Article

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Toward model-based control of non-linear hydraulic networks

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Abstract

Water leakage is an important component of water loss. Many methods have emerged from urban water supply systems (WSSs) for leakage control, but it still remains a challenge in many countries. Pressure management is an effective way to reduce the leakage in a system. It can also reduce the power consumption. To have a better understanding of leakage in WSSs, to control pressure and leakage effectively, and for optimal design of WSSs, suitable modeling is an important prerequisite. In this paper a model with the main objective of pressure control and consequently leakage reduction is presented. Following an analogy to electric circuits, first the mathematical expression for pressure drop over each component of the pipe network (WSS) such as pipes, pumps, valves and water towers is presented. Then the network model is derived based on the circuit theory and subsequently used for pressure management in the system. A suitable projection is used to reduce the state vector and to express the model in standard state-space form.

Keywords

Model-based control, nonlinear systems, hydraulic networks

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1. Introduction

Today's technological world with the unlimited demand for industrial growth and with the world population explosion is facing the reality of limited energy sources and a shortage of water supplies. To cope with these problems and to make the rapid development possible and less expensive, the world is moving towards more efficient use of resources and optimization of infrastructures. The water supply systems (WSSs) and the distribution infrastructures are among the most vital for human and industrial growth. However, fresh water is limited and global changes, such as population growth and urbanization, are placing new strains on water supply (Water Loss Reduction, 2009).

In the last few decades, several surveys concluded that the WSSs and the distribution networks need to be improved, due to leakages, high cost for maintenance and high energy consumption (Mays, 1991). Despite many advances in this context, there is a growing demand in industry for methods leading to more efficient WSSs while maintaining tractability. The huge amount of fresh water which has been produced consists of nonrevenue water. This amount of water is lost before it reaches the customers due to leaks in distribution systems, noninvoiced customers and unbilled authorized consumption (Kingdom et al., 2006).

This huge amount of leakage can be reduced by reducing the pressure in the pipe network, which is achieved by proper pressure management of the network (Ulanicki et al., 2000). Pressure management is a cost-effective solution for leakage reduction. However, leakage reduction also has many benefits such as power consumption reduction. There exist several methods for pressure management but the basic

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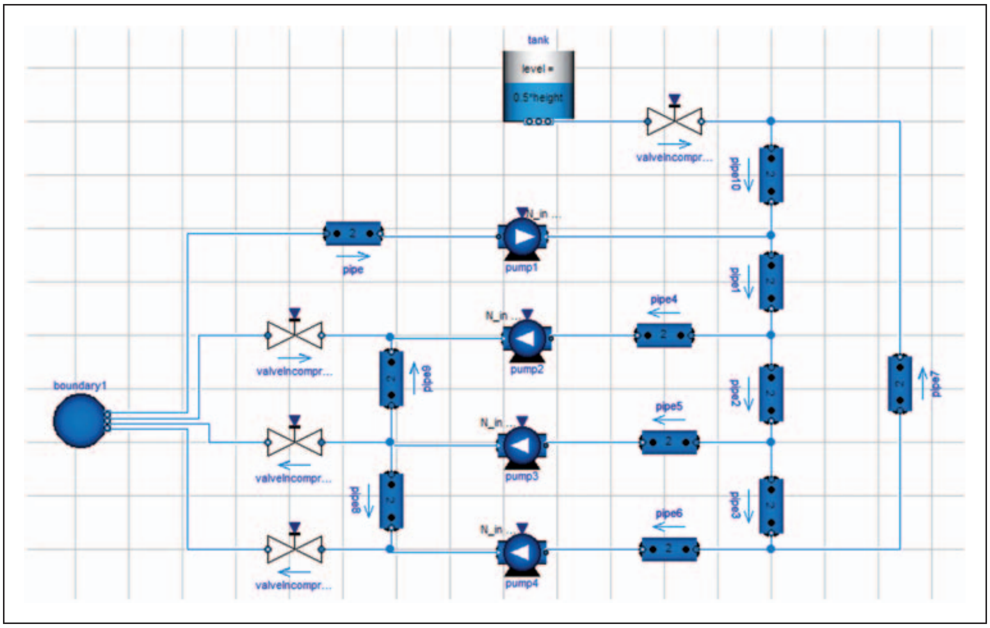

Figure 1. A water supply system (WSS) as modeled in Dymola with four pumps, four valves, a water tower, and pipes connecting these. The component named boundary accounts for ground level.

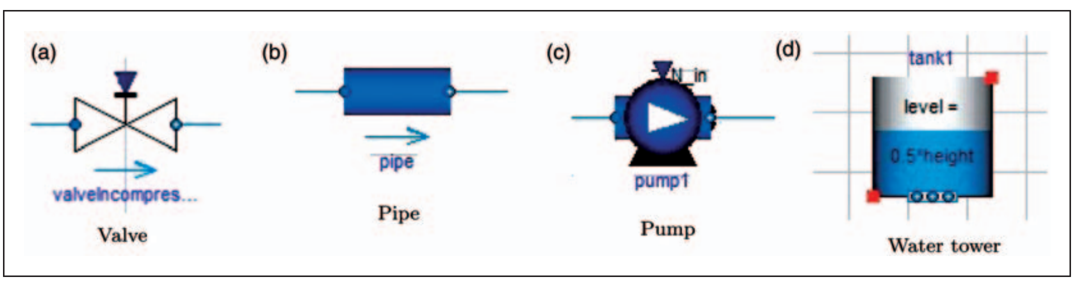

Figure 2. Models of WSS components.

steps and installations are the same in all of the approaches (Kingdom et al., 2006).

In water distribution systems, companies and manufacturers are becoming more interested in the computation of flows and pressure in pipe networks. Many methods have been used for these computations. Such methods range from graphical methods and use of physical analogies to the use of mathematical models. Suitable modeling is an important prerequisite for optimal design of WSSs and control pressure and leakage effectively. Ormsbee (2006) reviewed the network model analysis methods which have been developed and implemented on the computer over the last 50 years.

Control of hydraulic networks can be divided into two categories: control of open networks and control of closed-loop networks. One of the examples of open networks is irrigation networks. Cantoni et al. (2007) considered modeling and closed-loop control of open-water channels. The water quality control in drinking water distribution is considered and studied for another example of open networks by Polycarpou et al. (2002) and Wang et al. (2006). Mine ventilation networks, which are considered by Hu et al. (2003), are an example of closed networks. In this work nonlinear model-based feedback control of the air quality in mines is considered. The method proposed in this paper can be used also for water distribution networks. This work is extended by Koroleva et al. (2006), where decentralized feedback control of more general fluid flow networks is considered. The dynamics of these networks are closely related to the dynamics of the district heating system.

De Persis and Kallesøe (2008) considered a simple district heating system with two end-users. In this paper a proportional and a proportional-integral controller are designed which guarantee semi-global practical output regulation. De Persis and Kallesøe (2009a) derived the general network model which has been repeated in this section. The paper also provides a proof of semi-global practical output regulation when using the proportional control actions and constraining the control actions to nonnegative values only. De Persis and Kallesøe (2009b) extended the result from De Persis and Kallesøe (2009a) to showing semi-global output regulation when using nonnegative constrained binary control actions. Lastly, the result of De Persis and Kallesøe (2009b) is extended by De Persis and Kallesøe (2010) to show semi-global practical output regulation when using nonnegative constrained and quantized proportional control actions. All of the results mentioned here have been collected in the recent paper of De Persis and Kallesøe (2011). Global stabilization of large-scale hydraulic networks (district heating system) with different controllers was studied by Jensen (2012).

In this paper first, the mathematical expression for pressure drop over each component of the pipe network (WSS) such as pipes, pumps, valves, and water towers is presented. One of the differences of our model in comparison with its previous counterparts is that we consider a water tower in our network. We are dealing with an open hydraulic systems. The pressure delay in such a system is approximately 1 second for each 1.5 kilometer pipe line. Hence, we have delays below 5 seconds, we want to control the system such that this delay does not affect our control. This we do to avoid creating pressure waves in the network, which can destroy the piping if the shocks becomes too large (without pressure shock, i.e. incompressible flow). This means that the pressure at all open ends (the outlet of the valve, and the top of the water tower) in the network are connected to the atmospheric pressure. In this paper we define this pressure as a common vertex in the network, see Figure 3, and the equivalent node in Figure 4. This translates the open network to a closed network, which enables the use of network analysis for the model derivation. The inlet to the pipe network from the source and the outlet to the atmospheric pressure both create additional pressure losses (Swamee and Sharma, 2008). These losses are quadratic in nature as is the case for the piping and the valves. Here these losses are treated by adding an additional loss term to the outlet valves. Then the network model is derived based on the circuit theory for pressure management in the system. A suitable projection is used to reduce the state vector and to express the model in standard state-space form.

The notation used in this paper is as follows: h denotes the pressure at some given point, Δh denotes pressure difference over a given component, and q denotes the flow through a given component.

2. Hydraulic components modeling

The water supply system consists of a sequence of pumping stations that deliver water through pipelines to consumers. We consider a WSS as depicted in Figure 1, consisting of four (centrifugal) pumps, four valves, a water tower, and pipes connecting these. We tried to simplify our model as much as we can based on the reality. Subsequently we present the components of the WSS and their characteristics, see De Persis and Kallesøe (2009a) for details.

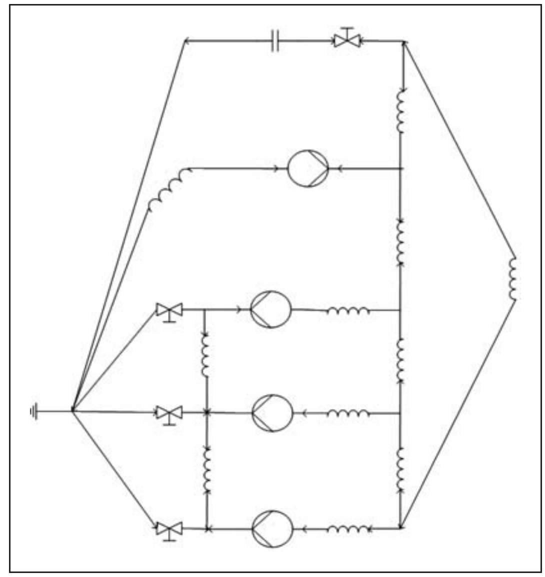

Figure 3. Electrical system model of the WSS depicted in Figure 1.

2.1. Valve

The pressure drop over a valve, depicted in Figure 2(a), is modeled as a quadratic function of the flow through the valve

$$\Delta h = K_v|q|q,$$

with K_v a constant describing the hydraulic resistance of the valve.

2.2. Pipe

The pressure drop over a pipe, depicted in Figure 2(b), with diameter D and of length l is modeled as

$$\Delta h = J\dot{q} + K_p|q|q + \rho g \Delta \mathcal{Z}, \tag{1}$$

with

$$\dot{q} = dq/dt,$$

and where J is the volume mass in the pipe, i.e.

$$J = l\rho/A_p$$

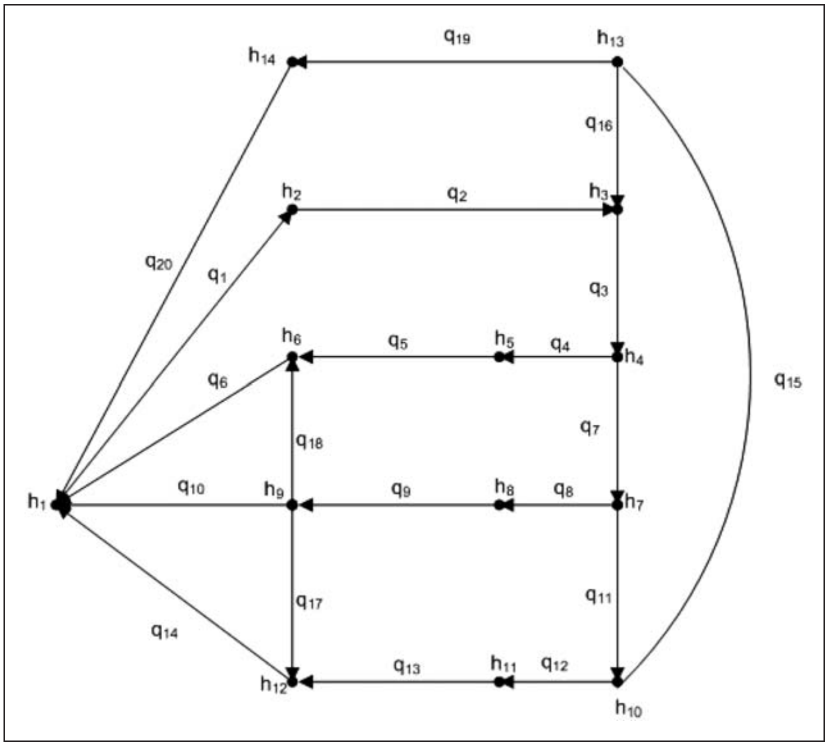

Figure 4. Graph representation of the WSS.

with

$$A_p = \pi D^2/4$$

the cross-sectional area of the pipe and ρ the mass density of water, and

$$K_p = fl\rho/(2DA_p^2)$$

a constant describing the hydraulic resistance of the pipe with friction f. In the pipe model, the term $\rho g \Delta \mathcal{Z}$ accounts for the topology of the location of the system with $\Delta \mathcal{Z}$ a height change (in meters) of a pipe in the system and g the gravity acceleration. The model (1) is derived under the assumption that the flow is uniformly distributed along a cross-section of the pipe, and the flow is turbulent De Persis and Kallesøe (2009a).

2.3. Pump

The pressure drop over a (centrifugal) pump, depicted in Figure 2(c), is modeled as

$$\Delta h = a_{h2}q^2 - a_{h1}qw - a_{h0}w^2,$$

with a_{h0}, a_{h1} and a_{h2} constants describing the pump, and w the rotational speed of the pump in radians per second. Moreover, the power consumption, P, of the pump is given by

$$P(q, w) = -a_{p2}q^2w + a_{p1}qw^2 + a_{p0}w^3,$$

with a_{p0}, a_{p1} and a_{p2} constants.

2.4. Water tower

The water towers are always placed in a very special way in the network. That is, the water surface is connected to the common vertex, and the output is connected to the network. The infinitesimal pressure drop over a water tower (open tank) is modeled as

$$\dot{h} = \frac{\rho g}{A_T} q,$$

where A_T is the cross-section of the tank and q is the flow to the tank.

3. WSS model

3.1. Network model

To obtain a control system model of WSS we will start by using the correlation between electrical and hydraulic circuits to obtain a model of the WSS. The valves are regarded as nonlinear resistors, the pipes are regarded as nonlinear inductors, the water tower is regarded as a capacitor, the pressures are regarded as voltages, the flows are regarded as currents, the pumps are regarded as voltage sources. These correlations are summarized in Table 1. The resulting electrical network is depicted in Figure 3.

This way of modeling has the advantage of making use of tools from circuit theory. Most of these tools are developed based on graph theory. These methods can be used to model the WSS as a weighted directed graph, where components of the system, i.e. valves, pipes, water tower, and pumps, correspond to edges (Desoer and Khu, 1969) and the weights are specified by the flows q. For each edge a reference direction is arbitrarily chosen. The direction of an edge is positive if the edge direction agrees with the flow direction through the corresponding component, and is negative otherwise. The graph corresponding to Figure 3 is depicted in Figure 4.

We use standard notation and terminology from graph theory (Desoer and Khu, 1969). Let G denote the graph corresponding to our WSS, hence the number of vertices $\nu = \#V(G)$ is 14 and the number of edges $\epsilon = \#E(G)$ is 20.

As a first step, to derive a model of the WSS, a set of independent flow variables (a set of flow variables which can be set independently from other flows in the network) is identified. These independent flows coincide with the flows through the so-called chords of the graph (Desoer and Khu, 1969). To achieve this, we assume the following.

Assumption 1. *We assume that G is a connected graph* (*De Persis and Kallesøe*, *2009a*).

Now with $H = [h_{ij}] \in \mathbb{R}^{\nu \times \epsilon}$ denoting the incident matrix of G, i.e.

$$h_{ij} = \begin{cases} 1 & \text{if the flow is entering edge } j \text{ at vertex } i\,, \\ -1 & \text{if the flow is leaving edge } j \text{ at vertex } i\,, \\ 0 & \text{if edge } j \text{ is not incident on vertex } i\,, \end{cases}$$

we can formulate Kirchoff's current law (KCL) as

$$Hq = 0, \tag{2}$$

where $q \in \mathbb{R}^{\epsilon}$ now is a vector containing the individual flows. It should be noted that when $H^{\top}$ is applied to the vector $h \in \mathbb{R}^{\nu}$ containing the individual pressures it will produce a vector $\Delta h \in \mathbb{R}^{\epsilon}$ containing the pressure differences over the various components

$$\Delta h = H^{\top} h. \tag{3}$$

For the WSS, (2) implies, not surprisingly, that the total water flow into a vertex equals the total flow out of a vertex.

Table 1. Correlation between electrical and hydraulic networks

Hydraulic component	Electrical component
Valve	Nonlinear resistor
Pipe	Linear inductor with a nonlinear drift term
Water Tower	Capacitor
Pressure	Voltage
Flow	Current
Pumps	Voltage source

The incident matrix $H \in \mathbb{R}^{\nu \times \epsilon}$ can be written as

$$H = \begin{bmatrix} H_1 & H_0 \end{bmatrix} \tag{4}$$

with $H_1 \in \mathbb{R}^{\nu \times \epsilon - 1}$ the incident matrix of the graph without the edge corresponding to the water tower and $H_0 \in \mathbb{R}^{\nu \times 1}$ the column of the incident matrix corresponding to the water tower. Therefore,

$$\Delta h = \begin{bmatrix} H_1 & H_0 \end{bmatrix}^{\top} h = \begin{bmatrix} H_1^{\top} \\ H_0^{\top} \end{bmatrix} h = \begin{bmatrix} \Delta h_1 \\ \Delta h_0 \end{bmatrix}. \tag{5}$$

Assumption 2. *There exists a spanning tree $\mathcal{T}$ of G such that the chords of $\mathcal{T}$ coincides with pipe components* (*Gross and Yellow*, *2003*, *page 536*).

To state Kirchoff's voltage law (KVL), let $\mathcal{T}$ denote the spanning tree of G, and recall that by adding to $\mathcal{T}$ any edge of G not contained in $E(\mathcal{T})$, i.e. a chord, a fundamental loop is obtained. Each fundamental loop has a reference direction given by the direction of the chord which completes the fundamental loop. Let l denote the number of fundamental loops and

$$l = \epsilon - \nu + 1$$

(e.g. $l = 7$ in Figure 4), then along any fundamental loop KVL holds and can be expressed by means of the fundamental loop matrix B as

$$B \Delta h = 0, \tag{6}$$

with $B = [b_{ij}] \in \mathbb{R}^{l \times \epsilon}$ and

$$b_{ij} = \begin{cases} 1 & \text{if edge } j \text{ is in loop } i \text{ and their} \\ & \text{reference directions agree}\,, \\ -1 & \text{if edge } j \text{ is in loop } i \text{ and their} \\ & \text{reference directions do not agree}\,, \\ 0 & \text{if edge } j \text{ is not in loop } i. \end{cases}$$

It should be noted that it is always possible to choose the numbering such that B has the form $B=[1\ F]$ with 1 the identity matrix of order l and F a suitable $\{l\times(\nu-1)$ matrix. Hence for the WSS, (6) implies that the total pressure of water in any fundamental loop is zero. From (3) and (6) we conclude that $\mathrm{im}(H^\top)\subseteqq\ker(B)$. Hence,

$$\mathrm{im}(H^\top)=\ker(B).$$

The fundamental loop matrix $B\in\mathbb{R}^{l\times\epsilon}$ can be written as

$$B=\begin{bmatrix} B_1 & B_0 \end{bmatrix} \tag{7}$$

with $B_1\in\mathbb{R}^{l\times\epsilon-1}$ the fundamental loop matrix of the graph without the edge corresponding to the water tower and $B_0\in\mathbb{R}^{l\times 1}$ the column of the fundamental loop matrix corresponding to the water tower.

As we mentioned above, q is a vector containing the individual flows (through a given component) and

$$q=\begin{bmatrix} q_1 \\ q_0 \end{bmatrix} \tag{8}$$

with $q_1\in\mathbb{R}^{\epsilon-1\times 1}$ the flow through all components except water tower and $q_0\in\mathbb{R}$ the flow through the water tower. The vector h contains the pressure at vertices and

$$h=\begin{bmatrix} h^1 \\ h^0 \end{bmatrix} \tag{9}$$

with $h^0=(h_{14})\in\mathbb{R}$ the pressure at the water tank and $h^1=(h_1,\ldots,h_{13})\in\mathbb{R}^{\nu-1\times 1}$ the pressures at the other vertices.

By the above discussion we can write an overall model for the WSS as

$$J\dot{q}_1=\tilde{f}(q_1,w,K_v)+H_1^\top h \tag{10}$$

$$\frac{d\Delta h_0}{dt}=\frac{\rho g}{A_T}q_0 \tag{11}$$

where now $w\in\mathbb{R}^s$ is a vector containing the individual pump rotational speeds, s is the number of pumps (in the case of our particular example $s=4$), $\Delta h_0=h_{14}-h_1=h^0-h_1=H_0^\top h$ (see Figure 4), and the function

$$\tilde{f}=(\tilde{f}_1,\ldots,\tilde{f}_{\epsilon-1})$$

given by

$$\tilde{f}_i=-K_{p_i}|q_i|q_i-\rho g\Delta\mathcal{Z}_i,\quad i=2,3,4,7,8,11,12,15,16,17,18 \tag{12a}$$

$$\tilde{f}_i=-K_{v_i}|q_i|q_i,\quad i=6,10,14,19 \tag{12b}$$

$$\tilde{f}_i=-a_{h2_i}q_i^2+a_{h1_i}q_iw_i+a_{h0_i}w_i^2,\quad i=1,5,9,13. \tag{12c}$$

3.2. State-space modeling and projection

To have a better understanding of leakage in WSSs, to control pressure and leakage effectively, and for optimal design of WSS, suitable modeling is an important prerequisite (Pfeiffer and Borchsenius, 2004). To this end a well-known modeling framework is the state-space modeling. In this framework, the dynamics of the system can be expressed as a set of first-order differential equations. A state-space representation is a mathematical model of a dynamical system which consists of state variables, inputs and outputs. The usage of this method enables us to:

- study more general models with multiple inputs and outputs;
- use the ideas of geometry for differential equations;
- relate internal and external descriptions; and
- use advanced control strategies which are based on state-space modeling, e.g. De Cuyper and Verhaegen (2002), Mann and Patel (2010), and Long-Xiang and Guo-Ping (2009).

The general standard form of the state-space representation is

$$\dot{x}=\varphi(x,u) \tag{13}$$

$$y=\psi(x,u) \tag{14}$$

where $u\in\mathbb{R}^m$ is control input, $y\in\mathbb{R}^k$ is output, and $x\in\mathbb{R}^n$ is a state vector. Furthermore, we would like the derived model to be in standard state-space form (13). It should be noted that, the model which is obtained is not in standard state-space form as it is clear from (10) that J is singular. On the other hand, the order of our system is high and needs to be reduced to maintain numerical tractability. In the following, a suitable projection is introduced which not only transforms our model into standard state-space form but also reduces the order of the system.

To this end, we would like to formulate (10) as a control system with input w, disturbance K_v, and output h. We introduce a new state $z \in \mathbb{R}^l$ as

$$q_1 = B_1^\top z, \tag{15}$$

as in De Persis and Kallesøe (2009a) and Desoer and Khu (1969, p. 482), hence (10) becomes

$$JB_1^\top \dot{z} = f(z, w, K_v) + H_1^\top h, \tag{16}$$

with

$$f(z, w, K_v) = \tilde{f}(B_1^\top z, w, K_v).$$

The physical meaning of the new state z is the flow in chosen chords of the graph G (De Persis and Kallesøe, 2009a). By multiplying (16) from the left by B_1:

$$B_1 JB_1^\top \dot{z} = B_1 f(z, w, K_v) + B_1 H_1^\top h, \tag{17}$$

Now recall that $\mathcal{J} = B_1 JB_1^\top$ is nonsingular and $\mathcal{J} = \mathcal{J}^\top > 0$ (De Persis and Kallesøe, 2009a; Kallesøe, 2007). However, choosing spanning tree and chords of the connected graph G is not unique and we can have some freedom in choice of independent flow variables.

Proposition 1. *Let $\epsilon^1 = (e_1^1, e_2^1, \ldots, e_l^1)$ be a set of chords of the graph G, such that $\mathcal{T}' = G - \epsilon^1$ is a spanning tree of the graph G and $\widetilde{B}$ is the fundamental loop matrix of the graph G with e_i^1 as chords. Then the following statements are equivalent*:

(i) $\widetilde{B}J\widetilde{B}^\top > 0$;
(ii) *There exists a set of edges $\epsilon_p = (e_1, e_2, \ldots, e_l)$ such that e_i coincides with a pipe component and $\mathcal{T} = G - \epsilon_p$ is a spanning tree of G.*

Proof. To prove (i) $\Rightarrow$ (ii), let

$$\mathcal{J} = \widetilde{B}_1 J \widetilde{B}_1^\top = \begin{bmatrix} \widetilde{B}_{1_p} & \widetilde{B}_1^1 \end{bmatrix} \times \begin{bmatrix} J_p & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} \widetilde{B}_{1_p}{}^\top & \widetilde{B}_1^{1\top} \end{bmatrix},$$

where p is the number of pipe components, $J = \mathrm{diag}(J_i) \in \mathbb{R}^{l \times l}$, J_i is positive definite for pipe components, $J_i = 0$ for nonpipe components, $\widetilde{B}_{1_p}$ is a sub matrix of $\widetilde{B}_1 \in R^{l \times p}$ corresponding to pipe components and J_p is a sub matrix of J corresponding to pipes.

Assume that $\mathcal{J}$ is positive definite ($\mathcal{J} > 0$), therefore $\mathcal{J}$ is full rank and it follows that

$$rank(\widetilde{B}_{1_p}) = l,$$

$$rank(J_p) \geq l$$

Since rank $(\widetilde{B}_{1_p}) = l$, there exists $\bar{B}_p$ invertible, such that, $\widetilde{B}_{1_p} = \begin{bmatrix} \bar{B}_p & S \end{bmatrix}$ where $\bar{B}_p \in \mathbb{R}^{l \times l}$ is an invertible matrix.

Let q_p be the vector of flows through pipe components and $q_p = \widetilde{B}_{1_p}^\top z$, where z is the vector of chosen independent flow variables,

$$q_p = \begin{bmatrix} \bar{B}_p^\top \\ S^\top \end{bmatrix} \times z = \begin{bmatrix} \bar{z} \\ \bar{q}_p \end{bmatrix}$$

where $\bar{z} = \bar{B}_p^\top z$. Since $\bar{B}_p$ is invertible,

$$z = (\bar{B}_p^\top)^{-1} \bar{z}$$

$$q_p = \begin{bmatrix} \bar{B}_p^\top \\ S^\top \end{bmatrix} \times (\bar{B}_p^\top)^{-1} \bar{z}. \tag{18}$$

From (18), $\bar{z}$ can be used as a vector of independent flow variables. Since $\bar{z}$ is a vector of flow through pipes, we can conclude that there exists a set of pipe edges which can be used as chords.

To prove (ii) $\Rightarrow$ (i), let $\bar{z}$ denote the independent flow variables based on Assumption 2, and let z denote the independent flow variables based on the desired set of chords. There exist fundamental loop matrices $B \in \mathbb{R}^{l \times \epsilon}$ and $\widetilde{B} \in \mathbb{R}^{l \times \epsilon}$, respectively, such that $\epsilon > l$. From (15),

$$q_1 = B_1^\top \bar{z} = \widetilde{B}_1^\top z, \tag{19}$$

and

$$rank(B) = rank(\widetilde{B}) = l.$$

Let

$$z = A\bar{z}$$

where $A \in \mathbb{R}^{l \times l}$, therefore

$$q_1 = B_1^\top \bar{z} = \widetilde{B}_1^\top A \bar{z},$$

$$B_1^\top = \widetilde{B}_1^\top A,$$

and since

$$dim(z) = dim(\bar{z}) = l,$$

therefore A is nonsingular

$$\widetilde{B}_1^\top = B_1^\top A^{-1}, \tag{20}$$

$$\widetilde{B}_1 = A^{-1\top} B_1. \tag{21}$$

Recall that $B_1JB_1^\top$ is nonsingular and $B_1JB_1^\top > 0$ (De Persis and Kallesøe, 2009a; Kallesøe, 2007). We should show that $\widetilde{B}_1J\widetilde{B}_1^\top > 0$. From (20):

$$\widetilde{B}_1J\widetilde{B}_1^\top = {A^{-1}}^\top B_1JB_1^\top A^{-1},$$

since

$$B_1JB_1^\top > 0$$

and A^{-1} is nonsingular, therefore $\widetilde{B}_1J\widetilde{B}_1^\top > 0$. □

As mentioned above $\mathcal{J} = B_1JB_1^\top$ is nonsingular and $\mathcal{J} = \mathcal{J}^\top > 0$ (De Persis and Kallesøe, 2009a; Kallesøe, 2007) and since

$$B_1H_1^\top = -B_0H_0^\top$$

and

$$\Delta h_0 = h^0 - h_1 = H_0^\top h$$

we obtain

$$\dot{z} = F(z, w, K_v) \tag{22}$$

with

$$F(z, w, K_v) = (B_1JB_1^\top)^{-1}(B_1f(z, w, K_v) - B_0\Delta h_0).$$

On the other hand, we know that

$$Hq = 0 \tag{23}$$

which can be written as

$$\begin{bmatrix} H_1 & H_0 \end{bmatrix} \times \begin{bmatrix} q_1 \\ q_0 \end{bmatrix} = 0,$$

therefore

$$H_0q_0 = -H_1q_1. \tag{24}$$

Let $\bar{H}$ denote the left pseudo-inverse of H_0. Subsequently from (11), (15) and (24),

$$\frac{d\Delta h_0}{dt} = -\frac{\rho g}{A_T}\bar{H}H_1B_1^\top z.$$

The control system will be

$$\dot{z} = F(z, w, K_v) \tag{25}$$

$$\frac{d\Delta h_0}{dt} = -\frac{\rho g}{A_T}\bar{H}H_1B_1^\top z \tag{26}$$

Since $H_1^\top$ is not full column rank it is not possible to find all pressures in the system (it is only possible to find $\nu - 1$ pressures). To obtain a full column rank matrix, we choose a reference pressure (pressure at the common vertex (h_1)) and we set it equal to zero. Then we remove the column in $H_1^\top$, which corresponds to the common vertex. Let this new matrix be denoted by H^*. To obtain the output expression let $L \in \mathbb{R}^\nu$ denote the left pseudo-inverse of H^*. Subsequently from (16), and replacing $H_1^\top$ by H^*, and (22) we obtain the output expression

$$\bar{h} = K(z, w, K_v),$$

where $\bar{h} = (h_2, \ldots, h_{14}, h_{h15})$ is the pressure at all vertices except the pressure at the common vertex and

$$K(z, w, K_v) = L(JB_1^\top(B_1JB_1^\top)^{-1}(B_1f(z, w, K_v) - B_0\Delta h_0) \\ - f(z, w, K_v)).$$

4. Conclusion

In this paper the nonlinear model of the water supply system has been obtained. Our model derivation is based on the circuit theory, and it is appropriate for pressure management and leakage control of the system. Futures work will be to design a set of optimal controllers which minimize the power consumption in pumps and also regulates each output (the pressure drop at the end-user valve) to a desired point value.

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