Set Theory Lecture Note

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1 ZFC

Actually, there exists a 0th axiom.

$$\exists x(x = x)$$

It means there 'is something'.

1. Extensionality

$$\forall x \forall y(\forall z(z \in x \leftrightarrow z \in y) \rightarrow x = y)$$
$$A \subseteq B, B \subseteq A \rightarrow A = b$$

2. Foundation / Regularity

$$\forall x(\exists y(y \in x) \rightarrow \exists y(y \in x \wedge \neg \exists z(z \in x \wedge z \in y)))$$

3. Seperation Schema
   For each ϕ with free variables among $x, z, w_1, \cdots, w_n$

$$\forall w_1, \cdots, \forall w_n \forall x \exists y \forall z(z \in y \leftrightarrow z \in x \wedge \phi(x, z, w_1, \cdots, w_n))$$

4. Pairing

$$\forall x \forall y \exists z(x \in z \wedge y \in z)$$

5. Union

$$\forall x \exists y \forall z \forall w ((z \in w \land w \in x) \to z \in y)$$

$$\bigcup x = \cup_{w \in x} w$$

6. Replacement scheme
   For each ϕ whose free variables among $x, z, w_1, \cdots, w_n$

$$\forall w_1 \cdots \forall w_n \forall x \forall z ((z \in x \to \exists ! y \phi) \to \exists u (\forall z \exists y (z \in x \to y \in u \land \phi)))$$

7. Infinity

$$\exists x (0 \in x \land \forall y (y \in x \to S(y) \in x))$$

8. Power set

$$\forall x \exists y \forall z (z \subseteq x \to z \in y)$$

9. Choice

$$\forall X \exists R (R \text{ well orders } X)$$

   The following statments are equivalent to axiom of choice

   - Every commutative ring with 1 has a maximal ideal.
   - Every vector space over a field has a basis.
   - Tychonoff's theorem