Aalborg University

# Optimal Control for Water Distribution

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Group:
CA-830

Student report

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**AALBORG UNIVERSITY**
STUDENT REPORT

**Topic:**

Optimal Control for Water Distribution

**Project:**

P8-project

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**Synopsis:**

This project covers the modelling and predictive control of a water distribution network with the aim of minimizing energy and economic cost.
At first the non-linear model of the components and the dynamics of the system are modelled based on a graph-based approach which leads to a state space representation of the whole network. Then system identification is carried out due to the uncertain parameters of the pipe components.
A model predictive controller is applied to the linearized model of the water distribution system extended with an elevation reservoir. The controller follows certain constraints to maintain consumer pressure-desire in two pressure management areas and to optimize the use of water tower such that the cost of pumping effort is minimized. The controller is implemented in a cascade system along with PI controllers and is based on the model of the network, the cost of electricity and the characteristics of end-user water usage.
Implementation carried out ...
The results show that ...

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# Preface

This project comprises of implementing a functional controller system for

Aalborg University, th of May 2017


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# Explanation of Notation

## Acronyms

| | |
|---|---|
| PMA | Pressure Management Area |
| CP | Critical Point |
| WT | Water Tower |
| MP | Minimization Problem |
| OD | Opening Degree |
| KVL | Kirchhoff´s Voltage Law |
| KCL | Kirchhoff´s Current Law |
| MPC | Model Predictive Control |
| GT | Graph Theory |
| ZOH | Zero Order Hold |

## Symbols

| Symbol | Description | Unit |
|---|---|---|
| $A$ | Cross sectional area | $[m^2]$ |
| $C_k$ | The $k^{th}$ component of the distribution network | $[\cdot]$ |
| $C$ | Electric capacitance | $[F]$ |
| $C_H$ | Hydraulic capacitance | $[m^3/(N/m^2)]$ |
| $D$ | Diameter | $[m]$ |
| $f$ | Moody friction factor | $[\cdot]$ |
| $F$ | Force | $[N]$ |
| $g$ | Acceleration due to gravity | $[m/s^2]$ |
| $h_f$ | Pressure given in head | $[m]$ |
| $h_m$ | Form loss | $[m]$ |
| $J_k$ | Water inertia of the $k^{th}$ component | $[kg/m^4]$ |
| $k_f$ | Form loss coefficient | $[\cdot]$ |
| $L$ | Length | $[m]$ |
| $m$ | Mass of body | $[kg]$ |
| $M$ | Linear momentum | $[kgm/s]$ |
| $n_i$ | The $i^{th}$ node of the distribution network | $[\cdot]$ |
| $n_{gl}$ | Valve characteristic curve factor | $[\cdot]$ |
| $p_a$ | Atmospheric pressure | $[bar]$ |
| $\Delta p_k$ | The pressure drop across the $i^{th}$ component | $[bar]$ |
| $q_k$ | Flow through the $k^{th}$ component | $[m^3/h]$ |
| $\boldsymbol{Re}$ | Reynolds Number | $[\cdot]$ |
| $T$ | Temperature | $[°]$ |
| $v$ | Velocity | $[m/s]$ |
| $V_t$ | Volume of the water in the water tower | $[m^3]$ |
| $\alpha_k(.)$ | The pressure boost given by the $k^{th}$ pump | $[bar]$ |
| $\epsilon$ | Average roughness | $[\cdot]$ |
| $\zeta$ | Pressure drop from elevation difference across the $k^{th}$ component | $[bar]$ |
| $\theta_{max}$ | Maximum angle of the opening degree | $[°]$ |
| $\theta_{off}$ | Minimum angle where the valve closes | $[°]$ |
| $\theta_{OD}$ | Angle of opening degree | $[°]$ |
| $\lambda_k(.)$ | Function of hydraulic resistance in the $k^{th}$ pipe | $[bar]$ |
| $\mu_k(.)$ | Function of hydraulic resistance in the $k^{th}$ valve | $[bar]$ |
| $\nu$ | Kinematic viscosity | $[kg/ms]$ |
| $\rho$ | Density | $[kg/m^3]$ |
| $\omega_r$ | Impeller angular velocity | $[rad/s]$ |

## Mathematical tools

vectorfields
time derivatives
vectors
matrices
derivative of vector fields
Jakobi matrix
chain rule in derivation
pseudo inverse

# Mathematical notation

This section will explain how the mathematical notation of this report.

### Upper and lower bounds

$$\underline{x} < x < \overline{x} \tag{1}$$

Where x is a variable and $\overline{x}$ and $\underline{x}$ are the upper and lower bounds.

# Contents

# 1 Introduction

Water pressure management is a vital part of the water supply infrastructure all over the world. It ensures that a positive water pressure is present such that the consumers are supplied with water at all time. Maintaining a minimum pressure in the network is an important task as it ensures the end user a decent water pressure and also minimizes the risk of contamination in the water system[1].

In the U.S alone 4 % of the national energy consumption is used on moving and treating water/wastewater[2]. With an increasing focus on green energy, more and more renewable energy sources are added to the grid. Nevertheless, the intermittent behavior of renewable energy sources and time-dependent consumer preferences result in fluctuation in the available power. This means that the price for electric power also varies [3]. To minimize the cost of running a water distribution network, potential energy can be used to maintain a minimum pressure. When electric prices are low, water can be pumped to a higher altitude and stored in a water tower (WT), and thereby energy is stored for future use. The potential energy of the water stored in the WT can then be used to maintain a minimum pressure that is required at the end consumer. However when a WT is included in a water distribution network, the pressure in the system is defined by the water level and hight of the WT. This means that to control pressure, the water level of the WT should be controlled.

Maximum allowed pressure in water distribution networks should also be considered as the risk of water leakage increases when pressure is increased[4], thus increased water losses due to leakage will lead to a higher energy consumption. In [4] it is stated that the estimated world wide water loss is at 30 %, so the energy used on cleaning the water for filth, bacteria and pressurizing it is lost. Another problem that should be highlighted regarding high pressure is that a high pressure will increase the wear on the pipes in a system[5], this leads to higher maintenances costs as pipes and fittings have to be replaced more frequently. Additionally, maintenances is not always an easy task, since the pipes usually are placed under ground and need to be dug up. Thereby the expense of maintenance is increased, especially in a city, were the operation also can have a negative impact on significant infrastructures. Based on these facts, the maximum pressure in a water distribution network is a vital parameter of the systems profitability. In a system with a WT the maximum allowed pressure will likely be defined by the maximum allowed water level in the WT, as the WT in most situations will be able to provide a dominant pressure compared to the desired network pressure.

Some constraints regarding a solution that implements a WT are still necessary to be taken into account. One of them being the quality of the water in the tower. If stored for too long the quality of the water will start to decrease due to a decreasing oxygen level [6, 7], thus the water should not be stored for to long. The oxygen level of the water also depends on the water temperature and therefor the water should not be too warm. Furthermore it is undesirable that the water remains stagnant in the tower or pipe as it also effects the water quality.

This leads to the following problem statement:

- *How can a water tower, implemented in a water distribution network, be controlled to minimize the cost of running a water distribution network without compromising the water quality.*

# Part I

# Analysis

# 2 System Description

This section will give an introduction to the available test system, including structure and components overview.

## 2.1 System overview

To develop and test different control methods for a water distribution system a test setup is required. Such a setup is available at Aalborg university which is based on a real water distribution system, though as a 1:20 downscaled version.

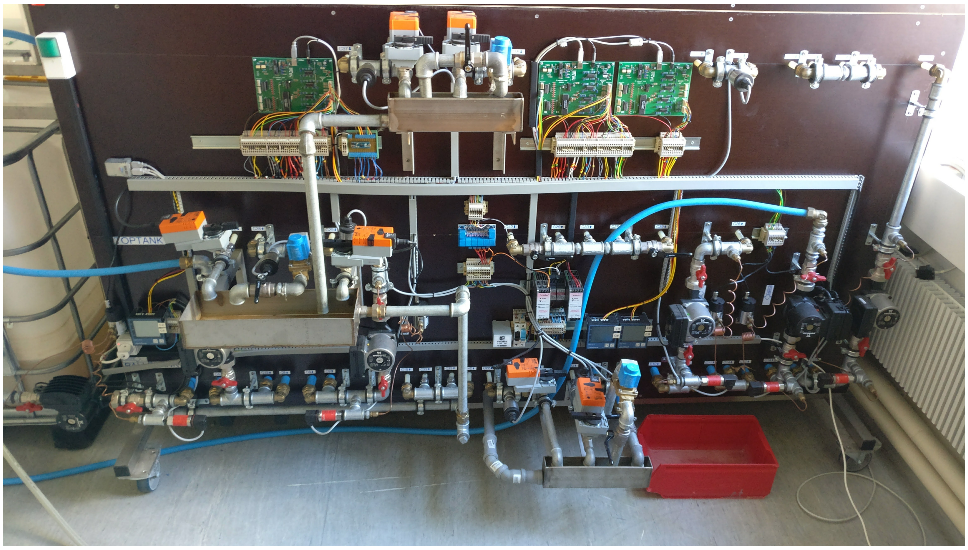


***Figure 2.1.*** The available test setup used to represent a real water distribution system.

The test setup represents a real system, thus the same structure concerning piping, leveling and all the other components. To achieve different elevation levels between system parts, the setup is mounted on a wall. This also allows for a quick overview of the complete setup and eases access to the components. As the system is used for various test scenarios other equipment is also present in the test setup shown in *Figure 2.1*, enabling the test system to mimic a variety of different system types and scenarios. A simplified diagram representing the structure of the test setup that will be used in this project is shown in *Figure 2.2*.

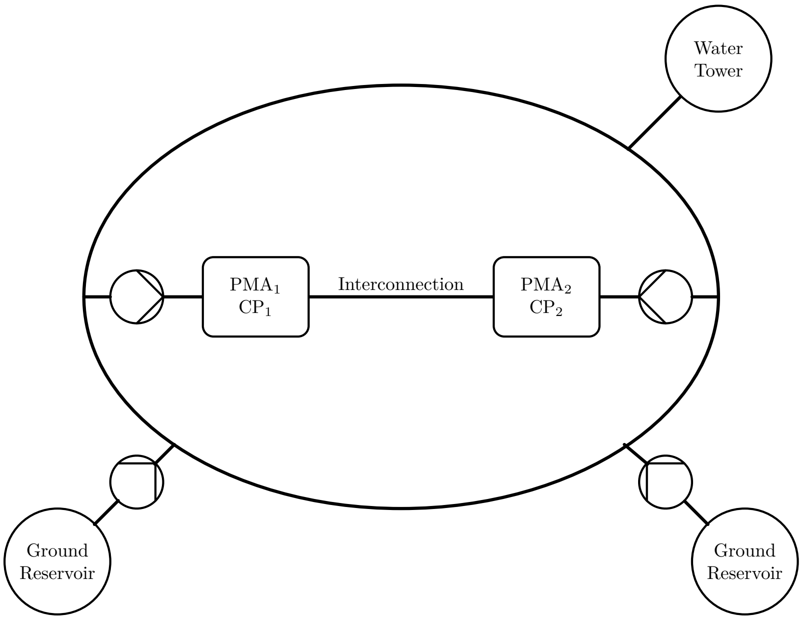



***Figure 2.2.*** Overview of the reduced system that fulfills the scenario of this project.

The system can be split up into different parts, where the main part is a water reservoir placed at ground level, used to supply the system. Two pumps are connected to the reservoir and they supply water to a main water ring formed around the PMA's. A water tower is also connected to the main water ring, and will act as an additional water reservoir and furthermore pressurize the ring due to the elevation of the tower. The direction of water flow, with respect to the tower, will depend on the pressure in the main ring. The tower can thus be filled by pressurizing the ring or be used to pressurize and supply water to the ring instead of the pumps. From the water ring, two PMA's are connected, each via their own pump. In each PMA a measuring point, called the critical point (CP), is placed and the pressure at this point shall be kept to accommodate supply demands of the consumers. Furthermore two consumers are placed in each PMA, these are simulated by valves with a variable opening degree where the water flows back to the main reservoir.

As the test setup consist of different components as valves, pumps and pipes, a basic water distribution network is shown in *Figure* **??** which will be used to illustrate and explain the individual components in the system and their functionality.

In the system two different types of Grundfos pumps are used. For supplying the water ring, two pumps, of the type UPMXL GEO 25-125[8], are used. Whereas the pumps used in each PMA are of the type UPM2 25-60[9], which is a smaller pump and typically used at the end-user.

In order to close off parts of the system that will not be used for a specific scenario or to simulate faulty behavior, manual rotary ball valves are placed trough out the system. To simulate a consumer, an electronically controlled belimo valve is used. Thereby is it possible to vary the opening degree of the valve over time according to a specific consumer behavior. For the pipes there are used two different material types. The pipes used in the main ring, which connect the reservoir and the water tower, are made of polyethylene grade 80 called PE80[10]. The pipes used to connect the PMA's to the ring and the internal connections in the PMA's are made of polyethylene with cross-links called PEX. In addition to the pipes, fittings, bends, and elbows are also present and found in various metals as iron and brass.

The pressure measuring in each PMA is done with a Jumo pressure sensor. The pressure is measured relative to a reference called Gnd, for the test system, Gnd is atmospheric pressure. Furthermore both the differential pressure over each pump and the absolute pressure at the pump is measured with a Grundfos direct sensor DPI v.1 and a Danfoss mbs32/33 pressure sensor, respectively.

The main reservoir has a volume of 700 L and the WT a volume of 200 L. The volume of the WT in this report is denoted as $V_t$. A system diagram of the entire test setup, including pipe dimensions, naming etc., can be seen in *Appendix: C.2*.

# Requirements and Constraints 3

Adding a WT to an existing water distribution network will introduce new constrains, which needs to be taken into account.

As mentioned in Section *1: Introduction*, a minimum pressure must be maintained at the end user. Furthermore the pressure can not exceed a maximum level as this will increase the possibility of water leakage and increase the wear on the pipes in the system. The system described in Section *2.1: System overview*, is designed to operate at a pressure around 0.1 bar, relative to the environment [11]. For the purpose of this project the interval for which the pressure should be within, is chosen to be between $0.08 < \boldsymbol{p_{cp}} < 0.14$ [Bar], where $p_{cp}$ is the pressure at a critical point.

Another important aspect when implementing a WT is water quality. If the water is stored, in the WT for too long the quality will decrease due to decreasing oxygen level, thus a requirement for water quality has to be formulated. As described in Section *2.1: System overview*, the WT has one combined input/output connection. Therefore a requirement only for flow is hard to formulate as the direction will change dependent on the usage. This could result in a flow based constraint being fulfilled by rapidly changing flow direction without actually replacing any significant water volume in the tower. Instead, a requirement for how often the content of the WT should be exchanged per time unit is proposed. For the purpose of this project the minimum requirement to volume exchange, is chosen to 30% of the volume of $V_T$ per day. This can be written as $\bar{q}_{wt} > 0.3 \cdot V_T \; [\frac{m^3}{day}]$. As stated in Section *2.1: System overview* $V_T = 200\;L$ so therefor $\bar{q}_{wt} > 0.06 \; [\frac{m^3}{day}]$.

This results in the following requirements:

- Pressure at CP, $0.08 < \boldsymbol{p_{cp}} < 0.14$ [bar]
- Minimum water exchange , $\bar{q}_{wt} > 0.06 \; [\frac{m^3}{day}]$
- Minimizing the total cost of running the system

# Modelling 4

## 4.1 Hydraulic modelling

All matrices should be bold font

We have to be consequent with the indexes - i or k?

Water distribution networks are designed to deliver water to consumers in terms of sufficient pressure and appropriate chemical composition. Distribution systems as such are generally consisting of four main components: pipes, pumps, valves and reservoirs. The common property is that they are all two-terminal components, therefore they can be characterized by the dynamic relationship between the pressure drop across the two endpoints and the flow through the element [12]. *Equation: (C.2)* shows the dual variables which describes one component.

$$\begin{bmatrix} \Delta P \\ q \end{bmatrix} = \begin{bmatrix} P_{in} - P_{out} \\ q \end{bmatrix} \tag{4.1}$$

Where

| | | | |
|---|---|---|---|
| | $\Delta p$ | is the pressure drop across the two endpoints, | [Pa] |
| and | $q$ | is the flow through the element. | $\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$ |

In the following chapter the hydraulic model of the system is derived by control volume approach [13]. The relationship between the two variables are introduced for each component in the hydraulic network.

### 4.1.1 Pipe model

Pipes are important components of water distribution systems since they are used for carrying pressurized and treated fresh water. A detailed model of pipes has to be derived in order to describe the relationship of pressure and flow for each pipe component. The dynamic model of a pipe can be originated from Newton's second law. *Equation: (4.2)* describes the proportionality between the rate of change of the momentum of the water and the force acting on it.

$$\frac{d}{dt} M = \sum_i F_i \tag{4.2}$$

Where

| | | | |
|---|---|---|---|
| | $M$ | is the linear momentum of the water flow, | $\left[\frac{\mathrm{kgm}}{\mathrm{s}}\right]$ |
| and | $F_i$ | is the set of forces acting on the water. | [N] |

The dynamic model of a pipe component is derived under the assumption that the flow of the fluid is uniformly distributed along the cross sectional area of the pipe. In other words, all pipes in the system are filled up fully with water all the time. Thus the density of water and the volume of the fluid is constant in time, as is the mass of the water.
Rewriting *Equation: (4.2)*, because of the above-mentioned asumptions, the mass of the water can be taken out in front of the derivative.

$$\frac{d}{dt}M = \frac{d(m_w v)}{dt} = m_w \frac{dv}{dt} = \sum_i F_i \tag{4.3}$$

Where

| | | | |
|---|---|---|---|
| | $m_w$ | is the mass of the water, | [kg] |
| and | $v$ | is the value of the velocity of the water at each point of the pipe. | $[\frac{m}{s}]$ |

The sum of the forces acting on the control volume can be seen as input forces, acting on the inlet of the pipe, output forces, acting on the outlet, resistance forces and gravitational force effect. These forces are expressed in terms of pressure in order to obtain the model of the pressure drop in the pipes. In *Figure 4.1* all forces acting on a pipe segment are shown:

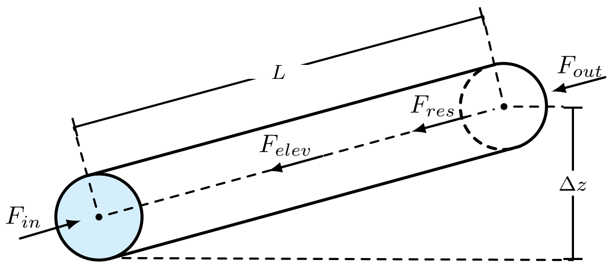



***Figure 4.1.*** Free-body diagram describing the forces acting on a segment of a pipe.

The pipe is assumed to have a cylindrical structure. Furthermore, the cross section of the pipe, $A(x)$, is constant for every $x \in [0, L]$, where $L$ is the length of the pipe:

$$A_{in} = A_{out} = \frac{1}{4}\pi D^2 \tag{4.4}$$

Where

| | | | |
|---|---|---|---|
| | $A$ | is the cross sectional area of a pipe, | $[m^2]$ |
| and | $D$ | is the diameter of the pipe. | [m] |

Water flow, $q$, can be expressed in terms of velocity, $v$, and cross sectional area, $A$, resulting in:

$$q = A \cdot v \tag{4.5}$$

In *Equation: (4.6)* the forces acting on the pipe are included. The difference between $F_{in}$ and $F_{out}$ is the pressure drop between two endpoints, the resistance forces $F_{res}$ and the gravitational force effect due to change in elevation $F_{elev}$.

$$m_w \frac{dv}{dt} = F_{in} - F_{out} - F_{res} - F_{elev} \tag{4.6}$$

In order to obtain an equation consisting of only pressure variables, the relationship between forces and pressures is used.

$$AL\rho\frac{dv}{dt} = A(p_{in} - p_{out}) - F_{res} - F_{elev} \tag{4.7}$$

Where

| | | | |
|---|---|---|---|
| | $A$ | is the cross sectional area of a pipe, | $[\mathrm{m}^2]$ |
| | $L$ | is the length of a pipe, | $[\mathrm{m}]$ |
| | $\rho$ | is the density of water, | $\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$ |
| and | $p$ | is the pressure at a point. | $[\mathrm{Pa}]$ |

Rewriting the velocity in terms of volumetric water flow and cross sectional area.

$$AL\rho\frac{d}{dt}\frac{q}{A} = A(p_{in} - p_{out}) - F_{res} - F_{elev} \tag{4.8}$$

By dividing the equation with the cross sectional area it can be seen that the equation is dependant on the pressure difference between two endpoints.

$$\frac{L\rho}{A}\frac{dq}{dt} = p_{in} - p_{out} - \frac{F_{res} - F_{elev}}{A} \tag{4.9}$$

Thus the desired pressure drop between two endpoint is obtained. The differential equation in *Equation: (4.10)*, describes the change in flow as a function of the pressure drops in the system.

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - \frac{F_{res} - F_{elev}}{A} \tag{4.10}$$

In *Equation: (4.10)*, the term $F_{res}$ is the resistance force acting on the pipe, which consists of two parts: surface resistance($h_f$) and the form resistance($h_m$) due to the fittings.

**Surface resistance ($h_f$)**
The flow of a liquid through a pipe suffers resistance from the turbulence occurring along the internal walls of the pipe, caused by the roughness of the surface. This surface resistance is given by the Darcy-Weisbach equation [14].

$$h_f = \frac{fLv^2}{2gD} \tag{4.11}$$

Should we find another formulation then "pressure given in head" - Kind of wierd to to have pressure given as 'm'

Where

| | | | |
|---|---|---|---|
| | $f$ | is the Moody friction factor, | $[\cdot]$ |
| | $h_f$ | is the pressure given in head, | $[\mathrm{m}]$ |
| | $g$ | is acceleration due to gravity, | $\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$ |
| and | $D$ | is the diameter of the pipe. | $[\mathrm{m}]$ |

*Equation: (4.11)* is under the assumption that $v > 0$. Assuming that the flow is not unidirectional and substituting the velocity by the volumetric flow and pipe area:

$$h_f = \frac{8fL}{\pi^2 g D^5}|q|q \tag{4.12}$$

The unknown parameter in 4.12 is the Moody friction factor which is non-dimensional and is a function of the Reynold´s number, **Re**. This friction factor depends on whether the flow is laminar, transient or turbulent, and the roughness of the pipe [15].

The Reynold´s number can be used to determine the regime of the flow [15]. When $\boldsymbol{Re} < 2300$ as laminar, if $2300 < \boldsymbol{Re} < 4000$ as transient and if $\boldsymbol{Re} > 4000$ as turbulent.

$$\boldsymbol{Re} = \frac{vD}{\nu} \tag{4.13}$$

Where

$\nu$ is the kinematic viscosity. $\left[\frac{\text{kg}}{\text{ms}}\right]$

The kinematic viscosity in [14] is given by :

$$\nu = 1.792 \cdot 10^{-6} \left[ 1 + \left( \frac{T}{25} \right)^{1.165} \right]^{-1} \tag{4.14}$$

Where

$T$ is the water temperature. [°C]

In order to estimate the range of Reynolds numbers in a common water distribution, typical values for the temperature, velocity and the radius of the pipes are considered [16].

- $0.5 \leq v \leq 1.5 \quad \frac{m}{s}$
- $50 \leq D \leq 1500 \quad mm$
- $10 \leq T \leq 20 \quad ^\circ C$

this is not how we have writen intervals other places ex. v ¡ [0.5,1.5]

These values result in a Reynold's number between 19000 and 225000, which yields to consider a turbulent fluid flow through the pipes. For turbulent flow the Moody friction factor is given by [14]:

$$f = 1.325 \left( ln \left( \frac{\epsilon}{3.7D} + \frac{5.74}{\boldsymbol{Re}^{0.9}} \right) \right)^{-2} \tag{4.15}$$

Where

$\epsilon$ is the average roughness of the wall inside the pipe. [m]

**Form resistance ($h_m$)**

Form resistance losses, appears at any time the flow changes direction, due to elbows, bends, or due to enlargers and reducers. It is a particular frictional resistance due to the fittings of a pipe. Form loss can be expressed as:

$$h_m = k_f \frac{v^2}{2g} \tag{4.16}$$

Applying the definition of volumetric flow:

$$h_m = k_f \frac{8}{\pi^2 g D^4} |q| q \tag{4.17}$$

Where

$k_f$ is the form-loss coefficient. [·]

The form-loss coefficient can be split into different losses depending on the fitting of the pipes.
Pipe bends are principally determined by the bend angle $\alpha$ and bend radius $r$, this is given by the following expression [14]:

$$k_f = \left[ 0.0733 + 0.923 \left( \frac{D}{r} \right)^{3.5} \right] \alpha^{0.5} \tag{4.18}$$

Pipe elbows are also used to change the direction of the flow but providing sharp turns in pipelines. The coefficient for the losses in elbows is determined by the angle of an elbow $\alpha$ and is given by:

$$k_f = 0.442\alpha^{2.17} \tag{4.19}$$

**Complete pipe model**

In *Equation: (4.12)* and *Equation: (4.17)*, the head loss of the friction losses are determined. These terms are introduced in *Equation: (4.10)* in terms of pressure. The friction factors are multiplied by the water density and gravity. Nevertheless, the head loss due to elevation has to be added in the model, yielding the final expression:

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - h_f \rho g - h_m \rho g - \Delta z \rho g \tag{4.20}$$

Substituting the terms $h_f$ and $h_m$ with their respective values:

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - \frac{8fL}{\pi^2 g D^5}\rho g|q|q - k_f \frac{8}{\pi^2 g D^4}\rho g|q|q - \Delta z \rho g \tag{4.21}$$

*Equation: (4.21)* describes the rate of flow in terms of pressure losses due to pressure change, frictions and elevation. A more compact form can be expressed for the $k$th component as such:

$$J_k \dot{q_k} = \Delta p_k - \lambda_k(q_k) - \zeta_k \tag{4.22}$$

Where

| | | |
|---|---|---|
| | $J_k$ | is an analogous parameter as inertia for the water, |
| | $\lambda_k(q_k)$ | is the friction as a function of flow, |
| and | $\zeta_k$ | is the pressure drop due to the elevation. |

As can be seen in *Equation: (4.22)*, the flow dynamics of the $k$th pipe is described by $J_k$, which is an analogous parameter as inertia in mechanical systems. $J_k$ is a diagonal matrix with zeros for the diagonal elements not related to a pipe, $\boldsymbol{J} = diag(J_i)$.

It is assumed, prior to the tests carried out on the system, that the presence of the water tower in the system has a slow effect on the flow due to slow integration behavior. This means that the water tower might have a relatively big time constant compared to the time constant of the pipes. Due to this consideration, a fair assumption that the parameter $J_k$ does not influence the flow in the system significantly, it could be neglected. However, the parameter is kept until this assumption is verified by tests. The complete model of a pipe yields:

$$\Delta p_k = \lambda_k(q_k) + J_k \dot{q_k} + \zeta_k \tag{4.23}$$

### 4.1.2 Valve model

Valves in the water distribution system are modeled according to the assumption that the length of each valve, $L$, and the change in elevation, $\Delta z$, are zero. Therefore it is assumed that the length of the valve does not influence the flow and the pressure between the endpoints. The fact that the length of a valve is considerably smaller than the length of a pipe makes this a fair assumption. Another assumption is that in case of a valve,

elevation is not present.
In the given system, valves are considered as end-user components since they are placed only in the PMAs. These user valves have a variable Opening Degree(OD) which influences the pressure drop across the endpoints.

In case of valves, manufacturers provide a parameter which indicates the valve capacity. This coefficient is called the $k_{v100}$- factor that describes the conductivity of the valve at maximum OD. This parameter sets the relationship between the water flow through the valve in $m^3$ in one hour. The experiments were carried out with a pressure drop of $\Delta p = 1[bar]$ at a fully open state of the valve. According to [17], the properties of water fulfil the requirements which allows to write up the following expression for flow and pressure:

$$q = k_{v100}\sqrt{\Delta p} \tag{4.24}$$

Where

$k_{v100}$ is the valve maximum capacity factor. $\left[\frac{\mathrm{m}^3}{\mathrm{h}}\right]$

*Equation: (4.24)* can be derived in detail using the law of continuity for each endpoint of the valve, however the exact derivations can be found in the datasheet [17]. In the further description and derivations, the coefficients and all the technical considerations are based on this datasheet.

### Valve conductivity function $k_v(OD)$

Instead of $k_{v100}$, more generally $k_v(OD)$ can be used which is a function of the opening degree, where $OD \in [0, 1]$. In case of user-operated valves, $k_v$ does not remain constant, it ranges over a compact set of values as the opening degree varies. [12].

All valves in the system share the same characteristics, therefore the following characteristics of $k_v$ are valid for all of them.

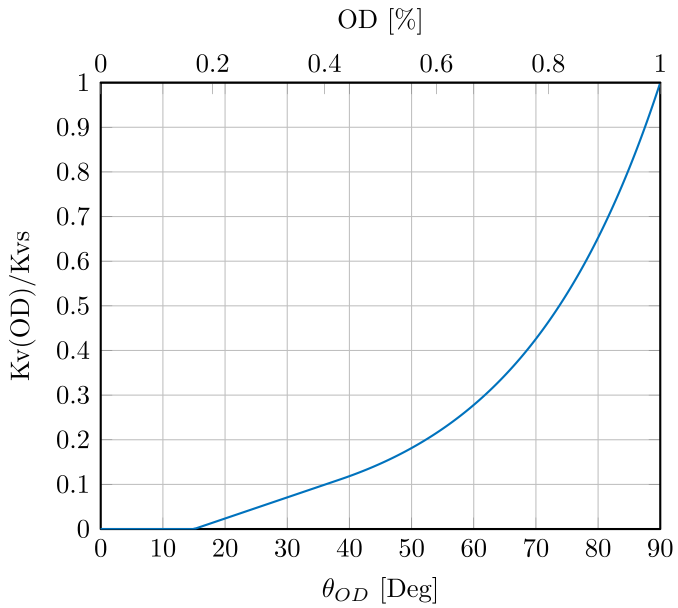



***Figure 4.2.*** Valve characteristics - Valve conductivity in the function of OD.

According to [18], the following definition can be written up generally for the conductivity function, $k_v(OD)$:

$$k_v(OD) = \begin{cases} k_{v100}\, \frac{\theta_{OD}}{\theta_{max}}\, n_{gl}\, e^{(1-n_{gl})}, & \text{if } \frac{\theta_{OD}}{\theta_{max}} \leq \frac{1}{n_{gl}}; \\ k_{v100}\, e^{(n_{gl}\,(\frac{\theta_{OD}}{\theta_{max}}-1))}, & \text{if } \frac{\theta_{OD}}{\theta_{max}} \geq \frac{1}{n_{gl}} \end{cases} \tag{4.25}$$

Where

| | | | |
|---|---|---|---|
| | $\theta_{OD}$ | is the opening degree, | [°] |
| | $\theta_{max}$ | is the upper interval of the opening degree where the conductivity does not change $\in [90°, 100°]$, | [°] |
| and | $n_{gl}$ | is the valve characteristic curve factor. | [·] |

A new parameter, $\theta_{max}$, is introduced which describes the maximum angle where the actuator closes the valve. The same can be stated for a minimum angle. The valve is closed when the position of the actuator $\in [0°, 15°]$. As a consequence, there is an offset in the curve as it is shown in *Figure 4.2.* Introducing the following angle:

$$\gamma = \frac{\theta_{OD} - \theta_{off}}{\theta_{max} - \theta_{off}} \tag{4.26}$$

Where

| | | |
|---|---|---|
| $\theta_{off}$ | is the minimum angle where the valve opens. | [°] |

In case of the water distribution system *Equation: (4.25)* modifies to:

$$k_v(OD) = \begin{cases} k_{v100}\, \gamma\, n_{gl}\, e^{(1-n_{gl})}, & \text{if } \gamma \leq \frac{1}{n_{gl}}; \\ k_{v100}\, e^{(n_{gl}\, \gamma)}, & \text{if } \gamma \geq \frac{1}{n_{gl}} \end{cases} \tag{4.27}$$

As it is shown, the conductivity function of the valve consists of two types of functions:

$$k_v(OD) = \begin{cases} k_v(\theta_{OD}) \sim linear(), & \text{if } \gamma \leq \frac{1}{n_{gl}}; \\ k_v(\theta_{OD}) \sim exponential(), & \text{if } \gamma \geq \frac{1}{n_{gl}} \end{cases} \tag{4.28}$$

Since exponential functions never cross the zero point, it is reasonable to use linear characteristics in the lower range. The transition from linear to exponential has to be continuously differentiable and predetermined by $n_{gl}$ [12, 18]

**Complete valve model**

Using *Equation: (4.24)* with the conductivity function $k_v(OD)$ and expressing $\Delta p$ yields:

$$\Delta p = \frac{1}{k_v(OD)^2}|q|q \tag{4.29}$$

Describing it in a compact form for the $k^{th}$ valve in the network yields:

$$\Delta p_k = \mu_k(q_k, k_v(OD)) \tag{4.30}$$

### 4.1.3 Pump model

In order to move water from the reservoirs to the consumers, pumping is required. To guarantee that the water reaches every end-user with the appropriate pressure, different pumps can be used in the water distribution system.
Centrifugal pumps are ideal for this purpose, as the output is steady and consistent. A model describing the pressure drop is derived which is presented in detail in [19]. The pressure provided by the pump is given by:

$$\Delta p = -a_{h2} q_i{}^2 + a_{h1} \omega_r q_i + a_{h0} \omega_r{}^2 \tag{4.31}$$

Where

| | | | |
|---|---|---|---|
| | $\Delta p$ | is the head produced by the pump, | [Bar] |
| | $a_{h2}$, $a_{h1}$, $a_{h0}$ | are constants describing the pump, | $[\cdot]$ |
| | $q_i$ | is the volume flow through the impeller, | $\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$ |
| and | $\omega_r$ | is the impeller speed. | $\left[\frac{\mathrm{rad}}{\mathrm{s}}\right]$ |

**Hydraulic power and efficiency**

To minimize the running cost of the plant, it is necessary to know the power consumption of the pumps to describe how much energy these consume. The electricity consumption of the pump, as a function of flow $q$, is described in [11]. This expression can be used to calculate the efficiency $\eta$, as a function of flow $q$ of the pump. By assuming $\eta$ to be constant the expression becomes simpler, however $\eta$ must be chosen within the operating area of the pump. The electrical power can then be calculated by taking the hydraulic power created by the pumps and the efficient of the pumps into account.

The hydraulic power created by a pump can be described by an equivalent to Joule's law, that is in terms of the pressure difference across the pump, multiplied with the flow through it, see *Equation: (4.32)*.

$$P_h = \Delta p \cdot q \tag{4.32}$$

Where

| | | |
|---|---|---|
| $P_h$ | is the hydraulic power | [W] |

The electrical power used for running the pump can then be described through the relation between the hydraulic power and the efficiency, $\eta$. This can be seen in *Equation: (4.33)*.

$$P_e = \frac{1}{\eta} \cdot \Delta p \cdot q \tag{4.33}$$

Where

| | | |
|---|---|---|
| $P_e$ | is the power consumption of the pump | [W] |
| $\eta$ | is the efficiency of the pump. | $[\cdot]$ |

*Equation: (4.33)* will be used in a later section to minimize this cost of running the system.

### 4.1.4 Water Tower

Water towers are used to maintain the correct pressure level in different systems, ensure reliability and to improve the optimality of the water supply. The WT plays a determinative role in the flow control. Therefore its dynamic model must be derived.

Similarly to the modelling of the other components, the relation between the two dual variables, pressure difference and flow are derived. The structure of the WT is illustrated in *Figure 4.3*.

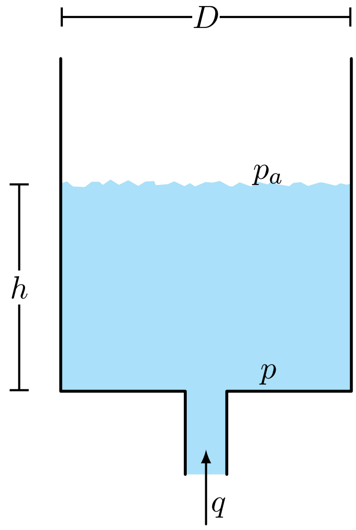



***Figure 4.3.*** Sketch of the open water tower

In *Figure 4.3*, $p_a$ represents the pressure on the surface of the water, therefore it is the atmospheric pressure at all time. $p$ equals to the pressure value on the bottom of the tank. The rate of change of the fluid volume in the water tower is proportional to the volumetric flow at which water enters or leaves the tank.

$$q = \frac{dV_t}{dt} = A\frac{dh}{dt} \tag{4.34}$$

Where

| | | | |
|---|---|---|---|
| | $h$ | is the height of the fluid in the WT, | [m] |
| | $V_t$ | is the volume of the WT, | $[\mathrm{m}^3]$ |
| | $A$ | is the cross section of the WT which is assumed to be constant for $y \in [0, h]$, | $[\mathrm{m}^2]$ |
| and | $q$ | is the volumetric flow. | $\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$ |

The force on the bottom of the WT is due to the weight of water. According to Newton's second law:

$$F = mg = \rho g V_t \tag{4.35}$$

Where

| | | |
|---|---|---|
| $\rho$ | is the density of water. | $\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$ |

*Equation: (4.35)* can be rewritten in terms of pressure such as:

$$\frac{F}{A} = \rho g h = p - p_a = \Delta p \tag{4.36}$$

The total pressure on the bottom of the WT is a result of the pressure difference due to the fluid ($p$) and the atmospheric pressure ($p_a$). However, the model is derived in such a way that the atmospheric pressure is set to zero. Therefore, if the water is assumed to be incompressible(density does not change with pressure), *Equation: (4.34)* can be written as:

$$q = \frac{dV}{dt} = \frac{A}{\rho g}\frac{d}{dt}\Delta p = C_H \Delta \dot{p} \tag{4.37}$$

Where

| | | |
|---|---|---|
| $C_H$ | is the hydraulic capacitance. | $\left[\frac{\text{m}^3}{\text{N/m}^2}\right]$ |

This equation shows proportionality between pressure and the volume of water, which is exactly the defining characteristic of a fluid capacitor. When the fluid capacitance is large, corresponding to a tank with a large area, a large increase in volume is accompanied by a small increase in pressure.

An analogy can be made between an electronic circuit and the hydraulic system, where the WT acts as a capacitor. Deriving the relationship between the voltage and the charge of the capacitor:

$$I = C\frac{dU}{dt} \tag{4.38}$$

Where

| | | |
|---|---|---|
| $U$ | is the voltage, | [V] |
| $C$ | is the capacitance. | [F] |

In the *Equation: (4.37)* the volume flow rate, $q$, is equivalent to the current, $I$, in a circuit and the constant term, $\frac{A}{\rho g}$, is equivalent to the capacitance of a capacitor, $C$. The voltage drop is analogous to the pressure drop in the water system.

## 4.2 Component model

Gathering the pressure drops, caused by each type of component in the system, a complete system model can be obtained. This model includes the pipe, valve, pump elements and the WT.

The model of the WT is described by a first order differential equation, consisting of the first time derivative of the pressure drop. The final expression is shown in *Equation: (4.39)*:

$$\Delta\dot{p}_{WT;k} = \frac{1}{C_{H;k}}q_k \tag{4.39}$$

Although *Equation: (4.39)* includes indexing for the pressure drops across the WTs, it is worth mentioning that the water distribution system consists of only one WT.
The complete model consists of the pipe model, *Equation: (4.30)*, the valve model, *Equation: (4.31)*, the pump model and the WT, *Equation: (4.39)*. For the pressure drop across the $k^{th}$ component the following expression can be written:

$$\Delta p_k = \underbrace{\lambda_k(q_k) + \zeta_k + J_k\dot{q}_k}_{\text{Pipe}} + \underbrace{\mu_k(q_k, k_v)}_{\text{Valve}} - \underbrace{\alpha_k(u_k)}_{\text{Pump}} + \underbrace{\Delta p_{WT;k}}_{\text{Water tank}} \tag{4.40}$$

The complete component model, *Equation: (4.40)*, is used to represent the pressure loss or contribution across each component. In order to describe every part of the system by *Equation: (4.40)*, the parameters and functions corresponding to the specific part of the system are selected. The remaining expressions are set to zero if the model does not match the specific part of the network. In other words, if the $k^th$ element of the system is a pump, then only $\alpha_k(u_k)$ is taken into account and the rest of the expressions are set to zero. *Table: 4.1* shows the parametrization of the system:

| **Component** | $J_k$ | $\lambda_k$ | $\mu_k$ | $\alpha_k$ | $\zeta_k$ | $\Delta p_{WT;k}$ |
|---|---|---|---|---|---|---|
| Pipe | $J_k$ | $\lambda_k$ | 0 | 0 | $\zeta_k$ | 0 |
| Valve | 0 | 0 | $\mu_k$ | 0 | 0 | 0 |
| Pump | 0 | 0 | 0 | $\alpha_k$ | 0 | 0 |
| Water tower | 0 | 0 | 0 | 0 | 0 | 1 |

***Table 4.1.*** Complete model parametrization.

### 4.2.1 Unit transformation

During the derivation of the dynamic model, the unit of the physical variables are considered as pascals and seconds. However, it is concluded in a later chapter that the flow is significantly small compared to the pressure if the SI-units are kept. Therefore a unit conversion is carried out from Pascal[Pa] to [bar]s and from seconds[s] to hours[h]. Another reason which makes this conversion reasonable is that the conductivity function,$k_{v100}$ in Section *4.1.2: Valve model*, is derived under the condition that the pressure drop is one bar [18]. The time scaling is due to conventions. Among the research community in hydraulics, a convenient way to handle the volumetric flow is in $[m^3/h]$ instead of $[m^3/s]$. The detailed derivation of the unit conversion can be found in *Appendix: A*. The result is stated here:

$$\frac{L\rho}{A \cdot 10^5}\frac{d}{dt}\frac{q}{3600} = \Delta\frac{p}{10^5} - \Big(\frac{8fL}{\pi^2 g D^5 \cdot 10^5} + k_f\frac{8}{\pi^2 g D^4 \cdot 10^5}\Big)\rho g\frac{|q|}{3600}\frac{q}{3600} - \frac{\Delta z \rho g}{10^5} \quad (4.41)$$

## 4.3 System description

After deriving the dynamics of all elements in the network, the complete system can be drawn. In *Appendix: C.2*, the topology of the test system is described in detail. In the following section, all conclusions and notations are based in the system diagram placed in the appendix.
The way of modelling a hydraulic system is in some way analogous to an electric circuit. Most of the various hydraulic components can be represented as electronic equivalents and vice versa, however there are some differences too. It should be emphasized that in hydraulic networks there are not such phenomenon as magnetic flux.
In the block diagram of the system, nodes are introduced which represent different potential points in the system. This is equivalent to hydraulic pressures. Nodes represent points in the system where pressure might have different values due to the elements e.g pipes, valves and pumps, placed between them. These points represent interconnection between hydraulic components and take into account the fact that each individual component in the system has an effect on the pressure drop on their two corresponding endpoints. Therefore nodes are present at all places where the pressure value is different due to the components of the network.
In the network, volumetric flow rate is equivalent to current and the quantity of water has similar representation as charge in an electric circuit. Again, it should be noted that e.g. the water quantity cannot be affected by magnetic fields, therefore the word: similar.
Although nodes can be placed across all the component endpoints, some simplifications are introduced in the network. These simplifications do not change the way how the system is described. There are two different types of simplification in the network.

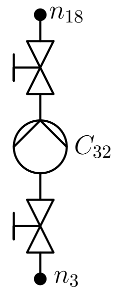



***Figure 4.4.*** Simplifications: The rotational speed of the pump, $\omega = 0$ and therefore this part is modelled differently.

The WT is connected to the rest of the system with three components, a pump, $C_{32}$, and two valves. This is shown in *Figure 4.3*, where the components are shown between $n_3$ and $n_{18}$. The latter is the node where the WT connects to the system. In this particular case the pump is turned off, however contributes to the pressure drop due to its resistance. The same can be said for the valves, except that they are fully open at all time but they modify the flow. Extra nodes are not introduced between the valves and the pump, instead the series connection is seen as one component. This can be modelled by lumping the resistance of the valve, *Equation: (4.29)*, into the model of the pump, *Equation: (4.31)*, when the rotational speed is zero. Thus the following model yields for the case when $\omega = 0$:

$$\Delta p = \Big(\frac{2}{k_{v100}^2} - a_{h2}\Big)|q|q \tag{4.42}$$

For the case where $\omega \neq 0$, a model including the pump and both valves can also be made. This is done in the same manner:

$$\Delta p = \Big(\frac{2}{k_{v100}^2} - a_{h2}\Big)|q|q + a_{h1}\omega_r q + a_{h0}{\omega_r}^2 \tag{4.43}$$

The case when $\omega = 0$ applies to one component between ($n_3$-$n_{18}$). The case when $\omega \neq 0$ applies to the components between ($n_1$-$n_2$), ($n_1$-$n_7$), ($n_4$-$n_8$) and ($n_5$-$n_{13}$). It should be mentioned however that all these subsystems inside this simplified model are modelled as described in Section *4.2: Component model*.

Since the components influence the pressure between the endpoints, they behave similarly as electric components. Valves are considered as nonlinear resistors, since the pressure is the quadratic function of the flow and they have a resistance depending on the OD. The model of the pipes is equivalent to a series connection of a linear inductor and a non-linear resistor. The pumps provide pressure and therefore drive flow in the system. They can be seen as voltage generators. The WT is a capacitor, as it is described in Section *4.1.4: Water Tower*. The equivalence between the hydraulic and electric system is summarized in *Table: 4.2*.

| **Hydraulic system** | **Electrical system** |
|---|---|
| Valve | Nonlinear resistor |
| Pipe | Linear inductor with a nonlinear drift term |
| WT | Capacitor |
| Pressure | Voltage |
| Flow | Current |
| Pumps | Voltage source |

***Table 4.2.*** Equivalence of an electrical and hydraulic network.

## 4.4 Graph representation

A graph is a formal mathematical way for representing a network, which can be applicable among others in engineering or scientific context such as in mechanical systems, electrical circuits and hydraulic networks [20].

The modelling of the water distribution network is done with the help of Graph Theory(GT). Each terminal of the network is associated with a node and the components of the system correspond to edges [21].

### 4.4.1 Incidence matrix

The incidence matrix, $\boldsymbol{H}$, of a graph with $n$ nodes and $e$ edges is defined by $\boldsymbol{H} = [a_{ij}]$. Where the number of rows and columns are defined by the amount of nodes and edges respectively. Additionally, the particular node and edge is denoted with the indices $i$ and $j$.

In case of a hydraulic network, the edges are directed in order to keep track of the direction of the flows in the system. It results in a directed incidence matrix as described below:

$$a_{ij} = \begin{cases} 1 & \text{if the } j^{th} \text{ edge is incident out of the } i^{th} \text{ node} \\ -1 & \text{if the } j^{th} \text{ edge is incident into the } i^{th} \text{ node} \\ 0 & \text{otherwise} \end{cases} \tag{4.44}$$

In *Appendix: C.5* the corresponding incidence matrix of the system is shown.

### 4.4.2 Cycle matrix

A spanning tree, $T \in \mathcal{G}$ is a subgraph which contains all nodes of $\mathcal{G}$ but has no cycles [22]. In order to obtain the spanning tree it is necessary to remove an edge from each cycle of the graph. The removed edges are called chords. The number of chords, $l$, are governed by the following expression:

$$l = e - n + 1 \tag{4.45}$$

By adding any additional chord to $T$, a new cycle is created which is called a fundamental cycle. A graph is conformed by as many fundamental cycles as many chords it has [22]. The set of fundamental cycles correspond to the fundamental cycle matrix $\boldsymbol{B}$, such as the number of rows and columns are defined by the amount of chords and edges, respectively.

The cycle matrix of a directed graph can be expressed with $\boldsymbol{B} = [b_{ij}]$ where $i$ and $j$ denote the chords and edges:

$$b_{ij} = \begin{cases} 1 & \text{if the edges } j^{th} \text{ is in the cycle } i^{th} \text{ and the directions match} \\ -1 & \text{if the edges } j^{th} \text{ is in the cycle } i^{th} \text{ and the directions are opposite} \\ 0 & \text{otherwise} \end{cases} \tag{4.46}$$

In *Appendix: C.6* the corresponding cycle matrix of the system is shown.

### 4.4.3 Kirchhoff's Law

In the same way as it is described in Section *4.1: Hydraulic modelling*, the graph of a hydraulic network assigns dual variables to every edge: the pressure, $\Delta p_k(t)$, and the flow, $q_k(t)$ in the function of time. These two variables are vectors containing the individual flows through the edges and the pressure drop across them:

$$\boldsymbol{\Delta p(t)} = \begin{bmatrix} \Delta p_c \\ \Delta p_f \\ \vdots \\ \Delta p_e \end{bmatrix} \text{ and } \boldsymbol{q(t)} = \begin{bmatrix} q_c \\ q_f \\ \vdots \\ q_e \end{bmatrix} \tag{4.47}$$

In order to derive a model for the hydraulic network, a set of independent flow variables are identified [23]. These flow variables have the property that their values can be set independently from other flows in the network and they coincide with the flows through the chords. Therefore it is convenient to choose the column indexing of the $\boldsymbol{H}$ and $\boldsymbol{B}$ matrix, such as:

$$\boldsymbol{H} = [\boldsymbol{H_c} \quad \boldsymbol{H_f}] \text{ and } \boldsymbol{B} = [\boldsymbol{B_c} \quad \boldsymbol{B_f}] = [\boldsymbol{I} \quad \boldsymbol{B_f}] \tag{4.48}$$

Where

$\boldsymbol{H_c}$ and $\boldsymbol{B_c}$ are the matrices corresponding to the chords,
$\boldsymbol{H_f}$ and $\boldsymbol{B_f}$ are the matrices corresponding to the spanning tree.

Since the edge variables are governed by elements interconnected in the network, they must obey the law of conservation of mass and pressure [22].

Kirchhoff´s Current Law (KCL) states that the net sum of all the flows leaving and entering a node is zero. Formulating this statement in matrix form:

$$\boldsymbol{H} \cdot \boldsymbol{q(t)} = 0 \tag{4.49}$$

Furthermore, regarding Kirchhoff´s Voltage Law (KVL) it is stated that at any time the net sum of the pressure drops in a cycle is zero. In terms of matrix form:

$$\boldsymbol{B} \cdot \boldsymbol{\Delta p(t)} = 0 \tag{4.50}$$

where the fundamental loops have a reference direction given by the direction of the chords.

## 4.5 Network model

Simon You are here!

Once the corresponding incidence and cycle matrices are identified and the analogy between hydraulic and electrical circuits is concluded, the whole hydraulic network can be described in a compact, generalized form as a set of differential equations describing the system. In this section an abstract and general form of the network model is derived using all the previously obtained expressions.
In Section *C.5: Incidence Matrix*, the form of the incidence matrix is shown. The last column of $\boldsymbol{H}$ represents the edge that belongs to the WT. (In case of the network, the number of edges representing the WT is one and in the further model description is

denoted with $w$.)
Hence, the $\boldsymbol{H}$ matrix can be written as

$$\boldsymbol{H} = [\boldsymbol{H_1} \quad \boldsymbol{H_0}] \tag{4.51}$$

Where

$\boldsymbol{H_1} \in \mathbb{R}^{n\times(e-w)}$ is the $\boldsymbol{H}$ matrix without the edge corresponding to the WT,
and $\boldsymbol{H_0} \in \mathbb{R}^{n\times w}$ is the $\boldsymbol{H}$ matrix with the column corresponding to the WT.

Similarly, the fundamental cycle matrix, B, is structured such as the last column agrees with the edge representing the WT.

$$\boldsymbol{B} = [\boldsymbol{B_1} \quad \boldsymbol{B_0}] \tag{4.52}$$

Where

$\boldsymbol{B_1} \in \mathbb{R}^{l\times(e-w)}$ is the $\boldsymbol{B}$ matrix without the edge corresponding to the WT,
and $\boldsymbol{B_0} \in \mathbb{R}^{l\times w}$ is the $\boldsymbol{B}$ matrix with the column corresponding to the WT.

As mentioned above, $\boldsymbol{q}$ is a vector containing all the individual flows, which can be structured as follows:

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{q_1} \\ \boldsymbol{q_0} \end{bmatrix} \tag{4.53}$$

Where

$\boldsymbol{q_1} \in \mathbb{R}^{(e-w)\times 1}$ is the flow through all edges expect for WT,
$\boldsymbol{q_0} \in \mathbb{R}^{w\times 1}$ is the flow through the edge belonging to the WT.

The vector containing the pressures at the nodes can be also structured as

$$\boldsymbol{p} = \begin{bmatrix} \boldsymbol{p_1} \\ \boldsymbol{p_0} \end{bmatrix} \tag{4.54}$$

Where

$\boldsymbol{p_1} \in \mathbb{R}^{(n-w)\times 1}$ is the pressure at all nodes expect for the WT,
$\boldsymbol{p_0} \in \mathbb{R}^{w\times 1}$ is the pressure in the WT.

In *Equation: (4.49)* KCL is applied to $\mathcal{G}$, which states that the sum of all the flows entering into a node must be equal to the sum of all the flows leaving the node.

By choosing independent set of flows corresponding to the chords of a spanning tree, the flow through every edge of the hydraulic system can be expressed in terms of the flow through the chords, $z$ [22]. The chord flows make it possible to deal with less variables, thus making the set of differential equations easier to handle. The elements of $\boldsymbol{z}$ are called the free flows of the system and are independent from each other, as mentioned previously [21].

$$\boldsymbol{q_i} = \boldsymbol{B_i}^T \boldsymbol{z} \tag{4.55}$$

Where

$z \in \mathbb{R}^{(1\times g)}$ is the chord flow vector and g is the number of elements.

As it is shown in *Equation: (4.55)*, the $i^{th}$ flow in the system is defined by the $i^{th}$ column of the cycle matrix and the vector of chord flows, $z$.
Before writing up an expression that describes all parts, the component model in *Equation: (4.40)* needs to be modified because of the simplifications introduced in Section *4.3: System description*. As it is mentioned in that section, there are four pumps in the system, two main and two PMA pumps, which provide pressure according to the input signals given to them. However there is one case between ($n_3$-$n_{18}$), where the model of the pump is plugged into the model of the valves and considered as an additional resistance without the pump being turned on. In this case the corresponding edge does not act as an input but can be described by *Equation: (4.42)*. Therefore *Equation: (4.40)* is structured in such a way that the edge corresponding to the connection between the WT and the system is represented respectively. The component-wise expression can be written as follows:

$$\Delta p_i = \underbrace{\lambda_i(q_i) + \zeta_i + J_i\dot{q}_i}_{\text{Pipe}} + \underbrace{\mu_i(q_i, k_{v;i})}_{\text{Valve}} - \underbrace{\tilde{\alpha}_i(u_i)}_{\text{Pump+valves}} + \underbrace{\Delta p_{WT;i}}_{\text{Water tank}} + \underbrace{\gamma_i(q_i)}_{\text{WT-connection}} \tag{4.56}$$

$$\tilde{f}_i(\boldsymbol{q_i}, \boldsymbol{\omega_i}, \boldsymbol{k_v}) = \lambda_i(\boldsymbol{q_i}) + \zeta_i + \mu_i(\boldsymbol{q_i}, \boldsymbol{k_v}) - \tilde{\alpha}_i(\boldsymbol{\omega_i}) + \gamma_i(\boldsymbol{q_i}) \tag{4.57}$$

Where

$$\tilde{f}_i = -\boldsymbol{C_{pi}q_i}|\boldsymbol{q_i}| \qquad \text{for } i = 2,3,4,5,6,7,10,11,12,14,17,18,19,21,23 \tag{4.58}$$

$$\tilde{f}_i = -\boldsymbol{C_{vi}q_i}|\boldsymbol{q_i}| \qquad \text{for } i = 13,15,20,22 \tag{4.59}$$

$$\tilde{f}_i = \Big(\frac{2}{k_{v100}^2} - a_{h2i}\Big)|\boldsymbol{q_i}|\boldsymbol{q_i} + a_{h1i}\boldsymbol{\omega_i q_i} + a_{h0i}\boldsymbol{\omega_i}^2 \qquad \text{for } i = 1,8,9,16 \tag{4.60}$$

$$\tilde{f}_i = \Big(\frac{2}{k_{v100}^2} - a_{h2i}\Big)|\boldsymbol{q_i}|\boldsymbol{q_i} \qquad \text{for } i = 24 \tag{4.61}$$

$$\tilde{f}_i = \Delta p_{WT} \qquad \text{for } i = 25 \tag{4.62}$$

The following hydraulic network model shows an overall model along with the above-mentioned considerations. Now recall that the inertia matrix, J, was defined in Section *4.1.1: Pipe model.*

$$\boldsymbol{\Delta p_1} = \boldsymbol{J\dot{q}_1} + \tilde{f}(\boldsymbol{q_1}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.63}$$

In *Equation: (4.63)* the hydraulic network model is described in terms of the flow through all the nodes. In order to reduce the order of the model and hence, the amount of unknowns, the chord flows according to *Equation: (4.55)* are applied.

$$\boldsymbol{\Delta p_1} = \boldsymbol{JB_1}^T\boldsymbol{\dot{z}} + f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.64}$$

Making use of the identity shown in *Equation: (4.50)*, the following is obtained

$$0 = \boldsymbol{B_1\Delta p_1} = \boldsymbol{B_1}[\boldsymbol{JB_1}^T\boldsymbol{\dot{z}} + f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v})] \tag{4.65}$$

Isolating the inertia matrix to the left side

$$-\boldsymbol{B_1}\boldsymbol{J}\boldsymbol{B_1}^T\dot{\boldsymbol{z}} = \boldsymbol{B_1}f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.66}$$

It is desired to know the value of the flow through the chords, hence the equation above is solved for $\dot{\boldsymbol{z}}$. In order to invert $(\boldsymbol{B_1}\boldsymbol{J}\boldsymbol{B_1}^T)$ it has to be non-singular i.e. invertible.

Setting $\boldsymbol{\mathcal{J}} = \boldsymbol{B_1}\boldsymbol{J}\boldsymbol{B_1}^T$, for $\boldsymbol{\mathcal{J}}$ to be positive-definite it has to be a square matrix and its determinant has to be non-zero. Note that $\boldsymbol{\mathcal{J}}$ is

$$\boldsymbol{\mathcal{J}} = \begin{pmatrix}\boldsymbol{I} & \boldsymbol{B_f}\end{pmatrix}\begin{pmatrix}\boldsymbol{J_c} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{J_f}\end{pmatrix}\begin{pmatrix}\boldsymbol{I} \\ \boldsymbol{B_f}^T\end{pmatrix} = \boldsymbol{J_c} + \boldsymbol{B_f}\boldsymbol{J_f}\boldsymbol{B_f}^T \tag{4.67}$$

Where

$\boldsymbol{J_c} \in \mathbb{R}^{l\times l}$ is the inertia in the chord components
$\boldsymbol{J_f} \in \mathbb{R}^{f\times f}$ is the inertia in the components of the spanning tree

$\boldsymbol{J_c}$ is a diagonal inertia matrix containing the chord elements. Since all the components corresponding to a chord in $\boldsymbol{\mathcal{G}}$ are pipes, all the diagonal terms are positive. Thus, $\boldsymbol{J_c} > 0$.

Nevertheless, if there is at least a chord corresponding to a non-pipe element, *Equation: (4.67)* would be positive-definite as long as it is possible to create a spanning tree containing all chords as pipe elements from $\boldsymbol{\mathcal{G}}$ [23].

For the remaining term $\boldsymbol{B_f}\boldsymbol{J_f}\boldsymbol{B_f}^T$, $\boldsymbol{J_f}$ is a non-negative matrix as all its elements are zero or describe the inertia of a pipe. Multiplying $\boldsymbol{B_f}\boldsymbol{J_f}\boldsymbol{B_f}^T$ by a non-zero vector column $\mathbf{x}$ and its transpose $\mathbf{x}^T$

$$\boldsymbol{x}^T\boldsymbol{B_f}\boldsymbol{J_f}\boldsymbol{B_f}^T\boldsymbol{x} \tag{4.68}$$

Creating a new variable $\boldsymbol{y} = \boldsymbol{B_f}^T\mathbf{x}$ and applying the definition of positive semi-definiteness [24]

$$\boldsymbol{y}^T\boldsymbol{J_f}\boldsymbol{y} \geqslant 0 \tag{4.69}$$

Thus, *Equation: (4.67)* is positive definite and it provides a sufficient condition for $\boldsymbol{\mathcal{J}}$ being invertible.

Therefore, the system can be described as follows

$$\dot{\boldsymbol{z}} = -(\boldsymbol{B_1}\boldsymbol{J}\boldsymbol{B_1}^T)^{-1}\boldsymbol{B_1}f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.70}$$

### 4.5.1 Pressure drop across the nodes

For ease of reading, the complete component model in *Equation: (4.63)* is restated:

$$\boldsymbol{\Delta p_1} = \boldsymbol{J}\dot{\boldsymbol{q}}_{\boldsymbol{1}} + \tilde{f}(\boldsymbol{q_1}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.71}$$

This equation describes the system by the pressure of across each elements except the part including the water tower. The dynamics are determined by the inertia of the pipes, while the pressure relations are described by f vectorfield. The same equation can be however expressed with a reduced set of equation system with the help of chord flows:

$$\boldsymbol{\Delta p_1} = \boldsymbol{J}\boldsymbol{B_1}^T\dot{\boldsymbol{z}} + f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.72}$$

The flow rate through the chords is found in *Equation: (4.70)*, thus the expression for $\boldsymbol{\Delta p_1}$ can be rewritten as

$$\boldsymbol{\Delta p_1} = \boldsymbol{J}\boldsymbol{B_1}^T[-(\boldsymbol{B_1}\boldsymbol{J}\boldsymbol{B_1}^T)^{-1}\boldsymbol{B_1}f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v})] + f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.73}$$

Writing in short form:

$$\boldsymbol{\Delta p_1} = (-\boldsymbol{J}\boldsymbol{B_1}^T(\boldsymbol{B_1}\boldsymbol{J}\boldsymbol{B_1}^T)^{-1}\boldsymbol{B_1} + \boldsymbol{\mathcal{I}})f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.74}$$

a general form of the network without the part corresponding to the WT ($\boldsymbol{p_0}$ and $\boldsymbol{q_0}$) is obtained. It should be noted however, that the same structure applies for the complete network which is extended with the WT. In the following sections this general model is used in a slightly different form that is suitable for the different estimation methods.

## 4.6 Nonlinear Parameter identification

The behavior of the complete water distribution system is governed by the previously derived model, however certain parameters of the system are either unknown or can vary significantly from the assumed design values. Furthermore, the obtained model of the system gives non-linear relations between flows and pressures in each individual components.
In case of the valves, the conductivity function is dependant on the OD, therefore the parameter of these elements are considered to be known. The centrifugal pumps are fully described by their models and by the coefficients provided by the manufacturer. The hydraulic capacity is also considered as known in case of the WT. However, certain parameters in the model of pipes are uncertain. Even though the necessary friction parameters can be found in the data sheet provided by the manufacturer, these values are only acceptable for new pipes, as over time material can build up on the inside of the pipes, since the laboratory set up to a large extent is built from PEX/PEM (plastic) pipes. On the other hand however, the physical parameters of the pipe volumes are assumed to be known to an accuracy where there is not any benefits from estimating it. Therefore the inertia matrix is known.
Consequently, the aim of the system identification in case of the water distribution system is to estimate the missing parameters which describe the frictions and form losses(which cause the major uncertainty in this case) in the pipes, therefore define the additional pressure losses. Due to these considerations, the importance of obtaining accurate parameters is essential in order to set up a simulation that represents the real test set up.

The parameter estimation is introduced for two different cases with different approaches. One being an estimation method with the model being kept non-linear, the other being a method where the model is linearized before the estimation.

The block diagram of a general parameter identification method is described in *Figure 4.5* below:

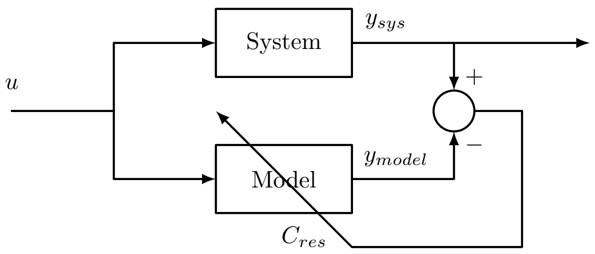



***Figure 4.5.*** Parameter identification block diagram.

As it is shown in the figure, the measurements from the real life system are compared to the output of the simulation. In general, the inputs are the same for both systems, however some modifications have to be introduced in the case of the linearized model. (This is explained in detail in the section covering the linear parameter identification method.)
In order to obtain measurements from the real life test set up, the system has to be excited by various input signals. In general, the inputs to the system are the input signals to the pumps, however in case of the parameter estimation the OD of the valves is also considered as an input. Therefore it is reasonable to reformulate the equation describing the network in *Equation: (4.70)* into such a form where the expressions for the inputs and states are isolated. Such an expression can be obtained for the whole network from the general model as follows:

$$\boldsymbol{BJB}^T\dot{\boldsymbol{z}} = \boldsymbol{B}g(\boldsymbol{B}^T\boldsymbol{z}) + \boldsymbol{B}u(\boldsymbol{\omega}, \boldsymbol{k_v}, \boldsymbol{B}^T\boldsymbol{z}) \tag{4.75}$$

where the vectorfield $u(.)$ contains all the functions for the elements which are dependant on the inputs and the vectorfield $g(.)$ describes the rest of the resistance terms which are responsible for the pressure drops in any part of the network. Although $u(.)$ is a function of $\omega$ and $k_v$, in the simulation however the inputs to the pumps are specified as pressure differences, $dp$, because the value of these pressures are available on the test set up.
In the system, output are defined as differential pressures according to the available sensors on the test set up. From the system setup 8 different relative pressures can be measured. Following the notation of *Figure C.2*, sensors are placed in: $n_2$ $n_4$ $n_5$ $n_7$ $n_{10}$ $n_{11}$ $n_{15}$ $n_{16}$.
During the parameter identification, these measurements are compared to the output from the simulation and the parameters are varied until they fit.
It is important to point out that the estimation is applied for steady-state which is reasonable, since the unknown parameters are the resistances and form losses and the inertia and the capacitance only affect the dynamics, therefore have no influence on the steady-state. The inertia of the pipes, and the capacitance of the WT is considered as known parameters.

Taking the steady-state into account, the system equation for the parameter estimation can be rewritten as:

$$0 = \boldsymbol{B}g(\boldsymbol{B}^T\boldsymbol{z}) + \boldsymbol{B}u(\boldsymbol{\omega}, \boldsymbol{k_v}, \boldsymbol{B}^T\boldsymbol{z}) \tag{4.76}$$

The aim of the parameter identification is therefore to obtain a minimum difference between the outputs by adjusting the parameters of the model. (The number of parameters are equal to the number of pipe elements in the network.) The general parameter estimation problem is the problem of the minimization of the following objective function over the variable $\boldsymbol{z}$:

$$\min_z \sum_{i=1}^{n} \left(g(z_i) + B_i^T u_i\right)^T \left(g(z_i) + B_i^T u_i\right) + \left(y_{sys;i} - y(z_i)\right)^T \left(y_{sys;i} - y(z_i)\right) \tag{4.77}$$

Where

| | | |
|---|---|---|
| $n$ | is the number of edges in the graph-based network, | [$\cdot$] |
| $g(z_i)$ | is the vectorfield with all resistance terms(parameters), | [bar] |
| $u$ | is the vectorfield with the inputs, | [bar] |
| $y_{sys;i}$ | is the pressure measurement over the $i^{th}$ edge on the system, | [bar] |
| $y_{sys}(z_i)$ | is the $i^{th}$ pressure in the output vector in the model . | [bar] |

## 4.7 Nonlinear parameter identification

In order to carry out the parameter estimation of the water distribution, Matlab NonLinear Grey Box toolbox is used [25]. This toolbox estimates previously defined coefficients of nonlinear differential equations, to fit with the desired data. Thereby, a nonlinear model has to be designed to complete the simulation.

### 4.7.1 NonLinear differential script

The nonlinear differential script contains all the information about the system and how it is constructed. The model needs 8 different inputs(u): the opening degree of the 4 PMA valves and the differential pressure across the main and PMA pumps. This inputs are obtained from the lab and are inserted directly into the model. Furthermore, it has been decided to include the dynamics of the system so the model remains as a nonlinear differential system.

First, the states of the system are set. Previously in the report, the states have been defined as the chord flows and denoted as $z = [z_1 z_2 z_3 z_4 z_5 z_6 z_7]$. The value of the states will be dependent on time, which will be specified by the data obtained from the Water Wall Setup in the lab. In this way, using the relation stated in *Equation: (4.55)* the flow through the edges is obtained.

The pressure across the edges has been defined in *Equation: (4.57)*, and in the same way has been done in Matlab.

- $\lambda$ and $\zeta$ denote the pressure in the pipes due to the friction and the elevation respectively.
- $\nu$ represents the pressure across the valves.
- $\alpha$ represent the pressure across the pumps, which is obtained from the inputs to the model.

In the system in total there are 15 pipes, the pressure across them is obtained as:

$$\lambda = (C_p * abs(q) * q/(10^5 * 3600^2)) + ((Z) * g * 1000/10^5) \tag{4.78}$$

Where, $C_p$ denotes the friction coefficient to be estimated by the NonLinear Grey Box and $Z$ is the height of the pipes. An initial guess is done for the value of $C_p$ to minimize the error as possible. Moreover, the unit conversion from Bar to Pas and seconds to hours is introduced.

$$Cp = f * ((8 * L * rho)/(pi^2 * D^5)) + k_f * ((8 * rho)/(pi^2 * D^4)) \tag{4.79}$$

The $f$ and $k_f$ values are obtained as described in *Equation: (4.15)* and *Equation: (4.18)* with the range of values considered in Section *4.1.1: Pipe model.*

In case of the valves the pressure across them is obtained as following

$$mu = (1/C_v^2) * abs(q) * q/3600^2; \tag{4.80}$$

The value of $C_v$ is obtained from the input data. When the valve is closed $\mu$ is set to 0.

For the pumps, the pressure across them is obtained from the lab and is set as $\alpha$.

As pointed out earlier, the inertia of the pipes is taken into account in the model and is calculated as

$$J = diag((4 * L * rho)/(D^2 * pi * 10^5 * 3600)); \tag{4.81}$$

Where, $L$ and $D$ are the pipes length and diameters respectively. The unit conversion is also added.

A new matrix variable is defined containing the pressure across the edges.

$$F = \lambda + \mu - \alpha \tag{4.82}$$

The derivative of the chords can be obtained isolating $\dot{z}$ in *Equation: (4.66)*

$$\dot{z} = -(B_1 J {B_1}^T)^{-1} B_1 F \tag{4.83}$$

Where, $B_1$ is the cycle matrix defined in *Appendix: C.6.*

Known the derivative of the chords, the derivative of the flow through the edges is obtained

$$\dot{q} = (B_1)^T * \dot{z} \tag{4.84}$$

*Equation: (4.64)* is now applied substituting $\dot{q}$ with the relations set in *Equation: (4.84)* and *Equation: (4.83)*, resulting in

$$\Delta P = [(J * (B_1)^T * (B_1 J {B_1}^T)^{-1} B_1(-\mu - \lambda + \alpha)] + \mu + \lambda - \alpha \tag{4.85}$$

Now the pressure across the edges are calculated, nevertheless, from the data of the lab the relative pressure at the nodes are obtained. This implies to have to calculate the pressure in the nodes so the comparison is done correctly.

The pressure at each node can be found by applying

$$\Delta P = (H_1)^T p \tag{4.86}$$

$\Delta P$ is known from *Equation: (4.85)*, the vector of pressure in the nodes is split up as $p = [p_o \quad p_r]$ where $p_o$ is the atmospheric pressure at node 1 and $p_r$ the pressure in the remaining nodes. Splitting $H_1$ matrix also

$$\Delta P = [{H_o}^T \quad {H_r}^T] \begin{bmatrix} p_o \\ p_r \end{bmatrix} \tag{4.87}$$

Isolating $p_r$ from the equation the pressure at each node is obtained

$$p_r = H_r^{\dagger}(\Delta p - (H_o)^T p_o) \tag{4.88}$$

$H_r^{\dagger}$ being the pseude-inverse of matrix ${H_r}^T$.

The outputs are set as node 2, node 4, node 5, node 7, node 10, node 11, node 15 and node 16 (see *Appendix: C.2* for the notation). Which will be the ones that Matlab will try to fit to the data obtained from the setup.

### 4.7.2 NonLinear grey box model

The objective of using this toolbox is to obtain an estimation of the value of $C_p$ given an initial guess. For this parameters a constraint to be positive is set.

The estimator compares the output.

Several test have been done, with different inputs and outputs from the lab.

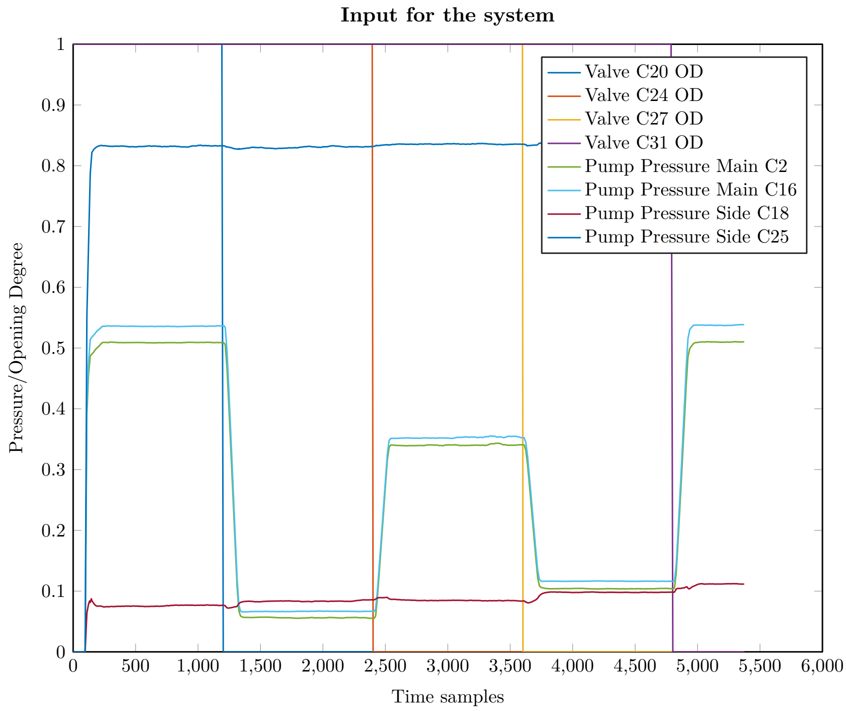



***Figure 4.6.*** Inputs to the parameter identification

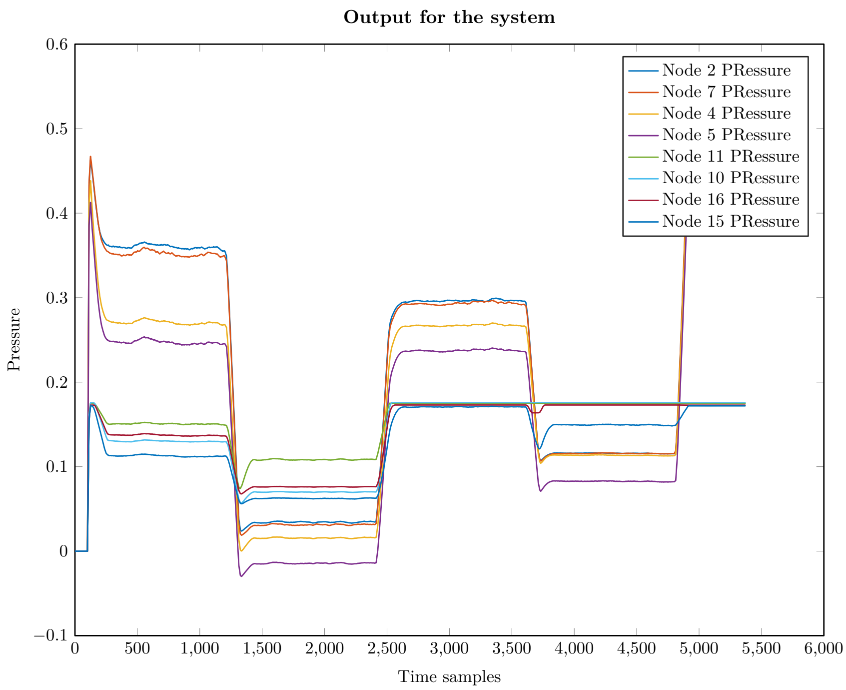



***Figure 4.7.*** Output pressure measurements

Together with the inputs from the lab and the nonlinear differential model the parameter estimation is done.

## 4.8 Linearization of the model

As it is shown in *Equation: (4.97)*, both $g(.)$ and $u(.)$ are vector-valued non-linear functions of the flow. Since the flows, the ODs and the differential pressure inputs, $dp$, are all functions of time, it can be stated that the differential equation describing the system is a first-order non-linear systems of differential equations. The number of equations are defined by the number of free variables, therefore the number of states.
In this section it is shown how the model describing the network is linearized with the help of Taylor-series. [source]

### 4.8.1 Taylor expansion on a simple example

The method of linearization is introduced on a simple one-state, one-input variable system. The consideration behind the example is analogous to the method applied for the water distribution system. In *Equation: (4.89)* the system with one state variable and one input can be seen:

$$\frac{d}{dt}x = f(x,u) \tag{4.89}$$

$f(x,u)$ can be written up with Taylor-series with the assumption that it is continuously differentiable, therefore the partial derivatives exist in the operating point:

$$f(x,u) = f(\bar{x},\bar{u}) + \frac{\partial f}{\partial x}_{|\bar{x},\bar{u}}\hat{x} + \frac{\partial f}{\partial u}_{|\bar{x},\bar{u}}\hat{u} + higher\ order\ terms \tag{4.90}$$

Where

| | | |
|---|---|---|
| $\bar{x}$ and $\bar{u}$ | is the operating point , |
| $\hat{x}$ and $\hat{u}$ | is the deviation from the operating point. |

The aim of the linearization is to describe the function $f(x,u)$ around an operating point as a linear function. However, it should be noted that the approximation around this point is only valid for cases when the deviation from this point is small. Therefore the linearized version of a dynamic model is often called the small-signal model of the system.
In *Equation: (4.90)*, the linearized term of $f(x,u)$ can be expressed. The operating point is chosen such that $f(\bar{x},\bar{u}) = 0$, hence an equilibrium for the system is given with input $\bar{u}$. The higher order terms are not taken into account in the approximation. Since the model is described by small-signals, quadratic and higher order terms result in very small values, therefore they are negligible.
The following expression in *Equation: (4.91)* gives the approximation of the function:

$$\frac{d}{dt}x = f(x,u) \approx \frac{\partial f}{\partial x}_{|\bar{x},\bar{u}}(x-\bar{x}) + \frac{\partial f}{\partial u}_{|\bar{x},\bar{u}}(u-\bar{u}) \tag{4.91}$$

In case of a pipe or a valve component, the pressure drop across the element is described by a quadratic function of the flow if steady-state is considered and the dynamics are neglected. For the sake of illustration, *Figure 4.8* describes a non-linear function, $f(q)$, and its linearized interpretation for the operating values of pressure and flow.

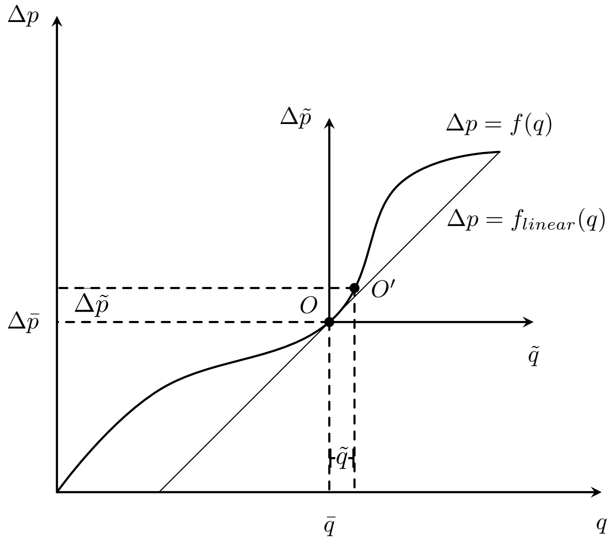



***Figure 4.8.*** Linearization of a non-linear function $f(q)$.

As can be seen, the line inserted in the operating point, $O$, describes the model accurately only if the deviation is very small from this point, for example $O'$. Therefore the linearized model describes the system behavior in the new coordinate system $(\bar{q}, \Delta\bar{p})$. It is important to mention that in *Figure 4.8* the function $f(q)$ is an illustration of a non-linear function and not the exact same as for a pipe element.

### 4.8.2 Linear system model

As it is shown in *Equation: (4.56)*, the vectorfield describing the pressure in the network consists of the pressure drops of each different elements, such as pipes$(\lambda(q) + \zeta + J\dot{q})$, valves$(\mu(q, OD))$, pumps$(\alpha(\omega, q))$, the WT $(\Delta p_{WT})$ and the WT connection $(\gamma(q))$.
Among these functions, the pipes, valves, the pumps and the edge describing the WT connection are non-linear functions, therefore they need to be linearized. The linearization is carried out according to Taylor-expansion [source].
The expression describing the pipes consists of three terms, one responsible for the resistances and form losses, one for the dynamics, and the last one for the elevation if there is any present.
The expression describing the pipes can be approximated by its linear model as follows:

$$\boldsymbol{B_1}\lambda(\boldsymbol{B_1^T z}) \approx \boldsymbol{B_1}\lambda(\boldsymbol{B_1^T \bar{z}}) + \boldsymbol{B_1}\left[\frac{\partial\lambda(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}} \boldsymbol{B_1^T \hat{z}} \tag{4.92}$$

where the partial derivative is a of all first-order partial derivatives of the vectorfield, $\lambda$, in the operating point $\bar{z}$. Since the derivation is according to $\boldsymbol{B_1^T z}$, due to the chain rule the derivative is multiplied by $\boldsymbol{B_1^T}$. The reason for only $\lambda(\boldsymbol{B_1^T z})$ being expressed is because the elevation is constant and the inertia term is a linear function of the flows.

In case of the valves, $\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})$ is not only the function of the independent flows, but also the opening degree. The conductivity function, $k_v$ is the function of OD, which can vary in time. Therefore the linearization has to be done according to the flow and the OD:

$$\boldsymbol{B_1}\mu(\boldsymbol{B_1^T z},\boldsymbol{OD}) \approx \boldsymbol{B_1}\mu(\boldsymbol{B_1^T}\bar{\boldsymbol{z}},\bar{\boldsymbol{OD}}) + \boldsymbol{B_1}\left[\frac{\partial\mu(\boldsymbol{B_1^T z},\boldsymbol{OD})}{\partial\boldsymbol{B_1^T z}}\right]_{(\bar{z},\bar{OD})}\boldsymbol{B_1^T}\hat{\boldsymbol{z}} + \boldsymbol{B_1}\left[\frac{\partial\mu(\boldsymbol{B_1^T z},\boldsymbol{OD})}{\partial\boldsymbol{OD}}\right]_{(\bar{z},\bar{OD})}\hat{\boldsymbol{OD}} \tag{4.93}$$

The Taylor-expansion is carried out in the same manner as in *Equation: (4.92)*, however the linearized valve model is in the function of two small-signal variables, the flows and the OD. Therefore the partial derivatives are calculated in the operating point defined by the operating value of $\boldsymbol{z}$ and $\boldsymbol{OD}$.
For the water tower connection, the same can be concluded as for the pipe model.

$$\boldsymbol{B_1}\gamma(\boldsymbol{B_1^T z}) \approx \boldsymbol{B_1}\gamma(\boldsymbol{B_1^T}\bar{\boldsymbol{z}}) + \boldsymbol{B_1}\left[\frac{\partial\gamma(\boldsymbol{B_1^T z})}{\partial\boldsymbol{B_1^T z}}\right]_{\bar{z}}\boldsymbol{B_1^T}\hat{\boldsymbol{z}} \tag{4.94}$$

The pumps are operating according to the model described in *Equation: (4.43)*, where the valves around each pump are taken into account. Although this model is both dependant on the OD of the valves and the flow through the pumps, it is unnecessary to linearize it for the following reason, explained with the help of *Figure 4.9* below:

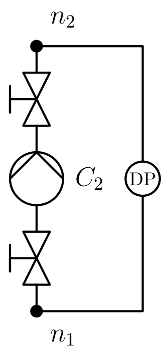



***Figure 4.9.*** Block representing the extended pump model.

As it is shown, there is a differential pressure sensor around every pump in the system. Angular velocity is therefore not used directly as input to the system, rather the differential pressure is set. It is shown in a later chapter, Chapter *II: Control Design*, that around the pumps cascade control is designed. The pumps are controlled by PI controllers in an inner loop with control variable chosen as the differential pressure. This inner control loop linearizes the pumps and therefore the differential pressure becomes the control input.
The input, in case of the parameter estimation, is defined by the four pumps and four valves. Therefore it is convenient to define an input vector which consists of the four opening degrees controlling the valves and the four differential pressures controlling the pumps:

$$\boldsymbol{u} = \begin{bmatrix} OD_{e13} \\ OD_{e15} \\ OD_{e20} \\ OD_{e22} \\ dP_{e01} \\ dP_{e08} \\ dP_{e09} \\ dP_{e16} \end{bmatrix} \tag{4.95}$$

It should be noted here, however, that this control input representation is valid only for the parameter estimation. It is shown in a later chapter, Chapter *II: Control Design*, that for the control, the structure of the model being described here and the input vector is structured differently. However, in this case it is convenient to handle all pumps and valves as input components.
Considering the input vector, the model of the pumps can be written up in the same manner as the linearized model of pipes and valves:

$$\boldsymbol{B_1}\alpha(DP) = \boldsymbol{B_1 G_p u} \tag{4.96}$$

Where
$\boldsymbol{G_p} \in \mathbb{R}^{(e \times u)}$ is a matrix representing a linear mapping where the dimension $u$ is the number of inputs and $e$ is the number of edges without the WT.

$\boldsymbol{G_p}$ is an extended matrix for the eight inputs which means that the first four columns consist of zeros, since the first four elements of the input vector are the valve opening degrees. $\boldsymbol{G_p}$ can be found in *Appendix: C.7*.

$G_p$ in the appendix should be corrected

### 4.8.3 State space model for linear parameter estimation

For the sake of clearance, the model describing the water distribution system in *Equation: (4.97)* is shown again:

$$\boldsymbol{BJB}^T\dot{\boldsymbol{z}} = \boldsymbol{B}g(\boldsymbol{B}^T\boldsymbol{z}) + \boldsymbol{B}u(\boldsymbol{\omega}, \boldsymbol{k_v}, \boldsymbol{B}^T\boldsymbol{z}) \tag{4.97}$$

The dynamics of the WT is described by the equation below:

$$\Delta\dot{p}_{WT} = \frac{1}{C_H}q_0 \tag{4.98}$$

These two differential equation systems give a full description of the pressures in the whole network, and describe the effect of the WT on the system. However, due to the linearization, the linear system is desired to formulate into the general state-space representation with inputs, outputs and states separated.
Before setting up the state-space form of the system, the following should be considered:

$$\boldsymbol{H_1 q_1} + \boldsymbol{H_0}q_0 = 0 \tag{4.99}$$

In *Equation: (4.99)*, the current law is shown for the WT and for the rest of the system. The two current laws sum up to zero taking into account the whole system. Expressing the flow in the WT yields:

$$q_0 = -\boldsymbol{H_0^\dagger H_1 q_1} \tag{4.100}$$

Inserting *Equation: (4.100)* into *Equation: (4.98)*, the original model of the WT, the following yields:

two letters are not the same

$$\Delta\dot{p}_{WT} = -\frac{1}{C_H}\boldsymbol{H_0^\dagger H_1 q_1} = \underbrace{-\frac{1}{C_H}\boldsymbol{H_0^\dagger H_1 B_1^T}}_{\mathbf{S}}\boldsymbol{z} \tag{4.101}$$

Therefore the dynamics of the WT can be expressed with the incidence matrix for the whole system in terms of the independent chord flow variables as follows:

$$\Delta \dot{p}_{WT} = -\boldsymbol{S}\boldsymbol{z} \tag{4.102}$$

Getting back to the original system model described in *Equation: (4.97)*, in order to formulate a state space representation, the linearized terms in vectorfields $g(.)$ and $u(.)$ should be separated according to the small signal values of the flows, the inputs and the pressure contribution from the WT. The representation is shown in *Equation: (4.103)* below:

$$\boldsymbol{B}\boldsymbol{J}\boldsymbol{B}^T\dot{\hat{\boldsymbol{z}}} = \boldsymbol{M}\hat{\boldsymbol{z}} + \boldsymbol{N}\hat{\boldsymbol{u}} + \boldsymbol{B_o}\Delta\hat{p}_{WT} \tag{4.103}$$

In *Equation: (4.103)*, the $\boldsymbol{M}$ matrix consists of the following terms:

$$\boldsymbol{M} = \boldsymbol{B_1}\left[\frac{\partial\lambda(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}}\boldsymbol{B_1^T} + \boldsymbol{B_1}\left[\frac{\partial\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})}{\partial \boldsymbol{B_1^T z}}\right]_{(\bar{z},\bar{OD})}\boldsymbol{B_1^T} + \boldsymbol{B_1}\left[\frac{\partial\gamma(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}}\boldsymbol{B_1^T} \tag{4.104}$$

And the $\boldsymbol{N}$ matrix consists of the following terms:

$$\boldsymbol{N} = \boldsymbol{B_1}\left[\frac{\partial\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})}{\partial \boldsymbol{u}}\right]_{(\bar{z},\bar{u})} + \boldsymbol{B_1 G_p} \tag{4.105}$$

$\boldsymbol{B_o}$ is the cycle matrix belonging to the WT. As can be seen, the linearized terms which are represented in the element-wise model description in *Equation: (4.56)*, are separated based on if they are multiplied by the small signal values of flow(state) or input.

In order to find a good state space representation for the system extended with the WT, first the dynamics has to be considered. As it is discussed in a previous section, two kind of elements have dynamics, the pipes and the WT. The pipes, compared to the WT, assumed to have very fast response time, which means that their time constants are small, therefore the decay time is short. Considering the control and also the parameter estimation, it means that they reach steady-state condition very quick compared to the WT. According to [source], in cases like this, the dynamics with the small time constant does not take part in the dynamics effectively, therefore they can be neglected. Due to this consideration, *Equation: (4.103)* is rewritten in steady-state form, where the derivative of the states are set to zero:

$$0 = \boldsymbol{M}\hat{\boldsymbol{z}} + \boldsymbol{N}\hat{\boldsymbol{u}} + \boldsymbol{B_o}\Delta\hat{p}_{WT} \tag{4.106}$$

The small signal value of the state vector can be expressed on the left side of the equation only if $\boldsymbol{M}$ is invertible. In *Equation: (4.104)*, the equation can be rewritten as follows:

$$\boldsymbol{M} = \boldsymbol{B_1}\left[\left[\frac{\partial\lambda(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}} + \left[\frac{\partial\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})}{\partial \boldsymbol{B_1^T z}}\right]_{(\bar{z},\bar{OD})} + \left[\frac{\partial\gamma(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}}\right]\boldsymbol{B_1^T} \tag{4.107}$$

In *Equation: (4.107)*, the $\boldsymbol{M}$ matrix is invertible for the same reason as it is described in Section *4.5: Network model*, for the inertia matrix. The proof of this statement can be found in that section.
Expressing the the state vector the following yields:

$$\hat{\boldsymbol{z}} = -(\boldsymbol{M^{-1}N})\hat{\boldsymbol{u}} - (\boldsymbol{M^{-1}B_o})\Delta\hat{p}_{WT} \tag{4.108}$$

Having the independent states expressed, *Equation: (4.108)* can be inserted in the previously-derived WT model with the $\boldsymbol{S}$ matrix in *Equation: (4.102)*.

$$\Delta\dot{p}_{WT} = (\boldsymbol{S M^{-1} B_o})\Delta\hat{p}_{WT} + (\boldsymbol{S M^{-1} N})\hat{\boldsymbol{u}} \tag{4.109}$$

*Equation: (4.109)* represents the linear system with the pressure drop across the water tank as a state and the input vector consisting of differential pressures from the pumps and OD values from the end-user valves. The general formulation of the state equation can be written as follows:

$$\Delta\dot{p}_{WT} = A_p\Delta\hat{p}_{WT} + \boldsymbol{B_p}\hat{\boldsymbol{u}} \tag{4.110}$$

Where

$A_p \in \mathbb{R}^{(1\times1)}$ is the system matrix for the parameter estimation, which in this case is a scalar,

$\boldsymbol{B_p} \in \mathbb{R}^{(1\times g)}$ is the input matrix for the parameter estimation.

An output equation is defined, which represents the pressure difference known from the system setup. In this way, the output equation can be compared to the data measured in the setup and proceed to estimate the unknown parameters.

$$\hat{\boldsymbol{y}} = \boldsymbol{C_1}\hat{z} + \boldsymbol{C_2}\hat{\boldsymbol{u}} \tag{4.111}$$

As in *Equation: (4.109)*, substituting the state vector, $\hat{\boldsymbol{z}}$, by the expression obtained in *Equation: (4.108)*, the equation above results in

$$\hat{\boldsymbol{y}} = \boldsymbol{C_1}(-(\boldsymbol{M^{-1}N})\hat{\boldsymbol{u}} - (\boldsymbol{M^{-1}B_o})\Delta\hat{p}_{WT}) + \boldsymbol{C_2}\hat{\boldsymbol{u}} \tag{4.112}$$

Reorganizing the terms

$$\hat{\boldsymbol{y}} = \boldsymbol{C_1}(-(\boldsymbol{M^{-1}B_o}))\Delta\hat{p}_{WT} + (\boldsymbol{C_1}(-(\boldsymbol{M^{-1}N})) + \boldsymbol{C_2})\hat{\boldsymbol{u}} \tag{4.113}$$

$$\hat{\boldsymbol{y}} = \boldsymbol{C_p}\Delta\hat{p}_{WT} + \boldsymbol{D_p}\hat{\boldsymbol{u}} \tag{4.114}$$

The equation above shows how the output equation includes a feedforward matrix, due to the outputs being affected directly by the inputs.

## 4.9 Model Parameters

In order to obtain a complete model of the physical setup, all the parameters describing the components have to be defined. Usually, these parameters can be obtained from the data sheets provided by the manufactures. Nevertheless, some components can change their characteristics over time, which introduces uncertainties into the model. Hence, in the current chapter a detailed compilation of the known/unknown parameters of the physical water distribution setup is carried out.

### 4.9.1 Known Parameters

The model parameters that are taken to be known are introduced below and can be found in *Appendix: C*.

- The parameters corresponding to the pumps such that: $a_{h0}$, $a_{h1}$, $a_{h2}$.
- In regards to the valves, the variable taking part on the modelling of a valve is the conductivity function $\boldsymbol{k_v(OD)}$. However, in order to describe the valve characteristic function, other parameters have to be introduced i.e. the curve factor $n_{gl}$, and the valve maximum, $\theta_{max}$, and minimum, $\theta_{off}$ opening degree.
- In the modelling of the pipes three parameters are considered as known. The length of the pipes $L$, their diameter $D$, and the change of elevation across each pipe $\Delta Z$.

### 4.9.2 Unknown Parameters

The unknown parameters are the ones related with the form losses, $k_f$, and form friction ,$f$, of the pipes. Despite they are provided by the manufactures they need to be estimated. On the one hand, the form losses depend on the fittings and bends of the pipes which they are not always known. On the other hand, the friction losses dependent on the inside average roughness of pipes, $\epsilon$, which can change its value due to passage of time and the rust generated inside pipes.

Furthermore, the operating points of the flow through the chords, $\bar{\boldsymbol{z}}$, is also unknown. These values, which correspond to the 8 flow chords, are introduced in the linearized expression of both pipes and valves, see *Equation: (4.92)* and *Equation: (4.93)*. Thus, not only pipe parameters introduce uncertainties into the system model but also the lack of knowledge of the chord operating points.

Consequently, it has been decided to estimate the total expression for the pressure across the pipes and valves in order to reduce the amount of unknowns in the system.

The system has 15 pipes in total, from *Equation: (4.92)* it can be seen that either tunning for $k_f$, $f$ or $\bar{\boldsymbol{z}}$ it will have the same result for the total value of the pressure across the pipes, $\lambda(\boldsymbol{B_1^T z})$. For this reason the pressure across the 15 pipes is estimated.

Valve linearized expression, see *Equation: (4.93)*, consists on the term depending on the chord flows and the one depending on the $OD$. Both terms include the operating point of the chord flows inside them, thus, the pressure difference given by both terms has to be estimated. In the system 4 valves take part, resulting in 8 unknowns in total.

The WT connection edge, see *Equation: (4.96)*, is conformed by two valves and one pump. Although the parameters corresponding to the pump are considered as known, the ones corresponding to the valves have to be estimated. Resulting in two more unknowns for the system.

All in all, the system has 24 unknown terms which will be calculated following the estimation process described in the next section.

## 4.10 Linear parameter estimation

The method that describes the linear parameter estimation is shown in *Figure 4.10* below:

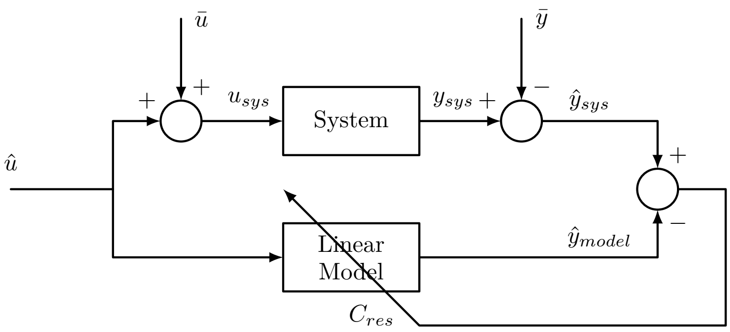



***Figure 4.10.*** Parameter identification block diagram for the linear system.

In this case small-signal inputs are applied to both the test setup and the model. Now the linearized model is compared to the real system, therefore the operating point is taken into account. Since the linear model is only valid for small deviations around the operating point, the real system has to be excited around the same operating point to obtain identical behavior. In order to achieve it, the operating values are added to both the input and the output such as:

$$u_{sys} = \bar{u} + \hat{u} \tag{4.115}$$

Where

| | | |
|---|---|---|
| $\hat{u}$ | is the small-signal input, | [bar] |
| $\bar{u}$ | is the operating value of the input, | [bar] |
| $u_{sys}$ | is the input to the real-life system. | [bar] |

$$y_{sys} = \bar{y} - \hat{y}_{sys} \tag{4.116}$$

Where

| | | |
|---|---|---|
| $\hat{y}_{sys}$ | is the small-signal output from the real-life system, | [bar] |
| $\bar{u}_{sys}$ | is the operating value of the output, | [bar] |
| $y_{sys}$ | is the output from the real-life system. | [bar] |

During the linear parameter estimation, the same problem is solved as it is shown in *Equation: (4.77)*. In this case, however, the parameters are varied according to the comparison of the small-signal outputs.

### 4.10.1 Measurements on the test setup

In order to verify the state-space model derived in Chapter *4.10: Linear parameter estimation* with the physical setup, an estimation for the parameters defined in Section *4.9: Model Parameters* is carried out.

From the system setup 9 different relative pressures can be measured, following *Figure C.2* notation the sensors are placed in: $n_2$ $n_4$ $n_5$ $n_7$ $n_{10}$ $n_{11}$ $n_{15}$ $n_{16}$ $n_{18}$. However, in the control problem of this project only the four PMA relative pressure sensor across the end-users and the pressure in the WT will take part. Thus, the estimation will be only focus on obtaining the best fit for those sensor outputs relevant in the control part.

The measurements obtained from the pressure sensors placed in these nodes, are relative to the atmospheric pressure. In order to compare the measurements from the system setup and the data obtained from the simulation in Matlab, an atmospheric pressure node, $n_1$, is set as reference point.

The relationship between pressures, where DpCXX describes the pressure difference for the XX component, can be defined as:

Node 10

$$y_5 = DpC20 + DpC21 \tag{4.117}$$

Node 11

$$y_6 = DpC24 \tag{4.118}$$

Node 15

$$y_7 = DpC28 + DpC20 \tag{4.119}$$

Node 15

$$y_8 = DpC31 \tag{4.120}$$

There is no need to define a relation with a referent point for the WT node, since the pressure across the WT is the state of the state-space model.

### 4.10.2 Estimation Data

As the parameter estimation is based on a linearized model an operating point for the system is chosen. This point is based on the tank level being approximately half way full which allows for an equal amount of deviation in both directions. For the chosen operating point, data is gather from the system while small steps are individually applied to the two main pumps and the opening degree of the PMA valves. In order to use the data for parameter estimation the operating point is subtracted, leaving only small signal values.

The operating point of the PMA valves is chosen to 63° and the small signal values for the estimation can be seen in *Figure 4.11*.

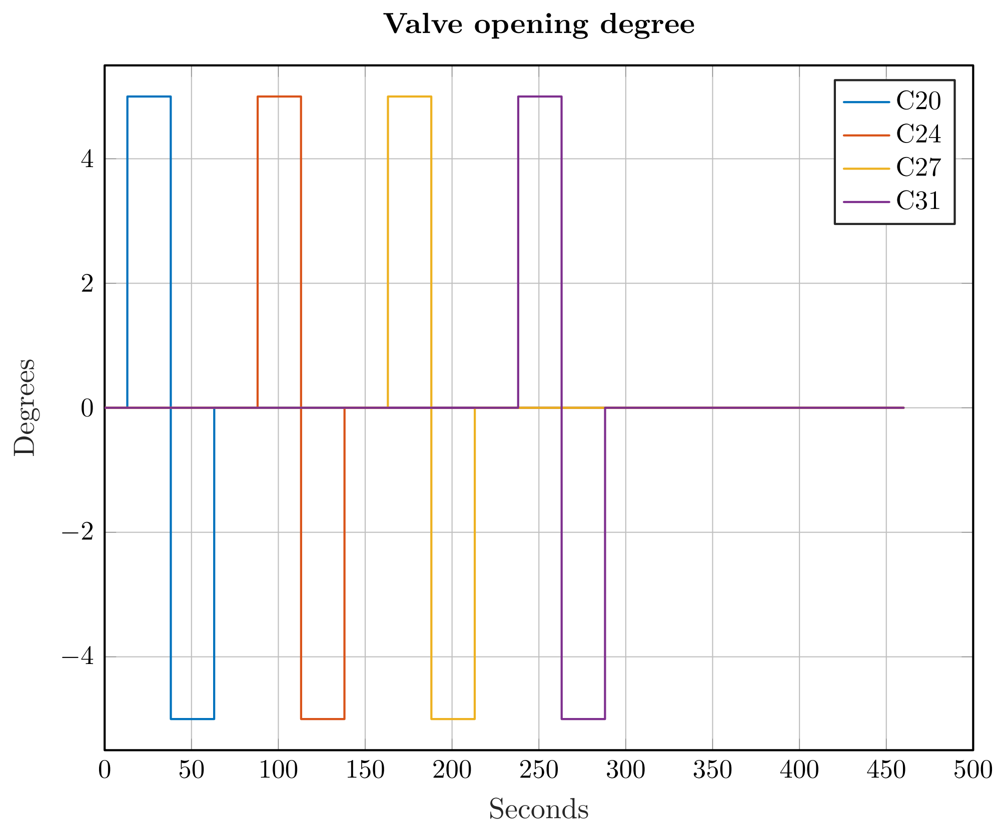



***Figure 4.11.*** Small signal values of the opening degrees of the pma valves.

Furthermore the operating point for the pumps is chosen at a speed of $\omega = 0.4$, where the small signal values for the estimation are shown in *Figure 4.12*.

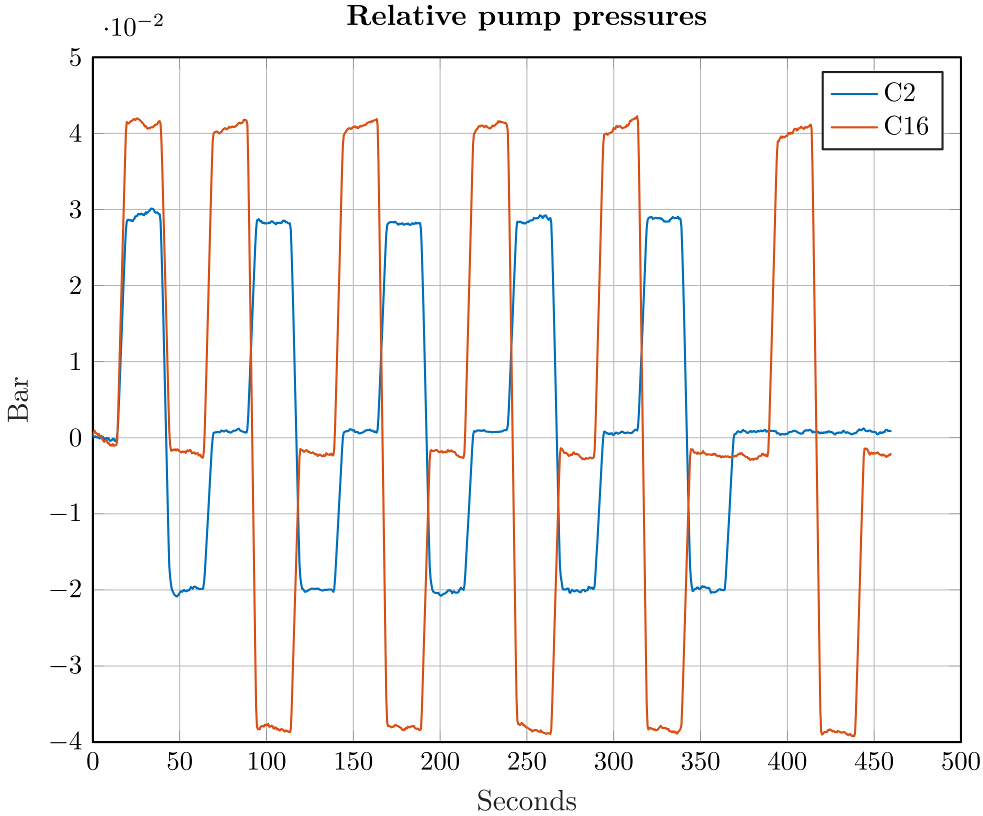



***Figure 4.12.*** Small signal values of the angular velocity of the two main pumps.

### 4.10.3 Estimation Method

The estimation of the unknown parameters is carried through Matlab Linear Grey-Box model estimation toolbox. This toolbox allows to estimate continuos-time grey-box models for differential equations using multiple input/output time-domain data [**?** ]. The numerical search method used for the estimation of the unknown parameters is the *Subspace Gauss-Newton least squares* search.

This method is automatically set by the parameter estimation process, thus any further understanding regarding the estimation method is considered out of scope of this project.

### 4.10.4 Estimation Result

The comparison between the test setup measurements and the estimated data is done with Matlab function *compare*. Together with the comparison plot, the normalized root mean square (NRMSE) measure is also added which measures the goodness of the fit. This fit is calculated as a percentage [**?** ] using:

$$fit = 100\left(1 - \frac{||y - \hat{y}||}{||y - mean(y)||}\right) \tag{4.121}$$

The following figures show the comparison between the data obtained from the lab and the estimated outputs of the model.

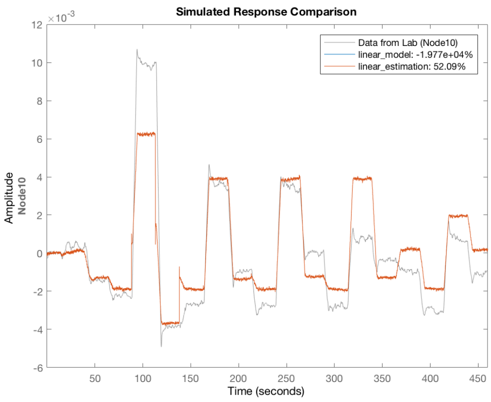



***Figure 4.13.*** Estimation comparison for node 10.

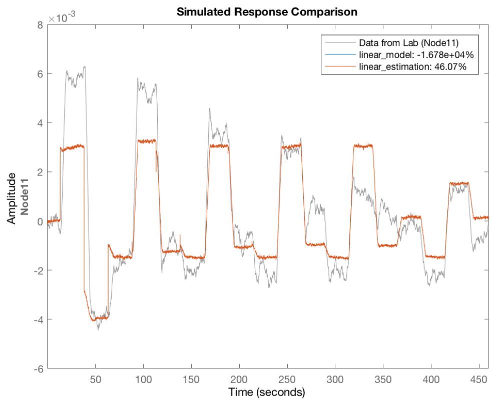



***Figure 4.14.*** Estimation comparison for node 11.

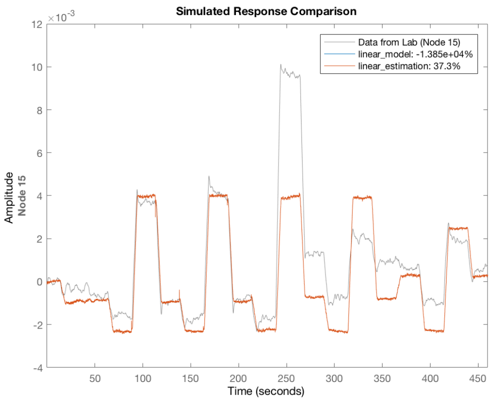



***Figure 4.15.*** Estimation comparison for node 15.

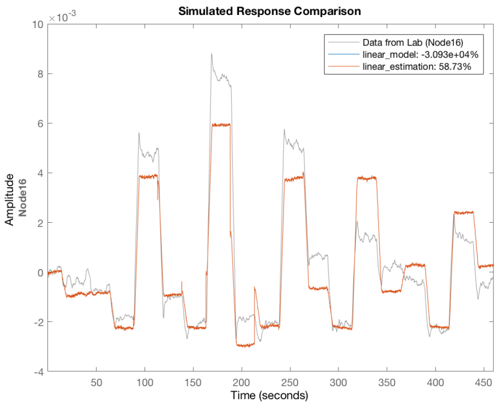



***Figure 4.16.*** Estimation comparison for node 16.

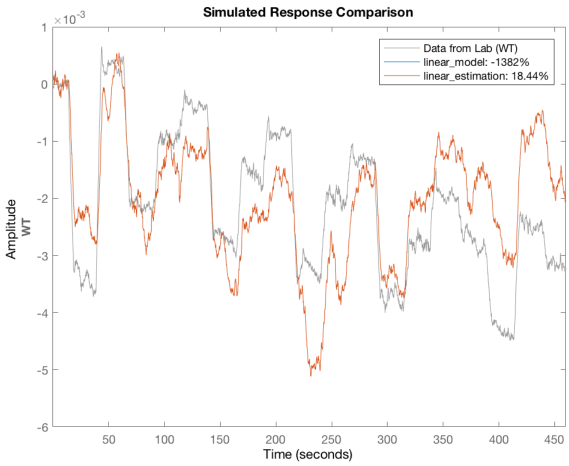



***Figure 4.17.*** Estimation comparison for WT.

## 4.11 State-space model for control

For a better clearance the steady-state, state-space representation for the parameter estimation for small-signal values is repeated here:

$$0 = \boldsymbol{M}\hat{\boldsymbol{z}} + \boldsymbol{N}\hat{\boldsymbol{u}} + \boldsymbol{B_o}\Delta\hat{p}_{WT} \tag{4.122}$$

and the equation for the outputs:

$$\hat{\boldsymbol{y}} = \boldsymbol{C_p}\Delta\hat{p}_{WT} + \boldsymbol{D_p}\hat{\boldsymbol{u}} \tag{4.123}$$

Although *Equation: (4.122)* and *Equation: (4.123)* include all the liniarized terms that are necessary to describe the system, for the control it is more convenient to restructure it. While the input vector for the parameter estimation consists of all the four differential pressures for the pumps along with the four valve opening degrees, in case of the control, only the two main pumps are considered as inputs. The control input for the system is therefore defined as $\hat{\boldsymbol{u}} \in \mathbb{R}^{(2\times1)}$ as follows:

$$\boldsymbol{u} = \begin{bmatrix} dP_{e01} \\ dP_{e08} \end{bmatrix} \tag{4.124}$$

The mapping yields:

$$\boldsymbol{B_1}\alpha(DP) = \boldsymbol{B_1}\boldsymbol{G_{c;1}}\boldsymbol{u} \tag{4.125}$$

minipage fix

Where

$\boldsymbol{G_{c;1}} \in \mathbb{R}^{(e\times u)}$ is a matrix representing a linear mapping between the vectorfield $\alpha(dP)$, describing the pressure contribution of the pumps, and between the input vector defined in *Equation: (4.124)* where the dimension $u$ is the number of inputs and $e$ is the number of edges without the WT.

$\boldsymbol{G_{c;1}}$ is now a mapping matrix for only the two inputs for the main pumps. $\boldsymbol{G_{c;1}}$ can be found in [appendix reference].

The input matrix for the control system can be written in the form:

$$\boldsymbol{N_c} = \boldsymbol{B_1}\boldsymbol{G_{c;1}} \tag{4.126}$$

Compared to the input matrix in the parameter estimation in *Equation: (4.105)*, it can be seen that now the linearized terms belonging to the valves are not part of the matrix since valves are not considered as control inputs.
However, the end-user valves and the two pumps in the PMAs are considered as measurable disturbances in the control system. Therefore the disturbance enters the model in such a way that it is defined as $\boldsymbol{d} \in \mathbb{R}^{(6\times1)}$ as follows:

$$\boldsymbol{d} = \begin{bmatrix} OD_{e13} \\ OD_{e15} \\ OD_{e20} \\ OD_{e22} \\ dP_{e09} \\ dP_{e16} \end{bmatrix} \tag{4.127}$$

Therefore between $\alpha(dP)$, the vectorfield describing the pump pressure, and the disturbance vector the mapping results in the following:

$$\boldsymbol{B_1}\alpha(DP) = \boldsymbol{B_1 G_{c;2} d} \tag{4.128}$$

minipage fix

Where

$\boldsymbol{G_{c;2}} \in \mathbb{R}^{(e \times u)}$ the dimension $d$ is the number of disturbances and $e$ is the number of edges without the WT.

The disturbance matrix for the state-space model consists of the pressure inputs of the two PMA pumps and the linearized terms of the end-user valves. The matrix can be formulated as follows:

$$\boldsymbol{Q_c} = \boldsymbol{B_1}\left[\frac{\partial\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})}{\partial \boldsymbol{d}}\right]_{(\bar{z},\bar{u})} + \boldsymbol{B_1 G_{c;2}} \tag{4.129}$$

Taking the same considerations into account as for the parameter estimation, the steady-state, state equation for small signals can be formulated as:

$$0 = \boldsymbol{M_c \hat{z}} + \boldsymbol{N_c \hat{u}} + \boldsymbol{Q_c \hat{d}} + \boldsymbol{B_o}\Delta\hat{p}_{WT} \tag{4.130}$$

where the system matrix, $\boldsymbol{M_c}$ is the same as the system matrix, $\boldsymbol{M_p}$, in case the parameter estimation.
$\boldsymbol{M_c}$ is invertible for the same reason as described in *Equation: (4.5)*, thus:

$$\boldsymbol{\hat{z}} = -(\boldsymbol{M_c^{-1} N_c})\boldsymbol{\hat{u}} - (\boldsymbol{M_c^{-1} Q_c})\boldsymbol{\hat{d}} - (\boldsymbol{M_c^{-1} B_o})\Delta\hat{p}_{WT} \tag{4.131}$$

Inserting the states into *Equation: (4.101)*:

$$\Delta\dot{\hat{p}}_{WT} = (\boldsymbol{S M_c^{-1} B_o})\Delta\hat{p}_{WT} + (\boldsymbol{S M_c^{-1} N_c})\boldsymbol{\hat{u}} + (\boldsymbol{S M_c^{-1} Q_c})\boldsymbol{\hat{d}} \tag{4.132}$$

Which in standard state-space form can be written as:

$$\Delta\dot{\hat{p}}_{WT} = A_c\Delta\hat{p}_{WT} + \boldsymbol{B_c \hat{u}} + \boldsymbol{E_c \hat{d}} \tag{4.133}$$

Where

$A_c = \boldsymbol{S M_c^{-1} B_o}$ is the system matrix for the control,
$\boldsymbol{B_c} = \boldsymbol{S M_c^{-1} N_c}$ is the input matrix for the control,
$\boldsymbol{E_c} = \boldsymbol{S M_c^{-1} Q_c}$ is the disturbance matrix for the control.

The output of the control model is defined as two measurement points around the valves in the two different PMAs. Therefore the output vector is defined as:

$$\boldsymbol{y} = \begin{bmatrix} dP_{e15} \\ dP_{e22} \end{bmatrix} \tag{4.134}$$

Since the outputs are pressures around two end-user valves, the output expression should be written in the form:

$$\boldsymbol{\hat{y}} = \boldsymbol{C_1 \hat{z}} + \boldsymbol{C_2 \hat{u}} \tag{4.135}$$

Where

| | |
|---|---|
| $\boldsymbol{C_1}$ | is the matrix consisting of the mapping between the vectorfield,$\mu$, and the output vector. Furthermore it includes the partial derivative matrix of the vectorfield according to the independent states, |
| $\boldsymbol{C_2}$ | is the matrix consisting of the mapping between the vectorfield, $\mu$, and the output vector. Furthermore it includes the partial derivative matrix of the vectorfield according to the ODs. |

Expressing the independent flow variables with *Equation: (4.131)*, the following yields:

$$\hat{\boldsymbol{y}} = \boldsymbol{C_1}\big[-(\boldsymbol{M_c^{-1}N_c})\hat{\boldsymbol{u}} - (\boldsymbol{M_c^{-1}Q_c})\hat{\boldsymbol{d}} - (\boldsymbol{M_c^{-1}B_o})\Delta\hat{p}_{WT}\big] + \boldsymbol{C_2}\hat{\boldsymbol{u}} \tag{4.136}$$

And the output equation in standard state-space form:

$$\hat{\boldsymbol{y}} = \boldsymbol{C_c}\Delta\hat{p}_{WT} + \boldsymbol{D_c}\hat{\boldsymbol{u}} + \boldsymbol{K_c}\hat{\boldsymbol{d}} \tag{4.137}$$

Where

| | |
|---|---|
| $C_c = -\boldsymbol{C_1M_c^{-1}B_o}$ | is the output matrix for the control, |
| $\boldsymbol{D_c} = \boldsymbol{C_2} - \boldsymbol{C_1M_c^{-1}N_c}$ | is the feed-forward matrix for the control, |
| $\boldsymbol{K_c} = -\boldsymbol{C_1M_c^{-1}Q_c}$ | is the disturbance matrix affecting the output. |

The continuous state space representation thus given by *Equation: (4.133)* and *Equation: (4.137)*.

### 4.11.1 Discretization of state space model

The dynamics of the water distribution system are now described. To use this linear continuous model to be subjected to MPC, the model needs to be discretized by assuming zero-order-hold(ZOH) of the variables at specified sampling points. This means that the variables are constant between these points. The aim is to have a linear discrete-time state-space model with piecewise constant $\Delta\hat{p}_{WT}[k]$, $\hat{\boldsymbol{u}}[k]$ and $\hat{\boldsymbol{d}}[k]$. The detailed method is presented in *Appendix: G*, however the final discretized state-space model is stated here with respect to the sampling time $T_s$:

$$\Delta\hat{p}_{WT}[k+1] = A_d\Delta\hat{p}_{WT}[k] + \boldsymbol{B_d}\hat{\boldsymbol{u}}[k] + \boldsymbol{E_d}\hat{\boldsymbol{d}}[k] \tag{4.138}$$

and the output equation:

$$\hat{\boldsymbol{y}}[k] = \boldsymbol{C_d}\Delta\hat{p}_{WT}[k] + \boldsymbol{D_d}\hat{\boldsymbol{u}}[k] + \boldsymbol{K_d}\hat{\boldsymbol{d}}[k] \tag{4.139}$$

Where

| | |
|---|---|
| $A_d \in \mathbb{R}^{(1\times1)}$ | is the discrete state matrix, |
| $\boldsymbol{B_d} \in \mathbb{R}^{(1\times2)}$ | is the discrete input matrix, |
| $\boldsymbol{E_d} \in \mathbb{R}^{(1\times6)}$ | is the discrete disturbance matrix, |
| $\boldsymbol{C_d} \in \mathbb{R}^{(2\times1)}$ | is the discrete output matrix, which is the same as in continuous case |
| $\boldsymbol{D_d} \in \mathbb{R}^{(2\times2)}$ | is the discrete feed-forward matrix, which is the same as in continuous case |
| $\boldsymbol{K_d} \in \mathbb{R}^{(4\times6)}$ | is the discrete disturbance matrix affecting the output, which is the same as in continuous case |

# Part II

# Control Design

# Controller 5

In this chapter the design of the controller is explained. The control problem, including optimality and control structure, is explained. The chosen control approach is then described and leads up to a section where the implementation is discussed.

## 5.1 Control Problem

The water distribution system described in Section *2.1: System overview* is to be controlled with respect to the requirements and constraints stated in Section *3: Requirements and Constraints* and the dynamics of the system described in Section *4.11: State-space model for control.* For a better clearance, the requirements and constraints are listed again below:

- Consumer pressure requirements, $0.08 < \boldsymbol{y} < 0.14$ [bar]
- Energy costs
- Performance constraints on the pumps

The system consists of four pumps and is fully controlled by the two main pumps. The two pumps in the PMAs influence only the operating point with a fixed pressure lift. The project deals with a control that optimizes the WT such that the pressure at the end-users is maintained and the cost of pumping effort in the main pumps is minimised. In other words, the use of the main pumps and the WT is controlled according to the constraints and requirements stated above.

In order to achieve such an optimal behavior, Model Predictive Control, MPC, is applied. With MPC, the dynamics of the model is used to predict the system behavior, therefore taking the future electrical prices into account. Apart from the MPC control, all pumps are controlled by PI controllers in order to deliver the required minimum pressure to the end-users. However, as it is explained above, the two main pumps are also controlled by MPC. As a consequence of the two control method, the following cascade structure can be formulated for the two main pumps as it is shown in*Figure 5.1*:

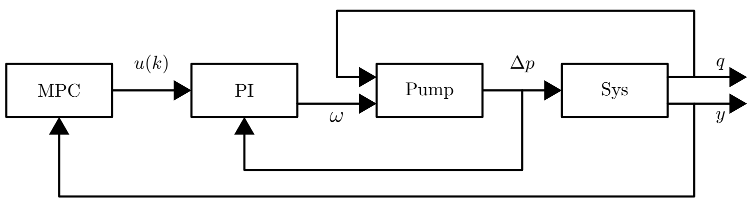



***Figure 5.1.*** Cascade control structure with MPC and PI controllers.

As it is shown, the PI controllers are responsible for setting the control inputs to the main pumps, using the feedback from the differential pressure output. It should be noted however, that typically the flow out of the pumping station is controlled instead of the pressure. Here the use of pressure feedback has the advantage that in case of system failure the pressure is important. The flow would typically either lead to too high or too low pressure or would make the WT overrun. On the other hand, the reference to the PI controllers is set by the MPC algorithm which takes the measurements from the output into account. These references are set as differential pressures. One of the advantage of such a cascade control is that the pump controllers are present for each pump, also including the PMA pumps. At the event of implementation or communication failure between the MPC and the pump controllers, the pumping remains operational.

In order to define an objective function for the MPC problem, the cost of running the system has to be taken into account along with the hydraulic power consumption of the pumps. Such an objective function can be written in the form:

$$\Upsilon(\boldsymbol{u}[k], \boldsymbol{q_p}[k], c[k]) = \sum_{i=0}^{H_p-1} \sum_{j=1}^{N} \Big(\frac{1}{\eta_j} \cdot u_j[k+i|k] \cdot q_{p;j}[k+i|k]\Big) \cdot c[k+i|k] \tag{5.1}$$

Where

| | | |
|---|---|---|
| $\boldsymbol{u}$ | is the differential pressure input to the main pumps, | [bar] |
| $\boldsymbol{q_p}$ | is the flow through the main pumps, | [$m^3/h$] |
| $\eta_i$ | is the efficiency of the $i^{th}$ main pump, | [$\cdot$] |
| $c[k]$ | is the electricity price cost sequence. | [DKK/MW] |

And the control problem can be formulated as a minimization problem on the form:

$$\begin{aligned}
\min_{\boldsymbol{u}} \Upsilon(\cdot) = \min_{\boldsymbol{u}} & \sum_{i=0}^{H_p-1} \sum_{j=1}^{N} \Big(\frac{1}{\eta_j} \cdot u_j[k+i|k] \cdot q_{p;j}[k+i|k]\Big) \cdot c[k+i|k] && (5.2)\\
s.t \quad & \Delta\hat{p}_{WT}[k+i+1|k] = A_d \Delta\hat{p}_{WT}[k+i|k] + \boldsymbol{B_d}\hat{\boldsymbol{u}}[k+i|k] && (5.3)\\
& + \boldsymbol{E_d}\hat{\boldsymbol{d}}[k+i|k] && (5.4)\\
& \hat{\boldsymbol{y}}[k+i|k] = \boldsymbol{C_d}\Delta\hat{p}_{WT}[k+i|k] + \boldsymbol{D_d}\hat{\boldsymbol{u}}[k+i|k] && (5.5)\\
& + \boldsymbol{K_d}\hat{\boldsymbol{d}}[k+i|k] && (5.6)\\
& \underline{y} < \boldsymbol{y} < \overline{y} && (5.7)\\
& \underline{\Delta p_{WT}} < \Delta p_{WT} < \overline{\Delta p_{WT}} && (5.8)\\
& \boldsymbol{u} < \overline{u} && (5.9)
\end{aligned}$$

Where

| | | |
|---|---|---|
| $Hp$ | is the control horizon, | [$\cdot$] |
| $N$ | is the number of main pumps, | [$\cdot$] |
| $\boldsymbol{y}$ | is the output vector with the PMA pressures, | [bar] |
| $\Delta p_{WT}$ | is the state which is the pressure in the WT. | [bar] |

As it is shown in *Equation: (5.2)* the optimization is about to obtain an input signal such as the cost of running the pumps is minimized. Therefore the objective function gives a price of all the consumed power over the control horizon, which is a specific future time interval. Furthermore, it should be noted that both the objective function and the constraints are linear, therefore the problem is a linear programming formulation. The minimization is subject to the dynamics of the water distribution network, and the constraints. As can be seen, there is a constraint on the output of the system, which is considered as the pressure demand for the end-users. There is a constraint on the pressure in the WT which represents the states in the dynamics and furthermore, there is a constraint on the input signal to the pumps.
The dynamic model of the system is explained in Chapter *4: Modelling* and linearized

in Section *4.10: Linear parameter estimation.* The electrical price, $c[k]$, is described in *Appendix: E.*

In the following sections the design of the described control system is explained.

## 5.2 Model predictive control

Model predictive control (MPC) is an advance control method that depends on a dynamic model of the plant. MPC allows to calculate an optimal control signal taking the future electrical pricing into account. The control structure of MPC is in general

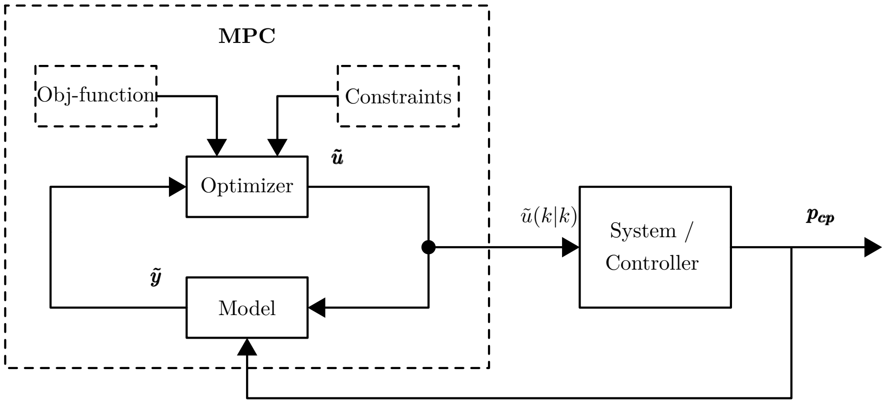



***Figure 5.2.*** The structure of the control with the MPC part specified [**?** ].

*Figure 5.2* is of the same structure as *Figure 5.1* but here the MPC block is specified. A MPC is a iterative process that can be setup as follows:

1: A model predict the future outputs, $\tilde{\boldsymbol{y}}$.
2: The future outputs are feed back to the optimizer that calculate the future inputs $\tilde{\boldsymbol{u}}$.
3: $\tilde{u}(k|k)$, the optimal control signal to the current time k, is sent as the control signal.
4: $\tilde{\boldsymbol{u}}$ are applied to the model, together with the old output.
5: Go back to step 1

The future outputs $\tilde{\boldsymbol{y}}$ and the future inputs $\tilde{\boldsymbol{u}}$ are defined by:

$$\tilde{\boldsymbol{y}} = \begin{bmatrix} \tilde{y}(k+H_w|k) \\ \vdots \\ \tilde{y}(k+H_p|k) \end{bmatrix} \tag{5.10}$$

$$\tilde{\boldsymbol{u}} = \begin{bmatrix} \tilde{u}(k|k) \\ \vdots \\ \tilde{u}(k+H_u|k) \end{bmatrix} \tag{5.11}$$

Where

| | | |
|---|---|---|
| | $\tilde{y}(k+H_w\|k)$ | is the predicted output to time $k+H_w$ based on the current output, |
| | $H_w$ | is the window horizon, |
| | $H_p$ | is the prediction horizon, |
| | $\tilde{u}(k\|k)$ | is the predicted optimal control signal to the current time k, |
| and | $H_u$ | is the control horizon.s |

To calculate the constraint matrices $H_p, H_u$and $H_w$ need to be determined. In Section *E: Electrical price* it can be seen that the price fluctuates a lot but some periodicity can be seen every 24 hours. Furthermore, water consumption can be seen as periodic, with a periodicity of 24 hours, due to the daily rhythm of the population. Therefor both $H_p$ and $H_u$ are chosen to 24. $H_w$ is used if deviation from the constraints are allow with in a transit period. Here $H_w$ is chosen to 1, because we want to be within the constraints at all times.

### 5.2.1 Extended system matrices

We want a way to describe the dynamics of the system during the prediction horizon. $\Delta\hat{p}_{wt}[0]$ is know from a measurement. The discrete system dynamics is stated in *Equation: (5.2.1)*.

$$\Delta\hat{p}_{wt}[1] = A_d\Delta\hat{p}_{wt}[0] + B_d\hat{u}[0] + E_d\hat{d}[0] \tag{5.12}$$

Now $\Delta\hat{p}_{wt}[2]$ is calculated in the same say, but $\hat{p}_{wt}[1]$ is substituted in.

$$\Delta\hat{p}_{wt}[2] = A_d[A_d\Delta\hat{p}_{wt}[0] + B_d\hat{u}[0] + E_d\hat{d}[0]] + B_d\hat{u}[1] + E_d\hat{d}[1] \tag{5.13}$$

This is done for all $\Delta\hat{p}_{wt}[k]$ until $\Delta\hat{p}_{wt}[H_p]$.

$$\begin{vmatrix} \Delta\hat{p}_{wt}[1] \\ \Delta\hat{p}_{wt}[2] \\ \vdots \\ \Delta\hat{p}_{wt}[H_p] \end{vmatrix} = \begin{vmatrix} A_d \\ A_d^2 \\ \vdots \\ A_d^{H_p} \end{vmatrix} \Delta p_{wt}[0] + \begin{vmatrix} B_d & 0 & \dots & 0 \\ A_dB_d & B_d & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ A_d^{H_p}B_d & A_d^{H_p-1}B_d & & B_d \end{vmatrix} \begin{vmatrix} \hat{u}[0] \\ \hat{u}[1] \\ \vdots \\ \hat{u}[H_p-1] \end{vmatrix} + \begin{vmatrix} E_d & 0 & \dots & 0 \\ A_dE_d & E_d & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ A_d^{H_p}E_d & A_d^{H_p-1}E_d & & E_d \end{vmatrix} \begin{vmatrix} \hat{d}[0] \\ \hat{d}[1] \\ \vdots \\ \hat{d}[H_p-1] \end{vmatrix} \tag{5.14}$$

This is written in short as:

$$\boldsymbol{\Delta\hat{p}_{wt}} = \boldsymbol{\Phi}\Delta\hat{p}_{wt}[0] + \boldsymbol{\Gamma U} + \boldsymbol{\Psi d} \tag{5.15}$$

### 5.2.2 Minimization problem

The minimization problem set up in *Equation: (5.1)* to *(??)*

We want to do this because we need to learn it.

The overall idea is to make everything depend on our state, that we can measure.

From *Equation: (5.1)* it can be seen that the energy consumption depends on the flow through the pumps, $\boldsymbol{q_{pump}}$, and the differential pressure created by the pumps, $\Delta p_i$. $\Delta p_i$ is a part of the input, which is know, but $\boldsymbol{q_{pump}}$ is an unknown term that have be expressed as a function of the sate, $\Delta\hat{p}_{wt}$.

The flow in any edges can be found using KCL.

$$\boldsymbol{q_1} = -\boldsymbol{H_1^\dagger H_0} q_0 \tag{5.16}$$

$\boldsymbol{q_{pump}}$ only consists of the flow in the edges representing the main pumps, edge 1 and 8, therefore a mapping matrix, $\boldsymbol{G}$, is introduced.

$$\boldsymbol{q_{pump}} = -\boldsymbol{G} \cdot \boldsymbol{H_1^\dagger H_0} q_0 \tag{5.17}$$

Where
$\boldsymbol{q_{pump}} \in \mathbb{R}^{2\times 1}$ is the flow through the pumps,
and $\boldsymbol{G} \in \mathbb{R}^{2\times 24}$ is a mapping matrix.

*Equation: (5.17)* depends in the flow through the WT, $q_0$. This can be transformed to the differential pressure at the bottom of the WT.

$$\Delta P_{wt}[k+1] = \frac{1}{C_H} \cdot q_0 \tag{5.18}$$

Now *Equation: (5.18)* can be substituted into *Equation: (5.17)*.

$$\boldsymbol{q_{pump}} = -C_H \boldsymbol{G H_1^\dagger H_0} \Delta P_{wt}[k+1] \tag{5.19}$$

*Equation: (5.19)* can then be substituted into *Equation: (5.1)*.

$$J = \sum_{i=1}^{N} \left( \frac{1}{\eta_i} \cdot u[k] \cdot -C_H \boldsymbol{G H_1^\dagger H_0} \Delta P_{wt}[k] \right) \cdot c[k] \tag{5.20}$$

From this the dynamic of the system, described in can be included. This will make it possible to describe the cost of running the system by only looking at the input signal. This is done by substituting the extended system description from to *Equation: (5.20)*.

add secref

add eqref

$$J = \sum_{i=1}^{N} \frac{-C_H}{\eta_i} \cdot \boldsymbol{G H_1^\dagger H_0} \cdot u[k] \cdot (\Phi \Delta P_{wt}[0] + \Gamma u[k] + \Psi d[k]) \cdot c[k] \tag{5.21}$$

## 5.3 PI controller

As described in Section *5.1: Control Problem* the two PMA pumps should generate a constant differential pressure. Furthermore Section *5.1: Control Problem* conclude that a simple PI controller, reacting to a reference calculated by the MPC, should be used to control the pressure generated by the main pumps.

In *Appendix: F* a linearized transfer function is derived. Whit the operating point of the four pumps and their respective parameters shown in *Appendix: C*, four different models are derived as

include the models

The controllers are design through the Matlab toolbox *Control System Designer App*[**?**], with the approach of the same control characteristics. Since the time constant of the water system is large, the settling time of the PI controllers has been chosen to 5 seconds.

With the settling time in mind the following controllers has been designed

include the PI controllers

# Implementation of controller 6

This chapter will explain how the controller designed in Chapter *5: Controller* is implemented in MATLAB simulink.

# Part III

# Conclusion and verification

# 7 Accepttest

# 8 Discussion

# Conclusion 9

# Part IV

# Appendices

# A Unit Conversion

Due to the large difference between the SI-units of flow, [m$^3$/s], and pressure, [Pa], a conversion from seconds to hours and pascal to Bar is made.

The final pipe model from *Equation: (4.21)*, is shown below.

$$\begin{aligned}\frac{L\rho}{A}\frac{dq}{dt} &= \Delta p - \frac{8fL}{\pi^2 g D^5}\rho g|q|q - k_f\frac{8}{\pi^2 g D^4}\rho g|q|q - \Delta z\rho g \\ &= \Delta p - (\frac{8fL}{\pi^2 g D^5} + k_f\frac{8}{\pi^2 g D^4})\rho g|q|q - \Delta z\rho g\end{aligned} \tag{A.1}$$

1[bar] = 10$^5$[Pa]. Therefore we can rewrite *Equation: (A.1)* to:

$$\begin{aligned}\frac{L\rho}{A\cdot 10^5}\frac{dq}{dt} &= \Delta\frac{p}{10^5} - (\frac{8fL}{\pi^2 g D^5\cdot 10^5} + k_f\frac{8}{\pi^2 g D^4\cdot 10^5})\rho g|q|q - \frac{\Delta z\rho g}{10^5} \\ \frac{L\rho}{A\cdot 10^5}\frac{dq}{dt} &= \Delta p_{bar} - (\frac{8fL}{\pi^2 g D^5\cdot 10^5} + k_f\frac{8}{\pi^2 g D^4\cdot 10^5})\rho g|q|q - \frac{\Delta z\rho g}{10^5}\end{aligned} \tag{A.2}$$

The conversion from $[\frac{m^3}{s}]$ to $[\frac{m^3}{h}]$ is $\frac{m^3}{s}3600 = \frac{m^3}{s}$. *Equation: (A.1)* can be written as:

$$\frac{L\rho}{A\cdot 10^5}\frac{d}{dt}\frac{q}{3600} = \Delta\frac{p}{10^5} - (\frac{8fL}{\pi^2 g D^5\cdot 10^5} + k_f\frac{8}{\pi^2 g D^4\cdot 10^5})\rho g\frac{|q|}{3600}\frac{q}{3600} - \frac{\Delta z\rho g}{10^5} \tag{A.3}$$

There is no need to apply the unit conversion to the final valve model from *Equation: (4.29)*, due to the parameter $k_v$ being designed for the water flow in $m^3$ through the valve in one hour and at a pressure drop across the valve of 1 Bar.

In the pump final model *Equation: (4.31)* the constants are scaled so the pump equation has the units in Bar and the flow has the units in $m^3/h$.

# Assumption List B

| Number | Assumptions | Section reference |
|---|---|---|
| 1 | The fluid in the network is water. | Section *4.1.1: Pipe model* |
| 2 | All pipes in the system are filled up fully with water at all time. | Section *4.1.1: Pipe model* |
| 3 | The pipes have a cylindrical structure and the cross section, $A(x)$, is constant for every $x \in [0, L]$. | Section *4.1.1: Pipe model* |
| 4 | The flow of water is uniformly distributed along the cross sectional area of the pipe and the flow is turbulent. | Section *4.1.1: Pipe model* |
| 5 | $\Delta z$, the change in elevation only occurs in pipes. | Section *4.1.2: Valve model* |
| 6 | The pumps in the network are centrifugal pumps. | Section *4.1.3: Pump model* |
| 7 | The storage of the WT has a constant diameter. In other words, the walls of the WT are vertical. | Section *4.1.4: Water Tower* |
| 8 | Valves in the water distribution system are modelled according to the assumption that the length,$L$, is zero. | Section *4.1.2: Valve model* |
| 9 | $\mathcal{G}$ is a connected graph. | Section *4.4: Graph representation* |
| 10 | The pipe volume is assumed to be known to an accuracy where there is not any benefit from estimating it. Thereby the estimation problem is simplified. | Section *4.6: Nonlinear Parameter identification* |
| 11 | Functions describing the pressure drops across the elements of the system are continuously differentiable. Therefore can be approximated with their Taylor-series. | Section *4.8.1: Taylor expansion on a simple example* |
| 12 | The operating point for the system is chosen such that $f(\bar{x}, \bar{u}) = 0$. | Section *4.8.1: Taylor expansion on a simple example* |

***Table B.1.*** List of assumptions

# System Description C

## C.1 Components of the System

Here all necessary data (from the datasheets) and notations should be listed about the components (pipes, pumps, valves etc)

| Part | Component | Length | Diameter | Material | $\epsilon$ | $\Delta z$ | Fittings | $\Sigma k_f$ |
|---|---|---|---|---|---|---|---|---|
| Ring | $C_4$ | 5m+0.3m | 20 mm | 25 mm PEM | 0.01 mm | 0 m | b,c,c,a,a | 4.42 |
| | $C_8$ | 10 m | 20 mm | 25 mm PEM | 0.01 mm | 0 m | c,b,a,c,b | 3.92 |
| | $C_9$ | 10 m | 20 mm | 25 mm PEM | 0.01 mm | 0 m | c | 0.51 |
| | $C_{10}$ | 10 m | 20 mm | 25 mm PEM | 0.01 mm | 0 m | c,a,a | 3.11 |
| | $C_{11}$ | 10 m | 20 mm | 25 mm PEM | 0.01 mm | 0 m | c,a | 1.81 |
| | $C_{12}$ | 10 m | 20 mm | 25 mm PEM | 0.01 mm | 0 m | c,c,a,c,b | 3.63 |
| | $C_{13}$ | 10 m | 20 mm | 25 mm PEM | 0.01 mm | 0 m | c | 0.51 |
| | $C_{14}$ | 5m+4m | 20 mm | 25 mm PEM | 0.01 mm | 0 m | a,c | 1.81 |
| PMA1 | $C_{19}$ | 2 m | 10 mm | 15 mm PEX | 0.007 mm | 0 m | b,c,d,c,e,a | 3.57 |
| | $C_{21}$ | 1 m | 10 mm | 15 mm PEX | 0.007 mm | 0 m | c,d,b | 1.46 |
| | $C_{22}$ | 1 m | 10 mm | 15 mm PEX | 0.007 mm | 0 m | c,d,b,e,b | 7.68 |
| | $C_{23}$ | 2 m | 10 mm | 15 mm PEX | 0.007 mm | 0 m | a,b,d,e | 2.55 |
| PMA2 | $C_{23}$ | 3 m | 10 mm | 15 mm PEX | 0.007 mm | 0.5 m | d,c,a,c,e | 2.77 |
| | $C_{23}$ | 1 m | 10 mm | 15 mm PEX | 0.007 mm | 0 m | c,e | 0.81 |
| | $C_{23}$ | 1 m | 10 mm | 15 mm PEX | 0.007 mm | 0 m | b,d,c,b | 2.26 |
| | $C_{23}$ | 2 m | 10 mm | 15 mm PEX | 0.007 mm | 0 m | b,a | 2.10 |
| - | $C_{42}$ | 2 m | 10 mm | 15 mm PEX | 0.007 mm | 0.5 m | c,c,a,d,e | 2.77 |
| - | | | | | | | | |
| - | | | | | | | | |

***Table C.1.*** Table with details about the pipes in the water system, shown on *Figure C.2.* Note that $\Sigma k_f$ is an initial guess for the parameter estimation in Section *4.6: Nonlinear Parameter identification.*

| Fitting | Symbol | $k_f$ |
|---|---|---|
| Tee - Over all loss | $k_{f,a}$ | 1.3 |
| Tee - Straigh through | $k_{f,b}$ | 0.8 |
| 90° bend - Diameter/radious ration 1:1 | $k_{f,c}$ | 0.51 |
| Sudden enlarger - Diameter ratio 1:2 | $k_{f,d}$ | 0.15 |
| Sudden contractor - Diameter ratio 1:2 | $k_{f,e}$ | 0.3 |

***Table C.2.*** Table with details about the fittings in the water system. The finttings are not shown in *Figure C.2.* The values are found in [**? ?** ].

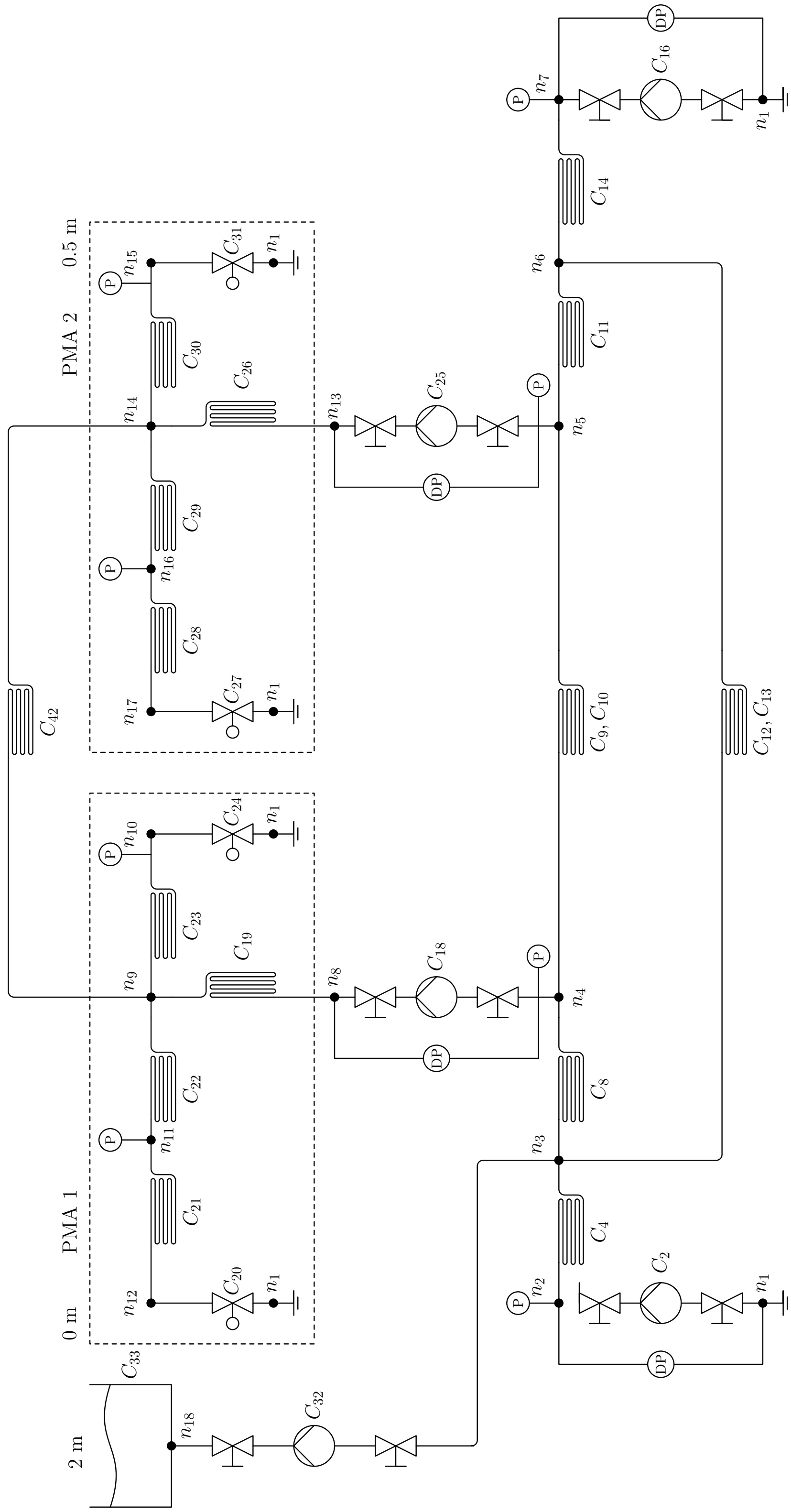
| Part | Component | Valve fitting | $k_{vs}$ | $n_{gl}$ | $\theta_{off}$ | $\theta_{max}$ | Valve motor |
|---|---|---|---|---|---|---|---|
| PMA1 | $C_{20}, C_{24}$ | Belimo R2015-1-S1 | 1 | 3.2 | 15° | 90° | Belimo LRQ24A-SR |
| PMA2 | $C_{27}, C_{31}$ | Belimo R2015-1-S1 | 1 | 3.2 | 15° | 90° | Belimo LRQ24A-SR |

***Table C.3.*** Table with details about the valves in the water system, shown on *Figure C.2.* The parameters are found in [**? ?** ]

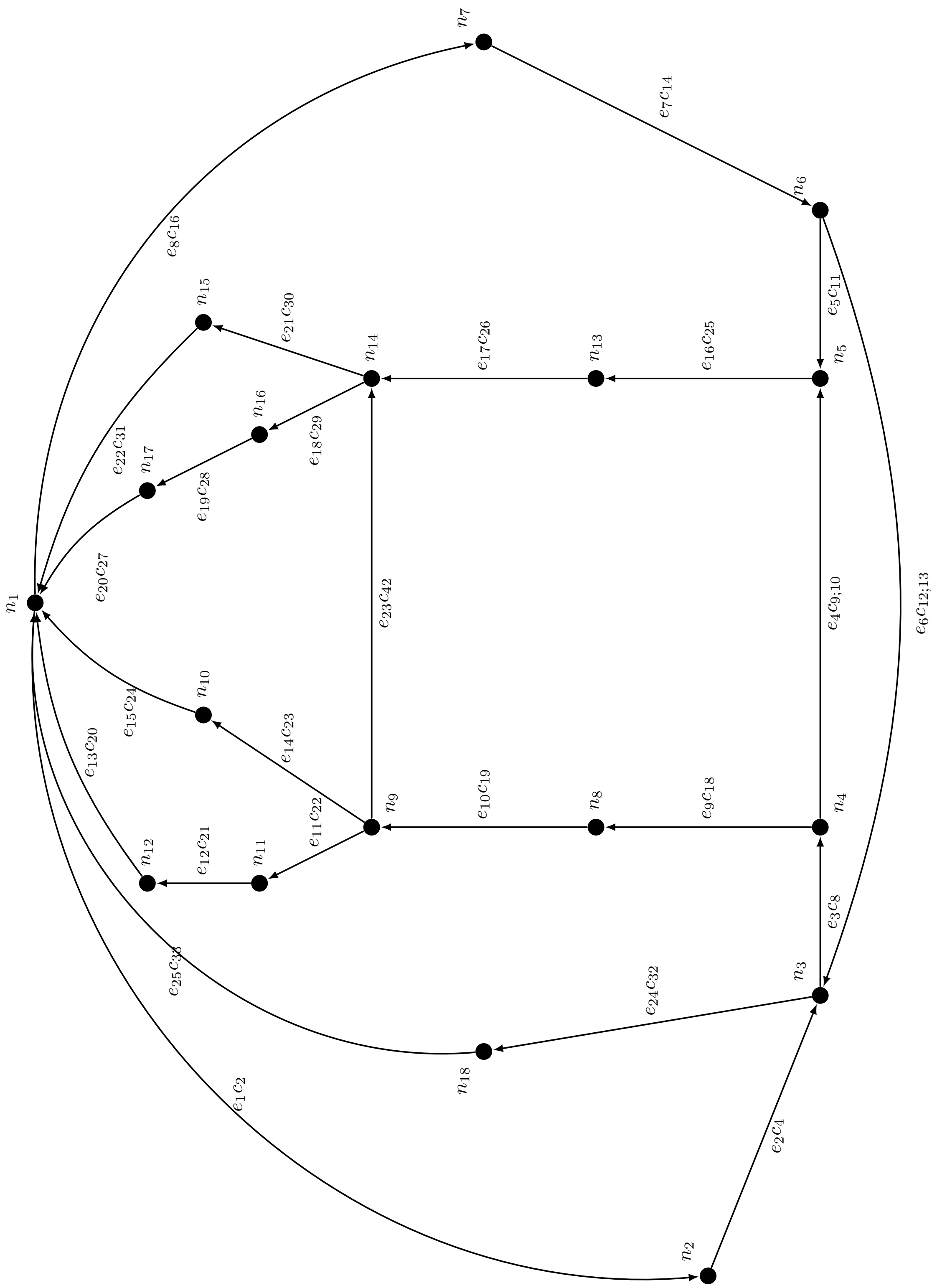
| Part | Component | Pump type | Constants |
|---|---|---|---|
| Ring | $C_2, C_{16}$ | Grundfors UPMXL GEO 25-125 180 | $a_{h0}$ = 1.2024<br>$a_{h1}$ = 0.0098<br>$a_{h2}$ = 0.0147<br>$B_0$ = 9.8924 |
| PMA(1,2) | $C_{18}, C_{25}, C_{32}$ | Grundfors UPM2 25-60 180 | $a_{h0}$ = 0.6921<br>$a_{h1}$ = -0.0177<br>$a_{h2}$ = 0.0179<br>$B_0$ = 0.0698 |

***Table C.4.*** Table with details about the pumps in the water system, shown on *Figure C.2.* The parameters are provided by Grundfos.

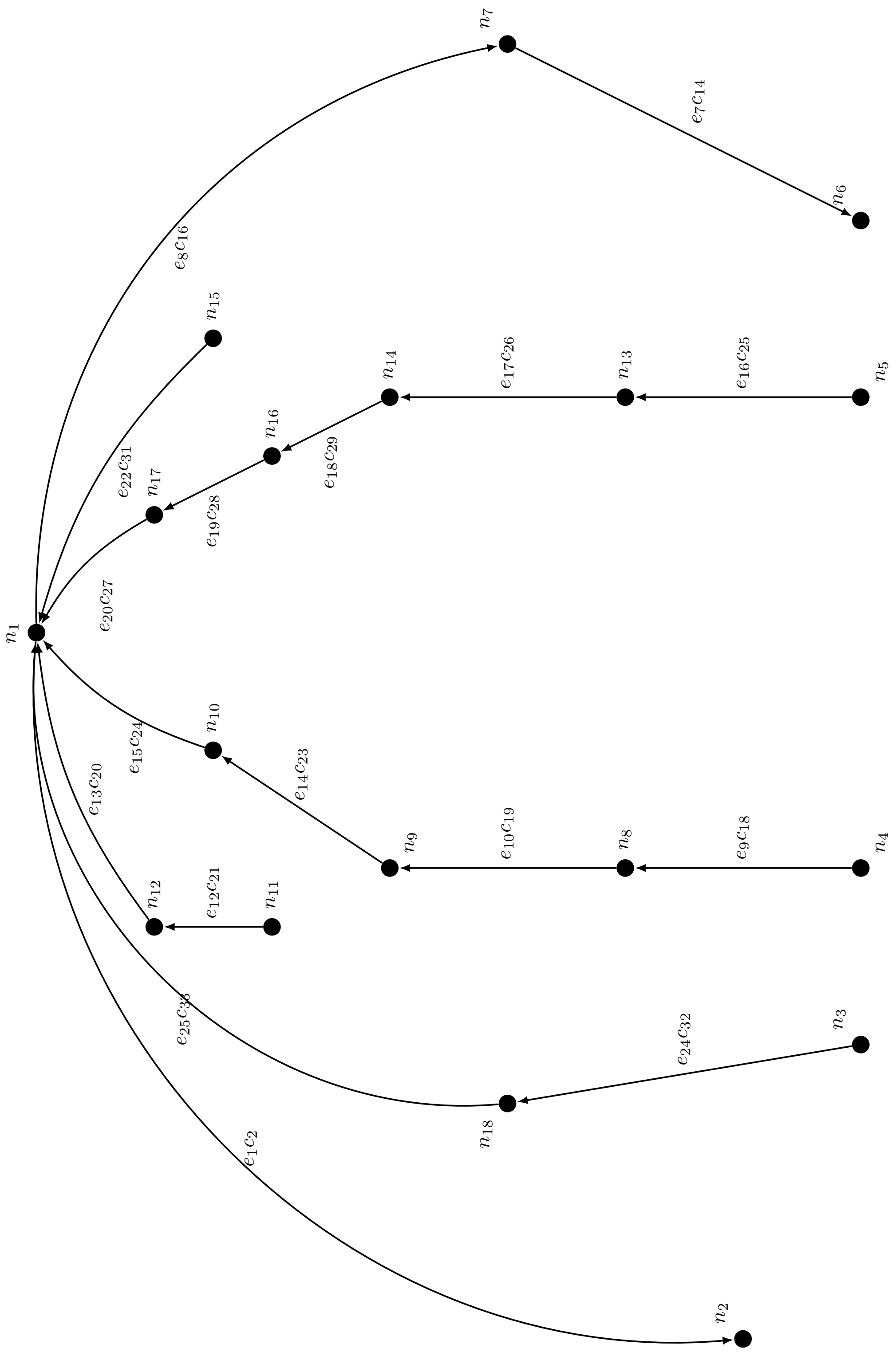
## C.2 System Topology



## C.3 System Graph



## C.4 Spanning Tree



## C.5 Incidence Matrix

## C.6 Cycle Matrix

## C.7 G mapping matrix

$$
G = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{C.3}
$$

$$
H = \left[\begin{array}{c|ccccccccccccccccccccccccc}
18x24 & e2 & e3 & e4 & e5 & e6 & e11 & e21 & e23 & e1 & e7 & e8 & e9 & e10 & e12 & e13 & e14 & e15 & e16 & e17 & e18 & e19 & e20 & e22 & e24 & e25 \\
n1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 \\
n2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n3 & -1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
n4 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n5 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n6 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n9 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n11 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
n14 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
n15 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
n16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
n17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
n18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{array}\right] \tag{C.1}
$$

$$B = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0
\end{bmatrix} \tag{C.2}$$

# Linearization D

**Valve equation**

The Kv value as a function of the opening degree is for a valve in the system given by:

$$Kv(OD) = kv_{100}e^{(n_{gl}\gamma)} \tag{D.1}$$

The pressure across the valve as a function of the flow is given by:

$$\mu(q) = \frac{1}{(Kv)^2}q|q| \tag{D.2}$$

Gathering the two previous equations allows to describe the pressure pressure across the valve as a combined function of both the flow and the opening degree.

$$M(q, OD) = \frac{1}{(kv_{100}e^{(n_{gl}\gamma)})^2}q|q| \tag{D.3}$$

The linerization of the function $M(q, OD)$ by multi variable Taylor expansion in the operating points $\bar{q}$ and $\bar{OD}$ is given by the form

$$\begin{aligned} M(q, OD) &\approx M(a, b) + \frac{\partial}{\partial x}(M(a, b))(x - a) + \frac{\partial}{\partial y}(M(a, b))(y - b) \\ &\approx e^{\frac{2(\theta_{off}-\bar{OD})n_{gl}}{\theta_{max}-\theta_{off}}+2}\bar{q}|\bar{q}| - 2\frac{e^{\frac{2(\theta_{off}-\bar{OD})n_{gl}}{\theta_{max}-\theta_{off}}+2}n_{gl}\hat{OD}\bar{q}|\bar{q}|}{\theta_{max}-\theta_{off}} + 2e^{\frac{2(\theta_{off}-\bar{OD})n_{gl}}{\theta_{max}-\theta_{off}}+2}\hat{q}|\bar{q}| \end{aligned} \tag{D.4}$$

Where

| | | |
|---|---|---|
| | $a = \bar{q}$ | $\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$ |
| | $x = \bar{q} + \hat{q}$ | $\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$ |
| | $b = \bar{OD}$ | $[°]$ |
| and | $y = \bar{OD} + \hat{OD}$ | $[°]$ |

**Pipe equation**

The pressure across a pipe as a function of the flow is given by:

$$\mu(q) = C_p q|q| \tag{D.5}$$

The first order linear Taylor expansion in the operating point $\bar{q}$ is given as:

$$\begin{aligned} \mu(x) &\approx \mu(a) + \frac{\partial}{\partial x}\mu(a)(x - a) \\ &\approx C_p\bar{q}|\bar{q}| + 2C_p\bar{q}\hat{q} \end{aligned} \tag{D.6}$$

### Pump equation

Concerning the pump that connects the WT with the remaining system the rotational speed is zero. Therefore will the pumps influence be described by a resistive term which is gives a differential pressure drop as a function of the flow.

$$\Delta p = (\frac{2}{kv_{100}^2} - a_{h2})q|q| \tag{D.7}$$

The first order linear Taylor expansion in the operating point $\bar{q}$ is given as:

$$\begin{aligned}\Delta p(x) &\approx \Delta p(a) + \frac{\partial}{\partial x}\Delta p(a)(x-a) \\ &\approx (\frac{2}{kv_{100}^2} - a_{h2})\bar{q}|\bar{q}| + 2(\frac{2}{kv_{100}^2} - a_{h2})\bar{q}\hat{q}\end{aligned} \tag{D.8}$$

# Electrical price E

To minimize the running cost of the system the power consumption of the pumps, $P_e$ Cf. Section *4.1.3: Pump model*, and the electrical price, $c[k]$, is needed. Predicting future prices is an extensive task that depends on many factors e.g user consumption and weather conditions. Due to the fact that the learning goals of this project is not to derive a high precision predictive model that describe future electrical prices, data gathered from [26] is used instead. The pricing can be seen on *Figure E.1*.

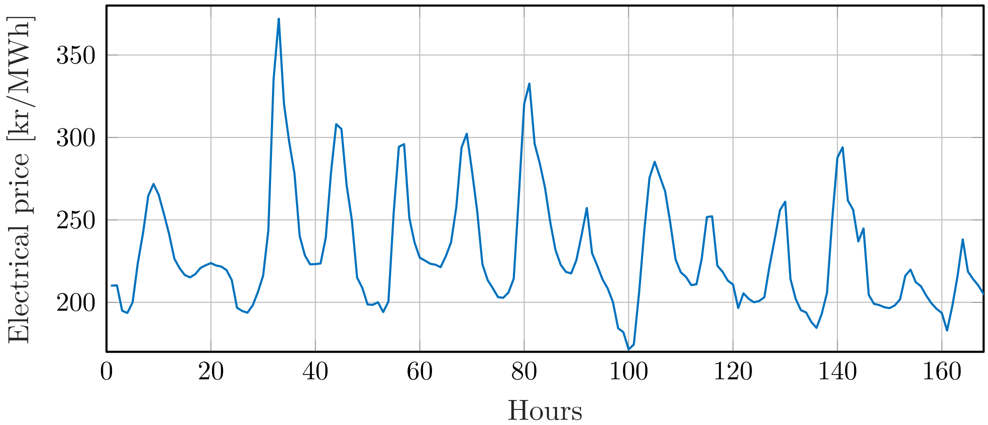



***Figure E.1.*** $c[k]$, describing the electricity prices in Denmark from the 27-03-2017 to 02-04-2017.

As this is real data from a given period, it will most likely not fit the pricing in any other given week, as the pricing is fluctuating a lot from day to day. However the data indicates the pricing is higher in the morning and evening which is applicable for any given week and thereby a general property of the time dependent pricing. This behavior can be seen as the periodicity of the data with two peaks a day. The chosen data will thus give a realistic idea of the improvement the controller can achieved in a real world scenario based on the week the data is recorded.

# F Pump linearizion and PI controller

The control structure chosen in Chapter *5.1: Control Problem* and shown in *Figure 5.1* includes a PI controller, which is to designed. The PI controllers purpose is to use the optimized control output from the MPC as a control reference.

The control focus of this project has been on the model predictive control, see Section *5.2: Model predictive control*. Therefore a simple PI controller has been design, where the control structure can be seen on *Figure F.1*.

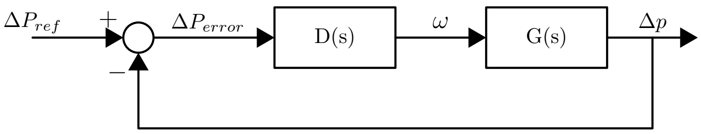



***Figure F.1.*** The structure of the PI controller.

The output from the MPC is a differential pressure, which is to be controlled through the rotational speed of the pumps. The general model for the pumps is given from *Equation: (4.31)* as

$$\Delta p = -a_{h2}{q_i}^2 + a_{h1}\omega_r q_i + a_{h0}{\omega_r}^2$$

which is a nonlinear model. By the assumption that the flow is constant, the expression can through a Taylor expansion be linearized, with respect to $\omega$, to a small signal model.

For simplification the pump model is separated into smaller chunks, see *Equation: (F.1)*, then Taylor approximated, see *Equation: (F.2)*, where $\omega = \hat{\omega} + \bar{\omega}$.

$$\begin{aligned} f_1(\bar{\omega}) &= a_{h1}\bar{q}\bar{\omega} \\ f_2(\bar{\omega}) &= a_{h0}\bar{\omega}^2 \end{aligned} \tag{F.1}$$

$$\begin{aligned} f_{t1}(\omega) &= f_1(\bar{\omega}) + f_1'(\bar{\omega}) \cdot (\omega - \hat{\omega}) \\ f_{t2}(\omega) &= f_2(\bar{\omega}) + f_2'(\bar{\omega}) \cdot (\omega - \hat{\omega}) \end{aligned} \tag{F.2}$$

From *Equation: (F.1)* an expression for the workspace of, $\bar{\Delta P}$, can be made.

$$\bar{\Delta P} = f_1(\bar{\omega}) + f_2(\bar{\omega}) + c \tag{F.3}$$

The linear pump model can then be expressed as seen in *Equation: (F.4)*

$$\begin{aligned} 0 &= -(\bar{\Delta P} + \hat{\Delta P}) + f_{t1}(\bar{\omega}) + f_{t2}(\bar{\omega}) + c \\ &= -(f_1(\bar{\omega}) + f_2(\bar{\omega}) + c + \hat{\Delta P}) + f_{t1}(\bar{\omega}) + f_{t2}(\bar{\omega}) + c \\ &= -\hat{\Delta P} + f_1'(\bar{\omega}) \cdot \hat{\omega} + f_2'(\bar{\omega}) \cdot \hat{\omega} \end{aligned} \tag{F.4}$$

*Equation: (F.4)* is then Laplace transformed and solved for the input output relationship as seen on *Figure F.1*

$$G(s) = \frac{\Delta P}{\omega} = f_1'(\bar{\omega}) + f_2'(\bar{\omega}) = a_{h1}\bar{q} + 2a_{h0}\bar{\omega} \tag{F.5}$$

## Controller

# G Discretization of state space model

Here the discretization of the continuous SS model is explained.

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## Rettelser

# Todo list