系统 I/O 模型;H_∞最优控制;鲁棒控制;多项式方程方法

1 Introduction

In digital control systems, the large sampling interval of some process outputs prevents early detection of the effect of load disturbances and results in large deviations from set point and long recovery time. To overcome the problem, the inferential control scheme estimate controlled output with other easily measured outputs[1]. Guilandoust[2] presented an inferential estimation algorithm and the formula given is effective, but no suitable controller is derived. Combining inferential estimation with model predictive control, Mao[3] proposed an inferential controller. Though it may be robust, no robustness analysis is given.

Robustness and its analysis are important for inferential controller[1]. State space based H_∞ optimal controller can not be applied to inferential control without modifications[4]. Grimble M. J.[5] proposed an H_∞ controller by polynomial equation method. It is much simpler.

Based on the proposed II/O model (i. e. I/O model of inferential control system), an H_∞ inferential controller is proposed by polynomial equation method in the present paper. First an II/O model is described. Then controller design is carried out and it is embedded within

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the GLQG problem framework, and sensitivity is also derived. Finally a digital example is given.

2 Description

State equation can express the controlled object

$$\begin{cases} x(k+1) = \Phi x(k) + \Gamma u(k) + L\xi(k), \\ y(k) = D_1 x(k) + L_1 \xi(k), \\ Z(k) = D_2 x(k), \end{cases} \tag{1}$$

where state $x \in \mathbb{R}^n$ can not be measured, u is control input, controlled output y can only be sampled at large interval of J, and secondary output $Z \in \mathbb{R}^m$ may be measured with rapid speed. And where ξ is disturbance, $\Phi, \Gamma, L, D_1, D_2$ are coefficient matrix or vector.

If state x is observable jointly from y and Z, the object (1) can also be described by I/O model (2)[6],

$$Ay(k) = Bu(k-1) + C_1 z_1(k) + \cdots + C_m z_m(k) + E\xi(k), \tag{2}$$

where z_i is the ith element of Z. The I/O model (2) can greatly simplify the design of inferential controller. The proposed H_∞ inferential controller can suppress the disturbance contained in secondary output.

When $V_i = C_i/A$, $W_1 = B/A$, $W_2 = E/A$ and setpoint r are defined, the structure of inferential control system is clear as shown in Fig. 1. Suppose that track error $r = e - y$, and that $n_2 = E\xi/A$ is measurable disturbance. Stochastic component is allowed to be involved in r, Z and ξ. $C_0 = G/H$ is the controller, where G and H are polynomials. Define sensitivity as $S = 1/(W_1 C_0 + 1)$.

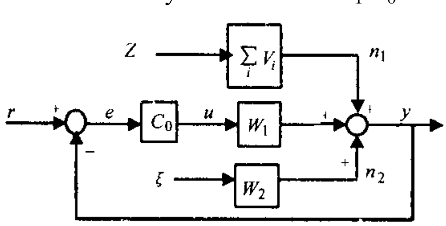

Fig. 1 Structure ofinferential control system(2)

To ensure closed loop stability, $C_0 = M/N$ should satisfy (3), where M and N are stable polynomial fractions[7]

$$AM + BN = 1. \tag{3}$$

3 H_∞ inferential controller design

In cost function of GLQG inferential controller[8]

$$J = \mathrm{E}\{m^2(k)\} = 1/(2\pi j)\oint_{|q|=1} \Phi_{mm}(q^{-1})\mathrm{d}q/q,$$

where

$$m(k) = \{P_c[r(k) - y(k)] + F_c u(k)\}.$$

Auxiliary problem[4] Consider a cost function:

$$J = 1/2\pi j \oint_{|q|=1} X(q^{-1})\Sigma(q-1)\mathrm{d}q/q,$$

supposing that for some real rational $\Sigma(q^{-1}) = \Sigma^*(q^{-1}) > 0$ and J is minimized by a function $X(q^{-1}) = X^*(q^{-1})$, for which $\sup_{|q|=1} X(q^{-1}) = \lambda^2$ (a real constant) on $|q| = 1$. Then $J^* = \sup_{|q|=1} \| X(q^{-1})\Sigma(q^{-1}) \|$ is minimized also by $X(q^{-1})$.

Embedding problem Embedding transform helps to link H_∞ and GLQG problems. According to the auxiliary problem, suppose $H_\sigma H_\sigma^* = \Sigma = A_\sigma^{*-1} B_\sigma^* B_\sigma A_\sigma^{-1}$, weightings P_c and F_c can be assumed to contain H_σ (a minimum phase and stable factory) by writing $P_c = P_d H_\sigma$ and $F_c = F_d H_\sigma$. New weightings P_d and F_d involve factor $H_\sigma = A_\sigma^{-1} B_\sigma$ (A_σ and B_σ is strictly Hurwitz). Thus the result of GLQG problem may be invoked if P_c, F_c, L_1, D_c are replaced respectively by P_d, F_d, L_{1d}, D_d which are defined below.

Theorem of H_∞ inferential controller Consider system (1), the optimal controller that can minimize the cost function $J_\infty = \|\Phi_{mm}(q^{-1})\|_\infty = \sup_{|z|=1} \|\Phi_{mm}(q^{-1})\|$ and guarantee the asymptotical stability of the feedback control system (1) is (4), where H_{00}, G_{00} satisfy Diophantine equations (5) and (6).

$$C_0 = H_{00}^{-1} G_{00}, \tag{4}$$

$$P_{dn} D_f F_{os}^- = P_{dd} A F_{00}^- \lambda + q^{-k} L_{2k} G_{00}, \tag{5}$$

$$F_{dn} D_f F_{os}^- = P_{dd} B F_{00}^- \lambda - L_{2k} H_{00}, \tag{6}$$

where

$$D_c = D_d B_\sigma, \quad L_{1d} = P_{dn} B - F_{dn} A,$$

$$D_d D_d^* = L_{1d} L_{1d}^*, \quad L_{1d} = L_{1k} L_{2k} q^{-k},$$

$$F_{os}^- = F_{00}^{-*} q^{-n1}, \quad n_1 = \deg F_{00}^{-*}.$$

B_σ is an embedding factor, D_f, D_c, D_d and L_{1k} are stable and satisfy $D_f D_f^* = (AA^* + \Sigma_i C_i C_i^* + EE^*)$ and $D_c D_c^* = (P_{cn} B - F_{cn} A)(P_{cn} B - F_{cn} A)^*$. L_{2k} is nonstable. J_∞ is minimized $J_{\infty\min} = \lambda^2$, and sensitivity

$$S = \frac{AH_{00}}{L_{1k} D_f F_{00}^-}. \tag{7}$$

Proof Making substitutions

$$P_{cd} = P_{dd} A_\sigma, \quad P_{cn} = P_{dn} B_\sigma, \quad F_{cn} = F_{dn} B_\sigma,$$

$$D_c = D_d B_\sigma, \quad L_1 = L_{1d} B_\sigma, \quad n_2 = \deg B_\sigma^*,$$

$$F_l = F_e B_\sigma^* q^{-n_2}, \quad g = n + n_2,$$

we obtain from (7)[8] and (8)[8]

$$L_{1d}^{*}P_{dn}D_fB_\sigma q^{-n} = P_{dd}A_\sigma AF_e + \bar{D}_dG_l,$$

$$L_{1d}^{*}F_{dn}D_fB_\sigma q^{-n} = P_{dd}A_\sigma B_1F_e - \bar{D}_dH_l.$$

Make the same substitutions on cost function (4)[8] of GLQG inferential controller

$$J_{\min} = \frac{1}{2\pi j}\oint_{|z|=1}\{P_cP_c^{*}Y_fY_f^{*} - \Phi_h\Phi_h^{*}(Y_cY_c^{*}Y_fY_f^{*})^{-1} + \frac{F_lF_l^{*}}{D_cD_c^{*}}\}\frac{dq}{q} = \frac{1}{2\pi j}\oint_{|z|=1}\frac{F_eF_e^{*}}{D_dD_d^{*}}\frac{dq}{q}.$$

By auxiliary problem, let $J_{\min} = (A_\sigma A_\sigma^{*})^{-1}B_\sigma B_\sigma^{*}\lambda^2$. So $(A_\sigma A_\sigma^{*})B_\sigma B_\sigma^{*} = (D_dD_d^{*}\lambda^2)^{-1}F_eF_e^{*}$. F_0 is strictly asymptotically stable polynomial, and let $F_eF_e^{*} = F_0F_0^{*}$. Thus $B_\sigma = F_0$ and $A_\sigma = D_d\lambda$. Now we have

$$P_{dd}AD_dF_e\lambda + \bar{D}_dG_l = P_{dn}L_{1d}^{*}D_fF_0q^{-n},$$

$$P_{dd}B_1D_dF_e\lambda - \bar{D}_dH_l = F_{dn}L_{1d}^{*}D_fF_0q^{-n}.$$

Also let

$$F_e = F_0^{+}F_0^{-},\quad G_l = G_0F_0^{+},$$

$$F_0 = F_0^{+}F_{os},\quad H_l = H_0F_0^{+},$$

where F_0^{+} and F_0^{-} are respectively the stable and nonstable factors of F_e, then $F_0^{-}F_0^{-*} = F_{os}F_{os}^{*}$. So $F_0^{-} = F_{os}^{*}q^{-n_1}$ and the two Diophantine equations become

$$P_{dd}AD_dF_0^{-}\lambda + \bar{D}_dG_0 = P_{dn}L_{ld}^{*}D_fF_{os}q^{-n},$$

$$P_{dd}B_1D_dF_0^{-}\lambda - \bar{D}_dH_0 = F_{dn}L_{ld}^{*}D_fF_{os}q^{-n}.$$

Define $L_{ld} = L_{1k}L_{2k}q^{-k}$, where L_{1k} and L_{2k} are respectively the stable and nonstable factors of L_{1d}, then

$$D_d^{*} = L_{1k}^{*}L_{2k}q^{-h_1},$$

$$D_d = D_cB_\sigma^{-1} = L_{1k}L_{2k}^{*}q^{-h_2},$$

$$\bar{D}_d = L_{1k}^{*}L_{2k}q^{-h_1-k},$$

where $h_2 = \deg L_{2k}^{*}$, $h_1 = \deg L_{1k}^{*}$.

Define

$$G_0 = L_{1k}L_{2k}^{*}q^{-h_2}G_{00},\quad H_0 = L_{1k}L_{2k}^{*}q^{-h_2}H_{00},$$

$$F_{os} = L_{1k}F_{os}^{-},\quad F_0^{-} = L_{1k}^{*}q^{-h_1}F_{00}^{-},$$

where

$$F_{00}^{-} = F_{os}^{-*}q^{h_3},\quad h_3 = \deg F_{os}^{-*},$$

then Diophantine equations (5) and (6) are deduced and $C_0 = H_{00}^{-1}G_{00}$. By embedding transform, $J_{\infty\min} = \lambda^2$.

From (5) and (6), the closed loop equation is derived as

$$L_{1k}D_fF_{os}^{-} = B_1G_{00} + AH_{00}.$$

Sensitivity S can be derived directly.

Robustness analysis Substitute sensitivity S into track $y(k)$, $e(k)$, $u(k)$ and cost function J_∞,

$$\begin{cases} y(k) = (1-S)r(k) + S[\sum_i V_iz_i(k) + n_2(k)], \\ e(k) = S[r(k) - \sum_i V_iz_i(k) - n_2(k)], \\ u(k) = SC_0[r(k) - \sum_i V_iz_i(k) - n_2(k)], \end{cases} \tag{8}$$

$$J_\infty = \|\Phi_{mm}(q^{-1}\|_\infty = \sup_{|q|=1}\|(P_cS+F_cSC_0)(\Phi_{rr}+\Phi_{ii}+\Phi_{nn})(P_cS+F_cSC_0)^{*}\|. \tag{9}$$

Equation (8) shows that the smaller the sensitivity S, the smaller the impact of unmeasurable disturbances on the track error $e(k)$ will be. At the same time, Equation (9) indicates that disturbance included in secondary output Z can be suppressed and measurable disturbance ξ can be offset by selecting weighting functions P_c and F_c and then tuning sensitivity S.

4 Conclusion

A robust method to design inferential controller is given, and the robustness are derived quantitatively. The controller needs to solve two spectral factorizations and one generalized eigenvalue problem. Meanwhile, the cost function includes sensitivity, so that capability of inferential control systems to suppress input disturbances is improved.

Input-output approach (I/O model) helps to avoid complex design and calculation needed by other robust inferential controller[2] or by H_∞ optimal control solved by state space approach[1]. This helps to realize self-tuning algorithm. The major advantage of the approach is that the effort in designing inferential control system can be reduced to some extent.

References

[1] Xu Zhe, Chai Tianyou and Wang Wei. A Review of inferential control [J]. Information & Control, 1998, 27(3): 206-214

[2] Guilandoust M T, Morris A J and Tham M T. Adaptive inferential control [J]. IEE Proceedings, Part D, 1987, 134(3): 171-179

[3] Mao Kezhi and Chai Tianyou. Inferential estimation and adaptive mo-

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