Kalman filter - for nonlinear systems
Execise

Torben Knudsen

Given the following scalar system

$$
\begin{aligned}
&x_k = a\sin(x_{k-1} + \varphi_f) + bu_{k-1} + w_{k-1} \\
&z_k = \sin(cx_k + \varphi_h) + v_k \\
&w \in \mathrm{NID}(0, Q)\ ,\ v \in \mathrm{NID}(0, R)\ ,\ x_0 \in \mathrm{N}(\hat{x}_0, P_0)
\end{aligned}
$$

assume $a, b, \varphi_f, c, \varphi_h, Q, R, x_0, P_0$ to be know.

1. For the system above, write the algorithm for the extended Kalman Filter.
2. The system can be simulated with the program NLSim.m. Here parameters are also given.
3. Try to program the EKF and make it work. If you don't succeed use the one in EKF.m. For comparision a linear KF is found in KF.m. To run these programs without errors you also need XCorrtk.m.
4. Compare and explain the performance of the EKF and linear KF for:
   (a) Initial parameters.[1]

   $$a = 0.95\ ,\ k = 1\ ,\ b = k(1-a)\ ,\ c = 1\ ,\ \varphi_f = 0\ ,\ \varphi_h = 0\ ,\ f_u = 0.02$$

   (b) $\varphi_h = \frac{\pi}{16}$.
   (c) $\varphi_f = \frac{\pi}{16}$.
   (d) $c = 10$.

[1] f_u is the frequency for the square wave used as input in the simulation in NLSim.m.