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Robust Dissipative Control for Nonlinear Systems

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Abstract: This paper is concerned with the problem of quadratic dissipative control for nonlinear systems with or without uncertainty. The uncertainty is described by bounded norm. We consider the design of feedback controllers to achieve quadratic dissipativeness. It is shown that the robust dissipative control problem can be resolved for all admissible uncertainty, if there exists a scaling function such that Hamilton-Jacobi inequality has a nonnegative solution.

Key words: nonlinear system; dissipative control; Hamilton-Jacobi inequality
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非线性系统鲁棒耗散控制

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摘要: 基于二次型供给率, 研究了不确定非线性系统的鲁棒耗散控制问题. 不确定项用有界范数来刻画. 基于 Hamilton-Jacobi 不等式, 得到了实现鲁棒耗散控制的充分条件及控制器的设计算法.

关键词: 非线性系统; 耗散控制; Hamilton-Jacobi 不等式

1 Introduction

Since the notion of dissipative system was introduced by Willems[1], and subsequently generalized by Hill and Moylan[2], it has played an important role in systems, circuits, and control. Dissipativeness is a generalization of the concept of passivity in electrical network and other dynamical systems which dissipate energy in some abstract sense. The application of dissipativeness in the stability analysis of linear systems with certain nonlinear feedback was first discussed by Willems[1,3]. Subsequently, dissipativeness was crucially used in stability analysis of nonlinear systems by Hill and Moylan[4]. The theory of dissipative systems generalizes basic tools including the passivity theorerm, bounded real lemma, Kalman-Yakubovich lemma and circle criterion.

In recent years, numerous papers have addressed the synthesis problem for L_2-gain, positive real or passivity (see [5] ~ [9]). For nonlinear systems, the general concept is the dissipativity which includes the L_2-gain, positive realness and passivity concepts as special cases. Many control system design problems such as disturbance attenuation[6], robust stabilization[5] and almost disturbance decoupling[8], for which many approaches have been investigated respectively can be reduced to the dissipative synthesis problem.

In this paper, we consider the problem of quadratic dissipative control for nonlinear systems with or without uncertainty. The uncertainty is described by bounded norm. It is shown that the robust dissipative control problem can be resolved for all admissible uncertainty, if there exists a scaling function such that Hamilton-Jacobi inequality has a nonnegative solution. A numerical example is also given to illustrate the design procedure and the effectiveness of the proposed approach.

2 Problem formulation

Consider the dynamical system

$$\begin{cases} \dot{x}(t) = f_0(x(t), \omega(t)), & x(0) = x_0, \\ z(t) = g_0(x(t), \omega(t)), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $\omega(t) \in \mathbb{R}^q$ is the input, $z(t) \in \mathbb{R}^m$ is the output and f_0 and g_0 are smooth real vector functions.

Definition 1 The system (1) with a $\mathbb{C}^1$ storage fun-

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ction $V(x)$ is said to be dissipative with respect to the supply rate $r(\omega,z)$, if there exists a nonnegative definite function $V(x)$ with $V(0)=0$ such that for all x_0 and all ω.

$$\dot{V}(x(t)) \leqslant r(\omega(t),z(t)), \quad \forall t \geqslant 0 \qquad (2)$$

holds. If the dissipation inequality (2) holds with strict inequality, then we say the system is strictly dissipative.

In this paper, we focus our attention on the quadratic supply rate

$$r(\omega(t),z(t)) = \frac{1}{2}\omega^{\mathrm{T}}Q\omega + \omega^{\mathrm{T}}Sz + \frac{1}{2}z^{\mathrm{T}}Rz, \qquad (3)$$

where Q, S and R are symmetric matrices with $R < 0$ and $Q > 0$.

3 Main results

3.1 Dissipativeness analysis

In this section, attention is focused on nonlinear systems described by:

$$\begin{cases} \dot{x} = f(x) + g_1(x)\omega, \\ z = h(x), \end{cases} \qquad (4)$$

where $x(t) \in \mathbb{R}^n$ is the state, $\omega(t) \in \mathbb{R}^q$ is the input, $z(t) \in \mathbb{R}^m$ is the output, and f, g_1 and h are known smooth functions.

Theorem 1 If the inequality

$$L_fV+\frac{1}{2}(L_{g_1}V-h^{\mathrm{T}}S)Q^{-1}(L_{g_1}V-h^{\mathrm{T}}S)^{\mathrm{T}}-\frac{1}{2}h^{\mathrm{T}}Rh \leqslant 0 \qquad (5)$$

has a nonnegative definite solution $V(x) \in \mathbb{C}^1$ with $V(0)=0$, then system (4) with $V(x)$ is dissipative with respect to the supply rate $r(\omega,z)$.

Proof Along the trajectories of system (4), we have

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} - r(\omega,z) = {} & V_xf + V_xg_1(x)\omega - \\ & \frac{1}{2}\omega^{\mathrm{T}}Q\omega - \omega^{\mathrm{T}}Sz - \frac{1}{2}z^{\mathrm{T}}Rz = \\ & L_fV + \frac{1}{2}(L_{g_1}V - h^{\mathrm{T}}S)Q^{-1}\cdot \\ & (L_{g_1}V - h^{\mathrm{T}}S)^{\mathrm{T}} - \frac{1}{2}h^{\mathrm{T}}Rh - \\ & \left(\frac{\sqrt{2}}{2}Q^{\frac{1}{2}}\omega - \frac{\sqrt{2}}{2}Q^{\frac{1}{2}}(L_{g_1}V - h^{\mathrm{T}}S)^{\mathrm{T}}\right)^{\mathrm{T}}\cdot \\ & \left(\frac{\sqrt{2}}{2}Q^{\frac{1}{2}}\omega - \frac{\sqrt{2}}{2}Q^{\frac{1}{2}}(L_{g_1}V - h^{\mathrm{T}}S)^{\mathrm{T}}\right).\end{aligned}$$

From condition (5), we obtain $\dot{V} \leqslant r(\omega,z)$.

Remark 1 If $Q = I$, $S = 0$ and $R = -I$, then Theorem 1 is reduced to

$$V_xf + \frac{1}{2}V_xg_1g_1^{\mathrm{T}}V_x^{\mathrm{T}} + \frac{1}{2}h^{\mathrm{T}}h \leqslant 0. \qquad (6)$$

This is Theorem 2 in [6].

3.2 Dissipative control

In this section, we shall design a feedback controller to render a nonlinear system dissipative. We consider nonlinear systems of the form

$$\begin{cases} \dot{x} = f(x) + g_1(x)\omega + g_2(x)u, \\ z = h(x) + k(x)u, \end{cases} \qquad (7)$$

where $x(t) \in \mathbb{R}^n$ is the state, $\omega(t) \in \mathbb{R}^q$ is the exogenous input, $z(t) \in \mathbb{R}^m$ is the controlled output, f, g_1, g_2, h and k are known smooth functions, and $u \in \mathbb{R}^r$ is the control input.

Theorem 2 If the inequality

$$\begin{aligned}& L_fV + \frac{1}{2}(L_{g_1}V - (h + k\alpha)^{\mathrm{T}}S)Q^{-1}(L_{g_1}V - \\ & (h + k\alpha)^{\mathrm{T}}S)^{\mathrm{T}} - \frac{1}{2}(L_{g_2}V - h^{\mathrm{T}}Rk)\hat{R}^{-1}(L_{g_2}V - \\ & h^{\mathrm{T}}Rk)^{\mathrm{T}} - \frac{1}{2}h^{\mathrm{T}}Rh \leqslant 0 \qquad (8)\end{aligned}$$

has a nonnegative definite solution $V(x) \in \mathbb{C}^1$ with $V(0) = 0$, where $\hat{R}^{-1} = -k^{\mathrm{T}}Rk > 0$, then the controller

$$u = \alpha(x) = -\hat{R}^{-1}(L_{g_2}V - h^{\mathrm{T}}Rk)^{\mathrm{T}}, \qquad (9)$$

makes system (7) with $V(x)$ dissipative with respect to the supply rate $r(\omega,z)$.

Proof Along the trajectories of system (7), we have

$$\begin{aligned}& \frac{\mathrm{d}V}{\mathrm{d}t} - r(\omega,z) = \\ & V_xf + V_xg_1(x)\omega + V_xg_2u - \\ & \frac{1}{2}\omega^{\mathrm{T}}Q\omega - \omega^{\mathrm{T}}Sz - \frac{1}{2}z^{\mathrm{T}}Rz = \\ & V_xf + V_xg_1(x)\omega + V_xg_2u - \frac{1}{2}\omega^{\mathrm{T}}Q\omega - \\ & \omega^{\mathrm{T}}S(h + ku) - \frac{1}{2}h^{\mathrm{T}}Rh - h^{\mathrm{T}}Rku - \frac{1}{2}u^{\mathrm{T}}k^{\mathrm{T}}Rku = \\ & L_fV + \frac{1}{2}(L_{g1}V - (h + ku)^{\mathrm{T}}S)Q^{-1}\cdot \\ & (L_{g_1}V - (h + ku)^{\mathrm{T}}S)^{\mathrm{T}} - \frac{1}{2}h^{\mathrm{T}}Rh - \\ & \frac{1}{2}(L_{g_2}V - h^{\mathrm{T}}Rk)\hat{R}^{-1}(L_{g_2}V - h^{\mathrm{T}}Rk)^{\mathrm{T}} +\end{aligned}$$

$$\left(\frac{\sqrt{2}}{2}\hat{R}^{\frac{1}{2}}u+\frac{\sqrt{2}}{2}\hat{R}^{-\frac{1}{2}}(L_{g_2}V-h^{\mathrm{T}}Rk)^{\mathrm{T}}\right)^{\mathrm{T}}\cdot$$
$$\left(\frac{\sqrt{2}}{2}\hat{R}^{\frac{1}{2}}u+\frac{\sqrt{2}}{2}\hat{R}^{-\frac{1}{2}}(L_{g_2}V-h^{\mathrm{T}}Rk)^{\mathrm{T}}\right)-$$
$$\left(\frac{\sqrt{2}}{2}Q^{\frac{1}{2}}\omega-\frac{\sqrt{2}}{2}Q^{-\frac{1}{2}}(L_{g_1}V-(h+ku)^{\mathrm{T}}S)^{\mathrm{T}}\right)^{\mathrm{T}}\cdot$$
$$\left(\frac{\sqrt{2}}{2}Q^{\frac{1}{2}}\omega-\frac{\sqrt{2}}{2}Q^{-\frac{1}{2}}(L_{g_1}V-(h+ku)^{\mathrm{T}}S)^{\mathrm{T}}\right).$$

Clearly, if $u=\alpha(x)=-\hat{R}^{-1}(L_{g_2}V-h^{\mathrm{T}}Rk)^{\mathrm{T}}$, then from condition (8), we obtain $\dot{V}\leqslant r(\omega,z)$.

Remark 2 If $Q=I, S=0, R=-I$ and $k^{\mathrm{T}}[h\quad k]=[0\quad I]$, then Theorem 2 is reduced to

$$\begin{cases}V_xf+\frac{1}{2}V_xg_1g_1^{\mathrm{T}}V_x^{\mathrm{T}}-\frac{1}{2}V_xg_2g_2^{\mathrm{T}}V_x^{\mathrm{T}}+\frac{1}{2}h^{\mathrm{T}}h\leqslant 0,\\ u=-g_2^{\mathrm{T}}V_x^{\mathrm{T}}.\end{cases} \tag{10}$$

This is Theorem 16 in [6].

3.3 Robust dissipativeness analysis

Motivated by the robust control design problem to be addressed in the next section, we shall analyze the dissipativeness of the following class of uncertain nonlinear systems:

$$\begin{cases}\dot{x}=f(x)+\Delta f(x)+g_1(x)\omega,\\ z=h(x),\end{cases} \tag{11}$$

where $x(t)\in\mathbb{R}^n$ is the state, $\omega(t)\in\mathbb{R}^q$ is the input, $z(t)\in\mathbb{R}^m$ is the output, f, g_1 and h are known smooth functions, and the smooth uncertain mapping $\Delta f(x)$ is assumed norm bounded as follows.

$$\Delta f(x)\in\Omega_f=\{e(x)\delta(x):\delta(x)^{\mathrm{T}}\delta(x)\leqslant m(x)^{\mathrm{T}}m(x)\},$$

where $e(x)$ and $m(x)$ are known smooth mappings.

Theorem 3 If there exists $\lambda_f(x)>0$ such that the inequality

$$L_fV+\frac{1}{2}(L_{g_1}V-h^{\mathrm{T}}S)Q^{-1}(L_{g_1}V-h^{\mathrm{T}}S)^{\mathrm{T}}+\frac{1}{4}\lambda_f^2(L_eV)(L_eV)^{\mathrm{T}}+\frac{1}{\lambda_f^2}m^{\mathrm{T}}m-\frac{1}{2}h^{\mathrm{T}}Rh\leqslant 0 \tag{12}$$

has a nonnegative definite solution $V(x)\in\mathbb{C}^1$ with $V(0)=0$, then system (11) with $V(x)$ is dissipative with respect to the supply rate $r(\omega, z)$.

Proof Along the trajectories of system (11), using the inequality

$$V_x\Delta f=V_xe(x)\delta(x)\leqslant\frac{1}{2}\lambda_f^2(L_eV)(L_eV)^{\mathrm{T}}+\frac{1}{2\lambda_f^2}m^{\mathrm{T}}m,$$

we can obtain Theorem 3.

Remark 3 If $Q=I, S=0$ and $R=-I$, then Theorem 3 is reduced to

$$V_xf+\frac{1}{2}V_x(g_1g_1^{\mathrm{T}}+\lambda_f^2ee^{\mathrm{T}})V_x^{\mathrm{T}}+\frac{1}{2}(h^{\mathrm{T}}h+\frac{1}{\lambda_f^2}m^{\mathrm{T}}m)\leqslant 0. \tag{13}$$

This is Corollary 1 in [11].

3.4 Robust dissipative control

Consider the dynamical system

$$\begin{cases}\dot{x}=f(x)+\Delta f(x)+g_1(x)\omega+g_2(x)u,\\ z=h(x)+k(x)u,\end{cases} \tag{14}$$

where $x(t)\in\mathbb{R}^n$ is the state, $\omega(t)\in\mathbb{R}^q$ is the exogenous input, $z(t)\in\mathbb{R}^m$ is the output, f, g_1, g_2, h and k are known smooth functions, $u\in\mathbb{R}^r$ is the control input and $\Delta f(x)\in\Omega_f$.

Theorem 4 If there exists $\lambda_f(x)>0$ such that the inequality

$$L_fV+\frac{1}{2}(L_{g_1}V-(h+k\alpha)^{\mathrm{T}}S)Q^{-1}(L_{g_1}V-(h+k\alpha)^{\mathrm{T}}S)^{\mathrm{T}}+\frac{1}{4}\lambda_f(L_eV)(L_eV)^{\mathrm{T}}-\frac{1}{2}(L_{g_2}V-h^{\mathrm{T}}Rk)\hat{R}^{-1}(L_{g_2}V-h^{\mathrm{T}}Rk)^{\mathrm{T}}+\frac{1}{\lambda_f^2}m^{\mathrm{T}}m-\frac{1}{2}h^{\mathrm{T}}Rh\leqslant 0 \tag{15}$$

has a nonnegative definite solution $V(x)\in\mathbb{C}^1$ with $V(0)=0$, where $\hat{R}^{-1}=-k^{\mathrm{T}}Rk>0$, then the controller

$$u=\alpha(x)=-\hat{R}^{-1}(L_{g_2}V-h^{\mathrm{T}}Rk)^{\mathrm{T}}, \tag{16}$$

makes system (14) with $V(x)$ dissipative with respect to the supply rate $r(\omega, z)$.

Proof Along system (14), using the inequality

$$V_x\Delta f=V_xe(x)\delta(x)\leqslant\frac{1}{2}\lambda_f^2(L_eV)(L_eV)^{\mathrm{T}}+\frac{1}{2\lambda_f^2}m^{\mathrm{T}}m,$$

we can obtain Theorem 4.

Remark 4 If $Q=I, S=0, R=-I$ and

$k^{\mathrm{T}}[h \quad k] = [0 \quad I]$, then Theorem 4 is reduced to

$$\begin{cases} V_x f + \dfrac{1}{2} V_x (g_1 g_1^{\mathrm{T}} + \lambda_f^2 ee^{\mathrm{T}} - g_2 g_2^{\mathrm{T}}) V_x^{\mathrm{T}} + \\ \dfrac{1}{2}(h^{\mathrm{T}} h + \dfrac{1}{\lambda_f^2} m^{\mathrm{T}} m) \leqslant 0, \\ u = - g_2^{\mathrm{T}} V_x^{\mathrm{T}}. \end{cases} \tag{17}$$

This is Theorem 2 in [11].

4 An example

Consider the following uncertain nonlinear system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} - x_1 x_2^2 - x_1^2 x_2^2 - \dfrac{3}{4} x_1^3 - x_1 \\ - x_1 x_2^3 - 2x_2 \end{bmatrix} + \begin{bmatrix} p(x) x_1 x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} x_2^2 \\ 0 \end{bmatrix} \omega + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} u,$$

$$z = u,$$

where $| p(x) | \leqslant 1$, $e(x) = [x_1 \quad 0]^{\mathrm{T}}$ and $m(x) = x_2$. By taking $\lambda_f = 1$ and supply rate $r(\omega, z) = \frac{1}{2}\omega^{\mathrm{T}}\omega + \omega^{\mathrm{T}} z - \frac{1}{2} z^{\mathrm{T}} z$, we can solve equation (15) and get the following solution:

$$V(x) = \frac{1}{2} x_1^2 + \frac{1}{2} x_2^2,$$

$$u = - x_1^2 - x_2^2.$$

5 Conclusion

This paper has addressed the problem of quadratic dissipative control for nonlinear systems with or without uncertainty. It has been shown that the robust dissipative control problem can be resolved for all admissible uncertainty, if there exists a scaling function such that Hamilton-Jacobi inequality has a nonnegative solution. Our results provide a more flexible and less conservative control design method.

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