Kalman filter - for linear systems
Execise

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Consider the system below. Try to use symbolic parameters instead of specific figures as far as possible.

$$x_{k+1} = \phi x_k + w_k$$
$$y_k = hx_k + v_k$$
$$\mathrm{E}(x_0) = \hat{x}_0 \ , \ \mathrm{Var}(x_0) = p_0 \ , \ \mathrm{E}(w_k) = 0 \ , \ \mathrm{E}(w_k^2) = q$$
$$\mathrm{E}(v_k) = 0 \ , \ \mathrm{E}(v_k^2) = r$$

1. Find The stationary solution i.e. k, $p^- \triangleq p_{k|k-1}$ and $p^+ \triangleq p_{k|k}$ for $\phi = 0.9, h = 1, q = 1$ and three (or more) different measurement noise levels $r = 0.1, 1, 10$.
2. Try to understand and explain why k, p^- and p^+ changes with the system parameters as they do.
3. Find the transfer function from measurement y to estimates $\hat{x}^+ \triangleq \hat{x}_{k|k}$.
4. Find the gains and poles for the three noise levels above.
5. Try to understand and explain why the stationary KF gain and pole changes with the system parameters as they do.