Miniproject
Optimality and Robustness
CVX Exercise - Scheduling Power Plant Production
Group 830

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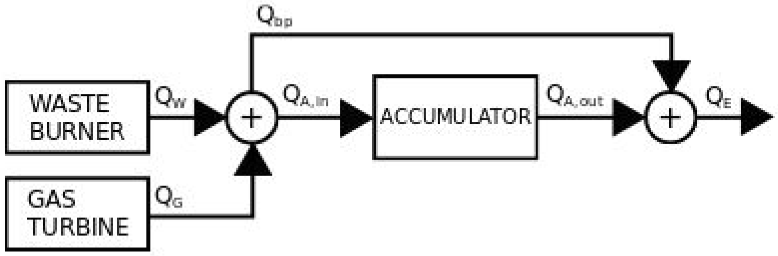
Object: To solve an MPC problem by the use of CVX.

The figure below shows the power plant, on which the optimization problem is based. Q_G and Q_W denotes the power produced by the gas turbine and the waste burner respectively. $Q_{A,in}$ is the power fed to the accumulator and Q_{bp} is the power that bypasses the accumulator, $Q_{A,out}$ is the power leaving the accumulator and Q_E is the power leaving the plant. The unit of power is in [MW].



The power flow of the plant is constrained by

$$Q_W + Q_G = Q_{bp} + Q_{A,in} \qquad Q_E = Q_{bp} + Q_{A,out}$$
$$0 \le Q_W \le 40 \quad 0 \le Q_{A,in} \le 50 \quad 0 \le Q_{A,out} \le 25$$

and the dynamics of the accumulator is

$$E_A[k+1] \;=\; E_A[k] + (Q_{A.in}[k] - Q_{A,out}[k]) * Ts$$

with Ts the sampling time in hours [h] and $E_A[k]$ the energy stored at the accumulator at sample k. The accumulator is assumed to be constrained by

$$0 \le E_A \le 200$$

The objective is to optimize the profit from running the plant, by scheduling the power production using knowledge of the dynamics and future energy prices. P_G, P_E and P_W denotes the (known) price pr. MWh, [DKK/MWh], of gas, electricity and waste burning respectively. The profit over the horizon L can be expressed as

$$\sum_{k=1}^{L}(P_E[k]Q_E[K] - (P_G[K]Q_G[K] + P_W[K]Q_W[K]))T_s \quad \text{[DKK]}$$

Solution

Matlab - Missing parts and explanation

CVX is a Matlab-based modeling system for convex optimization. It allows constraints and objective functions to be specified which are automatically transformed to canonical form and thereby solved. In case of the Power Scheduling problem, the objective function is given as a cost function which is desired to be maximised over a finite horizon(L). The constraints are given as the dynamics of the accumulator, constraints on the power flow in the whole system and as equilibrium equations between input and output power flows.
In order to solve the minimization problem for the whole control horizon, all variables must be defined before they are used in constraints or in the profit function. They are all treated as vectors with size of the window parameter(L) except the energy stored in the accumulator E_A , since the dynamics of the accumulator also requires the (k+1)th. future value. Therefore the variables are treated as follows:

variables Q_E(L) Q_G(L) Q_W(L) E_A(L+1) Q_A_in(L) Q_A_out(L) Q_bp(L)

In order to solve the optimal maximization problem of the profit function over the horizon, it is transformed to a minimization problem instead. The cost function in a call to minimize must be convex, however in a call to maximize must be concave. Since the objective function is stated as a convex optimality problem, the following is a reasonable solution:

*minimize(- (P_E(k:L+k-1)'*Q_E - (P_G(k:L+k-1)'*Q_G + P_W(k:L+k-1)'*Q_W))*Ts)*

The model of the system is constrained by equalities on the power flows:

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Q_W + Q_G == Q_bp + Q_A_in;
Q_E == Q_bp + Q_A_out;
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Constraints on the predefined variables:

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Q_W_min*onesL       <= Q_W <=      Q_W_max*onesL;
Q_G_min*onesL       <= Q_G <=      Q_G_max*onesL;
E_A_min*ones(L+1,1) <= E_A <=      E_A_max*ones(L+1,1);
Q_A_in_min*onesL    <= Q_A_in <=   Q_A_in_max*onesL;
Q_A_out_min*onesL   <= Q_A_out <= Q_A_out_max*onesL;
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And on the dynamics of the accumulator:

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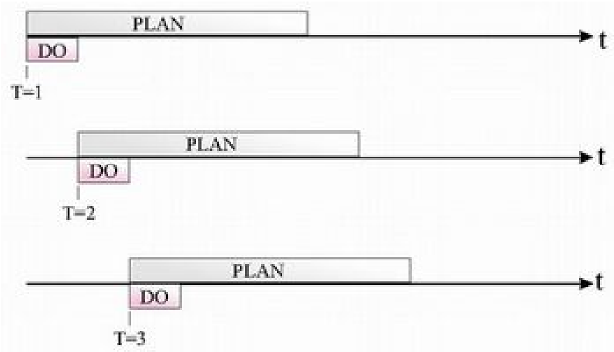
E_A(2:L+1) == E_A(1:L)+ (Q_A_in - Q_A_out)*Ts;
```

The energy stored in the accumulator depends on the power flow difference over one time step and the energy stored in the previous time step. Therefore the elements of the future accumulator energy is iterated from k = 2. This is due to the consideration, that in each iteration step the first value of the E_A vector is saved to the final $E_{A;sys}$ vector as follows:

E_A(1) == E_A_sys(k)

The optimization part of the problem in CVX is preceded by the command *cvx_begin* and terminated with the command *cvx_end*. All the variables, constraints and the cost function should fall in between. However, the optimization is done over the control horizon(L), therefore a for loop is used for iteration, until the horizon is shifted to the end of the control process.
The figure[1] below illustrates how the control horizon is shifted over the time steps:



After the minimization, according to the Model Predictive Control algorithm, only the first change in each variable is implemented, therefore the first element of the horizon variables are saved in the final xy_sys variables. The calculation is repeated in the next iteration when the next change is required.

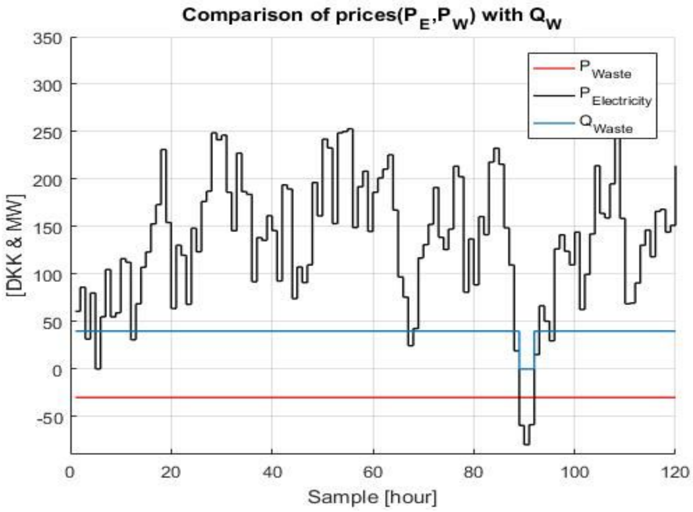
[1] In the figure "Plant" represents the control horizon (window parameter), while "Do" shows when the new calculation starts after each iteration. Axis- x represents the time and it is divided into time steps.

Solution

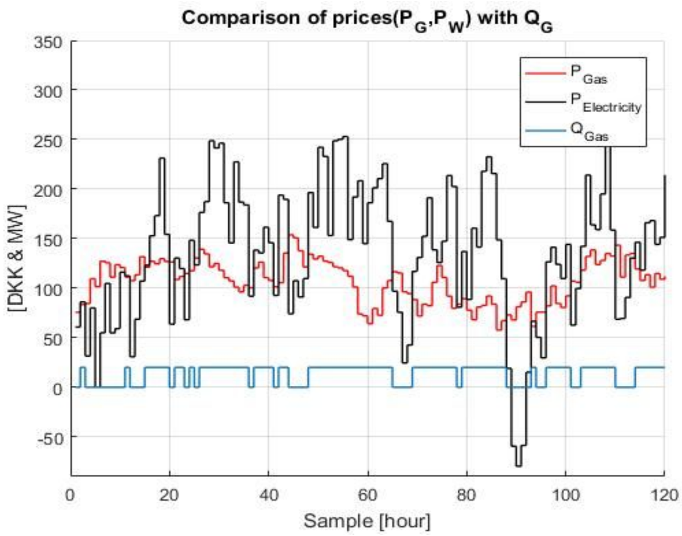
Graph analyzation and explanation

Note: The y-axis on the following graphs contains information about the price of energy in the unit of 10^0 and the power output in the unit of 10^6. Furthermore when the price goes beneath zero, it is defined as money earned by the system.

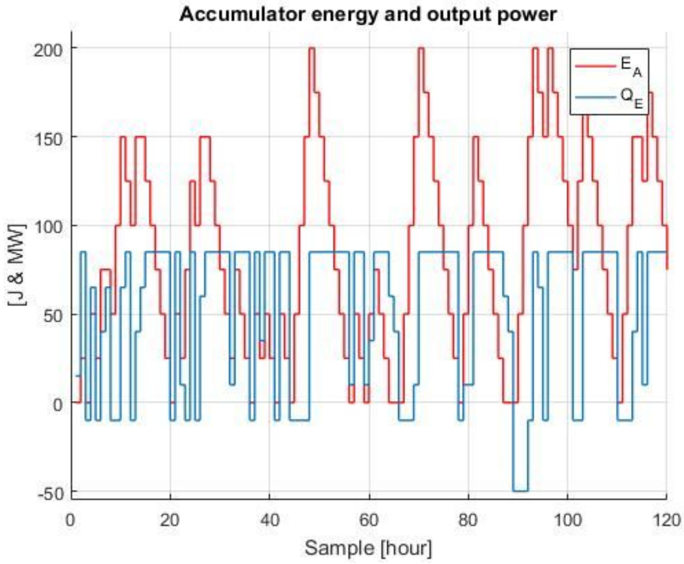
In the first figure, the curves for the electricity and waste prices along with the power from waste burning is shown. It can be seen that the waste burner (Q_W) provides maximum and 'constant' power to the system except from time between hour 89 to 92. In this interval the price of the electricity goes to minus, below the price of waste burning which is constant all the time. Therefore during this period, it is more profitable to shut down the waste burning and provide all the power by using the accumulator and the available energy stored in it.



In the second figure, a graph which includes the price of gas and electricity along with the power from gas turbine is shown. It can be clearly seen how the power coming from the gas turbine is dependant on the price of electricity and and the price of the gas. When the price of the gas is high compared to the electricity, the power provided by the turbines is completely shut down. When the price of the gas is below of electricity prices, the gas turbines provide the maximum amount.



Another interesting scenario is when the energy in the accumulator and the power output of the system is illustrated:



As it can be seen, the energy stored in the accumulator varies according to its constraints, dynamics and the future prices of the energy sources and waste burning. However, it is shown that the power flow on the output is between -50 and 80 [MW]. It means that at certain intervals power is taken from the grid in order to achieve a more profitable way of power production. During these time intervals, according to the last figure, energy is stored back in the accumulator. This is in accordance to the system specifications, since constraints are only defined for the energy stored in the accumulator, not on the power output. Therefore it seems a fair assumption, that power can flow in the other direction back from the grid to the plant when it is considered as profitable.

Conclusion

CVX- MPC

As it was shown, the convex optimization problem according to the cost function can be solved with the CVX matlab toolbox. It is also shown that the interpretation of the system dynamics and the constraints on the system variables are easy to implement. However CVX requires more computational time when the window parameter(L) is increased due to the increased constraints and system variables.