Exercises:

1. Energy storages.

2. Electric water heater.

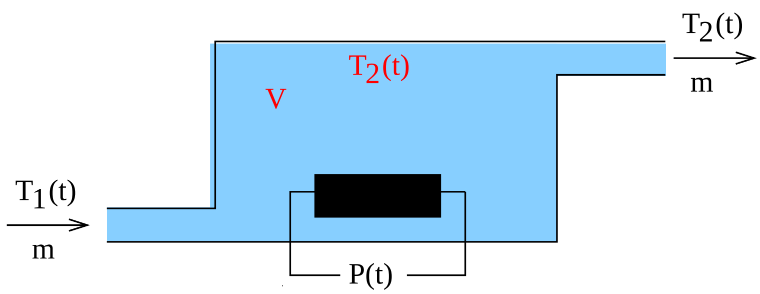

Figure 1: *Electric water heater*

The energy balance

$$\mathcal{P}(t) + cm(t)T_1(t) - cm(t)T_2(t) - K[T_2(t) - T_o(t)] = cV\rho\dot{T}_2(t)$$

Laplace transform

$$\mathcal{P}(s) + cmT_1(s) - cmT_2(s) - K[T_2(s) - T_o(s)] = cVs\rho\dot{T}_2(s)$$

Transfer functions

$$T(s) = [\mathcal{P}(s) + cmT_1(s) + KT_o(s)] \cdot \frac{1}{Vc\rho s + mc + K}$$

3. Model of building.

On the figure is shown a simplified model of a room with a heater. It is assumed that the room temperature is T_r, wall temperature is T_w, ambient temperature is T_a and the heater temperature is T_h.

- In steady state the heater gives 2000 W, $T_h = 80\ ^oC$, $T_r = 25\ ^oC$, $T_w = 18\ ^oC$ and $T_a = 10\ ^oC$. It is assumed that the heat transfers are convective. Find expressions for $\mathcal{P}_{hr}$, $\mathcal{P}_{rw}$ and $\mathcal{P}_{wa}$.

$$A_{hr}\alpha_{hr} = \frac{2000}{80 - 25} = 36.4 \Rightarrow \mathcal{P}_{hr} = A_{hr}\alpha_{hr}(T_h - T_r)$$

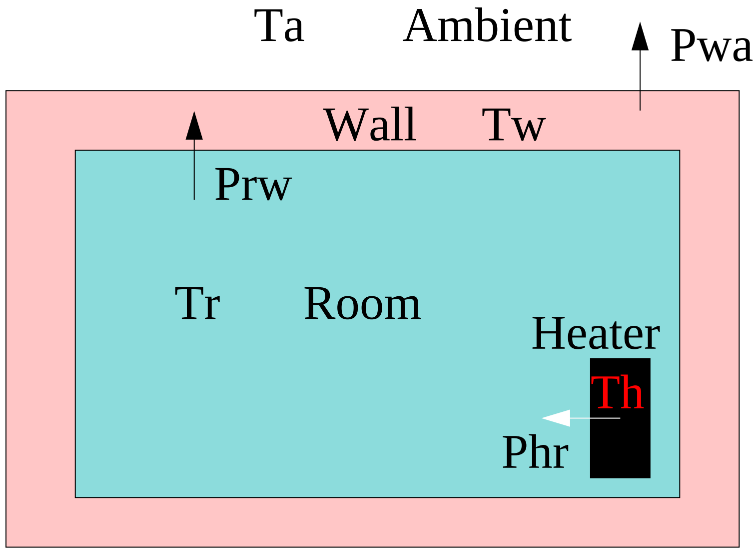

Figure 2: *Room model*

$$A_{rw}\alpha_{rw} = \frac{2000}{25-18} = 286 \Rightarrow \mathcal{P}_{rw} = A_{rw}\alpha_{rw}(T_r - T_w)$$
$$A_{wa}\alpha_{wa} = \frac{2000}{18-10} = 250 \Rightarrow \mathcal{P}_{wa} = A_{wa}\alpha_{wa}(T_w - T_a)$$

- Write the energy balances.

  Room:
  $$c_{air}M_{air}\dot{T}_r(t) = \mathcal{P}_{hr}(t) - A_{rw}\alpha_{rw}(T_r(t) - T_w(t))$$
  Wall:
  $$c_w M_w \dot{T}_w(t) = A_{rw}\alpha_{rw}(T_r(t) - T_w(t)) - A_{wa}\alpha_{wa}(T_w(t) - T_a)$$

- It is assumed that the ambient temperature is constant, and it is possible to control $\mathcal{P}_{hr}$. The heat capacity of the wall is 100 times the heat capacity of the room. Find a (linear) transfer function from $T_h(s)$ to $T_r(s)$. How do the step response look and why?

  Linearisation and Laplace transform gives (the variables are now small signals):
  $$c_{air}M_{air}sT_r(s) = \mathcal{P}_{hr}(s) - A_{rw}\alpha_{rw}(T_r(s) - T_w(s))$$
  $$c_w M_w sT_w(s) = A_{rw}\alpha_{rw}(T_r(s) - T_w(s)) - A_{wa}\alpha_{wa}T_w(s)$$
  Eliminating $T_w(s)$ from the equations we find:
  $$T_r(s) = \frac{c_{air}M_{air}s + A_{rw}\alpha_{rw} + A_{wa}\alpha_{wa}}{c_{air}M_{air}c_w M_w s^2 + [c_{air}M_{air}(A_{rw}\alpha_{rw} + A_{wa}\alpha_{wa}) + c_w M_w A_{rw}\alpha_{rw}]s + A_{rw}\alpha_{rw}A_{wa}\alpha_{wa}}\mathcal{P}_{hr}($$

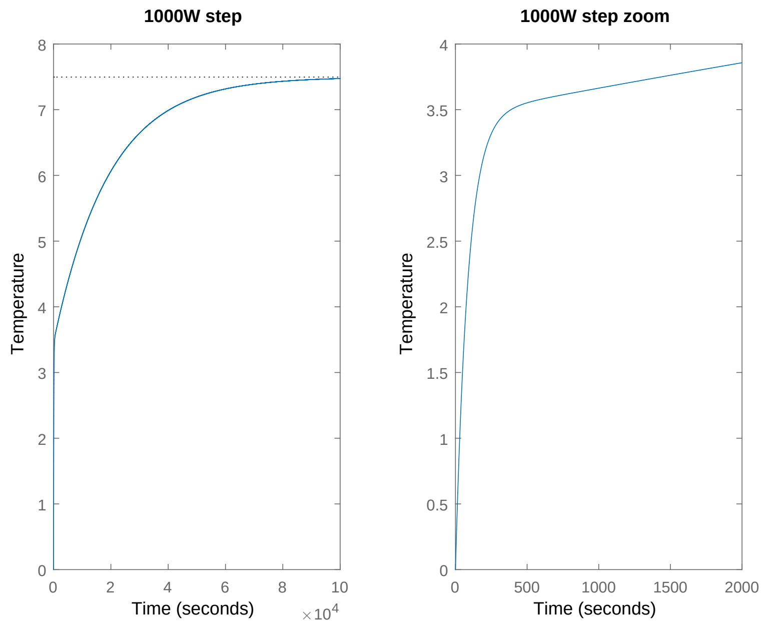

Figure 3: *Step response*

It is seen that the room temperature raises very quickly due to the low air heat capacity; then the room temperature must heat up the wall.

- Formulate a linear model in state space, $T_h(t)$ and $T_a(t)$ are inputs.

$$\begin{bmatrix} \dot{T}_r \\ \dot{T}_w \end{bmatrix} = \begin{bmatrix} -\frac{A_{rw}\alpha_{rw}}{c_{air}M_{air}} & \frac{A_{rw}\alpha_{rw}}{c_{air}M_{air}} \\ \frac{A_{rw}\alpha_{rw}}{c_w M_w} & -\frac{A_{rw}\alpha_{rw}+A_{wa}\alpha_{wa}}{c_w M_w} \end{bmatrix} \begin{bmatrix} T_r \\ T_w \end{bmatrix} + \begin{bmatrix} \frac{1}{c_{air}M_{air}} & 0 \\ 0 & \frac{A_{wa}\alpha_{wa}}{c_w M_w} \end{bmatrix} \begin{bmatrix} \mathcal{P}_{hr} \\ T_a \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} T_r \\ T_w \end{bmatrix}$$

4. <u>Model of a boiler.</u>

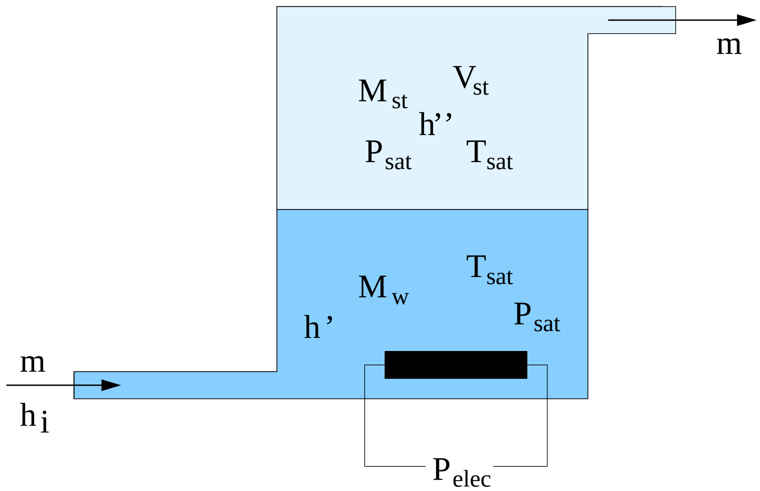

Figure 4: *Electric boiler*

- Explain why the energy balance is

$$\frac{d(M_w h' + M_{st} h'')}{dt} = m(h_i - h'') + \mathcal{P}_{elec} \tag{1}$$

- Put $M_{st} = \rho_{st} \cdot V_{st}$ in the equation. Which terms are dependent on the saturation temperature?
- Use the chain rule and the rule

$$\frac{d[f(T(t))]}{dt} = \frac{d[f(T(t))]}{dT(t)} \frac{dT(t)}{dt} \tag{2}$$

to find a non–linear state description of the system, where $T_{sat}(t)$ is the state.

$$
\begin{aligned}
&\frac{d(M_w h' + M_{st} h'')}{dt} = \frac{d(M_w h' + V_{st}\rho'' h'')}{dt} = M_w \frac{d(h')}{dt} + V_{st}\frac{d(\rho_{st} h'')}{dt} \\
&= M_w \frac{d(h'(T_{sat}(t))}{dt} + V_{st}\rho''(T_{sat}(t))\frac{d(h''(T_{sat}(t)))}{dt} + V_{st}h''(T_{sat}(t))\frac{d(\rho''(T_{sat}(t)))}{dt} \\
&= M_w \frac{d(h'(T_{sat}(t))}{dT_{sat}(t)}\frac{dT_{sat}(t)}{dt} + V_{st}\rho''(T_{sat}(t))\frac{d(h''(T_{sat}(t))}{dT_{sat}(t)}\frac{dT_{sat}(t)}{dt} \\
&+ V_{st}h''(T_{sat}(t))\frac{d(\rho''(T_{sat}(t)))}{dT_{sat}(t)}\frac{dT_{sat}(t)}{dt} \\
&= \underbrace{[M_w \frac{d(h'(T_{sat}(t))}{dT_{sat}(t)} + V_{st}\rho''(T_{sat}(t))\frac{d(h''(T_{sat}(t))}{dT_{sat}(t)} + V_{st}h''(T_{sat}(t))\frac{d(\rho''(T_{sat}(t)))}{dT_{sat}(t)}]}_{=\mathrm{E}}\frac{dT_{sat}(t)}{dt} \\
&= m(t)(h_i - h''(T_{sat}(t))) + \mathcal{P}_{elec}(t) \Rightarrow \\
&E\frac{dT_{sat}(t)}{dt} = m(t)(h_i - h''(T_{sat})(t)) + \mathcal{P}_{elec}(t) \Rightarrow \\
&\frac{dT_{sat}(t)}{dt} = m(t)(h_i - h''(T_{sat}(t)))\frac{1}{E} + \frac{1}{E}\mathcal{P}_{elec}(t)
\end{aligned}
$$

In an operating point E is constant and $h_i - h''(T_{sat}(t))$ is a negative constant so the equation gives

$$
\frac{dT_{sat}(t)}{dt} = -m(t)K_1 + K_2\mathcal{P}_{elec}(t)
$$