5. Model Matching and Virtual Sensors and Actuators

Exercise 7. *Consider the two-cart system illustrated in the figure below.*

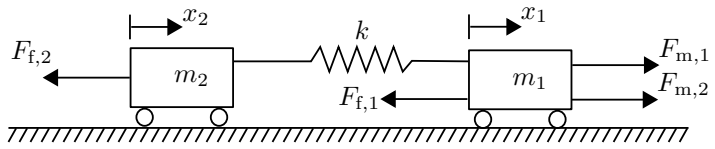

Figure 4: Illustration of two carts with masses m_1 and m_2 coupled by a spring, with spring constant (stiffness) k. The carts are subject to friction, and the forces $F_{\mathrm{m},1}$ and $F_{\mathrm{m},2}$ are applied to the front cart.

The nominal dynamics of the system are given as

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{b_1}{m_1} & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & -\frac{b_2}{m_2} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ \frac{1}{m_1} & 0 & 0 \\ 0 & \frac{2}{m_2} & 0 \end{bmatrix}, C = I_4.$$

The parameters for the model are assumed to be $m_1 = 2$ kg, $m_2 = 1$ kg, $k = 2$ N/m, $b_1 = 3$ N/(m/s), and $b_2 = 4$ N/(m/s).
Design a fault-tolerant controller using the following steps

1. *Design a nominal controller for the system.*
2. *Design appropriate accommodation strategies for the following sensor fault scenarios*

$$C_{\mathrm{f}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, C_{\mathrm{f}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, C_{\mathrm{f}} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. *Design appropriate accommodation strategies for the following actuator fault scenarios*

$$B_{\mathrm{f}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; B_{\mathrm{f}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_1} & 0 & 0 \\ 0 & \frac{2}{m_2} & 0 \end{bmatrix}, \; B_{\mathrm{f}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

4. *Simulate one of the fault scenarios for which you have designed a virtual sensor.*