Aalborg University

Nonlinear stuff is going down

Electronic & IT:
Control & Automation

Group:
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Student report

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STUDENT REPORT

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Synopsis:

Bla bla bla ...

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Preface

bla bla bla ...

Aalborg University, 30^{th} of May 2017

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Nomenclature

This needs to be updated!

Acronyms

MP	Minimization Problem
KVL	Kirchhoff´s Voltage Law
KCL	Kirchhoff´s Current Law
MPC	Model Predictive Control
NRMSE	Normalized Root Mean Square Error
MNGB	Matlab Nonlinear Grey Box

Symbols

Symbol	**Description**	**Unit**
test	test	test

Glossary of mathematical notation

This section sums up the mathematical notation and terminology used in this report.

Upper and lower bounds of a variable

$$\underline{x} < x < \overline{x} \tag{1}$$

Where $x \in \mathbb{R}$ and $\overline{x}$ and $\underline{x}$ are the upper and lower bounds, respectively.

Intervals

$$[a, b] = \{x \in \mathbb{R} | a \leq x \leq b|\}\underline{x} < x < \overline{x} \tag{2}$$

Where $\overline{x}$ and $\underline{x}$ are the upper and lower bounds, respectively.

Vectors and matrices

Vectors and matrices are noted with bold fonts, such that $\boldsymbol{v}$ is a vector:

$$\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \in \mathbb{R}^{(n \times 1)} \tag{3}$$

and $\boldsymbol{M}$ is a matrix:

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{12} & \dots & m_{1k} \\ m_{21} & m_{22} & \dots & m_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \dots & m_{nk} \end{bmatrix} \in \mathbb{R}^{(n \times k)} \tag{4}$$

Continues vector variables are noted with $\boldsymbol{v(t)}$ such that:

$$\boldsymbol{v(t)} = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_n(t) \end{bmatrix} \in \mathbb{R}^{(n \times 1)} \tag{5}$$

While discrete vector variables are referred to as sequences and noted with $\boldsymbol{v[k]}$, such that:

$$\boldsymbol{v[k]} = \begin{bmatrix} v_1[k] \\ v_2[k] \\ \vdots \\ v_n[k] \end{bmatrix} \in \mathbb{R}^{(n \times 1)} \tag{6}$$

is a sequence, where k is the time step between two entries.

The pseudo inverse of a matrix is noted with $\boldsymbol{M}^{\dagger}$.

Small-signal and operating point values

Small-signals are noted with $\hat{u}$ and the operating point values are noted with $\bar{u}$.

Derivatives

The partial derivative of a function is noted with

$$\frac{\partial f(x,y)}{\partial x} \tag{7}$$

The derivative of a vector by vector is noted with:

$$\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{w}} = \begin{bmatrix} \frac{\partial v_1}{\partial w_1} & \frac{\partial v_1}{\partial w_2} & \cdots & \frac{\partial v_1}{\partial w_n} \\ \frac{\partial v_2}{\partial w_1} & \frac{\partial v_2}{\partial w_2} & \cdots & \frac{\partial v_2}{\partial w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial v_k}{\partial w_1} & \frac{\partial v_k}{\partial w_2} & \cdots & \frac{\partial v_k}{\partial w_n} \end{bmatrix} \tag{8}$$

If the size of vector $\boldsymbol{v}$ and $\boldsymbol{w}$ are the same, the resulting matrix is referred to as a Jacobian.

The time derivative of a function is noted with

$$\dot{f} = \frac{df(t)}{dt} \tag{9}$$

Vector fields

Vector fields are introduced, and represent vector valued functions such that the mapping is the following:

$$\alpha(\boldsymbol{v}) : \mathbb{R}^{(n)} \rightarrow \mathbb{R}^{(n)} : [v_1, v_2, \ldots, v_n] \rightarrow [\alpha(v_1), \alpha(v_2), \ldots, \alpha(v_n)] \tag{10}$$

Contents

1 Introduction

Intercontinental transportation of cargo over sea is a growing industry which typically is referred to as "Freight transportation". This way of transportation utilize large freight ships, also known as container ships *Figure 1.1*, which can store thousands of containers and sail over seas with these[1].

Figure 1.1: Maresk contriner ship[2].

This industry handles 90% of the global goods transportation e.g transportation of medicine, oil, gasoline, perishables and foodstuff[3]. When transporting some types of cargo, certain temperatures has to be kept such that the cargo does not decay. Special designed containers has therefore been developed to make sure that the cargo has the correct temperature under transport, these are typically called refrigerated (reefer) containers.

1.1 Reefer Container

A reefer, is essentially a big rectangular container, equipped with a refrigeration unit at one end opposite to its door, see *Figure 1.2*. A reefer, that is generally either 20 or 40 foot long, is used to transport temperature controlled cargo at long distances and can be set to maintain a specific temperature or pre-programmed to follow a determined temperature sequence.

Figure 1.2: Star Cool Container[4].

Construction

The reefer in question for this project are from Maersk[5]. Reefers are formed by a steel frame and its walls are composed by aluminum sheets with foam insulation between them. The aluminum docking have a T shaped form and is responsible for the cold air supply to the cargo as mentioned in [6]. The air is cooled down by the refrigeration unit and pushed through air channels at the bottom of the reefer, which handles an even distribution of cold air throughout the reefer. After the air channels, the air goes in an upwards direction in between the pallets in the cargo, thus cooling the cargo. The air, after cooling the cargo, is warmed up by heat transfer from the cargo and outside environment, ascends to the ceiling of the container and goes back to the refrigeration unit as depicted in figure *Figure 1.3*.

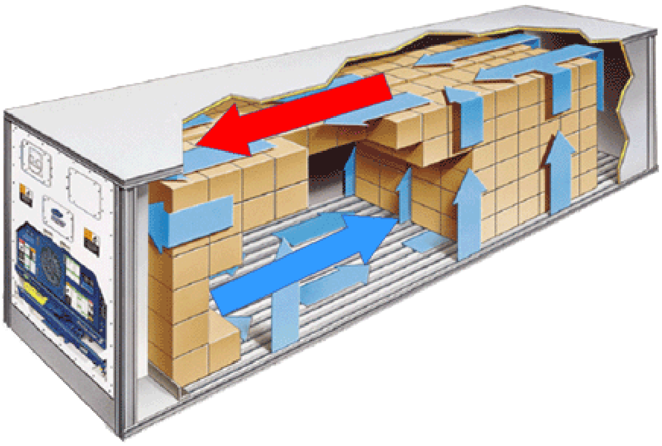


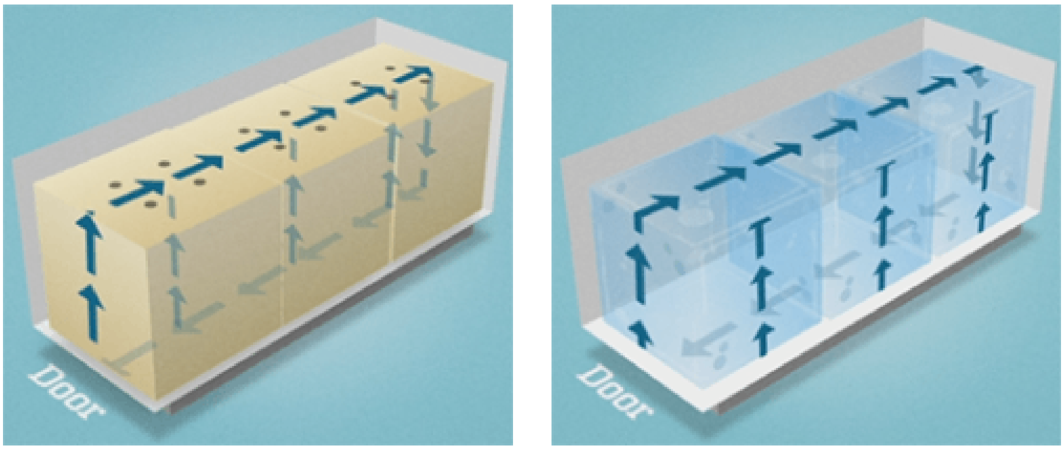
Figure 1.3: Air flow through the container[7].

Operation modes

Reefers are capable of operating in environments with temperatures ranging from -30°C to +50°C and its set point of temperature can be in between -30°C to +40 As described in [6], there are several modes for which the temperature can be operated inside the reefers:

- Chilled Mode: In this mode, air has to flow through the cargo, as depicted in *Figure 1.4a*, so that heat and gases evolved from the cargo can be removed. The cartons used should therefore have ventilation. This mode is used for high set points.

- Frozen Mode: In this case, air has to flow around the cargo, as seen in *Figure 1.4b*, which means that there should be no gaps between the cargo and the walls.



(a) Chilled Mode (b) Frozen Mode

Figure 1.4: Air Flow in chilled and frozen mode[8].

Other modes which focus on more than just temperature also exist, where some of them are listed below:

- Automatic Ventilation (AV+): This mode has the particularity of controlling CO_2 levels within the container. This is important for when the cargo is fruits and vegetables, since due to their respiration, there is a gradual increase in CO_2 level and after a certain threshold, it becomes damaging for the cargo itself. This is regulated through a fresh air valve.
- Controlled Atmosphere (CA): It is an improvement on the AV+ mode where both CO_2 and O_2 levels can be controlled using the same fresh air valve and vacuum pump that removes CO_2.
- Multiple Temperature Set-point (MTS): It is a mode where the temperature set point in the reefer can change over time according to a pre-programmed temperature sequence.
- Cold Treatment (CT): Mode used for killing insects in the cargo by lowering the the cargo temperature for a fixed period.
- QUEST: An energy saving program that uses the fact that most cargo can have its temperature regulated through a pulsed cooling below the set-point, which enables a more efficient utilization of the compressor and the fans, resulting in a lower energy consumption.

Depending on the cargo, the mode which meet the demand of the cargo can be chosen. These modes are made for different purposes, where an example could be transportation of fruit. Fruit is usually harvested before being fully riped, and then stored in a reefer. By controlling the climate inside the reefer the riping process can be controlled such that when the cargo reaches the destination the fruit is fully or almost riped. This makes it possible to transport exotic fruit to areas where the fruit can not be grown because of the climate, and thereby make a bigger marked for the fruit. The problem transporting fruit is that they are very delicate to temperature changes. Too high will increase the riping process and the fruit may perish before reaching destination, too cold will lead to not riped fruit, thus not ready to be sold.

1.2 Refrigeration Unit

The refrigeration unit consist of multiple components for changing the temperature inside the reefer. The unit shown in *Figure 1.5*, is the cooling unit for a star cooling reefer. It can be divided into several stages, each responsible for altering the thermodynamical properties of the refrigerant, that passes through the whole unit. The refrigerant used in this refrigeration unit is the r134a, see[9].

new figure

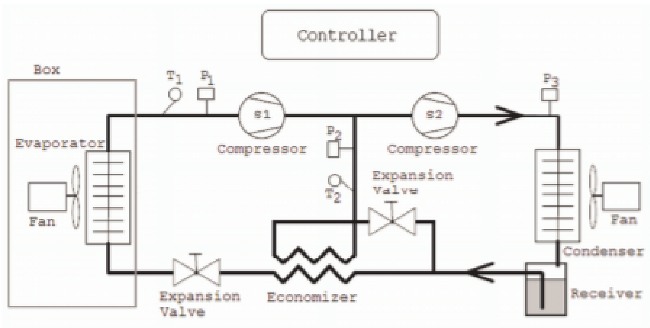


Figure 1.5: Schematic of the refrigeration unit

There are also several temperature (T) and pressure (p) sensors in the system, between the evaporator stage and compressor stage, between the two compressors and economizer and between the compressor and condenser stage. There are three main pressures in this system: p_1 is the suction pressure, p_2 is the intermediate pressure and p_3 is the discharge pressure. Below is a description of each component.

Compressor

The compressor stage is divided into a two stage compressors. At the entry of the compressor, the refrigerant is in a low-pressure, moderate-temperature gaseous form. It is important that the refrigerant is in gas form since liquid will damage the compressor by washing away an oil film that protects the compressor's pistons. This stage is responsible for compressing the refrigerant, increasing its pressure and temperature before it enters the condenser.

Condenser

The condenser is responsible for extracting heat from the refrigerant to the surroundings. When heat is extracted, the refrigerant passes from a gas stage to a liquid stage, mantaining its high pressure.

Receiver

The receiver is essencially a buffer tank for any excess refrigerant. After leaving the condenser, the refrigerant is led to the top of the receiver and it can be extracted from the bottom in order to go to the expansion valves.

Economizer

The economizer consists of a counterflow plate heat exchanger with two plates, which can be seen as two sides: the hot side and the cold side, both filled with refrigerant. The refrigerant in the cold side receives heat from the one in the hot side, therefore getting warmer and evaporating to the intermediate pressure. This also results in the sub-cooling of the refrigerant that is to enter the evaporator. This process allows more cooling capacity in the sense that since the refrigerant is colder, less mass flow is necessary to achieve a set temperature variation. The resultant evaporated refrigerant in the cold side then joins the refrigerant that got out from the first compressor through a T section joining pipe, which also reduces the necessary mass flow in the first compressor.

Expansion valve

The expansion valve is responsible for expanding the refrigerant. The high pressure liquid enters the expansion valve and the latter will control the quantity of refrigerant that passes to the evaporator. By limiting the amount of high-pressure liquid that passes, it forces the refrigerant to expand resulting in a decrease in pressure and temperature.

Evaporator

In this stage, the air that came from the inside the container flows around the cold refrigerant that came from the expansion vale. This results in an transfer of heat from the warm air to the cold refrigerant which in turn results in a temperature decrease of the air that is then blown once again into the cargo hold. The refrigerant that had its temperature raised, enters its gaseous form and proceeds to the compressor. A problem that is important to note here is the superheat. Superheat is the difference between the saturated suction temperature and the real suction temperature, so it is basically the excess energy added to the refrigerant after evaporation. As mentioned before it is important that the refrigerant reaching entering the compressor is dry as not to damage it, so the superheat must allways be above zero but the closer to zero it is, more efficient the system becomes. Therefore a careful control over it must be made.

1.3 Refrigeration cycle

For transferring heat from inside the reefer to outside, it is necessary to look at the refrigeration cycle in the refrigeration unit.

By utilizing the property of the r134a refrigerant, having a boiling point below zero degree Celsius, and the second law of thermodynamics which states that heat always goes towards cold, thermal energy can be transfered from inside the reefer into ambients[10].

rephrase

A basic compression refrigeration system can be seen in *Figure 1.6*

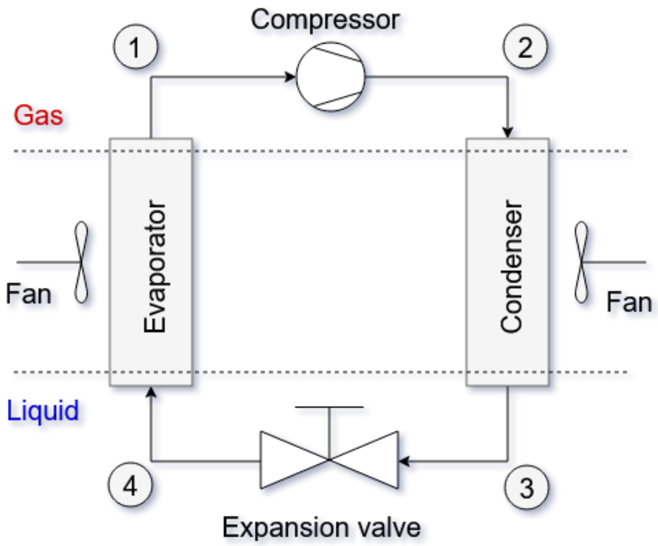


Figure 1.6: Single stage refrigeration system, where the colored text indicates which state the refrigerant is in. Between the stippled lines the refrigerant is in a mixed state of gas and liquid. The arrows indicates which way the refrigerant moves in the system

The first law of thermodynamic states that the internal energy in a system(ΔU_{int}) is a summation of the energy supplied(Q) and the work performed on the system(W) as seen in *Equation: (1.1)*.

$$\Delta U_{int} = Q + W \tag{1.1}$$

The work added to a refrigeration system is given by the compressor, and thereby increasing the internal energy in the system. What is obtained by this added work, is a change in pressure of the gaseous refrigerant which then increases the temperature as given by the ideal gas equation seen in *Equation: (1.2)*.

$$pV = nRT \tag{1.2}$$

The Volume V, the mole of gas n and the gas constant R is static for an ideal system, meaning that in a real world system refrigerant could leak from the system over time. An ideal system will be assumed for the rest of this section. In *Figure 1.10* a pressure enthalpy(PH) diagram is shown, with the refrigeration cycle given with the corresponding corner points of *Figure 1.6* in red.

increase numbers! or change to better color(read next todo Simon)

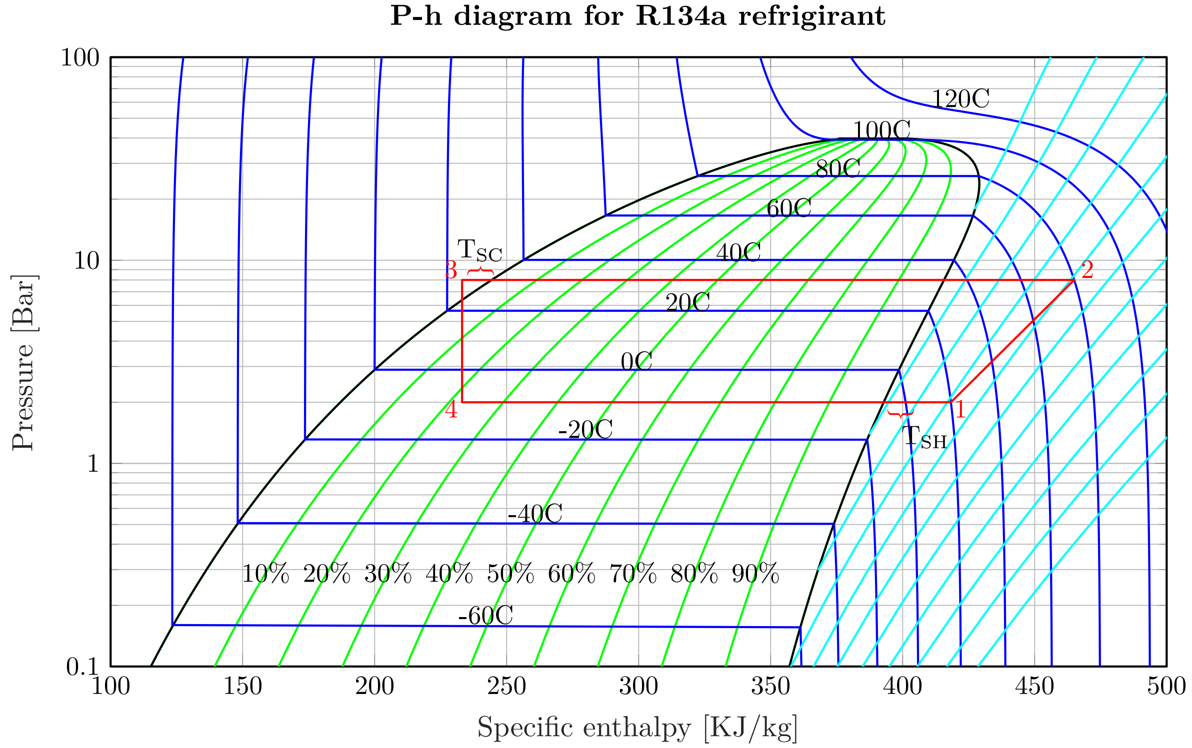


Figure 1.7: PH diagram for single stage refrigeration system, where the blue lines are temperature of the refrigerant, green lines are percentage refrigerant in gas form, black line is the saturation curve and the light blue curves are the isentropic lines.

Figure is tempo-rary

To avoid the different component failures mentioned in Section *1.2: Refrigeration Unit*, sub-cooling(T_{sc}) and super-heat(T_{sh}) is important factors in a refrigeration system. Sub-cooling is given as the temperature below the saturation of the refrigerant in liquid form. Furthermore it is affected by the fan speed, temperature of the ambients and the surface area of the condenser. Super-heat relates to the amount of liquid inside the evaporator. For most efficient absorption of heat energy in the evaporator, ideally super-heat should be zero degree Celsius. Some degree of super-heat is necessary to avoid compressor failure because of flooding. In *Figure 1.8* an example is given of how super-heat is related to opening degree of the expansion valve, and the load on the refrigeration system.

So we can't go below zero degree?

elaborate picture

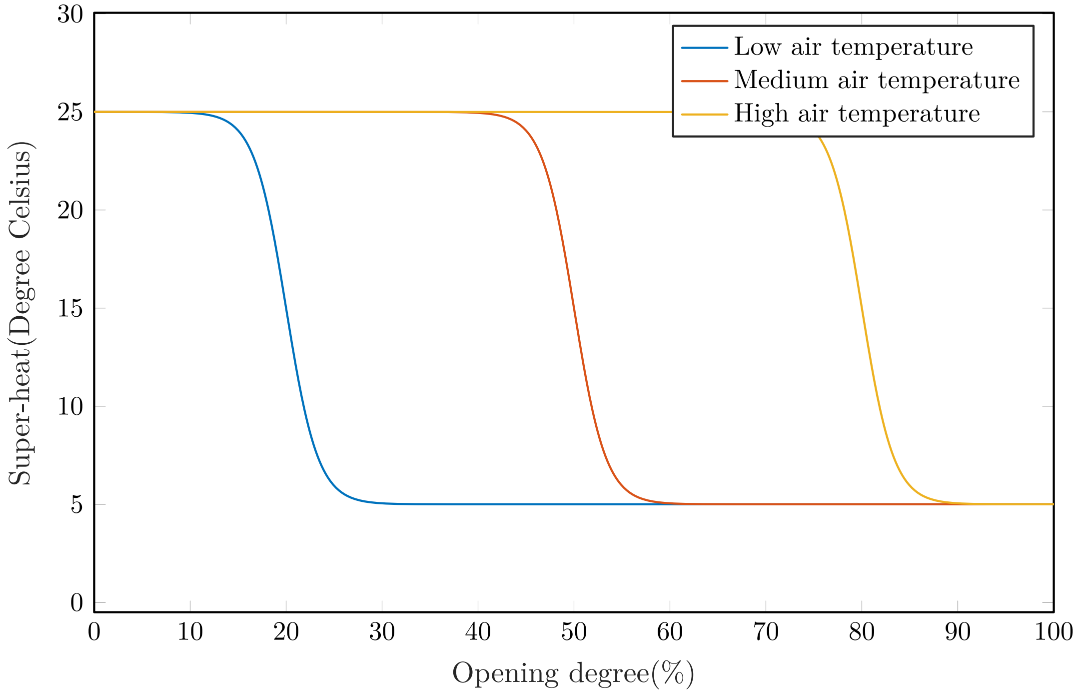


Figure 1.8: Illustration of how SH is related to OD and air temperature inside the reefer container. Static ambient air temperature is assumed

The load is dependent on the cargo inside the reefer, surface area of the evaporator, the air temperature and the mass flow of the air inside the reefer. This gives a non-linear sigmoid function which be difficult to apply control for.

In *Figure 1.9* a diagram of the refrigeration system utilized on the star cool reefers can be seen.

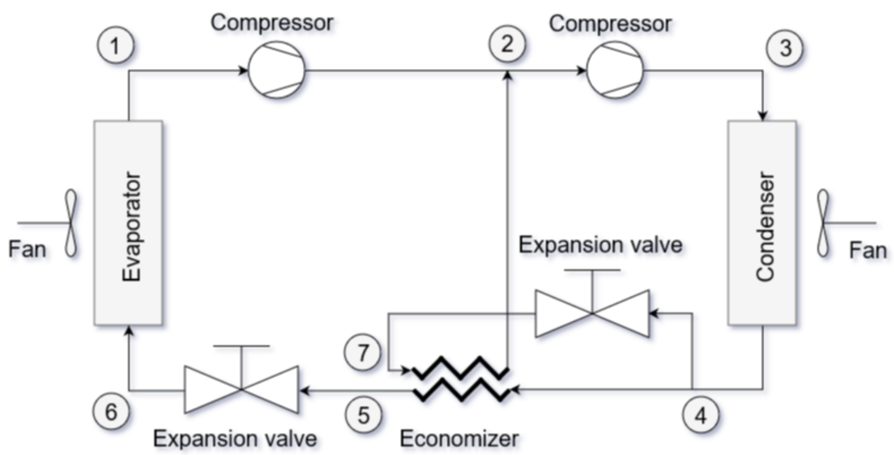


Figure 1.9: Refrigeration unit utilized on the star cool reefers

This system utilizes the double compressor setup to sub-cool the liquid refrigerant further by the use of an economizer.

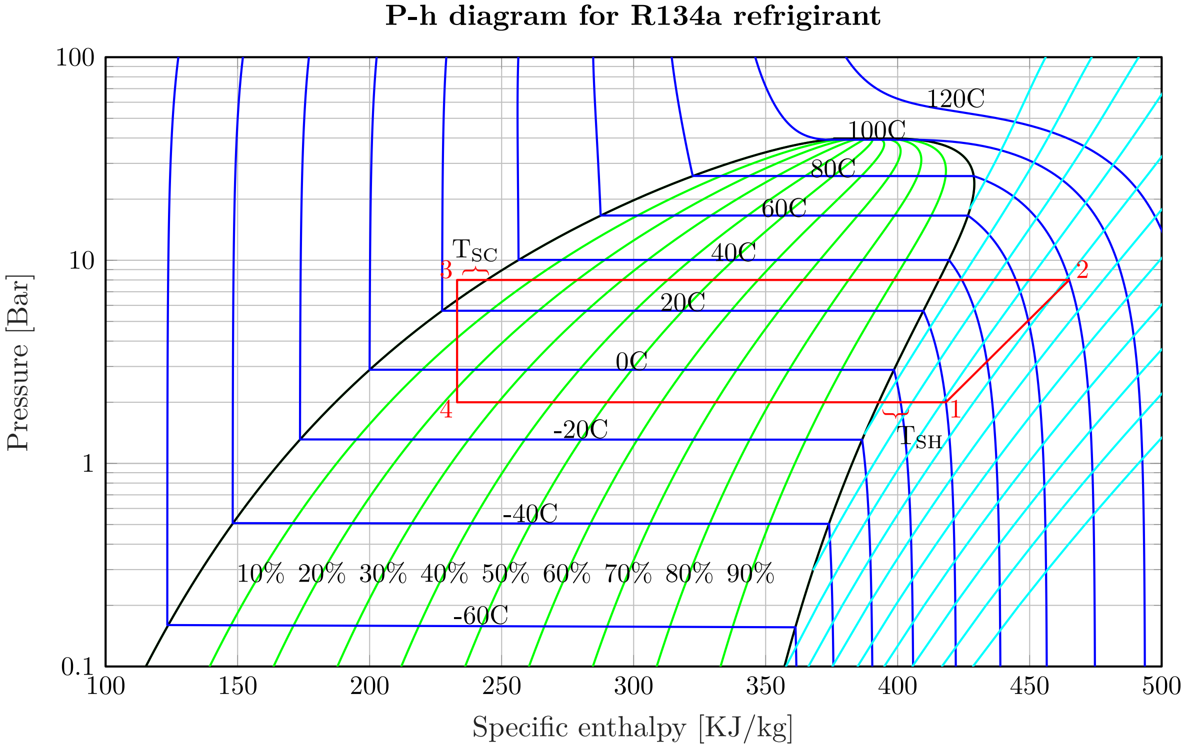


Figure 1.10: PH diagram for two stage refrigeration system, where the blue lines are temperature of the refrigerant, green lines are percentage refrigerant in gas form, black line is the saturation curve and the light blue curves are the isentropic lines.

Figure is temporary(imagine two stage system with additional sub-cooling)

The thing to be noted of the added sub-cooling is that it gives an increase in cooling ability.

Efficiency

To cool down the reefers, electricity is used to power the refrigeration unit. The electricity is generated on the ship by large generators, which is driven by its own fuel supply or directly attached to the drive shaft of the ship engine[11]. This have an impact on the fuel consumption used on the ship, which leads to a higher transportation cost and an increase in pollution, due to the increase of fuel. With an average power consumption of 3.6 kW for a twenty foot reefer and the amount of reefers in use, a small increase in efficiency could have a considerable impact on the global power consumption of reefers, and thus decrease the global fuel consumption[11].

The coefficient of performance (COP) is a way of determine the efficiency, i.c how well a system cools down compared to power usage, of the refrigeration system. Therefore it is desired to have as high as possible COP as this means a higher cooling capacity, with a lower power consumption.
The COP can be calculated in different ways, two of them shown below.

$$COP_1 = \frac{\text{Cooling capacity}}{\text{Work performed by compressor}} = \frac{|Q_c|}{W_c} \tag{1.3}$$

where Q_c is the cooling capacity and W_c is the work done by the compressor, thus *Equation: (1.3)* describe the relationship between the cooling capacity made in respect to the power consumption.

The second

$$COP_2 = \frac{T_e}{T_c - T_e} \tag{1.4}$$

where T_e is the temperature at the evaporator and T_c is the temperature at the condenser, see *Figure 1.9*.

From *Equation: (1.3)* it can be seen that a more efficient compressor, would result in a higher COP factor, thus more efficient system. In comparison to *Equation: (1.4)*, by increasing the evaporator temperature T_e the COP factor will increase. This can be done by lowering the amount of work done by the compressor, which leads to an increase in pressure at the evaporator, thus higher evaporating temperature for the refrigerant, and a decrease of pressure at the condenser, thus lower temperature, see *Figure 1.9* and *Figure 1.10*. Therefore *Equation: (1.3)* and *Equation: (1.4)* is closely related.

Summarizing the problems leads to the following problem formulation:

How to derive an optimal temperature controller for a reefer container without violating the temperature constraints for the cargo, while decreasing the power consumption.

2 Requirements and Constraints

For the purpose of this project the cargo is chosen to be bananas, which should be transported at 13 °C according to Lodam.

3 Modelling

e the goal of this project is the control focused on energy optimization for the reefer, a new model won't be made and an existent model will instead be used. The model used in this project was created by Kresten Sørensen on that proved to model the dynamics of the several components of the reefer accurately. Some small adjustments and additions will be made to it regarding energy optimization. Namely the model of the fans and the power usage models of said fans and the compressor.

include reference and correct name

Some of the components in the refrigeration unit have very fast dynamics, compared to the rest of the components and therefore these could be replaced by simple algebraic equations which reduces the model order, whilst preserving a correct description of the physical behavior of the model. The components in question are the expansion valves and the compressor. The remainder of the components were modeled through a combination of differential and algebraical equations. All thermodynamic processes are assumed adiabatic.

The resultant model comprises several smaller models that model each component of the refrigeration unit individually, plus the container box. The state in each control volume is given by pressure and enthalpy. The interface between component models is, in most cases, the pipes carrying the refrigerant and this type of interface has three states: pressure, mass flow and specific enthalpy.

The models are then described below.

Things that could still be mentioned before describing the model are(at least I didn't know what they were): what is the dew point; what is the bubble point; what is gas quality;

3.1 Compressor

The compressor is actually a two stage compressor which means that the gas will be compressed twice using two compressors. Therefore the model will be the same for both since the only difference between them is the displacement volume, which is twice as large in the first one.

Since a single piston stroke in the compressor has little influence over the evaporator temperature, its dynamics can be overlooked and replaced by algebric equations with the assumption that the mass flows are equal on the input and output. The model also includes adiabatic compression, clearance volume and valve pressure loss. This influences the values for the input and output pressure, that must be calculated taking in account the compressor speed and the valve loss constant as follows:

$$p_1 = p_{in} - kl_1 \cdot \omega; \tag{3.1}$$

$$p_2 = p_{out} + kl_2 \cdot \omega. \tag{3.2}$$

Where ω is the compressor speed and kl is the valve loss constant. From here on out in the compressor model, p_1 will be used instead of p_{in} and p_2 instead of p_{out}. The mass flow and output enthalpy are given by

$$\gamma = \frac{C_{cp}}{C_{cv}}, \tag{3.3}$$

$$v_2 = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{\gamma}} \cdot v_1, \tag{3.4}$$

$$\dot{m} = \left(\frac{V_1}{v_1} - \frac{V_C}{v_2}\right) \cdot \frac{\omega}{2}, \tag{3.5}$$

$$T_{out} = T_{in} \cdot \left(\frac{p_{out}}{p_{in}}\right)^{\frac{\gamma-1}{\gamma}}, \tag{3.6}$$

$$h_{out} = HTP(T_{out}, p_{out}). \tag{3.7}$$

Equation: (3.3) gives the ratio between the heat capacity at constant pressure C_{cp} and at constant volume C_{cv}. In *Equation: (3.4)* is calculated the specific refrigerant volume at discharge pressure p_2 from the specific volume and input pressure. Equation *Equation: (3.5)* yields the algebric equation for mass flow. It consists of a difference between the ratio of volumes before and after the stroke, being V_1 the internal volume of the cylinder before the stroke, v_1 and v_2 are the specific volume of the refrigerant before and after the stroke, respectively, and V_C is the internal cylinder volume after the stroke (clearance volume). This difference in masses before and after the stroke is then multiplied by the compressor speed to get the mass flow. The output temperature is calculated as in *Equation: (3.6)* and the output enthalpy is obtained through a table lookup in *Equation: (3.7)*.

3.2 Pipe Joining Junction

The pipe joining junction is responsible for combining the refrigerant coming from the first compressor and the economizer and passing it to the second compressor. It is modeled by two states, the refrigerant mass and the output enthalpy, calculated as follows

$$\frac{dM}{dt} = \dot{m}_{in1} + \dot{m}_{in2} - \dot{m}_{out}, \tag{3.8}$$

$$h_{out} = \frac{h_{in1} \cdot \dot{m}_{in1} + h_{in2} \cdot \dot{m}_{in2}}{\dot{m}_{in1} + \dot{m}_{in2}}, \tag{3.9}$$

where $\dot{m}_{in1}$, h_{in1}, $\dot{m}_{in2}$ and h_{in2} are the mass flows and enthalpy of the refrigerant coming in from the first compressor and economizer and $\dot{m}_{out}$ is the refrigerant mass leaving for the second compressor. *Equation: (3.8)* describes the mass flow of the refrigerant and *Equation: (3.9)* its output enthalpy.

3.3 Pipe Splitting Junction

The pipe splitting junction is located between the receiver and the economizer and contrary to the joining junction where two refrigerants with different thermodynamical propreties are mixed which results in a more complicated model, the splitting junction separates the same refrigerant into two so the model simply models the division of mass flows, while all the remaining properties remain the same and since there are no slow internal dynamics, this model is implemented as an algebric function. The model is described by the following equations:

$$\dot{m}_{in} = \dot{m}_{out1} + \dot{m}_{out2} \tag{3.10}$$

$$p_{out1} = p_{in} \tag{3.11}$$

$$p_{out2} = p_{in} \tag{3.12}$$

$$h_{out1} = h_{in} \tag{3.13}$$

$$h_{out2} = h_{in} \tag{3.14}$$

3.4 Condenser

The energy consumption of the compressor as well as the mass flow in the expansion valves are dependent on the discharge pressure which is, as mentioned before, the refrigerant's pressure before entering the condenser. This discharge pressure is modeled by the condenser model, therefore an accurate model of it is necessary in order to reduce overall energy consumption. The dominant dynamics in the condenser are determined by the temperature of the model which eases the process as only a simple model with a single refrigerant control volume is necessary. The state equations are then given below.

$$h_{out} = h_{in} + \frac{Q_{rm}}{\dot{m}_{in}}. \tag{3.15}$$

$$\frac{dM_r}{dt} = \dot{m}_{in} - \dot{m}_{out}. \tag{3.16}$$

$$\frac{dT_m}{dt} = \frac{Q_{rm} - Q_{ma}}{M_m \cdot Cp_m}. \tag{3.17}$$

Equation: (3.15) and *Equation: (3.16)* give, respectively, the energy and mass balance. *Equation: (3.17)* models the metal's internal energy through its temperature derivative.

The decrease in pressure is assumed to be a linear function of the mass flow, given by *Equation: (3.18)*, where p_{out} is the refrigerant's pressure after the condenser stage and λ is the pressure drop constant, determined experimentally.The actual output high pressure of the condenser will only be modeled in the receiver component and this is due to the fact that the receiver has a large internal volume which allows the condenser to be then modeled as a refrigerant cooler that follows the pressure of the receiver, even though the condenser as a direct influence on the output pressure. Generally, the pressure is given by the state of the refrigerant in the control volume, but since pressure is here given by the receiver, the state of the refrigerant must follow the pressure. Therefore the output mass flow of the condenser must ensure that the volume of the refrigerant converges to the internal volume of the condenser, which is achieved in *Equation: (3.19)*.

I don't really why the receiver is the one that models the output pressure, so probably the text isn't very good

$$p_{in} = p_{out} - \lambda \cdot \dot{m}_{in} \tag{3.18}$$

$$\dot{m}_{out} = \dot{m}_{in} + \frac{M_r - \frac{V_i}{v}}{1s} \tag{3.19}$$

In order to calculate the variation in metal temperature, the energy flow from the refrigerant to metal, Q_{rm} and from metal to air, Q_{ma} are calculated as follows:

$$Q_{rm} = UA_{rm} \cdot (T_r - T_m), \tag{3.20}$$

$$Q_{ma} = UA_{ma} \cdot (T_m - T_a) \cdot U_{fan}. \tag{3.21}$$

T_r is the refrigerant temperature calculated from h_{out} and p_{out}, T_m is the metal temperature and T_a is the air temperature. The energy flow from the metal to air is assumed to evolve linearly with the air flow provided by the fans, as such the energy flow is multiplied by the fan speed, U_{fan}. The fan has three set speeds: stopped, low and high speed. Since there is still some natural convection when the fan is stopped, a value greater than zero for U_{fan} is used. The value is 0.05, which was the best match for the data used for the verification of the model. The values for low and high speed are 1 and 2, respectively. $UA_{rm} = 650J/K$ and $UA_{ma} = 1500J/K$ are the heat transfer coefficients. The heat transfer coefficients used from here on out were all obtained experimentally in a special test unit.

3.5 Receiver

Even though the receiver is often neglected in refrigeration models, it has some influence over the system dynamics, particularly during rapid pressure variations, as the liquid refrigerant inside the receiver has a dampening effect over pressure and temperature transients. When the condenser fan is turned on, there is a pressure drop in the receiver. At this time, if the refrigerant is near boiling-point, as the pressure drops, it will begin to boil and this can be a problem since there should not be any gas in the expansion valves. However, in the receiver, the boiling liquid will halt the pressure drop, which causes a hybrid behavior.

There is both liquid and gaseous refrigerant inside the receiver and as such, two control volumes are necessary for which there are four possible sets of equations dependent on the quality of the refrigerant entering and inside the control volumes.

The states of the model are the pressure, enthalpy h_l and mass m_l of the liquid control volume, and enthalpy h_v and mass m_v of the vapor control volume. The inputs of the model are the mass flow $\dot{m}_{in}$ and enthalpy h_{in} of the refrigerant coming from the condenser and the mass flow $\dot{m}_{out}$ of refrigerant leaving the receiver.

The previously mentioned hybrid behavior is described from *Equation: (3.22)* to *Equation: (3.25)*.

$$\dot{m}_{lin} = \dot{m}_{in} \cdot (1 - X_{in}) \mid_{X_{in}=[0,1]} \tag{3.22}$$

$$\dot{m}_{vin} = \dot{m}_{in} \cdot X_{in} \mid_{X_{in}=[0,1]} \tag{3.23}$$

$$h_{vin} = HDewP(p_{in}) \tag{3.24}$$

$$h_{lin} = \begin{cases} HBubP(p_{in}) & if \quad X_{in} > 0; \\ h_{in} & otherwise \end{cases} \tag{3.25}$$

Equation: (3.22) and *Equation: (3.23)* give the mass flows in the two ontrol volumes. *Equation: (3.24)* and *Equation: (3.25)* give the enthalpy of the refrigerant flowing into the two control volumes. These equations are used under the assumption that the vapor entering the vapor control volume is at the dew point and that the refrigerant into the liquid control volume is either sub-cooled or at the bubble point.

The mass flow from the liquid control volume to vapor control volume is given by *Equation: (3.26)*.

$$\dot{m}_{blv} = \begin{cases} X_l \cdot M_l & if \quad h_l > HBubP(p); \\ 0 & otherwise \end{cases}, \tag{3.26}$$

where X$_l$ is the liquid refrigerant quality and M$_l$ the total mass of refrigerant. The inverse transaction, that is, the mass flow from vapor to liquid is given below

$$\dot{m}_{clv} = \begin{cases} (1 - X_v) \cdot M_v & if \quad h_v < HDewP(p); \\ 0 & otherwise \end{cases}, \tag{3.27}$$

where X$_v$ is the vapor quality and M$_l$ is the total vapor mass. These two mass flow equations assume instant mass flow for simplification. The control volumes are also connected through the surface of the liquid that acts as a boundary between them and the heat flow through this boundary is given by:

$$Q_{vl} = UA \cdot (T_v - T_l). \tag{3.28}$$

The final equations for mass and energy balance are given below:

$$h_l = h_{lin} + \frac{Q_{vl}}{\dot{m}_{lin}}, \tag{3.29}$$

$$h_v = h_{vin} + \frac{Q_{vl}}{\dot{m}_{vin}}, \tag{3.30}$$

$$\dot{m}_{vl} = \dot{m}_{cvl} - \dot{m}_{bvl}, \tag{3.31}$$

$$\frac{dM_l}{dt} = \dot{m}_{lin} + \dot{m}_{vl} - \dot{m}_{out}, \tag{3.32}$$

$$\frac{dM_v}{dt} = \dot{m}_{vin} - \dot{m}_{vl}, \tag{3.33}$$

where $\dot{m}_{vl}$ is the mass flow from vapor to liquid control volumes.

3.6 Economizer

As described before, the economizer provides an extra cooling for the refrigerant before it enters the evaporator, thus increasing the system's cooling capacity as well as decreasing the power consumption in the first compressor. Since the dynamics on the cold side of the economizer are slow, what matters in this model is to characterize the heat transfer from the hot side to the cold side, which has a great influence on the enthalpy of the refrigerant in the hot side. The equations describing the hot side will have the subscript "1" and the ones from the cold side, the subscript "2".

First of all the variation in temperature in the hot and cold side respectively are as follows:

$$\Delta T_1 = T_{in1} - T_{out2}, \tag{3.34}$$

$$\Delta T_2 = T_{out1} - T_{in2}. \tag{3.35}$$

The energy flow can then be calculated using these two temperature variations:

$$Q = UA \cdot \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}. \tag{3.36}$$

Now follows the model for both control volumes, separately, the hot and cold side. *Equation: (3.37)* to *Equation: (3.40)* are used to model the hot side:

$$h_{out1} = h_{in1} - \frac{Q}{\dot{m}_{in1}}, \tag{3.37}$$

$$p_{out1} = p_{in1} + \Delta p, \tag{3.38}$$

$$\frac{dM_1}{dt} = \dot{m}_{in1} - \dot{m}_{out1}, \tag{3.39}$$

$$\dot{m}_{in1} = \dot{m}_{out1} + \frac{\frac{V_1}{v_1} - M_1}{1s}. \tag{3.40}$$

The energy and mass balance are modeled, respectively by *Equation: (3.37)* and *Equation: (3.39)*. *Equation: (3.38)* gives the output pressure where δp is the pressure drop. Finally, *Equation: (3.40)* gives the input mass flow that, where v_1 is the specific volume of the refrigerant, obtained from h_{out1} and p_{out1}.

The model for the cold side is naturally similar and presented as follows:

$$h_{out2} = h_{in2} + \frac{Q}{\dot{m}_{in2}}, \tag{3.41}$$

$$\frac{dM_2}{dt} = \dot{m}_{in2} - \dot{m}_{out2}, \tag{3.42}$$

$$p_{out2} = p_{in2} + \Delta p, \tag{3.43}$$

$$\dot{m}_{out2} = \dot{m}_{in2} + \frac{M_2 - \frac{V_2}{v_2}}{1s}. \tag{3.44}$$

This time, the specific volume of the refrigerant is calculated using p_{out2} and a fixed refrigerant quality X_2.

3.7 Expansion Valve

The cooling capacity of the system is directly related to how much refrigerant passes through the evaporator, which in turn is directly connected with the mass flow of the expansion valve. Therefore, the expansion valve model is important for the overall effectiveness and efficiency of the system. The thermodynamic processes in the valve can be assumed to be adiabatic due to the low internal volume of the valve, which allows a simplification of the model, needing only algebraic equations to describe it. The model can also be assumed continuous since the pressure transients on the evaporator caused by the valves are relatively small if the evaporator is filled with refrigerant. There are two expansion valves in the system, as can be seen in *Figure 1.5*, one in the economizer and another in the evaporator. The model is the same for identical for both of them, but they are however part of the respective economizer and evaporator models instead of being a separate component. The mass flow for the expansion valve then comes as follows:

$$\dot{m} = C \cdot A \cdot \sqrt{\rho \cdot \Delta P}, \tag{3.45}$$

where C is the discharge coefficient, A is the cross-sectional area, ρ is the density of refrigerant entering the valve and δP is the pressure difference between the input and output of the valve. This is a textbook equation but as A and C were not available, they were combined into a constant K that was then experimentally obtained to be 1.18×10^{-5} for the evaporator expansion valve and $3.33 \times 10{-}6$ for the economizer expansion valve. The mass flow in the expansion valve can then be expressed as in *Equation: (3.46)*, where T_{on} is the fraction of time the valve is on.

$$\dot{m} = T_{on} \cdot K \cdot \sqrt{\frac{1}{v_{in}} \cdot (p_{in} - p_{out})} \tag{3.46}$$

$$h_{out} = h_{in} \tag{3.47}$$

3.8 Evaporator

The Evaporator model is slightly more complicated due to the problem of the superheat. It's important to be modeled correctly so that it is not below zero since it would damage the compressor and not too high above zero as it would be a waste regarding power consumption, which makes it rather difficult to model and control. As in the economizer and receiver model, the evaporator will be separated into two control volumes, one filled with a mixture of liquid and vapor and the other with only vapor, separated by a moving boundary σ. The boundary must also separate the metal that encapsulates the refrigerant since the heat transfer coefficient from metal to liquid and metal to air are different.

Assuming that the liquid-vapor mixture has a constant average quality X_e, in this case equal to 0.1 which was the found to be the most adequate value to match the verification data of the evaporator. The formula for the moving boundary is given by

$$\sigma = \frac{M_l \cdot v_1}{V_i}, \tag{3.48}$$

where v_1 is the specific volume of the refrigerant in the first control volume and V_i is the internal volume(of the evaporator?). The energy flows can then be described:

$$Q_{fan} = (155 \cdot U_{fan}^2 + 40 \cdot U_{fan}^2) \cdot 0.2, \tag{3.49}$$

$$T_{retfan} = T_{ret} + \frac{Q_{fan}}{\dot{m}_{air} \cdot Cp_{air}}, \tag{3.50}$$

$$Q_{amv} = Cp_{air} \cdot \dot{m}_{air} \cdot (T_{retfan} - T_{mv}), \tag{3.51}$$

$$T_{retsh} = T_{retfan} - \frac{Q_{amv}}{\dot{m}_{air} \cdot Cp_{air}}, \tag{3.52}$$

$$Q_{aml} = Cp_{air} \cdot \dot{m}_{air} \cdot (T_{retsh} - T_{ml}), \tag{3.53}$$

$$Q_{mvml} = UA_3 \cdot (T_{mv} - T_{ml}), \tag{3.54}$$

$$Q_{ml} = UA_1 \cdot (T_{ml} - T_l) \cdot \sigma, \tag{3.55}$$

$$Q_{mv} = UA_2 \cdot (T_{mv} - T_v) \cdot (1 - \sigma). \tag{3.56}$$

Since the evaporator is a counter-flow heat exchanger, the return air that enters it passes through the superheating section first and evaporation section second. For this reason, heat flows and air temperatures need to be calculated for both of these sections. As the return air enters the evaporator, it passes over the fans, which adds to heat to the hair. This added heat is described in *Equation: (3.49)*, where U_{fan} is the fan speed, which can be either 0, 1 or 2, as described before. *Equation: (3.50)* gives the air temperature after the heat received from the fan. *Equation: (3.51)* calculates the necessary energy to decrease the air temperature to the temperature of the metal encapsulating the vapor refrigerant, where $T_m v$ is the temperature of the metal around the vapor volume. *Equation: (3.52)* gives the temperature drop of the air passing over the superheat section of the evaporator, which is then used to obtain the necessary energy to drop the air temperature to the metal temperature around the liquid, T_{ml}. Despite being two separate control volumes, there is still heat transfer through the metal and so *Equation: (3.54)* gives the heat flow between the metal around the two control volumes. Finally *Equation: (3.55)* and *Equation: (3.56)* give the heat transfer from metal to liquid and vapor refrigerant, respectively. T_l is the saturated evaporation temperature of the liquid refrigerant and T_v the temperature of the refrigerant leaving the evaporator.

Due to the inertia of the air and the rotational inertia of the fan, the airflow through the evaporator does not immediately change after a change in fan speed. This behavior is described in *Equation: (3.57)* and *Equation: (3.58)*.

$$\bar{\dot{m}}_{air} = \frac{U_{fan}^2 \cdot 3400.5 + U_{fan}^3 \cdot (-1103.5)}{3600 \cdot \rho_{air}} \tag{3.57}$$

$$\frac{\delta \dot{m}_{air}}{\delta t} = \frac{\bar{\dot{m}}_{air} - \dot{m}_{air}}{10s} \tag{3.58}$$

The instant mass flow of the air ,$\bar{\dot{m}}_{air}$, was obtained through a polynomial fit in *Equation: (3.57)* and was then used in first-order difference with a time constant of 10 seconds in *Equation: (3.58)*. The remainder of the state equations are listed below.

$$\frac{dT_{ml}}{dt} = \frac{Q_{aml} - Q_{ml} + Q_{nvml}}{M_m \cdot Cp_m \cdot \sigma} \tag{3.59}$$

$$\frac{dT_{mv}}{dt} = \frac{Q_{amv} - Q_{mv} + Q_{mvml}}{M_m \cdot Cp_m \cdot (1-\sigma)} \tag{3.60}$$

$$p_{out} = PHV\left(hv, \frac{V_i - -V_l}{M_v}\right) \tag{3.61}$$

$$h_l = h_{in} + \frac{Q_{ml}}{\dot{m}_{in}} \tag{3.62}$$

$$h_v = h_{lv} + \frac{Q_{mv}}{\dot{m}_{lv}} \tag{3.63}$$

$$\frac{dM_l}{dt} = \dot{m}_{in} - \dot{m}_{lv} \tag{3.64}$$

$$\frac{dM_v}{dt} = \dot{m}_{lv} - \dot{m}_{out} \tag{3.65}$$

$$T_{sup} = T_{retfan} + \frac{Q_{aml} + Q_{amv}}{Cp_{air} \cdot \dot{m}_{air}} \tag{3.66}$$

Equation: (3.59) and *Equation: (3.60)* give, respectively, the variation in metal temperature surrounding the liquid and vapor volumes, where Cp_m is the specific heat of the metal and M_m is the mass of the metal. The output pressure in the evaporator is obtained through table lookup in *Equation: (3.61)* using the enthalpy and density of the vapor. *Equation: (3.62)* to *Equation: (3.65)* give the energy and mass balances for both control volumes, where $\dot{m}_{lv}$ is the mass flow between both volumes given by *Equation: (3.67)*.

$$\dot{m}_{lv} = \frac{Q_{ml}}{h_{dew} - h_{in}} \tag{3.67}$$

Finally, *Equation: (3.66)* gives the temperature of the supply air for the cargo hold.

3.9 Box

The model of the box and thermodynamic processes are divided between the air, the floor and the cargo. The air comes cooled through the floor which then cools the cargo, thus getting hotter. The variation in temperature of these three components is obtained by summing up all the heat transfers between them, as described from *Equation: (3.68)* to *Equation: (3.70)*:

$$\frac{dT_{air}}{dt} = \frac{Q_{ca} + Q_{aa} + Q_{fa} + Q_{fan} - Q_{cool}}{M_{air} \cdot Cp_{air}}, \tag{3.68}$$

$$\frac{dT_{floor}}{dt} = \frac{Q_{af} - Q_{fa}}{M_{floor} \cdot Cp_{floor}}, \tag{3.69}$$

$$\frac{dT_{cargo}}{dt} = \frac{-Q_{ca}}{M_{cargo} \cdot Cp_{cargo}}, \tag{3.70}$$

where Q_{ca} is the energy flow from cargo to air, Q_{aa} is the heat transfer from the inside air to the outside air, Q_{fa} is the heat transfer from the floor to the cargo, Q_{fan} is the heat added by the fans, Q_{cool} is the cooling from the evaporator. The calculations for these heat transfers are described below:

$$Q_{cool} = Cp_{air} \cdot \dot{m}_{air} \cdot (T_{ret} - T_{sup}), \tag{3.71}$$

$$Q_{aa} = (T_{amb} - T_{air}) \cdot UA_{amb} \cdot 0.81, \tag{3.72}$$

$$Q_{af} = (T_{amb} - T_{floor}) \cdot UA_{amb} \cdot 0.19, \tag{3.73}$$

$$Q_{ca} = (T_{cargo} - T_{air}) \cdot UA_{cargo}, \tag{3.74}$$

$$Q_{fa} = (T_{floor} - T_{air}) \cdot UA_{floor}, \tag{3.75}$$

$$Q_{fan} = (155 \cdot U_{fan}^2 + 40 \cdot U_{fan}^3) \cdot 0.8. \tag{3.76}$$

The heat transfer coefficients were obtained experimentally to be $UA_{cargo} = 500J/K$, $UA_{floor} = 1100J/K$, and UA_{amb} was specified by the manufacturer to be $43J/K$. Finally, and since the floor amounts to 0.19 of the total surface area of the container, *Equation: (3.72)* was multiplied by $1 - 0.19 = 0.81$ and *Equation: (3.73)* was multiplied by 0.19.

3.10 Fan

3.11 Compressor Power Usage

3.12 Fan Power Usage

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Appendices

Measurements A

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