第 17 卷第 3 期 控制理论与应用 Vol.17,No.3
2000 年 6 月 CONTROL THEORY AND APPLICATIONS Jun.,2000

Article ID:1000 - 8152(2000)03 - 0317 - 04

Bifurcation Analysis of the Permanent-Magnet Synchronous Motor Models Based on the Center Manifold Theorem *

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Abstract: The mathematical model of the permanent-magnet synchronous motor (PMSM) is formulated, and then the center manifold theorem is used to obtain a simplified center manifold equation of the PMSM. Finally, its stability and bifurcation are discussed.

Key words: permanent-magnet synchronous motor; center manifold theorem; bifurcation
Document code: A

基于中心流形定理的永磁同步电动机模型的分支分析

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摘要:在推导出永磁同步电动机数学模型的基础上,首次应用中心流形定理,得到了其简化的中心流形方程,并在此基础上讨论了其稳定性及其分支情形.

关键词:永磁同步电动机;中心流形;分支

1 Introduction

Over the past few years, chaos and bifurcation in nonlinear dynamic systems have been studied extensively. Some numerical or theoretical methods, such as Shil'nikov theorem and Poincare mapping, have been developed for analyzing chaotic and bifurcation phenomena in various nonlinear systems. But in general, it is difficult to study a nonlinear dynamics theoretically. A technique has been proposed to simplify dynamical systems, which is center manifold theorem[1,2]. The center manifold theorem provides a means for systematically reducing the dimension of the state space, which needs to be considered when analyzing bifurcations of a given type.

As shown in [3], the mathematical model of the permanent-magnet synchronous motor (PMSM) is nonlinear, which exhibits a variety of chaotic behavior, so it is difficult to analyze directly its stability and bifurcation. In this paper, we first formulate dynamic characteristics of the PMSM; second, we use the center manifold theorem to reduce a three-order equation to a first-order center manifold equation without changing its dynamic characteristics. Furthermore, we discuss its stability and bifurcation in a special case. It can be shown that a differential equation, when subject to a certain condition, can be greatly simplified in order to analyze its stability. Other systems, such as induction-motors or asynchronous motors, will be analogously discussed.

* Foundation item: supported by the National Natural Science Foundation of China (59777008) and the Guangdong Provincial Natural Science Foundation of China (98057)
Received date: 1999 - 07 - 23; Revised date: 2000 - 03 - 06.

2 The system model

The d-q axis, the dynamics of a PMSM can be modeled. Based on the d-q axis, as

$$\begin{cases} \dfrac{\mathrm{d}i_d}{\mathrm{d}t} = (u_d - R_1 i_d + \omega L_q i_q)/L_d, \\ \dfrac{\mathrm{d}i_q}{\mathrm{d}t} = (u_q - R_1 i_q - \omega L_d i_d - \omega \Psi_r)/L_q, \\ \dfrac{\mathrm{d}\omega}{\mathrm{d}t} = [n_p \Psi_r i_q + n_p(L_d - L_q) i_d i_q - T_L - \beta\omega]/J, \end{cases} \tag{1}$$

where i_d, i_q and ω are the state variables, $\tilde{u}_d$ and $\tilde{u}_q$ are the direct- and quadrature-axis stator voltage components, respectively, J is the polar moment of inertia, T_l is the external load torque, β is the viscous damping coefficient, R_1 is the stator winding resistance, L_d and L_q are the direct- and quadrature-axis stator inductors, respectively, i_d and i_q are currents, ω is the motor angular frequency, Ψ_r is the permanent-magnet flux, and n_p represents the number of pole-pairs.

By applying an affine transformation of the form

$$x = \tilde{\lambda}\tilde{x}, \tag{2}$$

and a time-scaling transformation

$$t = \tau\tilde{t}, \tag{3}$$

where $x = [i_d \quad i_q \quad \omega]^{\mathrm{T}}$, $\tilde{x} = [\tilde{i}_d \quad \tilde{i}_q \quad \tilde{\omega}]^{\mathrm{T}}$,

$$\tilde{\lambda} = \begin{bmatrix} \tilde{\lambda}_d & 0 & 0 \\ 0 & \tilde{\lambda}_q & 0 \\ 0 & 0 & \tilde{\lambda}_\omega \end{bmatrix} = \begin{bmatrix} bk & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & \dfrac{1}{\tau} \end{bmatrix},$$

$$b = \frac{L_q}{L_d}, k = \frac{\beta}{n_p \tau \Psi_r} \text{ and } \tau = \frac{L_q}{R_1},$$

we obtain a system of equations in the dimensionless form:

$$\begin{cases} \dfrac{\mathrm{d}\tilde{i}_d}{\mathrm{d}\tilde{t}} = -\tilde{i}_d + \tilde{\omega}\tilde{i}_q + \tilde{u}_d, \\ \dfrac{\mathrm{d}\tilde{i}_q}{\mathrm{d}\tilde{t}} = -\tilde{i}_q - \tilde{\omega}\tilde{i}_d + \gamma\tilde{\omega} + \tilde{u}_q, \\ \dfrac{\mathrm{d}\tilde{\omega}}{\mathrm{d}\tilde{t}} = \sigma(\tilde{i}_q - \tilde{\omega}) + \varepsilon\tilde{i}_d\tilde{i}_q - \tilde{T}_L, \end{cases} \tag{4}$$

where $\gamma = -\dfrac{\Psi_r}{kL_q}$, $\sigma = \dfrac{\beta\tau}{J}$, $\tilde{u}_q = \dfrac{1}{R_1 k}u_q$, $\tilde{u}_d = \dfrac{1}{R_1 k}u_d$, $\varepsilon = \dfrac{n_p b \tau^2 k^2 (L_d - L_q)}{J}$ and $\tilde{T}_L = \dfrac{\tau^2}{J}T_L$.

Now we just study the dynamic characteristics of a smooth-air-gap PMSM, with $L_d = L_q = L$ in the model. Thus, system (4) becomes

$$\begin{cases} \dfrac{\mathrm{d}\tilde{i}_d}{\mathrm{d}\tilde{t}} = -\tilde{i}_d + \tilde{\omega}\tilde{i}_q + \tilde{u}_d, \\ \dfrac{\mathrm{d}\tilde{i}_q}{\mathrm{d}\tilde{t}} = -\tilde{i}_q - \tilde{\omega}\tilde{i}_d + \gamma\tilde{\omega} + \tilde{u}_q, \\ \dfrac{\mathrm{d}\tilde{\omega}}{\mathrm{d}\tilde{t}} = \sigma(\tilde{i}_q - \tilde{\omega}) - \tilde{T}_L. \end{cases} \tag{5}$$

3 Center manifold theorem

We consider vector field depending on a vector of parameters, say $\varepsilon \in \mathbb{R}^p$, of the following form[1]

$$\begin{cases} \dot{x} = Ax + f(x, y, \varepsilon), \\ \dot{y} = By + g(x, y, \varepsilon), \end{cases} (x, y, \varepsilon) \in \mathbb{R}^c \times \mathbb{R}^s \times \mathbb{R}^p, \tag{6}$$

where

$$\begin{aligned} f(0,0,0) &= 0, \quad Df(0,0,0) = 0, \\ g(0,0,0) &= 0, \quad Dg(0,0,0) = 0. \end{aligned} \tag{7}$$

In the above, A is a $c \times c$ matrix having eigenvalues with zero real parts, B is an $s \times s$ matrix having eigenvalues with negative real parts, and f and g are C^r function ($r \geqslant 2$).

Definition 1 An invariant manifold will be called a center manifold for (6) if it can be locally represented as follows

$$W^c(0) = \{(x, y, \varepsilon) \in \mathbb{R}^c \times \mathbb{R}^s \times \mathbb{R}^p \mid y = h(x, \varepsilon), |x| < \delta, |\varepsilon| < \bar{\delta}, h(0,0) = 0, Dh(0,0) = 0\},$$

for δ and $\bar{\delta}$ sufficiently small.

The first result on center manifolds is an exitence theorem[1].

Theorem 1 There exists a C^r center manifold for (6). The dynamics of (6) which is restricted to the center manifold is, for u sufficiently small, given by the following c – dimensional vector field

$$\begin{cases} \dot{u} = Au + f(u, h(u, \varepsilon), \varepsilon), \\ \dot{\varepsilon} = 0, \end{cases} (u, \varepsilon) \in \mathbb{R}^c \times \mathbb{R}^p. \tag{8}$$

Theorem 2 i) Suppose the zero solution of (8) is stable (asymptotically stable) (unstable), then the zero solution of (6) is also stable (asymptotically stable) (unstable). ii) Suppose the zero solution of (8) is stable. Then if $(x(t), y(t), \varepsilon)$ is a solution of (6) with $(x(0), y(0), 0)$ sufficiently small, there is a solution $u(t)$ of (8) such that as $t \to \infty$

$$\begin{aligned} x(t) &= u(t) + O(\mathrm{e}^{-\lambda t}), \\ y(t) &= h(u(t)) + O(\mathrm{e}^{-\lambda t}), \end{aligned}$$

where $\lambda > 0$ is a constant.

Using invariance of $W^c(0)$ under the dynamics of (6), we derive a quasilinear partial differential equation that $h(x,\varepsilon)$ must satisfy. This is done as follows:

1) The (x,y,ε) coordinates of any points on $W^c(0)$ must satisfy

$$y = h(x,\varepsilon). \tag{9}$$

2) Differentiating (9) with respect to time imples that the $(\dot{x},\dot{y},\dot{\varepsilon})$ coordinates of any point on $W^c(0)$ must satisfy

$$\dot{y} = D_x h(x,\varepsilon)\dot{x} + D_\varepsilon h(x,\varepsilon)\dot{\varepsilon}. \tag{10}$$

3) Substituting (6) and (9) into (10) results in the following quasilinear partial differential equation that $h(x,\varepsilon)$ must satisfy in order for its graph to be a center manifold.

$$\begin{aligned} &D_x h(x,\varepsilon)[Ax + f(x,h(x,\varepsilon),\varepsilon)] = \\ &Bh(x,\varepsilon) + g(x,h(x,\varepsilon),\varepsilon) \end{aligned} \tag{11}$$

or

$$\begin{aligned} &N(h(x,\varepsilon)) \equiv \\ &D_x h(x,\varepsilon)[Ax + f(x,h(x,\varepsilon),\varepsilon)] - \\ &Bh(x,\varepsilon) - g(x,h(x,\varepsilon),\varepsilon) = 0. \end{aligned} \tag{12}$$

It is probably more difficult to solve (12) than our original problem; however, the following theorem gives us a method to compute an approximate solution of (12) to any desired degree of accuracy.

Theorem 3 Let $\phi: \mathbb{R}^c \times \mathbb{R}^p \to \mathbb{R}^s \times \mathbb{R}^p$ be a C^1 mapping with $\phi(0,0) = D\phi(0,0) = 0$ such that $N(\phi(x,\varepsilon)) = O(|(x,\varepsilon)|^q)$ as $x \to 0, \varepsilon \to 0$ for some $q > 1$. Then

$|h(x,\varepsilon) - \phi(x,\varepsilon)| = O(|(x,\varepsilon)|^q)$ as $x \to 0, \varepsilon \to 0$.

Proof for above theorems: see [4].

4 The stability of the PMSM model

In this section, we will discuss the stability in the case that after an operating period of the system, the external inputs are set to be zero, namely, $\tilde{u}_d = \tilde{u}_q = \tilde{T}_L = 0$. This system (5) becomes

$$\begin{cases} \dfrac{d\tilde{i}_d}{d\tilde{t}} = -\tilde{i}_d + \tilde{\omega}\tilde{i}_q, \\ \dfrac{d\tilde{i}_q}{d\tilde{t}} = -\tilde{i}_q + \tilde{\omega}\tilde{i}_d + \tilde{\omega} + \bar{\gamma}\tilde{\omega}, \\ \dfrac{d\tilde{\omega}}{d\tilde{t}} = \sigma(\tilde{i}_q - \tilde{\omega}), \end{cases} \tag{13}$$

where σ is viewed as fixed positive constant, $\bar{\gamma} = \gamma - 1$ is a parameter, $\bar{\gamma}\tilde{\omega}$ is a nonlinear term.

It should be clear that the origin is a fixed point of (13). Linearizing (13) about this fixed point, we obtain the associated matrix

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & \sigma & -\sigma \end{pmatrix}. \tag{14}$$

Since (14) is in block form, the eigenvalues are particularly easy to compute and are given by

$$0, -(\sigma - 1), -1 \tag{15}$$

with eigenvectors

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -\sigma \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{16}$$

Our goal is to determine the nature of the stability of the origin for $\bar{\gamma}$ near zero. Using the eigenbasis (16), we obtain the transformation

$$\begin{pmatrix} \tilde{i}_d \\ \tilde{i}_q \\ \tilde{\omega} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & -\sigma & 0 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{17}$$

which transforms (13) into

$$\begin{cases} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -(1+\sigma) & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \\ \qquad \dfrac{1}{1+\sigma} \begin{pmatrix} \sigma(u - \sigma v)(\bar{\gamma} - w) \\ (u - \sigma v)(\bar{\gamma} - w) \\ (1+\sigma)(u+v)(u - \sigma v) \end{pmatrix}, \\ \dot{\bar{\gamma}} = 0. \end{cases} \tag{18}$$

Thus, from center manifold theory, the stability of $(\tilde{i}_d, \tilde{i}_q, \tilde{\omega}) = (0,0,0)$ near $\bar{\gamma} = 0$ can be determined by studying a one-parameter family of first-order ordinary differential equations on a center manifold, which can be represented as a graph over the u and $\bar{\gamma}$ variables, i.e.,

$$\begin{cases} W^c(0) = \{(u,v,w,\bar{\gamma}) \in \mathbb{R}^4 \mid v = h_1(u,\bar{\gamma}), \\ w = h_2(u,\bar{\gamma}), h_i(0,0) = 0, Dh_i(0,0) = 0, i = 1,2\}, \end{cases} \tag{19}$$

for u and $\bar{\gamma}$ sufficiently small.

We are now to compute the center manifold and derive the vector field on the center manifold. Using Theorem 3, we assume

$$\begin{aligned} h_1(u,\bar{\gamma}) &= a_1 u^2 + a_2 u\bar{\gamma} + a_3\bar{\gamma}^2 + \cdots, \\ h_2(u,\bar{\gamma}) &= b_1 u^2 + b_2 u\bar{\gamma} + b_3\bar{\gamma}^2 + \cdots. \end{aligned} \tag{20}$$

Recall from (12) that the center manifold must satisfy, and x, y, ε and h are respectively

$$\begin{cases} x \equiv u,\ y \equiv (v,w),\ \varepsilon \equiv \bar{\gamma},\ h = (h_1,h_2), \\ A = 0,\ B = \begin{pmatrix} -(1+\sigma) & 0 \\ 0 & -1 \end{pmatrix}, \\ f(x,y,\varepsilon) = \dfrac{1}{1+\sigma}[\sigma(u+v)(\bar{\gamma}-w)], \\ g(x,y,\varepsilon) = \dfrac{1}{1+\sigma}\begin{pmatrix} (u-\sigma v)(\bar{\gamma}-w) \\ (1+\sigma)(u+v)(u-\sigma v) \end{pmatrix}. \end{cases} \tag{21}$$

Substituting (20) into (12) and using (21), and then equating terms of like powers to zero give

$$\begin{cases} u^2: a_1(1+\sigma) = 0 \Rightarrow a_1 = 0, \\ \quad b_1 - 1 = 0 \Rightarrow b_1 = 1; \\ u\bar{\gamma}: (1+\sigma)a_2 + \dfrac{1}{1+\sigma} = 0 \Rightarrow a_2 = \dfrac{-1}{(1+\sigma)^2}, \\ \quad b_2 = 0. \end{cases} \tag{22}$$

Then, using (22) and (20), we obtain

$$\begin{aligned} h_1(u,\bar{\gamma}) &= -\frac{1}{(1+\sigma)^2}u\bar{\gamma} + \cdots, \\ h_2(u,\bar{\gamma}) &= u^2 + \cdots. \end{aligned} \tag{23}$$

Finally, substituting (23) into (18) we obtain the vector field reduced to the center manifold

$$\begin{aligned} \dot{u} &= \frac{\sigma}{1+\sigma}u(\bar{\gamma} - u^2 + \cdots), \\ \dot{\bar{\gamma}} &= 0. \end{aligned} \tag{24}$$

In Fig.1 we plot the fixed points of (24). It should be clear that $u = 0$ is always a fixed point and is stable for $\bar{\gamma} < 0$ and unstable for $\bar{\gamma} > 0$. At the point of exchange of stability (i.e., $\bar{\gamma} = 0$) two new stable fixed points are created and are given by

$$\bar{\gamma} = u^2 \tag{25}$$

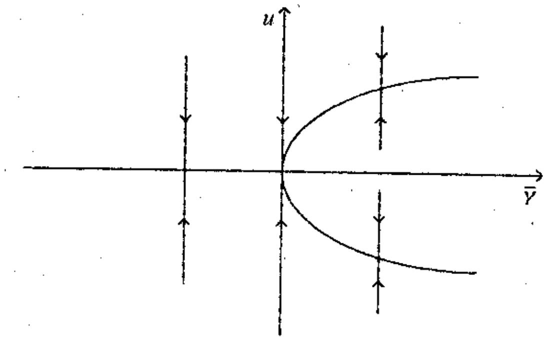

Fig.1 The stability of the center manifold equation

A simple calculation shows that these fixed points are stable and this is an example of a pitchfork bifurcation[1].

Here, we have not considered the effects of the higher order terms in (24). In fact, they do not qualitatively change the stability of any of the fixed points[1].

5 Conclusion

In this paper, we have formulated the mathematical model of the PMSM, and used the center manifold theorem to simplify it. Furthermore, we have discussed the stability and bifurcation of the model under a typical situation. It has been shown that when the model of the PMSM simplified by using the center manifold theorem, the stability and bifurcation of a typical PMSM model became easier to be analyzed. For the nonlinearity of the PMSM model and the complexity of its operating, more new techniques need to be developed for analyzing its more general situations.

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毛宗源　见本刊 2000 年第 1 期第 101 页.

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