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Nonlinear Systems Modeling via Fuzzy Logic Rules

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Abstract: We propose a new self-tuning fuzzy modeling by means of fuzzy clustering. Based on fuzzy clustering, the adaptive fuzzy inference is used to modify the fuzzy system. Moreover, based on this modified fuzzy system, the paper presents an on-line identifying algorithm with which the on-line parameter estimation of nonlinear system is realized. To demonstrate the applicability of the proposed method, simulation results relative to a few examples are presented in the end.

Key words: system identification; fuzzy systems; nonlinear system modeling; recursive fuzzy clustering; Kalman filtering algorithm; on-line identification

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基于模糊规则的非线性系统建模方法

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摘要:提出了一种基于模糊聚类自调整的模糊建模方法.基于模糊聚类通过自适应模糊推理来调整模糊系统.一种在线辨识算法是通过非线性系统参数的在线估计来进行的.为了证明所提出方法的适用性,给出了几个实例的仿真结果.

关键词:系统辨识;模糊系统;非线性系统建模;递推模糊聚类;卡尔曼滤波算法;在线辩识

1 Introduction

As for dynamic systems with complex, ill-conditioned, or nonlinear characteristics, the fuzzy model based on fuzzy sets is a very useful method to describe the properties of the dynamic systems using fuzzy inference rules. Many kinds of fuzzy models for modeling and control have been developed since Takagi-Sugeno's model (T-S model) was proposed[1]. These functional rule models allow us to describe analytically the input-output relation of fuzzy system. However, it is hard to determine the fuzzy rules and fuzzy space structure. For the complex system, the more the identifying accuracy of model is required, the more fuzzy input regions need to be partitioned. As a result, the number of fuzzy rules and identifying parameters are exponentially increased. In addition, these fuzzy modeling methods have some problems, for example, learning algorithms are complex, generalization performance of algorithms is bad and on-line identification algorithms are not used.

To solve these problems, we propose a new self-tuning fuzzy modeling by means of fuzzy clustering. Based on fuzzy clustering, the adaptive fuzzy inference is used to modify the fuzzy system. Moreover, based on this modified fuzzy system, the paper presents an on-line identification algorithm with which the on-line parameter estimation of nonlinear system is realized. To demonstrate the advantages of the proposed method, simulation results relative to a few examples are also presented.

2 Description of fuzzy model and adaptive fuzzy inference

In this section, we consider a system $P(U,Y)$ as a multi-input and multi-output system, $\mathrm{adap}_i(0) = r_i$ $(i = 1,2,\cdots,c)$, $l = 1$. For the multi-input and multi-output system, we can divide it into q multi-input and single output system. Hence we only discuss a multi-input and single output system.

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We consider the following format of fuzzy for a multi-input and single output system.

R^i : if z is $\bar{z}_i, r_i$, then $y^i = z^{\mathrm{T}}\theta_i$, $i = 1,2,\cdots,c$, (1a)

$$\begin{cases} \hat{y} = \dfrac{\sum_{i=1}^{c}\mu_i y^i}{\sum_{i=1}^{c}\mu_i} = \sum_{i=1}^{c}\beta_i y^i, \quad \dfrac{\mu_i}{\sum_{i=1}^{c}\mu_i} = \beta_i, \\ \mu_i = \begin{cases} 1 - \dfrac{\| z - \bar{z}_i \|}{r_i}, & \text{if } \| z - \bar{z}_i \| \leqslant r_i, \\ 0, & \text{if } \| z - \bar{z}_i \| > r_i. \end{cases} \end{cases} \tag{1b}$$

Where R^i is the ith rule. z is the input vector, $z = (1, x_1, x_2, \cdots, x_m)^{\mathrm{T}}$. $\bar{z}_i$ is the ith centroid vector, $\bar{z}_i = (\bar{x}_{i1}, \bar{x}_{i2}, \cdots, \bar{x}_{im})^{\mathrm{T}}(i = 1,2,\cdots,c)$. r_i is the radius of the corresponding input region. y^i is the local output of the ith rule. $\hat{z}$ is the input vector of conclusion part, $\hat{z} = (1, x_1, x_2, \cdots, x_m)^{\mathrm{T}}$. θ_i is the conclusion parameter vector of the ith rule, $\theta_i = (p_{i0}, p_{i1}, \cdots, p_{im})^{\mathrm{T}}$. c is the number of rules. $\hat{y}$ is the output of fuzzy model.

In essence, the local linearization of input and output of system is carried out by means of fuzzy model (1) expressing nonlinear systems. It is obvious that $\| \bar{z} - \bar{z}_i \| \leqslant r_i$ represents the local input region of rules. If an input sample point z belongs to a region or a few regions, satisfying $\| \bar{z} - \bar{z}_i \| \leqslant r_i$, then the union of input regions of rules can cover the input space. Otherwise, the union of input regions of rules does not have to contain the whole input space. In general, if the input region of cach rule is determined by the experience of skilled operators or experts according to the input ranges, the union of input regions may equal the whole input space. However, the identifying parameters $\{\bar{z}_i, r_i, \theta_i\}$ corresponding the input region of each rule are obtained by identification methods. Since the training sample points may not be sufficient to fill the whole input space or the number of clusters is not big enough, the union of the partitioned input regions may be inferior to the input space of system.

While a new input datum belongs to a region of rules or a few regions of rules, the general fuzzy reasoning is used by means of Eq.(1). On the other hand, while a new input datum does not belong to any region of rules, the adaptive fuzzy inference is proposed through the following steps[2]:

1) Select $\mathrm{adap}_i(0) = r_i(i = 1,2,\cdots,c)$, $l = 1$.

2) $\mathrm{adap}_i(l) = \mathrm{adap}_i(l-1) + \lambda r_i(i = 1,2,\cdots,c)$,
if $\| z(t) - \bar{z}_i \| < \mathrm{adap}_i(l)$,

then $\mu_i = 1 - \dfrac{\| z(t) - \bar{z}_i \|}{\mathrm{adap}_i(l)}$, else $\mu_i = 0$.

3) if $\sum_{i=1}^{c}\mu_i = 0$, then $l = l + 1$, go to 2)

if $\sum_{i=1}^{c}\mu_i \neq 0$, then $\hat{y} = \dfrac{\sum_{i=1}^{c}\mu_i(\bar{z}^{\mathrm{T}}\theta_i)}{\sum_{i=1}^{c}\mu_i}$

In the above process, the radius of each input region is gradually increased by the growth factor λr_i until a certain region or regions contain the new sample point $z(t)$, satisfying $\| \bar{z} - \bar{z}_i \| < (1 + \sum\lambda) r_i$[2].

3 On-line identification of fuzzy model

The goal of on-line identification for fuzzy model can on-line update the parameters $\{\bar{z}, r_i, \theta_i\}$. The premise parameters including $\bar{z}_i$ and r_i of fuzzy c-means algorithm (FCM) has been used for generation of the reference fuzzy set[3]. An advantage of this method is that it provides an automatic way of forming the reference fuzzy sets and does not require any initial knowledge about the structure in the data set. Unfortunately, the fuzzy clustering is quite time-consuming and may not be suitable for on-line modeling and control. In this paper, based on the original fuzzy clustering, the recursive fuzzy clustering is proposed to partition input space of system.

The recursive fuzzy clustering algorithm is shown as follows:

1) Select the number of clusters c $(2 \leqslant c \leqslant N)$ and initial centroid vectors $\bar{z}_i(1,2,\cdots,c)$.

2) Determine the membership degrees for any sample point

$$\mu_{ik} = \left[\sum_{j=1}^{c}\left(\frac{\| z(k) - \bar{z}_i \|}{\| z(k) - \bar{z}_j \|}\right)^2\right]^{-1}, \quad i = 1,2,\cdots,c. \tag{2}$$

3) Update centrioid vectors $\bar{z}_i(i = 1,2,\cdots,c)$

$$\bar{z}_i(k) = \frac{A_i(k)}{B_i(k)} = \frac{A_i(k-1) + (\mu_{ik})^2 z(k)}{B_i(k-1) + (\mu_{ik})^2}, \tag{3}$$

$i = 1,2,\cdots,c$, $k = 1,2,\cdots,n$,

$A_i(0) = \overline{\text{zero}}$ ($\overline{\text{zero}}$ represents zero vector), $B_i(0) = 0 (i = 1,2,\cdots,c)$.

Since the original fuzzy clustering method is that the centroid vectors are modified for all training sample points, it is quite time-consuming and may not be suitable for on-line identification. The recursive fuzzy clustering method is that the centroid vectors are updated for each training point. As a result, the calculating time becomes shorter and convergence speed becomes faster.

The radius r_i of each input region not only determines the size of each input region but also decides the overlapping degree between regions. For this reason, how to select the radius of each input region is of great importance. To update on-line the radius of each input region, first of all, the overlaping degree σ between two regions is given. The radius of each input region is calculated as follows:

$$r_i = \max_{\substack{j=1,2,\cdots,c \\ i \neq j}} \frac{\| \bar{z}_i - \bar{z}_j \|}{\sigma}, \quad 0 < \sigma < 1. \tag{4}$$

Based on the process, the fuzzy partitioning space of system and radius of each input region can on-line be modified.

The output of model (1) is shown through the following equations:

$$\hat{y}(k) = \sum_{i=1}^{c} \beta_i y_k^i = X(k)\theta, \tag{5}$$

$$X(k) = (\beta_1^k,\cdots,\beta_c^k,\beta_1^k x_1^k,\cdots,\beta_c^k x_1^k,\cdots,\beta_1^k x_m^k,\cdots,\beta_c^k x_m^k), \tag{6}$$

$$\theta = (p_{10},\cdots,p_{c0},p_{11},\cdots,p_{c1},\cdots,p_{1m},\cdots,p_{cm})^{\mathrm{T}}, \tag{7}$$

where k represents the kth sampling. To estimate the parameter vector θ, Kalman filtering algorithm is used. Here, we apply it to calculate the parameter vector θ as follows:

$$\theta_{k+1} = \theta_k + \frac{S_{k+1}^* X_{k+1}^{\mathrm{T}*}(y_{k+1} - X_{k+1}^* \theta_k)}{Q + X_{k+1}^* S_k^* X_{k+1}^{\mathrm{T}}}, \tag{8}$$

$$S_{k+1} = S_k - \frac{S_k^* X_{k+1}^{\mathrm{T}*} X_{k+1}^* S_k}{Q + X_{k+1}^* S_k^* X_{k+1}^{\mathrm{T}}}, k = 0,1,\cdots,n-1. \tag{9}$$

Where θ is the parameter vector, $S(k)$ is $[(m+1)\times c] \times [(m+1)\times c]$ matrix. $\theta_0 = \overline{\text{zero}}$ ($\overline{\text{zero}}$ represents zero vectors), $S_0 = \alpha I$ (I is an identity matrix and α is a large positive number), $Q = \exp(-\text{number}/N)$ (number represents the iteration counter and N is a constant) and y_{k+1} is the real output of the $k+1$ th sampling.

To sum up, the paramters $\bar{z}_i, r_i, \theta$ of fuzzy rules are on-line estimated by means of the following steps:

Step 1 The number of clusters c and the overlapping degree $\sigma (0 < \sigma < 1)$ are given.

Step 2 The initial centroid vectors $\bar{z}_i (i = 1,2,\cdots,c)$ are selected, $\theta_0 = \overline{\text{zero}}$. $S_0 = \alpha I$ (α is a large positive number).

Step 3 The membership degrees of any sample point are calculated by Eq.(2); after that, the centroid vectors $\bar{z}_i (i = 1,2,\cdots,c)$ are updated by Eq.(3).

Step 4 The radii $r_i (i = 1,2,\cdots,c)$ are updated by Eq.(4).

Step 5 The parameter vector θ is calculated by Eq.(8) to Eq.(9).

Step 6 if $k = n$, then go to Step 7, or else repeat Step 3, where n is the number of all the sample points.

Step 7 if $\frac{|J(c+1) - J(c)|}{|J(c)|} < \xi$ then stop else $c = c + 1$ go to Step 3.

4 Simulation examples

Example 1 Consider the gas furnace data of Box and Jenkins as a sample data[4]. This data set is well known and frequently used as a simulated example for test identification algorithms. We apply our approach to build fuzzy model based on the Box-Jenkins data set. $y(t-1)$ and $u(t-4)$ are considered as input variables. The sampling interval is 9s. The number of fuzzy rules is 7. Fig. 1 shows the identified model output in comparison with the output obtained from the real system.

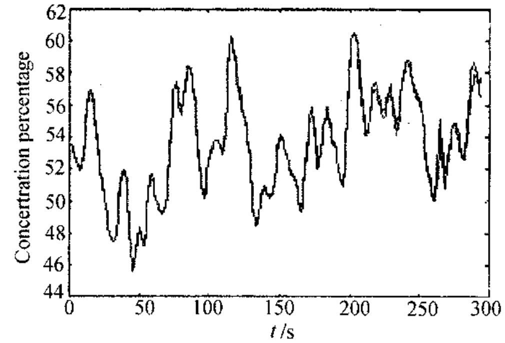

Fig.1 The idntifyign model output in comparison with the actual output

In Table 1, we compare our fuzzy model with other models identified from the same data. It can be seen that the performance of our model is superior to other models.

Table 1 Comparison of our model with other models

Name of System Model	Input	Number of Fuzzy Rules	Mean Squared Error
Tong 's model[5]	$y(t-1), u(t-4)$	19	0.469
Pedrye 's model[5]	$y(t-1), u(t-4)$	81	0.32
Xu 's model[5]	$y(t-1), u(t-4)$	25	0.328
Yoshinari 's model[6]	$y(t-1), u(t-3)$	6	0.299
Our model	$y(t-1), u(t-4)$	7	0.1427

Example 2 The example is taken from the literature[3], in which the plant to be identified is given by the second-order nonlinear difference equation:

$$y(t+1) = \frac{y(t)y(t-1)[y(t)+2.5]}{1+y^2(t)+y^2(t-1)} + u(t). \tag{10}$$

300 simulated data points are generated from the plant model (10). The 300 data points are obtained by a sinusoid input signal $u(t) = \sin(2\pi t/25)$ $(t = 1,\cdots,300)$. We select $y(t-1)$, $y(t)$ and $u(t)$ as the input variables. The number of rules is ten. Fig.2 shows the output of identifying model and the actual output of system.

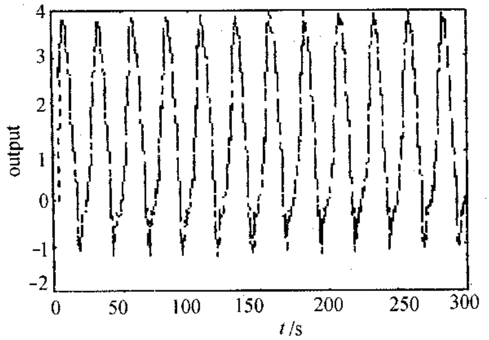

Fig.2 Identifying result

It can be seen that the output from the identified model attains a rather good match with that of the actual model.

5 Conclusion

This paper presents an approach to identifying a fuzzy model composed of fuzzy-logic rules for a multi-input/single output system. We propose a new self-tuning fuzzy modeling by means of fuzzy clustering. Based on fuzzy clustering, the adaptive fuzzy inference is used to modify the fuzzy system. Moreover, based on this modified fuzzy system, the paper presents an on-line identifying algorithm with which the on-line parameter estimation of nonlinear system is realized. This proposed approach has the advantages of simplicity, flexibility, and high accuracy.

References

[1] Takagi T and Sugeno M. Fuzzy identification of system and its application to modeling and control [J]. IEEE Trans. Systems, Man, and Cybernetics, 1985,15(1):16 – 32

[2] Chen Jianqin, Xi Yugeng and Zhang Zhongjun. On-line identification of nonlinear systems using fuzzy model [J]. Acta Automatic Sinica, 1998,24(1):90 – 94

[3] Liang Wang and Reza Langari. Complex system modeling via fuzzy logic [J]. IEEE Trans. Systems, Man, and Cybernetics, 1996,26(1):100 – 106

[4] Box E P. Time Series Analysis, Forecasting and Control [M]. San Franciso: Holden Day, 1970

[5] Sugeno M, et al. A fuzzy logic-based approach to qualitative modeling [J]. IEEE Trans. on Fuzzy Systems, 1993,1(1):7 – 31

[6] Yoshinari Y, et al. Construction of fuzzy model technique [J]. Fuzzy Sets and Systems, 1993,54(2):157 – 165

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