

Exercise 1

Consider the polynomial

$$12x^2 - 6.3x^4 + x^6 + 3xy - 12y^2 + 12y^4$$

Use yalmip to find the minimum value of the polynomial. Plot the polynomial to visually verify your result.

Exercise 2

Consider the dynamical system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 + \frac{1}{2}x_2 - x_1^3 \\ \frac{1}{2}x_1 - 2x_2 - x_2^3 \end{bmatrix}$$

Find a polynomial Lyapunov function for the system.

Exercise 3

Use dual decomposition to show the stability of the following system (this is the example from "Compositional Stability Analysis Based on Dual Decomposition" by Ufuk Topcu et.al.)

$$\begin{aligned} \dot{x}_1 &= \begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix} x_1 + I_2 \begin{bmatrix} w_2 \\ w_4 \end{bmatrix} \\ w_1 &= 0.5 \begin{bmatrix} 1 & 1 \end{bmatrix} x_1 \\ \dot{x}_2 &= \begin{bmatrix} -8 & 0 \\ 12 & -2 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w_1 \\ \begin{bmatrix} w_2 \\ w_3 \end{bmatrix} &= 0.5x_2 \\ \dot{x}_3 &= \begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w_3 \\ w_4 &= 0.4 \begin{bmatrix} 1 & 1 \end{bmatrix} x_3 \end{aligned}$$