Fault Detection, Isolation, and Modeling

Fault-Tolerant Control

Christoffer Sloth

`ces@es.aau.dk`

Department of Electronic Systems
Aalborg University
Denmark

AALBORG UNIVERSITY
DENMARK

Agenda

Relation to Curriculum

Introduction

Model Matching

Virtual Sensor and Actuator

Relation to Curriculum

Knowledge

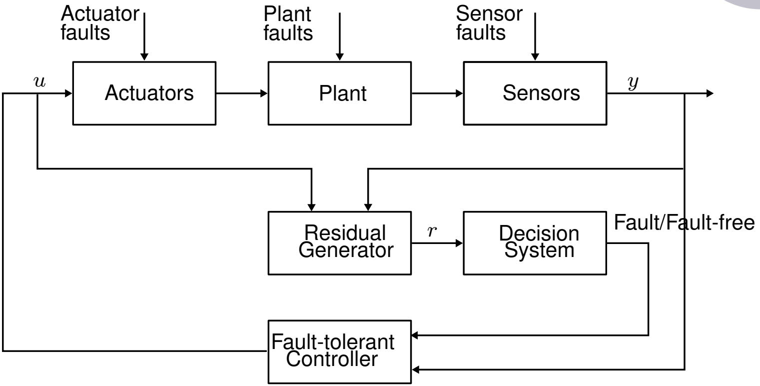
- The taxonomy of fault tolerant systems

Skills

- In designing strategies for handling faults
  - Active fault tolerance
    - Control strategy change

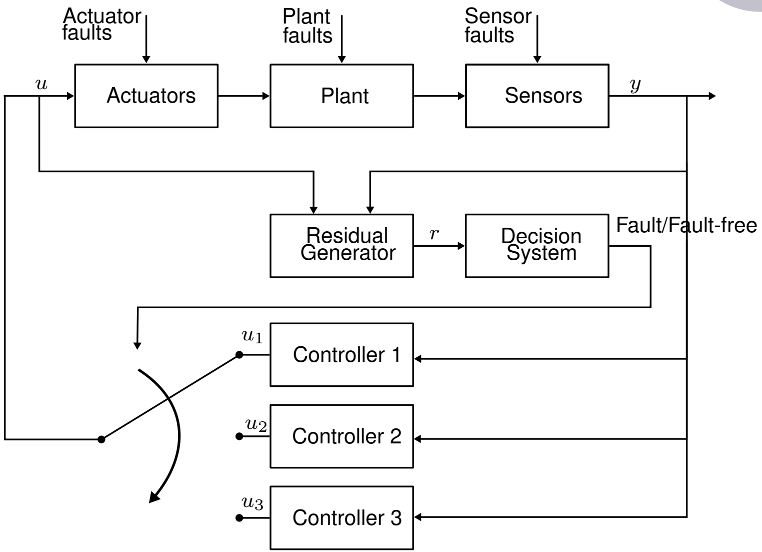
Competencies

- In designing fault tolerance strategies for a given system

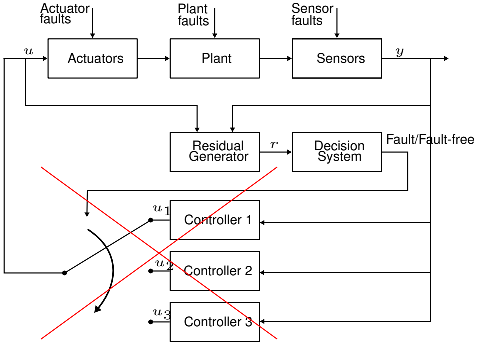


Actuator faults
Plant faults
Sensor faults
u
Actuators
Plant
Sensors
y
Residual Generator
r
Decision System
Fault/Fault-free
Fault-tolerant Controller

Introduction
Fault-Tolerant Control



When a passive fault-tolerant controller is used, the controller will not reconfigure; however, it will be designed to be robust against faults.



Henrik Niemann and Jakob Stoustrup. **Passive fault tolerant control of a double inverted pendulum - a case study**. *Control Engineering Practice*, 13(8):1047-1059, August 2005.

Method	Characteristics
Model Matching (Today)	Based on algebraic relations
Virtual Sensors and Actuators (Today)	Observer-based method
Fault-Tolerant LQRs	Switching between controllers

Given a system model

$$\dot{x} = g(x, u, f)$$
$$y = h(x, u, f),$$

and a nominal controller $u = k(y)$. Find a new control law $u = k_f(y)$ such that the faulty system has the required performance.

Given a system model

$$\dot{x} = g(x, u, f)$$
$$y = h(x, u, f),$$

and a nominal controller $u = k(y)$. Find a new control law $u = k_f(y)$ such that the faulty system has the required performance.

The following fault scenarios are considered

- **Sensor Fault:** The output matrix C is changed to C_f, where the i^{th} row is zero in case of i^{th} sensor is lost.
- **Actuator Fault:** The input matrix B is changed to B_f, where the i^{th} column is zero in case of i^{th} actuator is lost.

Model Matching

Given models

$$\text{Nominal System:} \quad \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}, \; u = -Ky$$

$$\text{Faulty System:} \quad \begin{cases} \dot{x} = A_{\mathsf{f}}x + B_{\mathsf{f}}u \\ y = C_{\mathsf{f}}x. \end{cases}$$

Find controller

$$u = -K_f y$$

such that

$$(A - BKC) = (A_f - B_f K_f C_f).$$

Given models

$$\text{Nominal System:} \quad \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}, \; u = -Ky$$

$$\text{Faulty System:} \quad \begin{cases} \dot{x} = A_{\mathsf{f}}x + B_{\mathsf{f}}u \\ y = C_{\mathsf{f}}x. \end{cases}$$

Find controller

$$u = -K_f y$$

such that

$$(A - BKC) = (A_f - B_f K_f C_f).$$

This is called the ***model matching problem***.

We say that the ***exact model matching problem*** is solved if

$$(A - BKC) = (A_f - B_f K_f C_f).$$

Model Matching
Problem Formulation

We say that the ***exact model matching problem*** is solved if

$$(A - BKC) = (A_f - B_f K_f C_f).$$

If this cannot be solved, we find a control $u = -K_f y$ that solves the following minimization problem

$$\min_{K_f} ||(A - BKC) - (A_f - B_f K_f C_f)||$$

Model Matching
Problem Formulation

We say that the ***exact model matching problem*** is solved if

$$(A - BKC) = (A_f - B_f K_f C_f).$$

If this cannot be solved, we find a control $u = -K_f y$ that solves the following minimization problem

$$\min_{K_f} ||(A - BKC) - (A_f - B_f K_f C_f)||$$

Case	**Faulty System Model**
State Feedback	$\begin{cases} \dot{x} = A_\mathsf{f} x + B_\mathsf{f} u \\ y = x \end{cases}$
Sensor Fault	$\begin{cases} \dot{x} = Ax + Bu \\ y = C_\mathsf{f} x \end{cases}$
Actuator Fault	$\begin{cases} \dot{x} = Ax + B_\mathsf{f} u \\ y = Cx \end{cases}$

Model Matching
Solution to State Feedback Case

The optimal solution K_{f}^*, i.e.,

$$K_{\mathsf{f}}^* = \arg \min_{K_{\mathsf{f}} \in \mathbb{R}^{m \times n}} ||(A - BK) - (A_{\mathsf{f}} - B_{\mathsf{f}} K_{\mathsf{f}})||$$

is given by

$$K_{\mathsf{f}}^* = \underbrace{(B_{\mathsf{f}}^T B_{\mathsf{f}})^{-1} B_{\mathsf{f}}^T}_{= B_{\mathsf{f}}^+} (A_{\mathsf{f}} - A + BK).$$

Model Matching
Solution to State Feedback Case

The optimal solution K_f^*, i.e.,

$$K_\mathsf{f}^* = \arg \min_{K_\mathsf{f} \in \mathbb{R}^{m \times n}} ||(A - BK) - (A_\mathsf{f} - B_\mathsf{f} K_\mathsf{f})||$$

is given by

$$K_\mathsf{f}^* = \underbrace{(B_\mathsf{f}^T B_\mathsf{f})^{-1} B_\mathsf{f}^T}_{=B_\mathsf{f}^+}(A_\mathsf{f} - A + BK).$$

Applying the control K_f^* to the faulty system gives

$$\dot{x} = (A_\mathsf{f} - B_\mathsf{f} B_\mathsf{f}^+ A_\mathsf{f} + B_\mathsf{f} B_\mathsf{f}^+ (A - BK))x.$$

Consider the two-cart system subject to a fault causing the friction of Cart 1 to double, i.e.,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{b_1}{m_1} & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & -\frac{b_2}{m_2} \end{bmatrix}$$

$$A_f = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{2b_1}{m_1} & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & -\frac{b_2}{m_2} \end{bmatrix}, B = B_f = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix}.$$

The parameters for the model are assumed to be $m_1 = 2$ kg, $m_2 = 1$ kg, $k = 2$ N/m, $b_1 = 3$ N/(m/s), and $b_2 = 4$ N/(m/s). The nominal control is given by

$$u = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} x.$$

Model Matching

Example: State Feedback Case

The controller for the faulty system is now calculated according to

$$K_\mathsf{f} = B_\mathsf{f}^{+}(A_\mathsf{f} - A + BK)$$

resulting in

$$K_\mathsf{f} = \begin{bmatrix} 0 & 0 & 2 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -3 & 0 \\ 2 & -2 & 0 & -4 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -1.5 & 0 \\ 2 & -2 & 0 & -4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \right)$$

giving a controller gain

$$K_\mathsf{f} = \begin{bmatrix} 1 & 1 & -2 & 1 \end{bmatrix}.$$

Model Matching
Problem Formulation: Sensor Fault

Consider the *model matching problem*, and assume that $A_f = A$ and $B_f = B$.

Model Matching
Problem Formulation: Sensor Fault

Consider the *model matching problem*, and assume that $A_f = A$ and $B_f = B$. Find a controller gain matrix K_f such that the nominal and faulty systems have the same dynamics, i.e.,

$$(A - BKC) = (A - BK_\mathsf{f}C_\mathsf{f}).$$

Model Matching
Solvability: Sensor Fault

A solution to the *exact model-matching problem* with $A_f = A$ and $B_f = B$ can be found only if

$$\ker(C_\mathsf{f}) \subseteq \ker(C).$$

Model Matching
Solvability: Sensor Fault

A solution to the *exact model-matching problem* with $A_f = A$ and $B_f = B$ can be found only if

$$\ker(C_{\mathsf{f}}) \subseteq \ker(C).$$

The controller

$$u = -\underbrace{KP}_{=K_{\mathsf{f}}} y_{\mathsf{f}},$$

where

$$P = CC_{\mathsf{f}}^{+} = CC_{\mathsf{f}}^{T}(C_{\mathsf{f}}C_{\mathsf{f}}^{T})^{-1}$$

gives the solution to *exact model-matching problem* with $A_f = A$ and $B_f = B$.

When is this condition

$$\mathsf{ker}(C_{\mathsf{f}}) \subseteq \mathsf{ker}(C)$$

satisfied? (Provide examples)

Consider the *model matching problem*, and assume that $A_f = A$ and $C_f = C$.

Consider the *model matching problem*, and assume that $A_f = A$ and $C_f = C$. Find a controller gain matrix K_f such that the nominal and faulty systems have the same dynamics, i.e.,

$$(A - BKC) = (A - B_\mathsf{f} K_\mathsf{f} C).$$

A solution to the *exact model-matching problem* with $A_f = A$ and $C_f = C$ can be found only if

$$\mathrm{Im}(B_{\mathrm{f}}) \supseteq \mathrm{Im}(B).$$

A solution to the *exact model-matching problem* with $A_f = A$ and $C_f = C$ can be found only if

$$\mathrm{Im}(B_\mathrm{f}) \supseteq \mathrm{Im}(B).$$

The controller

$$u = -\underbrace{NK}_{=K_\mathrm{f}} y,$$

where

$$N = B_\mathrm{f}^{+} B = (B_\mathrm{f}^T B_\mathrm{f})^{-1} B_\mathrm{f}^T B$$

gives the solution to the *exact model-matching problem* with $A_f = A$ and $C_f = C$.

When is this condition

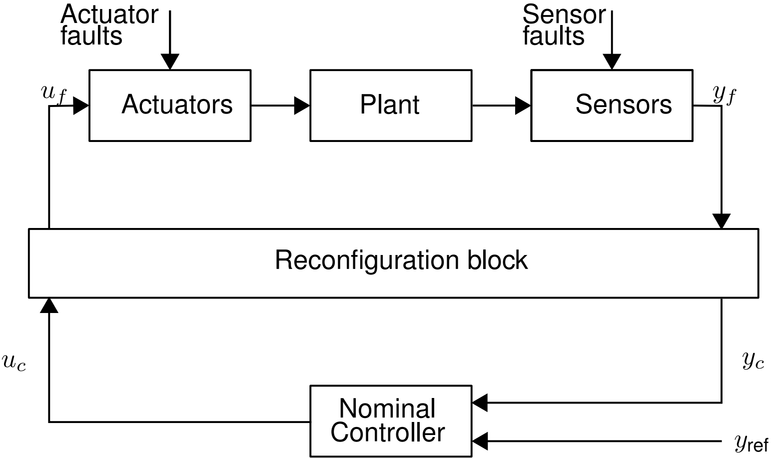
$$\mathrm{Im}(B_{\mathsf{f}}) \supseteq \mathrm{Im}(B)$$

satisfied? (Provide examples)

Virtual Sensor and Actuator

Virtual Sensor and Actuator

Problem Formulation



Given system models

Nominal System: $\begin{cases} \dot{x} = Ax + Bu + Ed \\ y = Cx \end{cases}, u = -Ky$

Faulty System: $\begin{cases} \dot{x} = A_\mathsf{f} x + B_\mathsf{f} u + Ed \\ y = C_\mathsf{f} x. \end{cases}$

Design reconfiguration block such that **(strong reconfiguration goal)**

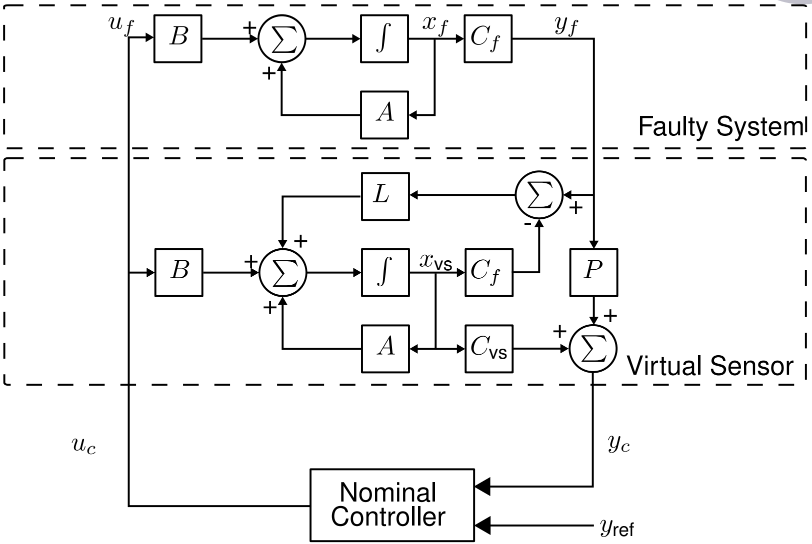
$$y_f = y$$

Otherwise, ensure that **(weak reconfiguration goal)**

$$y_f \to y \text{ for } t \to \infty$$

Virtual Sensor and Actuator

Virtual Sensor



Virtual Sensor and Actuator
Definition: Virtual Sensor

A virtual sensor for the faulty system is

$$\begin{cases} \dot{x}_{\mathsf{vs}} &= A_{\mathsf{vs}} x_{\mathsf{vs}} + B_{\mathsf{vs}} u_c + L y_{\mathsf{f}} \\ u_{\mathsf{f}} &= u_c \\ y_c &= C_{\mathsf{vs}} x_{\mathsf{vs}} + P y_{\mathsf{f}} \end{cases}$$

with

$$\begin{aligned} A_{\mathsf{vs}} &= A - L C_{\mathsf{f}} \\ B_{\mathsf{vs}} &= B \\ C_{\mathsf{vs}} &= C - P C_{\mathsf{f}}. \end{aligned}$$

The matrices P and L are freely chosen.

Lemma. A virtual sensor solves the weak reconfiguration problem if the eigenvalues of $A - LC_f$ are in the left half-plane.

Input: $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$, $C_{\mathsf{f}} \in \mathbb{R}^{p \times n}$.
Output: $P \in \mathbb{R}^{p \times p}$, $L \in \mathbb{R}^{n \times p}$, $C_{\mathsf{vs}} \in \mathbb{R}^{p \times n}$.
Procedure:

1. Check the solvability of the *exact model matching problem*. If $\ker(C_{\mathsf{f}}) \subseteq \ker(C)$ then design fault-tolerant control by exact model matching problem. Otherwise, define

$$P = CC_f^+$$

2. Check the detectability of the pair (C_{f}, A). If (C_{f}, A) is not detectable then a virtual sensor cannot be designed; otherwise, design L such that the eigenvalues of $A - LC_f$ are in the open left half-plane.
3. Compute the virtual sensor output matrix C_{vs} as

$$C_{\mathsf{vs}} = C - PC_{\mathsf{f}}.$$

Consider the two-cart system subject to a sensor fault, i.e.,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{b_1}{m_1} & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & -\frac{b_2}{m_2} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & \frac{2}{m_1} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, C_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Virtual Sensor and Actuator

Example: Virtual Sensor

Consider the two-cart system subject to a sensor fault, i.e.,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{b_1}{m_1} & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & -\frac{b_2}{m_2} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & \frac{2}{m_1} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, C_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Exact model matching is not possible, since

$$\mathsf{Ker}(C_\mathsf{f}) \nsubseteq \mathsf{Ker}(C).$$

Virtual Sensor and Actuator

Example: Virtual Sensor

Consider the two-cart system subject to a sensor fault, i.e.,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{b_1}{m_1} & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & -\frac{b_2}{m_2} \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & \frac{2}{m_1} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \; C_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Pick P similar as (feed trough all fault-free measurements)

$$P = CC_f^{+} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Virtual Sensor and Actuator

Example: Virtual Sensor

Consider the two-cart system subject to a sensor fault, i.e.,

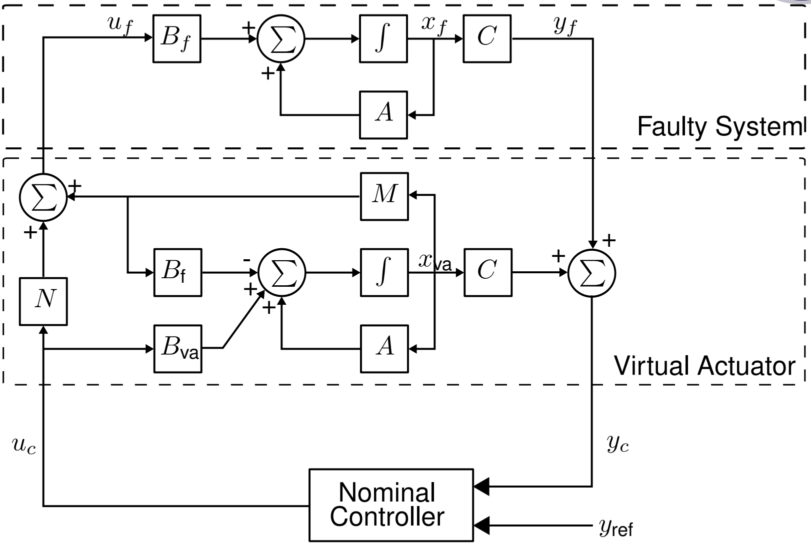
$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{b_1}{m_1} & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & -\frac{b_2}{m_2} \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & \frac{2}{m_1} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \; C_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Pick P similar as (feed trough all fault-free measurements)

$$P = CC_f^{+} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The pair (C_f, A) is observable; thus, the eigenvalues of $(A - LC_f)$ can be placed in the open left half-plane by pole placement.



u_f
B_f
$\sum$
$\int$
x_f
C
y_f
A
Faulty System
$\sum$
M
B_f
$\sum$
$\int$
x_{va}
C
$\sum$
N
B_{va}
A
Virtual Actuator
u_c
y_c
Nominal Controller
y_{ref}

A virtual actuator for the faulty system is given by

$$\begin{aligned}
\dot{x}_{\mathsf{va}} &= A_{\mathsf{va}} x_{\mathsf{va}} + B_{\mathsf{va}} u_c \\
u_{\mathsf{f}} &= C_{\mathsf{va}} x_{\mathsf{va}} + D_{\mathsf{va}} u_c \\
y_c &= C x_{\mathsf{va}} + y_{\mathsf{f}}
\end{aligned}$$

with

$$\begin{aligned}
A_{\mathsf{va}} &= A - B_{\mathsf{f}} M \\
B_{\mathsf{va}} &= B - B_{\mathsf{f}} N \\
C_{\mathsf{va}} &= M \\
D_{\mathsf{va}} &= N.
\end{aligned}$$

The matrices N and M can be freely chosen.

Theorem. A virtual actuator is a solution to the reconfiguration problem, such that the weak goal is reached, if the faulty process is controllable.

Theorem. A virtual actuator is a solution to the reconfiguration problem, such that the weak goal is reached, if the faulty process is controllable.

Lemma. A virtual actuator solves the weak reconfiguration problem if the eigenvalues $(A - B_f M)$ are in the open left-half plane.

Virtual Sensor and Actuator
Algorithm: Virtual Actuator

A virtual actuator is designed as follows

- Pick as $N = B_f^+ B$.
- Pick M such that $eig(A - B_f M)$ are in the open left half plane.

Input: $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $B_{\mathsf{f}} \in \mathbb{R}^{n\times m}$.
Output: $M \in \mathbb{R}^{n\times m}$, $N \in \mathbb{R}^{m\times m}$, $B_{\mathsf{va}} \in \mathbb{R}^{n\times m}$.
Procedure:

1. Check the solvability of the *exact model matching problem*. If $\mathrm{Im}(B_{\mathsf{f}}) \supseteq \mathrm{Im}(B)$ then design fault-tolerant control by exact model matching problem. Otherwise, define

$$N = B_f^{+} B$$

2. Check the controllability of the pair (A, B_{f}). If (A, B_{f}) is not controllable then a virtual actuator cannot be designed; otherwise, design M such that the eigenvalues of $(A - B_f M)$ are in the open left half-plane.
3. Compute the virtual actuator input matrix B_{va} as

$$B_{\mathsf{va}} = B - B_{\mathsf{f}} N.$$