Stability of LPV

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I will introduce and discuss stability notions for LPV including quadratic stability (the parameters are unknown but constant) and robust stability (the parameters change in time).

- Litterature [CR], Ch. 2.2.2 pp. 41-43, Ch. 2.3, pp. 43-51, and Ch. 2.4, pp. 51-56
- Exercises:
  1. Consider LPV
  $$A(\rho) = \begin{bmatrix} 1 & \rho \\ -\frac{4}{\rho} & -3 \end{bmatrix}$$
  with constant parameter $\rho \in [-1, -\frac{1}{2}] \cup [\frac{1}{2}, 1]$
  Using parameter-dependent (quadratic) Lyapunov function
  $$v(x, \rho) = x^T P(\rho) x$$
  with
  $$P(\rho) = \begin{bmatrix} 50 + 6\rho^2 & 16\rho \\ * & 1 + 7\rho^2 \end{bmatrix}$$
  show that the system is robustly stable. You might use Maple for this purpose (useful functions are: *Transpose, Determinant, realroot*).
- Consider the LPV
$$\dot{x}(t) = \left(\frac{\rho(t)}{\rho(t)^2 + 1} - 3 \right) x(t) \text{ for } \rho \in [-1, 1]$$
with the following descriptor equation
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} -3 & -1 & 0 & 1 \\ \rho & 1 & 0 & 0 \\ 0 & -\rho & 1 & \rho \\ 0 & 0 & -\rho & 1 \end{bmatrix} \begin{bmatrix} x \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}.$$
Verify that
$$-A(\rho)^T P - PA(\rho) > 0$$
with $P = \begin{bmatrix} 4 & 0 & 0 & 0 \\ -5 & -7 & -1 & 1 \\ -6 & -1 & -7 & -1 \\ -4 & 1 & -1 & -7 \end{bmatrix}$ is quadratic Lyapunov function.