2016 European Control Conference (ECC)
June 29 - July 1, 2016. Aalborg, Denmark

MPC Control of Water Supply Networks

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Abstract— **This paper investigates the modelling and predictive control of a drinking water supply network with the aim of minimising the energy and economic cost. A model predictive controller, MPC, is applied to a nonlinear model of a drinking water network that follows certain constraints to maintain consumer pressure desire. A model predictive controller, MPC, is based on a simple model that models the main characteristics of a water distribution network, optimizes a desired cost minimisation, and keeps the system inside specified constraints. In comparison to a logic (on/off) control design, controlling the drinking water supply network with the MPC showed reduction of the energy and the economic cost of running the system. This has been achieved by minimising actuator control effort and by shifting the actuator use towards the night time, where energy prices are lower. Along with energy cost reduction the MPC also achieves reduction in the amount of consumed water by keeping the pressure closer to the lower pressure constraint.**

I. Introduction

A drinking water supply network is one of the key points in the infrastructure of societies all over the world. Typically a network is working by making pumps transport water through the pipes in the network. For large cities these networks can be rather complex. For most water supply networks the pumps pumping water from waterworks are not able to deliver all of the demanded water in the peak periods of a day. Therefore, an elevated reservoir is installed in the network as a buffer to even out the demand differences as it will supply the network with the residual water during high demand periods. Also at the consumer site of the network the pressure needs to be higher than a predefined minimum level in order to provide a satisfying water pressure at the consumers.

Running a drinking water supply network requires electricity for the pumps to work. The aim is to reduce the electrical energy used by the system achieving a sustainable solution going towards a greener environment and economy. This can be done by having the pumping stations running the system as close to the minimum system specifications as possible and shifting some of the energy use to the night period where energy prices are low. An approach to accomplish this is Model Predictive Control. Model Predictive control (MPC) for reservoir control in water distribution has been studied in several papers, see for example [5], [8], [1], [4], [6]. These papers deal with economic optimal control

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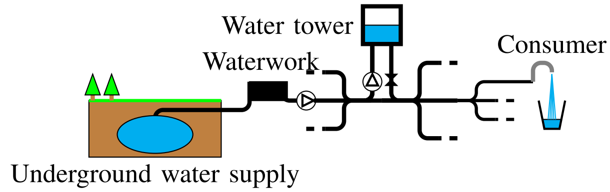

Fig. 1: Simple diagram of a generic drinking water system

of water networks with several reservoirs, and exemplifies their results on the water distribution network of Barcelona. The underlying network model deals, in all the cases, with mass conservation only. This means that the control does not take pipe pressure losses into account. In [8], [1], [4] a distributed MPC approach is proposed to make the solution easy scalable and reconfigurable. In [6] the uncertainty of the demand flow is considered, which is the main disturbance in water distribution networks. A stochastic model for demand forecast is developed for use in the MPC framework.

In this paper we study networks with one reservoir, a pump-valve station that controls the network pressure, and a supply pumping station. Energy consumption of the pumps and the cost of operating the pumps are studied. The control is done with respect to; the consumer pressure requirements, energy cost, and pressure losses in the pipe network. The topology with the pump-valve station between the network and the elevated reservoir makes it possible to control the consumer pressure independent of the water level in the reservoir. This opens up for a more active use of the reservoir for shifting the energy consumption to optimize the cost of operation with respect to variable energy prices.

The water network of this paper is modeled and simulated using the EPANET hydraulic simulator software [2].

This paper introduces the network model in Section II. This is followed by the implementation of the MPC for the model in Section III. In Section IV and V first some implementation issues are considered followed by a presentation of the simulation results obtained by the proposed controller.

II. Network Model

This section deals with the model of the water supply network. The structure of the water network under consideration is depicted in Fig. 1. This paper deals with a control that optimizes the use of elevated reservoirs in water supply network such that the pressure at the end-users is maintained, and the cost of the pumping effort is minimised. Therefore, the model developed here models the pumps, the reservoir, the end-user demands, and the pressure losses in the piping. The network diagram presented in Fig. 2 captures exactly these parts of the water supply system in Fig. 1.

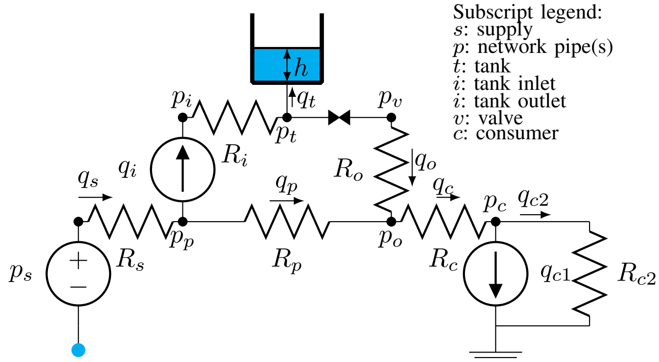

Fig. 2: The structure of the water supply network depicted partly as a resistance network.

The network shown in Fig. 2 has three control inputs; the supply pressure p_s, valve pressure p_v, and the tank input flow q_i. The pressure p_s is controlled by the main pump, the flow q_i is controlled by a reservoir pump that decouple the pressure in the network from the reservoir pressure. The flow q_o out from the tank and into the network and the corresponding pressure p_v is controlled by a valve.

The network is modelled using the relations between hydraulic and electrical networks, by letting the flow q correspond to currents, and the pressure p correspond to voltage. Here, the resistances R_s, R_p, R_c, R_i, and R_o represents pipes, which has the following relation between the inlet pressure p_{in}, the outlet pressure p_{out}, and flow q_k for the k^{th} pipe

$$\Delta p_k = p_{in} - p_{out} = r_k|q_k|q_k - (h_{in} - h_{out}) \tag{1}$$

where r_k is the non-linear resistor of the pipe and $h_{in} - h_{out}$ represents the change in elevation between the inlet and outlet of the pipe. The term $|q_k|$ is the absolute value og q_k The pipe resistance can be calculated from the dimensions of the pipe, see for example [7].

The water consumption of the end-users is represented by a pressure dependent term modelled by the resistance R_{c2} and a constant flow term modelled by a constant flow source, such that the end-user flow is

$$q_c = q_{c1} + k_v\sqrt{p_c} \tag{2}$$

where k_v is the non-linear conductivity of the valve denoted R_{c2}, and q_{c1} is the consumer demands. There are always more than one consumer in water distribution networks, but it is possible to model their flows by a single user flow using argument similar to the one used in [3].

The only dynamics in the network is due the elevated reservoir. This dynamics follows the equation

$$a_t\dot{h} = q_t \tag{3}$$

where a_t is the cross sectional area of the tank, h is the water level, and q_t is the water flow into the tank. In some cases a_t is a function of the level h. However, here it is assumed that the area a_t is constant over all levels. The reservoir is typically raised above the ground level. This elevation provides an increase in pressure, here denoted h_e. The total pressure of the reservoir p_t is then

$$p_t = h + h_e \tag{4}$$

The water level in the reservoir is constrained by its physical limitations. The lower and upper limit are $\underline{h}$ and $\overline{h}$ respectively, where

$$\underline{h} \leq h \leq \overline{h} \tag{5}$$

In this work economical MPC will be used for calculating optimal references for the pumps and valve control, that is; calculating optimal values for p_s, q_i, and p_v. For this, a linear version of the system is needed. The only non-linear elements are the model of the pipes and the end-user valve R_{c2}. Dividing the variables of the pipe model into an operating point marked with 0 and a variation around this valve marked with tilde, the approximation $\Delta p \approx \Delta p_{k,0} + \Delta\tilde{p}_k$ is obtained, where

$$\Delta p_{k,0} + \Delta\tilde{p}_k = r_k|q_{k,0}|q_{k,0} + 2r_k|q_{k,0}|\tilde{q}_k - (h_{in} - h_{out}) \tag{6}$$

Likewise, for the end-user valve

$$q_{c,0} + \tilde{q}_c = q_{c1,0} + \tilde{q}_{c1} + k_v\sqrt{p_{c,0}} + \frac{k_v}{2\sqrt{p_{c,0}}}\tilde{p}_c \tag{7}$$

This leads to the following model for the pipe elements

$$\Delta\tilde{p}_k = R_k\tilde{q}_k \tag{8}$$

where $R_k = 2r_k|q_{k,0}|$. Using the same approach for the model of the end-user valve the following model is obtained

$$\tilde{q}_c = \tilde{q}_{c1} + \frac{\tilde{p}_c}{R_{c2}} \tag{9}$$

where $R_{c2} = \frac{k_v}{2\sqrt{p_{c,0}}}$.

Using Kirchhoff's current and voltage laws with the small signal descriptions of the pipe and valve resistances used on the network in Fig. 2 leads to a model of the system.

The LTI model has the state $x = p_t$, the output vector $y = [q_s \quad p_i \quad p_p \quad q_o \quad p_c \quad q_{c2}]^\top$, $u = [p_s \quad q_i \quad p_v]^\top$, and the disturbance $d = q_c$, and is given by

$$\dot{x} = \mathbf{B}_{su}u + \mathbf{B}_{sd}d \tag{10a}$$
$$y = \mathbf{C}_s x + \mathbf{D}_{su}u + \mathbf{D}_{sd}d\ . \tag{10b}$$

where the three inputs to control are p_s the supply pump pressure, q_i the booster pump flow, and p_v the valve pressure.

The eigenvalue of the model is zero, $\lambda_s = 0$ hence the tank acts as a pure integrator in the model. The model is fully controllable as the controllability matrix is identical to the $\mathbf{B_{su}}$ matrix which has full rank. The model is also fully observable as the observability matrix is identical to the $\mathbf{C_s}$ matrix which has full rank.

III. MPC Setup

The MPC will be made as an instantaneous controller using the linearised discrete time model at the current sample for the control optimisation. It is linearised using a linearisation point of the state, inputs, and outputs at its current sample. The discrete time version of (10) is given by

$$x_{i+1} = \mathbf{A}x_i + \mathbf{B_u}u_i + \mathbf{B_d}d_i \tag{11a}$$

$$y_i = \mathbf{C}x_i + \mathbf{D_u}u_i + \mathbf{D_d}d_i \tag{11b}$$

A. Cost function

The cost function for the MPC describes the energy consumption of the pumps time the cost of energy c. The energy of the supply pump is given by $q_s p_s$. The energy of the flow pumps is given by $q_i(p_i(t) - p_p(t))$, as this pump has to provide the pressure $p_i(t) - p_p(t)$ to the flow q_i. The valve is not part of the cost function as the cost related to the control of the valve is negligible.

$$J = \sum_{i=0}^{\infty} c(t)\left(q_s(t)\,p_s(t) + (p_i(t) - p_p(t))\,q_i(t)\right) \tag{12}$$

With $q_s = \tilde{q_s} + q_{s,0}$, $p_s = \tilde{p_s} + p_{s,0}$, $p_i = \tilde{p_i} + p_{i,0}$, $p_p = \tilde{p_p} + p_{p,0}$, and $q_i = \tilde{q_i} + q_{i,0}$. Let $y_{qs} = q_s$, $y_{pip} = p_i - p_p$, and $u = \begin{bmatrix} p_s & q_i & p_v \end{bmatrix}^\top$ and let the weights be given by

$$w_{ps} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad , \quad w_{qi} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \tag{13}$$

Reorganizing the cost function into a vector equation gives

$$J = \frac{1}{2}\big(Y_{qs}^\top C\,W_{ps}\,U + U^\top W_{ps}^\top C^\top\,Y_{qs} + Y_{pi}^\top C\,W_{qi}\,U + U^\top W_{qi}^\top C^\top\,Y_{pi}\big) \tag{14}$$

where

$$W_{ps} = \mathrm{diag}\{w_{ps,0}\,,\,\dots\,,\,w_{ps,N}\} \tag{15}$$

and

$$W_{qi} = \mathrm{diag}\{w_{qi,0}\,,\,\dots\,,\,w_{qi,N}\} \tag{16}$$

The weighting $w_{ps,i}$ and $w_{qi,i}$ at each sample i are chosen to have the same values. The cost c changes between the samples, which leads to the following block diagonal cost matrix

$$C = \mathrm{diag}\{c_0\,,\,\dots\,,\,c_N\} \tag{17}$$

Note that $Y_{qs} = \tilde{Y}_{qs} + \tilde{Y}_{qs,0}$, $Y_{pip} = \tilde{Y}_{pip} + \tilde{Y}_{pip,0}$, and $U = \tilde{U} + U_0$, where $\tilde{Y}_{qs,0}$, $\tilde{Y}_{pip,0}$, and U_0 represent the operating point for the linearised model in (11).

The cost function will be solved for the deviation around the linearisation point. The output deviation vectors are defined as

$$\tilde{Y}_{qs} = \mathcal{O}_{qs}\tilde{x_0} + \mathcal{G}_{qsu}\tilde{U} + \mathcal{G}_{qsd}\tilde{D} \tag{18}$$

and

$$\tilde{Y}_{pip} = \mathcal{O}_{pip}\tilde{x_0} + \mathcal{G}_{pipu}\tilde{U} + \mathcal{G}_{pipd}\tilde{D} \tag{19}$$

where $\mathcal{O}_{qs}$ and $\mathcal{O}_{pip}$ are the extended observability matrices and $\mathcal{G}_{qsu}$, $\mathcal{G}_{qsd}$, $\mathcal{G}_{pipu}$, and $\mathcal{G}_{pipd}$ are the Toeplitz matrices. The QP problem is then

$$J = U^\top HU + f\,U \tag{20}$$

with

$$H = \frac{1}{2}\big(\mathcal{G}_{qsu}^\top C\,W_{ps} + W_{ps}^\top C^\top \mathcal{G}_{qsu} + \mathcal{G}_{pipu}^\top C\,W_{qi} + W_{qi}^\top C^\top \mathcal{G}_{pipu}\big) \tag{21}$$

and

$$f = x_0^\top \mathcal{O}_{qs} C\,W_{ps} + D^\top \mathcal{G}_{qsd}^\top C\,W_{ps} + x_0^\top \mathcal{O}_{pip} C\,W_{pi} + D^\top \mathcal{G}_{pipd}^\top C\,W_{qi} + Y_{qs,0}^\top C\,W_{ps} + U_0^\top W_{ps}^\top C^\top \mathcal{G}_{qsu} + Y_{pip,0}^\top C\,W_{qi} + U_0^\top W_{qi}^\top C^\top \mathcal{G}_{pipu} \tag{22}$$

B. Constraints

The constraints are due to performance limitations on the pumps and valve, and limitation on the reservoir volume. The volume restrictions are given by constraints on the water level in the reservoir. Here the water level is measured via the pressure p_t.

$$\underline{p_t} = (\underline{h} + h_e) \le p_t \le (\overline{h} + h_e) = \overline{p_t} \tag{23}$$

The difference between the tank pressure p_t and valve outlet pressure p_v drives the flow through the control valve and the flow through the valve cannot be negative, i.e. cannot flow from the network to the reservoir. This leads to the following constraints

$$\underline{p_v} \le p_v \le p_t \tag{24}$$

$$\underline{q_o} \le q_o\ . \tag{25}$$

The requirements on the end-user pressure p_c is stated as a minimum pressure requirement. This leads to the following constraint

$$\underline{p_c} \le p_c\ . \tag{26}$$

Finally the constraints on the pump capability are described by

$$\underline{p_s} \le p_s \le \overline{p_s} \tag{27}$$

$$\underline{q_s} \le q_s \le \overline{q_s} \tag{28}$$

$$\underline{q_i} \le q_i \le \overline{q_i} \tag{29}$$

All the constraints are linear and can therefore be put on the following form

$$z_i = \mathbf{C}_z x_i + \mathbf{D_{zu}}u_i + \mathbf{D_{zd}}d_i \le z_b$$

where z_b is a vector of the constant values. Collecting the constraints over the prediction horizon the following constraint equation is obtained

$$\underbrace{\mathcal{G}_{uz}}_{F}\,U \le \underbrace{Z_b - \mathcal{O}_z x_0 - \mathcal{G}_{dz}D}_{b} \tag{30}$$

The QP problem is given by the cost function of Equation (20) and the constraint equation

$$FU \leq b \tag{31}$$

C. Additional Features to the Control

As the model of the hydraulic network is highly non-linear there is a good chance of infeasibility of the MPC optimization problem. To overcome this problem additional features are added to the controller.

In the cases where the optimal solution makes the flow control signal $q_i = 0$, then $\mathbf{B_u}$ of the linearised system will have a zero element. This violates the controllability of the system, meaning that the $\mathbf{B_u}$ element remains zero and eventually the control empties the reservoir. Therefore the linearisation point of the flow $q_{i,0}$ will be forced to have a minimum value larger than zero, we chose $q_{i,0,min} = 0.2\, q_{c0}$ where q_{c0} is constant. Whenever the flow q_i gets lower than $q_{i,0,min}$ the linearisation point is set to $q_{i,0,min}$.

Besides the problem with controllability, simulations has shown that the MPC controls the consumer pressure to a too low value when the consumer demand is increasing. This is due the fact that consumer demand that dictates the linearization point for the underling linear model is too low. This is solved by linearizing the system around the estimated demand of the following sample, as this demand in general is closer to the true demand in the sample period. This linearization approach is only used in periods with increasing consumer demands.

The above described linearization approach only partly solves the problem with too low consumer pressure, as the deviation between the linear model and the nonlinear system still has a tendency to lead to a too low consumer pressure. The consumer pressure is lifted by changing the pressure constraint with time. In the current sample of the control horizon the consumer pressure constraint is at the desired value. In the following horizon samples the constraint is then slowly increased, forcing the system to increase the consumer pressure. A way of doing this is shown in Figure 3.

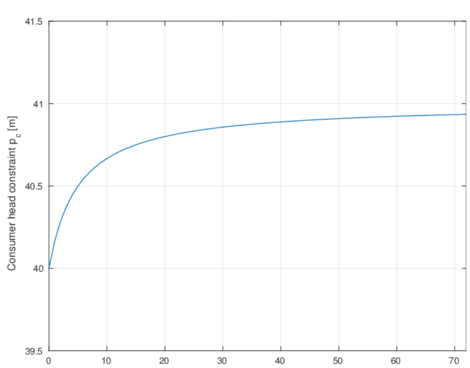

Fig. 3: Evolution of the consumer head constraint p_c on the horizon

IV. Control Algorithm

The flow of the control at each sample is shown in the following steps:

1) Estimate future consumer demand based on current consumer flow and forecastings.
2) Measure measurable variables and estimate remaining states and outputs.
3) Solve the QP problem for the model linearised around the current system state and set resulting optimal input as system input.
4) Let system run and start over from step 1 at next sample.

To test the performance of the proposed MPC an EPANET model of a water supply network is developed and executed from Matlab.

At each sample the demanded flow is updated from the prediction. Next the outputs are estimated to be used for the linearisation point. After this the actual QP problem, described in Section III, is solved, resulting in the optimal control input for the coming sample period. If the QP problem is not found feasible the optimal input vector $\hat{\tilde{U}}$ of the previous sample is used, where the applied input is then chosen as the control values of the second sample $\tilde{u} = \hat{\tilde{u}}_1$ instead of $\hat{\tilde{u}}_0$.

In theory the full control vector $\hat{\tilde{U}}$ found by the last feasible solution to the QP problem can be used as long as infeasibility appears. However, the more times this is done the model error becomes more dominant and might lead to bad behaviour. Tests have shown that the last found optimal solution vector can be used for three concurrent samples by applying $\hat{\tilde{u}}_0$, $\hat{\tilde{u}}_1$, and $\hat{\tilde{u}}_2$ respectively. After that it is better to hold the control constant until a feasible solution is found. Before applying the input to the real system it must be added to the input initial condition u_0. At the end of each sample the actual simulation is made where the model runs for the duration of the sample with the applied disturbance and input. At last the tank level is read to be used for next sample.

V. Simulation Results

The MPC control approach is in this section compared to typical solution for a network with the structure shown in Fig. 2. We will compare the energy consumption and the water consumption in three types of control: **Logical pressure control**, **Energy MPC**, and **Economic MPC**.

A typical approach **(Logical pressure control)** for controlling water networks with a structure similar to the one shown in Fig. 2 is to use the pump flow q_i to control the consumer pressure p_c. This is done by setting a reference p_s^* on the pressure p_s. If the consumer pressure gets to high $p_c > p_c^* + \epsilon$, where ϵ defines a hysteresis band, then the flow pump starts to pump water to the reservoir i.e. $q_i > 0$. If the pressure p_c drops below p_c^* the pump stops and the valve opens controlling the flow to the consumers by controlling the pressure p_v. This logical control is implemented on the simulation model and the result of the control is depicted in Fig. 4.

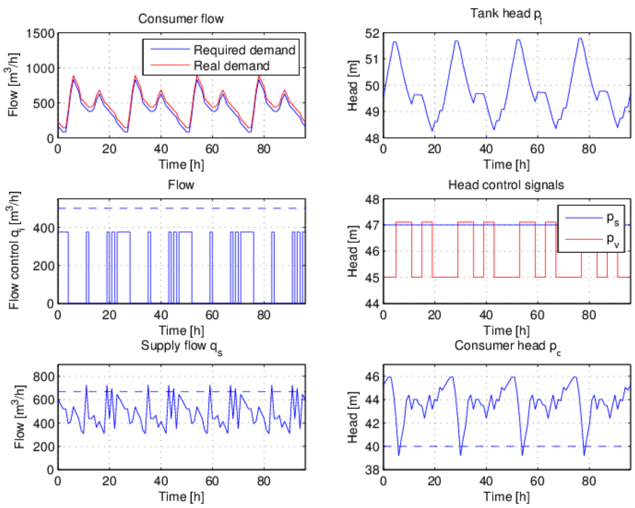

Fig. 4: Simulation results with the Logic controller.

The consumer pressure p_c is generally higher than 40 meters. This is to secure that the low peak is only a little below 40 meters. This of course increases the cost of the pumps a little, as will be discussed in the following section. It is seen that the valve pressure rises to let water out of the tank in peak periods where also the pump flow q_i is zero. In the periods of low consumer flow the tank is then filled with water. The consumer flow is shown with two graphs; the real and the required flow demand. These are q_c and q_{c1} respectively.

The results on the **Energy MPC** is obtained by setting the cost term c constant for all samples. The results obtained with the Energy MPC is depicted in Fig. 5. The Energy MPC minimizes the energy consumption by using the reservoir as little as possible. This is done by only using water from the reservoir when the water consumer demand is high, and filling the reservoir when the consumer demand is at its lowest. The consumer pressure p_c is kept just above the constraint limit of 40 m, except at one sample point. This results in an infeasible optimization problem and is the reason for introducing the features described in Section III-C.

To test the **Economic MPC** a variable cost is added to the MPC optimization problem. Future costs are expected to be known and for the test follows the profile shown in Fig. 6. The price function c is included in the cost minimisation and the upper limit constraints $\overline{q_s}$ and $\overline{q_i}$ needed to be slightly increased to solve the problem as the cost minimisations will be further stressed. The economic cost minimisation results in the control behaviour shown in Fig. 7.

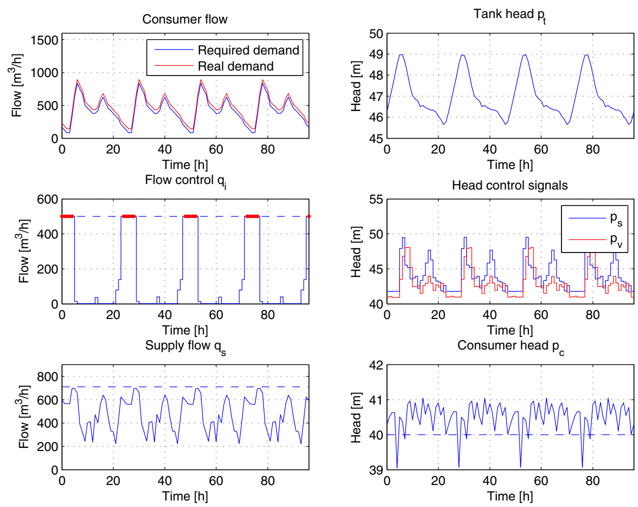

Fig. 5: Simulation results with the Energy MPC.

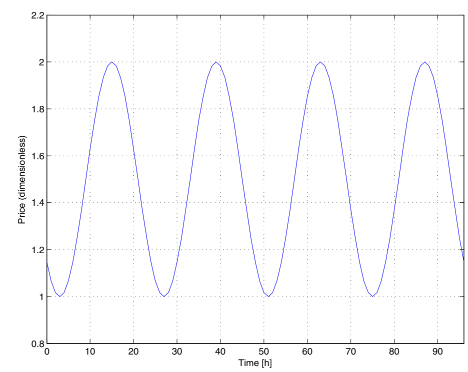

Fig. 6: Economic energy price variation

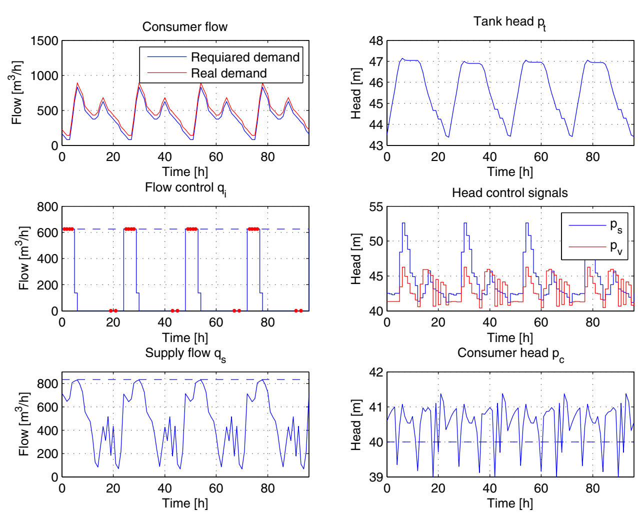

Fig. 7: Head dependant consumer simulation with economic cost

The consumer pressure still varies in between 40 and 41 meters with single low peak samples around 39 meters as before which results in infeasible solutions just before the peak. The reservoir steady state level has been decreased reducing the cost of pumping water into the tank.

The control signal at the peak energy price has decreased and the control has been shifted towards the less expensive energy price period at night and the valve takes over as the pressure supply in a short period where the price is high. The reservoir variation from the minimum level to the maximum level is approximately 3 meters. This is a little on the high side of typical reservoir variations that are between 1 and 2 meters.

A. Comparison of the Controllers

The costs of the three controllers have been accumulated over 10 days and plotted in Figure 8. As expected the Economic

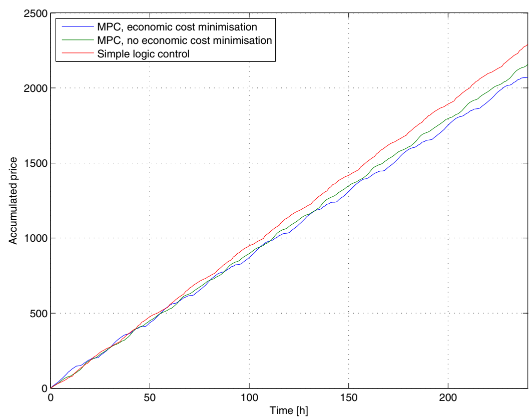

Fig. 8: Accumulated cost comparison

MPC minimize for the economic cost and therefore has the lowest accumulated cost. The Energy MPC has a little higher cost, and the simple logic controller has the highest cost. The main reason for the high cost of the logical controller is that the average pressure must be chosen a little higher compared to the MPC solutions to cope with the larger pressure variation obtained with this controller. The cost savings between the logical controller and the Economic MPC is around 10%.

The amount of consumed water for the three controllers are compared. The accumulated water volume is shown in Fig. 9. The dashed curve is the required demand of the consumers and is not controllable by the controllers compared here, but forms a lower limit on the water consumption. The water consumption with the three controllers are almost the same. The reason for this is that the water consumption is controlled by the pressure p_c, which only varies within a few meters in the three test cases. This indicates that the MPC control solution does not impose significant water savings.

VI. Conclusions

The aim of this work has been to test the savings potential by introducing an advanced control solution in drinking water supply networks. The potential with MPC control is

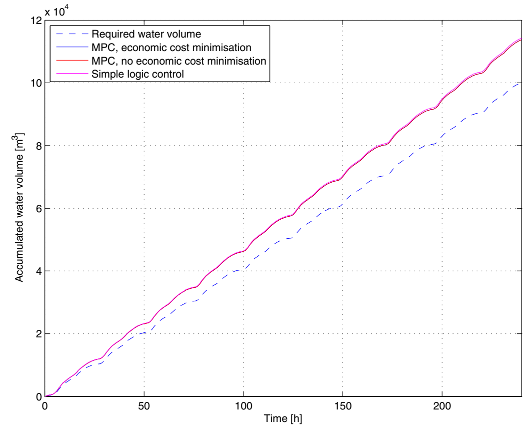

Fig. 9: Accumulated consumed water volume comparison

compared to a standard logical control solutions, reviling the saving potential on cost of energy and water. The supply network under consideration in this work has a reservoir with a pump and valve system that enables flow control to and from the reservoir.

The MPC has a great potential for minimising the costs of energy running the pumps in wider scale drinking water supply networks with reservoirs. In the actual simulation case the saving was approximately 10%. The saving on the water consumption is on the other hand minor, and cannot be used as an argument for introducing advance control in drinking water systems with a structure like the one analysed here.

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