Exercises:

1. Energy storages.

   Energy can be stored as kinetic energy, potential energy or heat. Some claim that the two first are hopeless for larger amount of energy, let us make a test.

   We have a granite stone where we can store energy as

   (a) potential energy by lifting the stone h [meter]

   (b) kinetic energy by giving the stone a velocity v [m/s]

   (c) heat energy by increasing the temperature ΔT [^oC]

   If the temperature of the stone is increased by 1 [^oC] we store a certain amount of energy. If we will store the same amount of energy as potential energy how much must we then lift the stone? If we will store the same heat energy as kinetic energy what must be the velocity of the stone (in km/hour)?

2. Electric water heater.

   Electricity is used as heating source for some small water heaters.

   The figure shows an electric water heater with volume V [m^3]. The flow through the water heater is m [kg/sek]. The tank is always filled with water. The water is stirred meaning that the water temperature is the same everywhere in the heater and the outlet. A heating element is supplying a power, $\mathcal{P}(t)$ [Watt], to the water.

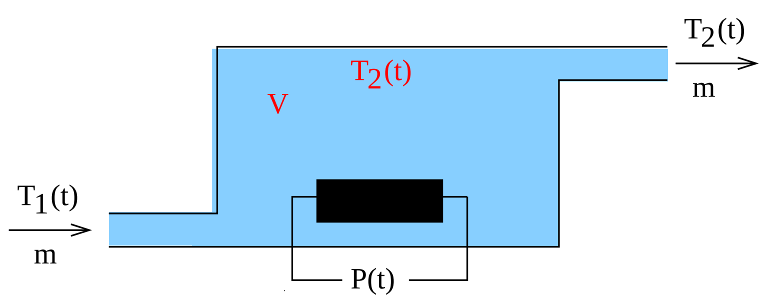

Figure 1: *Electric water heater*

The ambient temperature is T_o(t). The power, $\mathcal{P}_L$(t) [Watt], from the tank to the ambient is:

$$\mathcal{P}_L(t) = K[T_2(t) - T_o(t)]$$

The inlet water has the temperature, T_1(t).

b. Write the energy balance.

c. If m is constant is it possible to Laplace transform the equation. Do it and draw a block diagram where $T_2(s)$ is the output and $\mathcal{P}(s)$ is the controllable input. T_1 and T_o are disturbances (they are both input to the model).

d. Calculate $T_2(s)$ as a function of the inputs.

3. Model of building.

On the figure is shown a simplified model of a room with a heater. It is assumed that the room temperature is T_r, wall temperature is T_w, ambient temperature is T_a and the heater temperature is T_h.

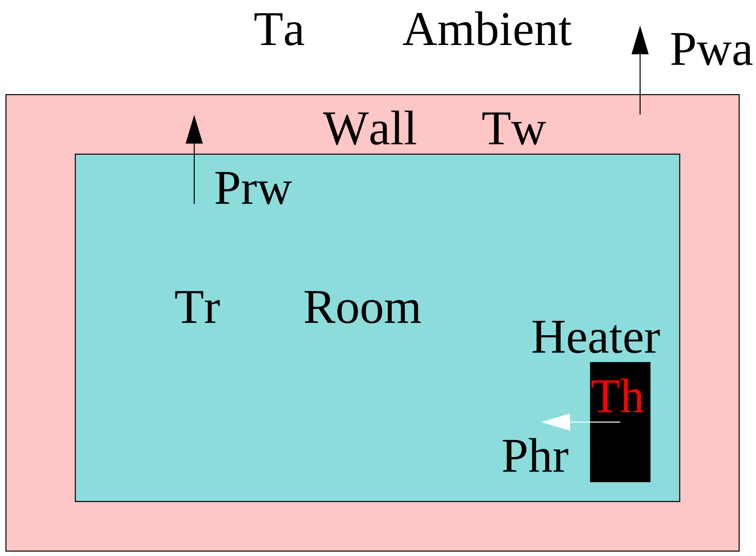

Figure 2: *Room model*

- In steady state the heater gives 2000 W, $T_h = 80\ ^oC$, $T_r = 25\ ^oC$, $T_w = 18\ ^oC$ and $T_a = 10\ ^oC$. It is assumed that the heat transfers are convective. Find expressions for $\mathcal{P}_{hr}$, $\mathcal{P}_{rw}$ and $\mathcal{P}_{wa}$.
- Write the energy balances.
- It is assumed that the ambient temperature is constant, and it is possible to control $\mathcal{P}_{hr}$. The heat capacity of the wall is $c_w M_w = 4.8 \cdot 10^6\ J/^oC$; the heat capacity of the room is $c_{air} M_{air} = 25000\ J/^oC$. Find a (linear) transfer function from $\mathcal{P}_{hr}(s)$ to $T_r(s)$. How do the step response look and why?
- Formulate a linear model in state space, $\mathcal{P}_{hr}(t)$ and $T_a(t)$ are inputs. $T_r(t)$ is output.

4. <u>Model of a boiler.</u>

It is assumed that there is saturation in the boiler and the pressure in the steam and

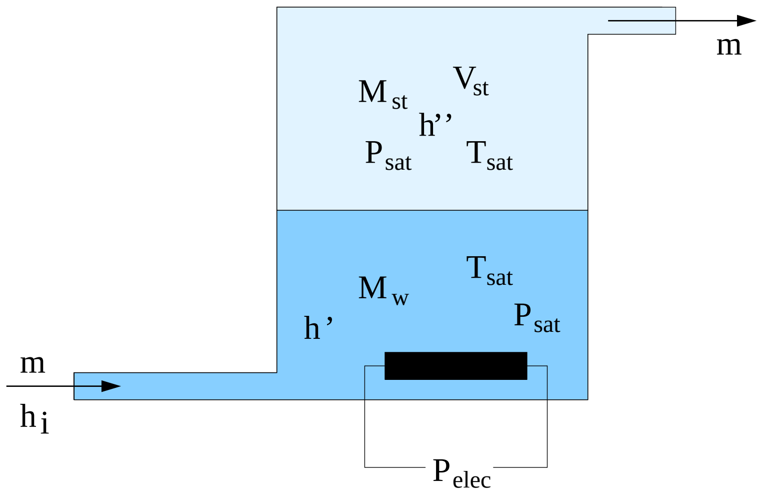

Figure 3: *Electric boiler*

in the water is the same; this implies that the pressure is the same all-over. The mass flow in and out are assumed equal, and it is also assumed that the mass of water in the boiler is constant, implying that the steam volume is constant. So the constants are M_w, V_{st} and h_i; all others are time varying.

- Explain why the energy balance is

$$\frac{d(M_w h' + M_{st} h'')}{dt} = m(h_i - h'') + \mathcal{P}_{elec} \tag{1}$$

- Put $M_{st} = \rho_{st} \cdot V_{st}$ in the equation. Which terms are dependent on the saturation temperature?
- Use the chain rule and the rule

$$\frac{d[f(T(t))]}{dt} = \frac{d[f(T(t))]}{dT(t)} \frac{dT(t)}{dt} \tag{2}$$

  to find a non–linear state description of the system, where $T_{sat}(t)$ is the state. Note that terms like

$$\frac{dh'(T_{sat})}{dT_{sat}} \tag{3}$$

  can be found in the steam table using numerical differentiation.