Aalborg University

Optimal Control for Water Distribution

Electronic & IT:
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Student report

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STUDENT REPORT

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Optimal Control for Water Distribution

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Synopsis:

This project covers the modelling and predictive control of a water distribution network with the aim of minimizing energy and economic cost.
At first the non-linear model of the components and the dynamics of the system are modelled based on a graph-based approach which leads to a state space representation of the whole network. Then system identification is carried out due to the uncertain parameters of the pipe components.
A model predictive controller is applied to the linearized model of the water distribution system extended with an elevation reservoir. The controller follows certain constraints to maintain consumer pressure-desire in two pressure management areas and to optimize the use of water tower such that the cost of pumping effort is minimized. The controller is implemented in a cascade system along with PI controllers and is based on the model of the network, the cost of electricity and the characteristics of end-user water usage.
Implementation carried out ...
The results show that ...

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Preface

This project comprises of implementing a functional controller system for

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Nomenclature

Acronyms

PMA	Pressure Management Area
CP	Critical Point
WT	Water Tower
MP	Minimization Problem
OD	Opening Degree
KVL	Kirchhoff´s Voltage Law
KCL	Kirchhoff´s Current Law
MPC	Model Predictive Control
GT	Graph Theory
ZOH	Zero Order Hold

Symbols

Symbol	Description	Unit
A	Cross sectional area	$[m^2]$
C_k	The k^{th} component of the distribution network	$[\cdot]$
C	Electric capacitance	$[F]$
C_H	Hydraulic capacitance	$[m^3/(N/m^2)]$
D	Diameter	$[m]$
f	Moody friction factor	$[\cdot]$
F	Force	$[N]$
g	Acceleration due to gravity	$[m/s^2]$
h_f	Pressure given in head	$[m]$
h_m	Form loss	$[m]$
J_k	Water inertia of the k^{th} component	$[kg/m^4]$
k_f	Form loss coefficient	$[\cdot]$
L	Length	$[m]$
m	Mass of body	$[kg]$
M	Linear momentum	$[kgm/s]$
n_i	The i^{th} node of the distribution network	$[\cdot]$
n_{gl}	Valve characteristic curve factor	$[\cdot]$
p_a	Atmospheric pressure	$[bar]$
Δp_k	The pressure drop across the i^{th} component	$[bar]$
q_k	Flow through the k^{th} component	$[m^3/h]$
$\boldsymbol{Re}$	Reynolds Number	$[\cdot]$
T	Temperature	$[°]$
v	Velocity	$[m/s]$
V_t	Volume of the water in the water tower	$[m^3]$
$\alpha_k(\cdot)$	The pressure boost given by the k^{th} pump	$[bar]$
ϵ	Average roughness	$[\cdot]$
ζ	Pressure drop from elevation difference across the k^{th} component	$[bar]$
θ_{max}	Maximum angle of the opening degree	$[°]$
θ_{off}	Minimum angle where the valve closes	$[°]$
θ_{OD}	Angle of opening degree	$[°]$
$\lambda_k(\cdot)$	Function of hydraulic resistance in the k^{th} pipe	$[bar]$
$\mu_k(\cdot)$	Function of hydraulic resistance in the k^{th} valve	$[bar]$
ν	Kinematic viscosity	$[kg/ms]$
ρ	Density	$[kg/m^3]$
ω_r	Impeller angular velocity	$[rad/s]$

Mathematical tools

vectorfields
time derivatives
vectors
matrices
derivative of vector fields
Jakobi matrix
chain rule in derivation
pseudo inverse
explain that $\boldsymbol{x}[k]$ is not iteration, it shows that x is a vector and a sequence-¿ this is very important to state !

Mathematical notation

This section will explain how the mathematical notation of this report.

Upper and lower bounds

$$\underline{x} < x < \overline{x} \tag{1}$$

Where x is a variable and $\overline{x}$ and $\underline{x}$ are the upper and lower bounds.

Intervals

$$[a, b] = \{x \in \mathbb{R} | a \leq x \leq b|\}\underline{x} < x < \overline{x} \tag{2}$$

Where x is a variable and $\overline{x}$ and $\underline{x}$ are the upper and lower bounds.

Contents

1 Introduction

Water pressure management is a vital part of the water supply infrastructure all over the world. It ensures that a positive water pressure is present such that the consumers are supplied with water at all time. Maintaining a minimum pressure in the network is an important task as it ensures the end user a decent water pressure and also minimizes the risk of contamination in the water system[1].

In the U.S alone 4 % of the national energy consumption is used on moving and treating water/wastewater[2]. With an increasing focus on green energy, more and more renewable energy sources are added to the grid. Nevertheless, the intermittent behavior of renewable energy sources and time-dependent consumer preferences result in fluctuation in the available power. This means that the price for electric power also varies [3]. To minimize the cost of running a water distribution network, potential energy can be used to maintain a minimum pressure. When electric prices are low, water can be pumped to a higher altitude and stored in a water tower (WT), and thereby energy is stored for future use. The potential energy of the water stored in the WT can then be used to maintain a minimum pressure that is required at the end consumer. However when a WT is included in a water distribution network, the pressure in the system is defined by the water level and hight of the WT. This means that to control pressure, the water level of the WT should be controlled.

Maximum allowed pressure in water distribution networks should also be considered as the risk of water leakage increases when pressure is increased[4], thus increased water losses due to leakage will lead to a higher energy consumption. In [4] it is stated that the estimated world wide water loss is at 30 %, so the energy used on cleaning the water for filth, bacteria and pressurizing it is lost. Another problem that should be highlighted regarding high pressure is that a high pressure will increase the wear on the pipes in a system[5], this leads to higher maintenances costs as pipes and fittings have to be replaced more frequently. Additionally, maintenances is not always an easy task, since the pipes usually are placed under ground and need to be dug up. Thereby the expense of maintenance is increased, especially in a city, were the operation also can have a negative impact on significant infrastructures. Based on these facts, the maximum pressure in a water distribution network is a vital parameter of the systems profitability. In a system with a WT the maximum allowed pressure will likely be defined by the maximum allowed water level in the WT, as the WT in most situations will be able to provide a dominant pressure compared to the desired network pressure.

Some constraints regarding a solution that implements a WT are still necessary to be taken into account. One of them being the quality of the water in the tower. If stored for too long the quality of the water will start to decrease due to a decreasing oxygen level [6, 7], thus the water should not be stored for to long. The oxygen level of the water also depends on the water temperature and therefor the water should not be too warm. Furthermore it is undesirable that the water remains stagnant in the tower or pipe as it also effects the water quality.

This leads to the following problem statement:

- *How can a water tower, implemented in a water distribution network, be controlled to minimize the cost of running a water distribution network without compromising the water quality.*

Part I

Analysis

System Description 2

This section will give an introduction to the available test system, including structure and components overview.

2.1 System overview

To develop and test different control methods for a water distribution system a test setup is required. Such a setup is available at Aalborg university which is based on a real water distribution system, though as a 1:20 downscaled version.

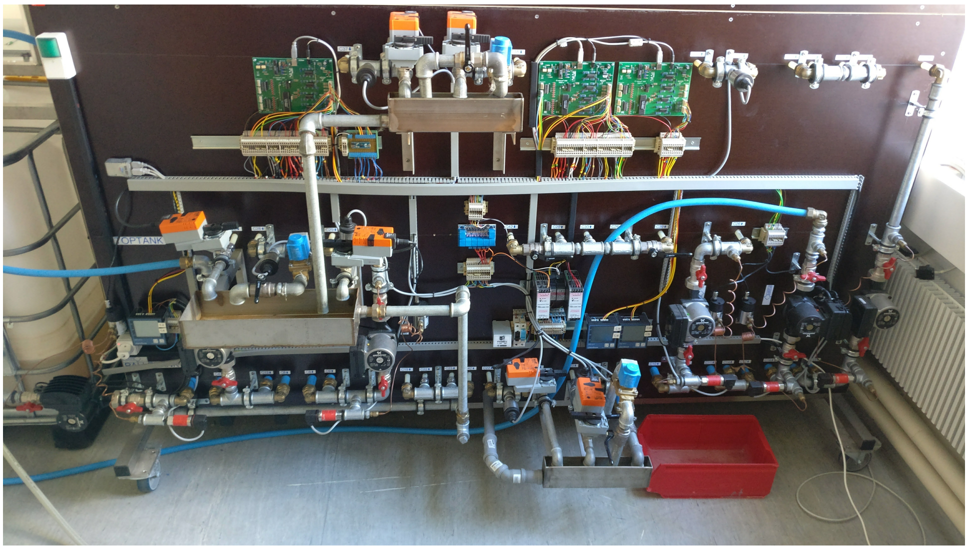


Figure 2.1. The available test setup used to represent a real water distribution system.

The test setup represents a real system, thus the same structure concerning piping, leveling and all the other components. To achieve different elevation levels between system parts, the setup is mounted on a wall. This also allows for a quick overview of the complete setup and eases access to the components. As the system is used for various test scenarios other equipment is also present in the test setup shown in *Figure 2.1*, enabling the test system to mimic a variety of different system types and scenarios. A simplified diagram representing the structure of the test setup that will be used in this project is shown in *Figure 2.2*.

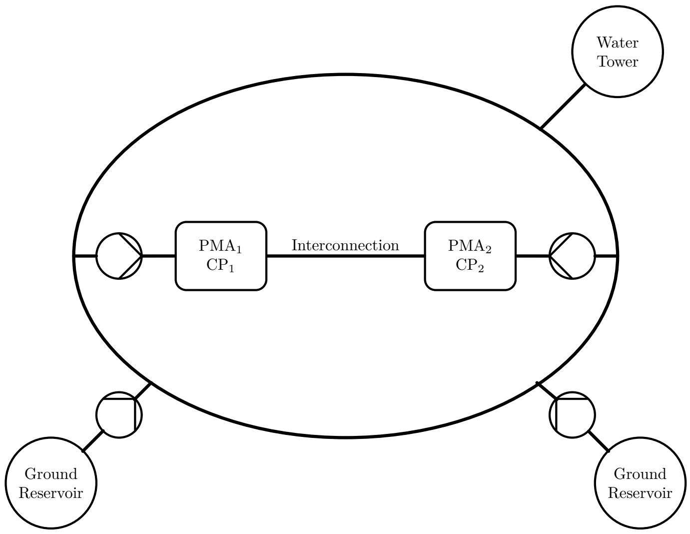


Figure 2.2. Overview of the reduced system that fulfills the scenario of this project.

The system can be split up into different parts, where the main part is a water reservoir placed at ground level, used to supply the system. Two pumps are connected to the reservoir and they supply water to a main water ring formed around the PMA's. A water tower is also connected to the main water ring, and will act as an additional water reservoir and furthermore pressurize the ring due to the elevation of the tower. The direction of water flow, with respect to the tower, will depend on the pressure in the main ring. The tower can thus be filled by pressurizing the ring or be used to pressurize and supply water to the ring instead of the pumps. From the water ring, two PMA's are connected, each via their own pump. In each PMA a measuring point, called the critical point (CP), is placed and the pressure at this point shall be kept to accommodate supply demands of the consumers. Furthermore two consumers are placed in each PMA, these are simulated by valves with a variable opening degree where the water flows back to the main reservoir.

As the test setup consist of different components as valves, pumps and pipes, a basic water distribution network is shown in *Figure* **??** which will be used to illustrate and explain the individual components in the system and their functionality.

In the system two different types of Grundfos pumps are used. For supplying the water ring, two pumps, of the type UPMXL GEO 25-125[8], are used. Whereas the pumps used in each PMA are of the type UPM2 25-60[9], which is a smaller pump and typically used at the end-user.

In order to close off parts of the system that will not be used for a specific scenario or to simulate faulty behavior, manual rotary ball valves are placed trough out the system. To simulate a consumer, an electronically controlled belimo valve is used. Thereby is it possible to vary the opening degree of the valve over time according to a specific consumer behavior. For the pipes there are used two different material types. The pipes used in the main ring, which connect the reservoir and the water tower, are made of polyethylene grade 80 called PE80[10]. The pipes used to connect the PMA's to the ring and the internal connections in the PMA's are made of polyethylene with cross-links called PEX. In addition to the pipes, fittings, bends, and elbows are also present and found in various metals as iron and brass.

The pressure measuring in each PMA is done with a Jumo pressure sensor. The pressure is measured relative to a reference called Gnd, for the test system, Gnd is atmospheric pressure. Furthermore both the differential pressure over each pump and the absolute pressure at the pump is measured with a Grundfos direct sensor DPI v.1 and a Danfoss mbs32/33 pressure sensor, respectively.

The main reservoir has a volume of 700 L and the WT a volume of 200 L. The volume of the WT in this report is denoted as V_t. A system diagram of the entire test setup, including pipe dimensions, naming etc., can be seen in *Appendix: C.2*.

3 Requirements and Constraints

Adding a WT to an existing water distribution network will introduce new constrains, which needs to be taken into account.

As mentioned in Section *1: Introduction*, a minimum pressure must be maintained at the end user. Furthermore the pressure can not exceed a maximum level as this will increase the possibility of water leakage and increase the wear on the pipes in the system. The system described in Section *2.1: System overview*, is designed to operate at a pressure around 0.1 bar, relative to the environment [11]. For the purpose of this project the interval for which the pressure should be within, is chosen to be between $0.08 < \boldsymbol{p_{cp}} < 0.14$ [Bar], where p_{cp} is the pressure at a critical point.

Another important aspect when implementing a WT is water quality. If the water is stored, in the WT for too long the quality will decrease due to decreasing oxygen level, thus a requirement for water quality has to be formulated. As described in Section *2.1: System overview*, the WT has one combined input/output connection. Therefore a requirement only for flow is hard to formulate as the direction will change dependent on the usage. This could result in a flow based constraint being fulfilled by rapidly changing flow direction without actually replacing any significant water volume in the tower. Instead, a requirement for how often the content of the WT should be exchanged per time unit is proposed. For the purpose of this project the minimum requirement to volume exchange, is chosen to 30% of the volume of V_T per day. This can be written as $\bar{q}_{wt} > 0.3 \cdot V_T \; [\frac{m^3}{day}]$. As stated in Section *2.1: System overview* $V_T = 200\ L$ so therefor $\bar{q}_{wt} > 0.06 \; [\frac{m^3}{day}]$. This results in the following requirements:

write that $q_w t$ is not used as a constraint

- Pressure at CP, $0.08 < \boldsymbol{p_{cp}} < 0.14$ [bar]
- Minimum water exchange , $\bar{q}_{wt} > 0.06 \; [\frac{m^3}{day}]$
- Minimizing the total cost of running the system

Modelling 4

4.1 Hydraulic modelling

All matrices should be bold font

We have to be consequent with the indexes - i or k?

Water distribution networks are designed to deliver water to consumers in terms of sufficient pressure and appropriate chemical composition. Distribution systems as such are generally consisting of four main components: pipes, pumps, valves and reservoirs. The common property is that they are all two-terminal components, therefore they can be characterized by the dynamic relationship between the pressure drop across the two endpoints and the flow through the element [12]. *Equation: (4.1)* shows the dual variables which describes one component.

$$\begin{bmatrix} \Delta P \\ q \end{bmatrix} = \begin{bmatrix} P_{in} - P_{out} \\ q \end{bmatrix} \tag{4.1}$$

Where

	Δp	is the pressure drop across the two endpoints,	[Pa]
and	q	is the flow through the element.	$\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$

In the following chapter the hydraulic model of the system is derived by control volume approach [13]. The relationship between the two variables are introduced for each component in the hydraulic network.

4.1.1 Pipe model

Pipes are important components of water distribution systems since they are used for carrying pressurized and treated fresh water. A detailed model of pipes has to be derived in order to describe the relationship of pressure and flow for each pipe component. The dynamic model of a pipe can be originated from Newton's second law. *Equation: (4.2)* describes the proportionality between the rate of change of the momentum of the water and the force acting on it.

$$\frac{d}{dt}M = \sum_i F_i \tag{4.2}$$

Where

	M	is the linear momentum of the water flow,	$\left[\frac{\mathrm{kgm}}{\mathrm{s}^2}\right]$
and	F_i	is the set of forces acting on the water.	[N]

The dynamic model of a pipe component is derived under the assumption that the flow of the fluid is uniformly distributed along the cross sectional area of the pipe. In other words, all pipes in the system are filled up fully with water all the time. Thus the density of water and the volume of the fluid is constant in time, as is the mass of the water.
Rewriting *Equation: (4.2)*, because of the above-mentioned asumptions, the mass of the water can be taken out in front of the derivative.

$$\frac{d}{dt}M = \frac{d(m_w v)}{dt} = m_w\frac{dv}{dt} = \sum_i F_i \tag{4.3}$$

Where

	m_w	is the mass of the water,	[kg]
and	v	is the value of the velocity of the water at each point of the pipe.	$\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$

The sum of the forces acting on the control volume can be seen as input forces, acting on the inlet of the pipe, output forces, acting on the outlet, resistance forces and gravitational force effect. These forces are expressed in terms of pressure in order to obtain the model of the pressure drop in the pipes. In *Figure 4.1* all forces acting on a pipe segment are shown:

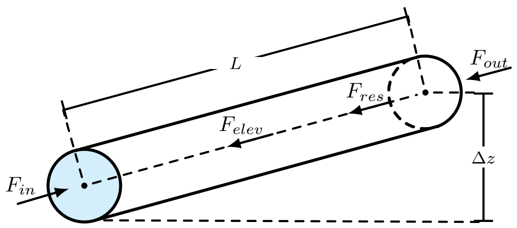


Figure 4.1. Free-body diagram describing the forces acting on a segment of a pipe.

The pipe is assumed to have a cylindrical structure. Furthermore, the cross section of the pipe, $A(x)$, is constant for every $x \in [0, L]$, where L is the length of the pipe:

$$A_{in} = A_{out} = \frac{1}{4}\pi D^2 \tag{4.4}$$

Where

	A	is the cross sectional area of a pipe,	[m^2]
and	D	is the diameter of the pipe.	[m]

Water flow, q, can be expressed in terms of velocity, v, and cross sectional area, A, resulting in:

$$q = A \cdot v \tag{4.5}$$

In *Equation: (4.6)* the forces acting on the pipe are included. The difference between F_{in} and F_{out} is the pressure drop between two endpoints, the resistance forces F_{res} and the gravitational force effect due to change in elevation F_{elev}.

$$m_w\frac{dv}{dt} = F_{in} - F_{out} - F_{res} - F_{elev} \tag{4.6}$$

In order to obtain an equation consisting of only pressure variables, the relationship between forces and pressures is used.

$$AL\rho\frac{dv}{dt} = A(p_{in} - p_{out}) - F_{res} - F_{elev} \tag{4.7}$$

Where

	m_w	is the mass of the water,	[kg]
	A	is the cross sectional area of a pipe,	$[m^2]$
	L	is the length of a pipe,	[m]
	ρ	is the density of water,	$\left[\frac{kg}{m^3}\right]$
and	p	is the pressure at a point.	[Pa]

Rewriting the velocity in terms of volumetric water flow and cross sectional area.

$$AL\rho\frac{d}{dt}\frac{q}{A} = A(p_{in} - p_{out}) - F_{res} - F_{elev} \tag{4.8}$$

By dividing the equation with the cross sectional area it can be seen that the equation is dependant on the pressure difference between two endpoints.

$$\frac{L\rho}{A}\frac{dq}{dt} = p_{in} - p_{out} - \frac{F_{res} - F_{elev}}{A} \tag{4.9}$$

Thus the desired pressure drop between two endpoint is obtained. The differential equation in *Equation: (4.10)*, describes the change in flow as a function of the pressure drops in the system.

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - \frac{F_{res} - F_{elev}}{A} \tag{4.10}$$

In *Equation: (4.10)*, the term F_{res} is the resistance force acting on the pipe, which consists of two parts: surface resistance(h_f) and the form resistance(h_m) due to the fittings.

Surface resistance (h_f)

The flow of a liquid through a pipe suffers resistance from the turbulence occurring along the internal walls of the pipe, caused by the roughness of the surface. This surface resistance is given by the Darcy-Weisbach equation [14].

$$h_f = \frac{fLv^2}{2gD} \tag{4.11}$$

Should we find another formulation then "pressure given in head" - Kind of wierd to to have pressure given as 'm'

Where

	f	is the Moody friction factor,	[·]
	h_f	is the pressure given in head,	[m]
	g	is acceleration due to gravity,	$\left[\frac{m}{s^2}\right]$
and	D	is the diameter of the pipe.	[m]

Equation: (4.11) is under the assumption that $v > 0$. Assuming that the flow is not unidirectional and substituting the velocity by the volumetric flow and pipe area:

$$h_f = \frac{8fL}{\pi^2 gD^5}|q|q \tag{4.12}$$

The unknown parameter in 4.12 is the Moody friction factor which is non-dimensional and is a function of the Reynold´s number, **Re**. This friction factor depends on whether the flow is laminar, transient or turbulent, and the roughness of the pipe [15].

The Reynold´s number can be used to determine the regime of the flow [15]. When $\boldsymbol{Re} < 2300$ as laminar, if $2300 < \boldsymbol{Re} < 4000$ as transient and if $\boldsymbol{Re} > 4000$ as turbulent.

$$\boldsymbol{Re} = \frac{vD}{\nu} \tag{4.13}$$

Where

ν is the kinematic viscosity. $\left[\frac{\text{kg}}{\text{ms}}\right]$

The kinematic viscosity in [14] is given by :

$$\nu = 1.792 \cdot 10^{-6}\left[1 + \left(\frac{T}{25}\right)^{1.165}\right]^{-1} \tag{4.14}$$

Where

T is the water temperature. [°C]

In order to estimate the range of Reynolds numbers in a common water distribution, typical values for the temperature, velocity and the radius of the pipes are considered [16].

- $v \in [0.5, 1.5] \quad \frac{m}{s}$
- $D \in [50, 1500] \quad mm$
- $T \in [10, 20] \quad ^\circ C$

These values result in a Reynold's number between 19000 and 225000, which is considered a turbulent fluid flow through the pipes. For turbulent flow the Moody friction factor is given by [14]:

$$f = 1.325\left(ln\left(\frac{\epsilon}{3.7D} + \frac{5.74}{\boldsymbol{Re}^{0.9}}\right)\right)^{-2} \tag{4.15}$$

Where

ϵ is the average roughness of the wall inside the pipe. [m]

Form resistance (h_m)

Form resistance losses, appears at any time the flow changes direction, due to elbows, bends, or due to enlargers and reducers. It is a particular frictional resistance due to the fittings of a pipe. Form loss can be expressed as:

$$h_m = k_f \frac{v^2}{2g} \tag{4.16}$$

Applying the definition of volumetric flow:

$$h_m = k_f \frac{8}{\pi^2 g D^4}|q|q \tag{4.17}$$

Where

k_f is the form-loss coefficient. [·]

The form-loss coefficient can be split into different losses depending on the fitting of the pipes.
Pipe bends are principally determined by the bend angle α and bend radius r, this is given by the following expression [14]:

$$k_f = \left[0.0733 + 0.923\left(\frac{D}{r}\right)^{3.5}\right]\alpha^{0.5} \tag{4.18}$$

Pipe elbows are also used to change the direction of the flow but providing sharp turns in pipelines. The coefficient for the losses in elbows is determined by the angle of an elbow α and is given by:

$$k_f = 0.442\alpha^{2.17} \tag{4.19}$$

Complete pipe model

In *Equation: (4.12)* and *Equation: (4.17)*, the head loss of the friction losses are determined. These terms are introduced in *Equation: (4.10)* in terms of pressure. The friction factors are multiplied by the water density and gravity. Nevertheless, the head loss due to elevation has to be added in the model, yielding the final expression:

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - h_f\rho g - h_m\rho g - \Delta z\rho g \tag{4.20}$$

Substituting the terms h_f and h_m with their respective values:

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - \frac{8fL}{\pi^2 gD^5}\rho g|q|q - k_f\frac{8}{\pi^2 gD^4}\rho g|q|q - \Delta z\rho g \tag{4.21}$$

Equation: (4.21) describes the rate of flow in terms of pressure losses due to pressure change, frictions and elevation. A more compact form can be expressed for the kth component as such:

$$J_k\dot{q_k} = \Delta p_k - \lambda_k(q_k) - \zeta_k \tag{4.22}$$

Where

	J_k	is an analogous parameter as inertia for the water,
	$\lambda_k(q_k)$	is the friction as a function of flow,
and	ζ_k	is the pressure drop due to the elevation.

As can be seen in *Equation: (4.22)*, the flow dynamics of the kth pipe is described by J_k, which is an analogous parameter as inertia in mechanical systems. J_k is a diagonal matrix with zeros for the diagonal elements not related to a pipe, $\boldsymbol{J} = diag(J_i)$.

It is assumed, prior to the tests carried out on the system, that the presence of the water tower in the system has a slow effect on the flow due to slow integration behavior. This means that the water tower might have a relatively big time constant compared to the time constant of the pipes. Due to this consideration, a fair assumption would be that the parameter J_k does not influence the flow in the system significantly and it could be neglected. However, the parameter is kept until this assumption is verified by tests. The complete model of a pipe yields:

$$\Delta p_k = \lambda_k(q_k) + J_k\dot{q_k} + \zeta_k \tag{4.23}$$

4.1.2 Valve model

Valves in the water distribution system are modeled according to the assumption that the length of each valve, L, and the change in elevation, Δz, is zero. Therefore it is assumed that the length of the valve does not influence the flow and the pressure between the endpoints. The fact that the length of a valve is considerably smaller than the length of a pipe makes this a fair assumption. Another assumption is that in case of a valve, elevation is not present.

In the given system, valves are considered as end-user components since they are placed only in the PMAs. These user valves have a variable Opening Degree(OD) which influences the pressure drop across the endpoints.

In case of valves, manufacturers provide a parameter which indicates the valve capacity. This coefficient is called the k_{v100}- factor that describes the conductivity of the valve at maximum OD. This parameter sets the relationship between the water flow through the valve in m^3 in one hour. The experiments were carried out with a pressure drop of $\Delta p = 1[bar]$ at a fully open state of the valve. According to [17], the properties of water fulfil the requirements which allows to write up the following expression for flow and pressure:

$$q = k_{v100}\sqrt{\Delta p} \tag{4.24}$$

Where
k_{v100} is the valve maximum capacity factor. $\left[\frac{\mathrm{m}^3}{\mathrm{h}}\right]$

Equation: (4.24) can be derived in detail using the law of continuity for each endpoint of the valve, however the exact derivations can be found in the datasheet [17]. In the further description and derivations, the coefficients and all the technical considerations are based on this datasheet.

Valve conductivity function $k_v(OD)$

Instead of k_{v100}, more generally $k_v(OD)$ can be used which is a function of the opening degree, where $OD \in [0, 1]$. In case of user-operated valves, k_v does not remain constant, it ranges over a compact set of values as the opening degree varies. [12].

All valves in the system share the same characteristics, therefore the following characteristics of k_v are valid for all of them.

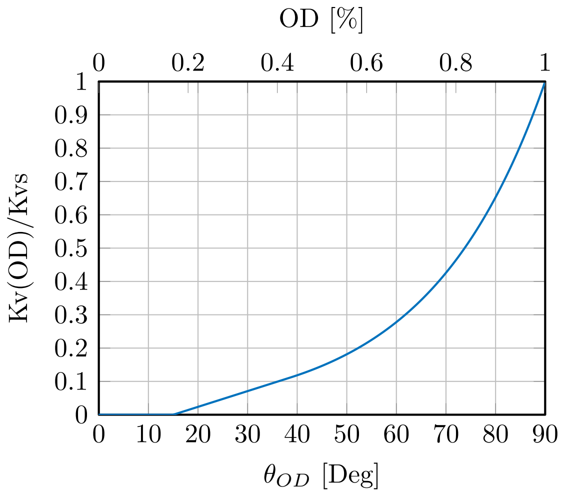


Figure 4.2. Valve characteristics - Valve conductivity in the function of OD.

According to [18], the following definition can be written up generally for the conductivity function, $k_v(OD)$:

$$k_v(OD) = \begin{cases} k_{v100}\,\frac{\theta_{OD}}{\theta_{max}}\,n_{gl}\,e^{(1-n_{gl})}, & \text{if } \frac{\theta_{OD}}{\theta_{max}} \leq \frac{1}{n_{gl}}; \\ k_{v100}\,e^{(n_{gl}\,(\frac{\theta_{OD}}{\theta_{max}}-1))}, & \text{if } \frac{\theta_{OD}}{\theta_{max}} \geq \frac{1}{n_{gl}} \end{cases} \tag{4.25}$$

Where

	θ_{OD}	is the opening degree,	[°]
	θ_{max}	is the upper opening degree,	[°]
and	n_{gl}	is the valve characteristic curve factor.	[·]

A new parameter, θ_{max}, is introduced which describes the maximum angle where the actuator closes the valve. The same can be stated for a minimum angle. The valve is closed when the position of the actuator $\in [0°, 15°]$. As a consequence, there is an offset in the curve as it is shown in *Figure 4.2*. Introducing the following angle:

$$\gamma = \frac{\theta_{OD} - \theta_{off}}{\theta_{max} - \theta_{off}} \tag{4.26}$$

Where

θ_{off}	is the minimum angle where the valve opens.	[°]

In case of the water distribution system *Equation: (4.25)* modifies to:

$$k_v(OD) = \begin{cases} k_{v100}\,\gamma\,n_{gl}\,e^{(1-n_{gl})}, & \text{if } \gamma \leq \frac{1}{n_{gl}}; \\ k_{v100}\,e^{(n_{gl}\,\gamma)}, & \text{if } \gamma \geq \frac{1}{n_{gl}} \end{cases} \tag{4.27}$$

As it is shown, the conductivity function of the valve consists of two types of functions:

$$k_v(OD) = \begin{cases} k_v(\theta_{OD}) \sim linear(), & \text{if } \gamma \leq \frac{1}{n_{gl}}; \\ k_v(\theta_{OD}) \sim exponential(), & \text{if } \gamma \geq \frac{1}{n_{gl}} \end{cases} \tag{4.28}$$

Since exponential functions never cross the zero point, it is reasonable to use linear characteristics in the lower range. The transition from linear to exponential has to be continuously differentiable and predetermined by n_{gl} [12, 18]

Complete valve model

Using *Equation: (4.24)* with the conductivity function $k_v(OD)$ and expressing Δp yields:

$$\Delta p = \frac{1}{k_v(OD)^2}|q|q \tag{4.29}$$

Describing it in a compact form for the k^{th} valve in the network yields:

$$\Delta p_k = \mu_k(q_k, k_v(OD)) \tag{4.30}$$

4.1.3 Pump model

In order to move water from the reservoirs to the consumers, pumping is required. To guarantee that the water reaches every end-user with the appropriate pressure, different pumps can be used in the water distribution system.
Centrifugal pumps are good for this purpose, as the output is steady and consistent. A model describing the pressure drop is derived which is presented in detail in [19]. The pressure provided by the pump is given by:

$$\Delta p = -a_{h2}{q_i}^2 + a_{h1}\omega_r q_i + a_{h0}{\omega_r}^2 \tag{4.31}$$

Where

	Δp	is the pressure produced by the pump,	[Bar]
	a_{h2}, a_{h1}, a_{h0}	are constants describing the pump,	$[\cdot]$
	q_i	is the volume flow through the impeller,	$\left[\frac{\mathrm{m}^3}{\mathrm{h}}\right]$
and	ω_r	is the impeller rotational speed.	$\left[\frac{\mathrm{rad}}{\mathrm{s}}\right]$

The pump model is based on the parameters a_{h2}, a_{h1}, a_{h0}, which are provided in the data sheet and are scaled to units of pressure in [Bar] and the unit for flow in $[\frac{m^3}{h}]$.

Hydraulic power and efficiency

To minimize the running cost of the plant, it is necessary to know the power consumption of the pumps to describe how much energy these consume. The electricity consumption of the pump, as a function of flow q, is described in [11]. This expression can be used to calculate the efficiency η, as a function of flow q of the pump. By assuming η to be constant the expression becomes simpler, however η must be chosen within the operating area of the pump. The electrical power can then be calculated by taking the hydraulic power created by the pumps and the efficient of the pumps into account.

The hydraulic power created by a pump can be described by an equivalent to Joule's law, that is in terms of the pressure difference across the pump, multiplied with the flow through it, see *Equation: (4.32)*.

$$P_h = \Delta p \cdot q \tag{4.32}$$

Where

P_h	is the hydraulic power.	[W]

The electrical power used for running the pump can then be described through the relation between the hydraulic power and the efficiency, η. This can be seen in *Equation: (4.33)*.

$$P_e = \frac{1}{\eta} \cdot \Delta p \cdot q \tag{4.33}$$

Where

P_e	is the power consumption of the pump,	[W]
η	is the efficiency of the pump.	$[\cdot]$

Equation: (4.33) will be used in a later section to minimize this cost of running the system.

4.1.4 Water Tower

Water towers are used to maintain the correct pressure level in different systems, ensure reliability and to improve the optimality of the water supply. The WT plays a determinative role in the flow control. Therefore its dynamic model must be derived.

Similarly to the modeling of the other components, the relation between the two dual variables, pressure difference and flow is derived. The structure of the WT is illustrated in *Figure 4.3*.

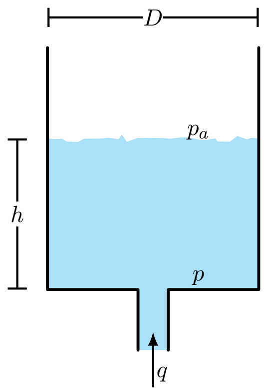


Figure 4.3. Sketch of the open water tower.

In *Figure 4.3*, p_a represents the pressure at the surface of the water, thus is it always describing the atmospheric pressure. The variable p is used to describe the pressure value on the bottom of the tank.
The rate of change of the fluid volume in the water tower is proportional to the volumetric flow at which water enters or leaves the tank.

$$q = \frac{dV_t}{dt} = A\frac{dh}{dt} \tag{4.34}$$

Where

	h	is the height of the fluid in the WT,	[m]
	V_t	is the volume of the WT,	[m^3]
	A	is the cross section of the WT which is assumed to be constant for $y \in [0, h]$,	[m^2]
and	q	is the volumetric flow.	$\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$

The force on the bottom of the WT is due to the weight of water. According to Newton's second law:

$$F = mg = \rho g V_t \tag{4.35}$$

Where

ρ	is the density of water.	$\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$

Equation: (4.35) can be rewritten in terms of pressure such as:

$$\frac{F}{A} = \rho g h = p - p_a = \Delta p \tag{4.36}$$

The total pressure on the bottom of the WT is a result of the pressure difference due to the fluid, p, and the atmospheric pressure, p_a. However, the model is derived in such a way that the atmospheric pressure is set to zero. Therefore, if the water is assumed to be incompressible, density does not change with pressure, *Equation: (4.34)* can be written as:

$$q = \frac{dV}{dt} = \frac{A}{\rho g}\frac{d}{dt}\Delta p = C_H \Delta\dot{p} \tag{4.37}$$

Where

C_H	is the hydraulic capacitance.	$\left[\frac{\mathrm{m}^3}{\mathrm{N/m}^2}\right]$

This equation shows proportionality between pressure and the volume of water, which is exactly the defining characteristic of a fluid capacitor. When the fluid capacitance is large, corresponding to a tank with a large area, a large increase in volume is accompanied by a small increase in pressure.

An analogy can be made between an electronic circuit and the hydraulic system, where the WT acts as a capacitor. Deriving the relationship between the voltage and the charge of the capacitor:

$$I = C\frac{dU}{dt} \tag{4.38}$$

Where

U	is the voltage,	[V]
C	is the capacitance.	[F]

In *Equation: (4.37)* the volume flow rate, q, is equivalent to the current, I, in a circuit and the constant term, $\frac{A}{\rho g}$, is equivalent to the capacitance of a capacitor, C. The voltage drop is analogous to the pressure drop in the water system.

4.1.5 Complete component model

Gathering the pressure drops, caused by each type of component in the system, a complete system model can be obtained. This model includes the pipe, valve, pump elements and the WT.

The model of the WT is described by a first order differential equation, consisting of the first time derivative of the pressure drop. The final expression is shown in *Equation: (4.39)*:

$$\Delta \dot{p}_{WT;k} = \frac{1}{C_{H;k}} q_k \tag{4.39}$$

Although *Equation: (4.39)* includes indexing for the pressure drops across the WTs, it is worth mentioning that the water distribution system consists of only one WT.
The complete model consists of the pipe model, *Equation: (4.30)*, the valve model, *Equation: (4.31)*, the pump model and the WT, *Equation: (4.39)*. For the pressure drop across the k^{th} component the following expression can be written:

$$\Delta p_k = \underbrace{\lambda_k(q_k) + \zeta_k + J_k \dot{q}_k}_{\text{Pipe}} + \underbrace{\mu_k(q_k, k_v)}_{\text{Valve}} - \underbrace{\alpha_k(u_k)}_{\text{Pump}} + \underbrace{\Delta p_{WT;k}}_{\text{Water tank}} \tag{4.40}$$

The complete component model, *Equation: (4.40)*, is used to represent the pressure loss or contribution across each component. In order to describe every part of the system by *Equation: (4.40)*, the parameters and functions corresponding to the specific part of the system are selected. The remaining expressions are set to zero if the model does not match the specific part of the network. In other words, if the k^th element of the system is a pump, then only $\alpha_k(u_k)$ is taken into account and the rest of the expressions are set to zero. *Table: 4.1* shows the parametrization of the system:

Component	J_k	λ_k	μ_k	α_k	ζ_k	$\Delta p_{WT;k}$
Pipe	J_k	λ_k	0	0	ζ_k	0
Valve	0	0	μ_k	0	0	0
Pump	0	0	0	α_k	0	0
Water tower	0	0	0	0	0	1

Table 4.1. Complete model parametrization.

Unit transformation

During the derivation of the dynamic model, the unit of the physical variables are considered as pascals and seconds. However, it is concluded in a later chapter that the flow is significantly small compared to the pressure if the SI-units are kept. Therefore a unit conversion is carried out from Pascal[Pa] to [bar]s and from seconds[s] to hours[h]. Another reason which makes this conversion reasonable is that the conductivity function,k_{v100} in Section *4.1.2: Valve model*, is derived under the condition that the pressure drop is one bar [18]. The time scaling is due to conventions. Among the research community in hydraulics, a convenient way to handle the volumetric flow is in $[m^3/h]$ instead of $[m^3/s]$. The detailed derivation of the unit conversion can be found in *Appendix: A*. The result is stated here:

we need to ref to the chapter where we conclude this or then we need a source

$$\frac{L\rho}{A \cdot 10^5}\frac{d}{dt}\frac{q}{3600} = \Delta\frac{p}{10^5} - \Big(\frac{8fL}{\pi^2 g D^5 \cdot 10^5} + k_f\frac{8}{\pi^2 g D^4 \cdot 10^5}\Big)\rho g\frac{|q|}{3600}\frac{q}{3600} - \frac{\Delta z \rho g}{10^5} \tag{4.41}$$

4.2 Simplification and electrical analogy

After deriving the dynamics of all elements in the network, the complete system can be drawn. In *Appendix: C.2*, the topology of the test system is described in detail. In the following section, all conclusions and notations are based on the system diagram placed in the appendix.

The way of modelling a hydraulic system is in some way analogous to an electric circuit. Most of the various hydraulic components can be represented as electronic equivalents and vice versa, however there are some differences too. It should be emphasized that in hydraulic networks there are not such phenomenon as magnetic flux.

In the block diagram of the system, nodes are introduced which represent different potential points in the system. This is equivalent to hydraulic pressures. Nodes represent points in the system where pressure might have different values due to the elements e.g pipes, valves and pumps, placed between them. These points represent interconnection between hydraulic components and take into account the fact that each individual component in the system has an effect on the pressure drop on their two corresponding endpoints. Therefore nodes are present at all places where the pressure value is different due to the components of the network.

In the network, volumetric flow rate is equivalent to current and the quantity of water has similar representation as charge in an electric circuit. Again, it should be noted that e.g. the water quantity cannot be affected by magnetic fields, therefore the word: similar.

Although nodes can be placed across all the component endpoints, some simplifications are introduced in the network. These simplifications do not change the way how the system is described. There are two different types of simplification in the network.

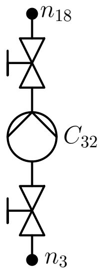


Figure 4.4. Simplifications: The rotational speed of the pump, $\omega = 0$ and therefore this part is modelled differently.

The WT is connected to the rest of the system with three components, a pump, C_{32}, and two valves. This is shown in *Figure 4.4*, where the components are shown between n_3 and n_{18} which also connects the WT to the system. In this particular case the pump is turned off, however contributes to the pressure drop due to its resistance. The same can be said for the valves, except that they are fully open at all time but they modify the flow. Extra nodes are not introduced between the valves and the pump, instead the series connection is seen as one component. This can be modelled by lumping the resistance of the valve, *Equation: (4.29)*, into the model of the pump, *Equation: (4.31)*, when the rotational speed is zero. Thus the following model yields for the case when $\omega = 0$:

$$\Delta p = \Big(\frac{2}{k_{v100}^2} - a_{h2}\Big)|q|q \tag{4.42}$$

For the case where $\omega \neq 0$, a model including the pump and both valves can also be made. This is done in the same manner:

$$\Delta p = \Big(\frac{2}{k_{v100}^2} - a_{h2}\Big)|q|q + a_{h1}\omega_r q + a_{h0}{\omega_r}^2 \tag{4.43}$$

The case when $\omega = 0$ applies to one component between (n_3-n_{18}). The case when $\omega \neq 0$ applies to the components between (n_1-n_2), (n_1-n_7), (n_4-n_8) and (n_5-n_{13}). It should be mentioned however that all these subsystems inside this simplified model are modelled as described in Section *4.1.5: Complete component model.*

Since the components influence the pressure between the endpoints, they behave similarly as electric components. Valves are considered as nonlinear resistors, since the pressure is the quadratic function of the flow and they have a resistance depending on the OD. The model of the pipes is equivalent to a series connection of a linear inductor and a non-linear resistor. The pumps provide pressure and therefore drive flow in the system. They can be seen as voltage generators. The WT is a capacitor, as it is described in Section *4.1.4: Water Tower.* The equivalence between the hydraulic and electric system is summarized in *Table: 4.2.*

Hydraulic system	**Electrical system**
Valve	Nonlinear resistor
Pipe	Linear inductor with a nonlinear drift term
WT	Capacitor
Pressure	Voltage
Flow	Current
Pumps	Voltage source

Table 4.2. Equivalence of an electrical and hydraulic network.

4.3 Graph representation

A graph is a formal mathematical way for representing a network, which can be applicable among others in engineering or scientific context such as in mechanical systems, electrical circuits and hydraulic networks [20].

The modelling of the water distribution network is done with the help of Graph Theory(GT). Each terminal of the network is associated with a node and the components of the system correspond to edges [21].

Incidence matrix

The incidence matrix, $\boldsymbol{H}$, of a graph with n nodes and e edges is defined by $\boldsymbol{H} = [a_{ij}]$. Where the number of rows and columns are defined by the amount of nodes and edges respectively. Additionally, the particular node and edge is denoted with the indices i and j.

In case of a hydraulic network, the edges are directed in order to keep track of the direction of the flows in the system. It results in a directed incidence matrix as described below:

$$a_{ij} = \begin{cases} 1 & \text{if the } j^{th} \text{ edge is incident out of the } i^{th} \text{ node} \\ -1 & \text{if the } j^{th} \text{ edge is incident into the } i^{th} \text{ node} \\ 0 & \text{otherwise} \end{cases} \tag{4.44}$$

In *Appendix: C.5* the corresponding incidence matrix of the system is shown.

Cycle matrix

A spanning tree, $T \in \mathcal{G}$ is a subgraph which contains all nodes of $\mathcal{G}$ but has no cycles [22]. In order to obtain the spanning tree it is necessary to remove an edge from each cycle of the graph. The removed edges are called chords. The number of chords, l, are governed by the following expression:

$$l = e - n + 1 \tag{4.45}$$

By adding any additional chord to T, a new cycle is created which is called a fundamental cycle. A graph is conformed by as many fundamental cycles as many chords it has [22]. The set of fundamental cycles correspond to the fundamental cycle matrix $\boldsymbol{B}$, such as the number of rows and columns are defined by the amount of chords and edges, respectively.

The cycle matrix of a directed graph can be expressed with $\boldsymbol{B} = [b_{ij}]$ where i and j denote the chords and edges:

$$b_{ij} = \begin{cases} 1 & \text{if the edges } j^{th} \text{ is in the cycle } i^{th} \text{ and the directions match} \\ -1 & \text{if the edges } j^{th} \text{ is in the cycle } i^{th} \text{ and the directions are opposite} \\ 0 & \text{otherwise} \end{cases} \tag{4.46}$$

In *Appendix: C.6* the corresponding cycle matrix of the system is shown.

Kirchhoff's Law

In the same way as it is described in Section *4.1: Hydraulic modelling*, the graph of a hydraulic network assigns dual variables to every edge: the pressure, $\Delta p_k(t)$, and the flow, $q_k(t)$ in the function of time. These two variables are vectors containing the individual flows through the edges and the pressure drop across them:

$$\boldsymbol{\Delta p(t)} = \begin{bmatrix} \Delta p_c \\ \Delta p_f \\ \vdots \\ \Delta p_e \end{bmatrix} \text{ and } \boldsymbol{q(t)} = \begin{bmatrix} q_c \\ q_f \\ \vdots \\ q_e \end{bmatrix} \tag{4.47}$$

In order to derive a model for the hydraulic network, a set of independent flow variables are identified [23]. These flow variables have the property that their values can be set independently from other flows in the network and they coincide with the flows through the chords. Therefore it is convenient to choose the column indexing of the $\boldsymbol{H}$ and $\boldsymbol{B}$ matrix, such as:

$$\boldsymbol{H} = [\boldsymbol{H_c} \quad \boldsymbol{H_f}] \text{ and } \boldsymbol{B} = [\boldsymbol{B_c} \quad \boldsymbol{B_f}] = [\boldsymbol{I} \quad \boldsymbol{B_f}] \tag{4.48}$$

Where

$\boldsymbol{H_c}$ and $\boldsymbol{B_c}$	are the matrices corresponding to the chords,
$\boldsymbol{H_f}$ and $\boldsymbol{B_f}$	are the matrices corresponding to the spanning tree.

Since the edge variables are governed by elements interconnected in the network, they must obey the law of conservation of mass and pressure [22].

Kirchhoff´s Current Law (KCL) states that the net sum of all the flows leaving and entering a node is zero. Formulating this statement in matrix form:

$$\boldsymbol{H} \cdot \boldsymbol{q(t)} = 0 \tag{4.49}$$

Furthermore, regarding Kirchhoff´s Voltage Law (KVL) it is stated that at any time the net sum of the pressure drops in a cycle is zero. In terms of matrix form:

$$\boldsymbol{B} \cdot \boldsymbol{\Delta p(t)} = 0 \tag{4.50}$$

where the fundamental loops have a reference direction given by the direction of the chords.

4.3.1 Network model

Once the corresponding incidence and cycle matrices are identified, and the analogy between hydraulic and electrical circuits is concluded, the whole hydraulic network can be described in a compact, generalized form as a set of differential equations. In this section an abstract and general form of the network model is derived using all the previously obtained expressions.
In *Appendix: C.5*, the form of the incidence matrix is shown. The last column of $\boldsymbol{H}$ represents the edge that belongs to the WT. The number of edges representing the WT is one and in the further model description is denoted with w.
Hence, the $\boldsymbol{H}$ matrix can be written as

$$\boldsymbol{H} = [\boldsymbol{H_1} \quad \boldsymbol{H_0}] \tag{4.51}$$

Where

	$\boldsymbol{H_1} \in \mathbb{R}^{n\times(e-w)}$	is the $\boldsymbol{H}$ matrix without the edge corresponding to the WT,
and	$\boldsymbol{H_0} \in \mathbb{R}^{n\times w}$	is the $\boldsymbol{H}$ matrix with the column corresponding to the WT.

Similarly, the fundamental cycle matrix, B, is structured such as the last column agrees with the edge representing the WT.

$$\boldsymbol{B} = [\boldsymbol{B_1} \quad \boldsymbol{B_0}] \tag{4.52}$$

Where

	$\boldsymbol{B_1} \in \mathbb{R}^{l\times(e-w)}$	is the $\boldsymbol{B}$ matrix without the edge corresponding to the WT,
and	$\boldsymbol{B_0} \in \mathbb{R}^{l\times w}$	is the $\boldsymbol{B}$ matrix with the column corresponding to the WT.

As mentioned in Section *4.3: Kirchhoff's Law*, $\boldsymbol{q}$ is a vector containing all the individual flows, which can be structured as follows:

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{q_1} \\ \boldsymbol{q_0} \end{bmatrix} \tag{4.53}$$

Where

$\boldsymbol{q_1} \in \mathbb{R}^{(e-w)\times 1}$	is the flow through all edges expect for WT,
$\boldsymbol{q_0} \in \mathbb{R}^{w\times 1}$	is the flow through the edge belonging to the WT.

The vector containing the pressures at the nodes can be also structured as

$$\boldsymbol{p} = \begin{bmatrix} \boldsymbol{p_1} \\ \boldsymbol{p_0} \end{bmatrix} \tag{4.54}$$

Where

$\boldsymbol{p_1} \in \mathbb{R}^{(n-w)\times 1}$	is the pressure at all nodes expect for the WT,
$\boldsymbol{p_0} \in \mathbb{R}^{w\times 1}$	is the pressure in the WT.

In *Equation: (4.49)* KCL is applied to $\mathcal{G}$, which states that the sum of all flows entering into a node must be equal to the sum of the flows leaving the node.

By choosing independent set of flows corresponding to the chords of a spanning tree, the flow through every edge of the hydraulic system can be expressed in terms of the flow through the chords, z [22]. The chord flows make it possible to deal with less variables, thus making the set of differential equations easier to handle. The elements of $\boldsymbol{z}$ are called the free flows of the system and are independent from each other[21].

$$\boldsymbol{q_i} = \boldsymbol{B_i^T z} \tag{4.55}$$

Where

$\boldsymbol{z} \in \mathbb{R}^{(1\times g)}$	is the chord flow vector and g is the number of elements.

As shown in *Equation: (4.55)*, the i^{th} flow in the system is defined by the i^{th} column of the cycle matrix and the vector of chord flows, z.

Before writing up an expression that describes all parts, the component model, *Equation: (4.40)*, needs to be modified with the simplifications introduced in Section *4.2: Simplification and electrical analogy*. As mentioned in Section *4.2: Simplification and electrical analogy*, there are four pumps in the system, two main pumps and two PMA pumps, which provide a pressure according to the input signals. However there is on case between (n_3-n_{18}), see *Appendix: C.2*, where the pump act as a resistance for the series connection. This is because the pump is inactive in the system. In this case the corresponding edge does not acts as an input but can be described by *Equation: (4.42)*. Therefore *Equation: (4.40)* is structured in such a way that the edge corresponding to the connection between the WT and the system is represented separately, thus *Equation: (4.40)* can be rewritten as *Equation: (4.56)*.

Kris or Ignacio: Half of this part has been rewritten due to

$$\Delta p_i = \underbrace{\lambda_i(q_i) + \zeta_i + J_i\dot{q}_i}_{\text{Pipe}} + \underbrace{\mu_i(q_i, k_{v;i})}_{\text{Valve}} - \underbrace{\tilde{\alpha}_i(u_i)}_{\text{Pump+valves}} + \underbrace{\Delta p_{WT;i}}_{\text{Water tank}} + \underbrace{\gamma_i(q_i)}_{\text{WT-connection}} \tag{4.56}$$

$$\tilde{f}_i(\boldsymbol{q_i}, \boldsymbol{\omega_i}, \boldsymbol{k_v}) = \lambda_i(\boldsymbol{q_i}) + \zeta_i + \mu_i(\boldsymbol{q_i}, \boldsymbol{k_v}) - \tilde{\alpha}_i(\boldsymbol{\omega_i}) + \gamma_i(\boldsymbol{q_i}) \tag{4.57}$$

Where

$$\tilde{f}_i = -\boldsymbol{C_{pi}}\boldsymbol{q_i}|\boldsymbol{q_i}| \qquad \text{for } i = 2,3,4,5,6,7,10,11,12,14,17,18,19,21,23 \tag{4.58}$$
$$\tilde{f}_i = -\boldsymbol{C_{vi}}\boldsymbol{q_i}|\boldsymbol{q_i}| \qquad \text{for } i = 13,15,20,22 \tag{4.59}$$
$$\tilde{f}_i = \Big(\frac{2}{k_{v100}^2} - a_{h2i}\Big)|\boldsymbol{q_i}|\boldsymbol{q_i} + a_{h1i}\boldsymbol{\omega_i}\boldsymbol{q_i} + a_{h0i}\boldsymbol{\omega_i}^2 \qquad \text{for } i = 1,8,9,16 \tag{4.60}$$
$$\tilde{f}_i = \Big(\frac{2}{k_{v100}^2} - a_{h2i}\Big)|\boldsymbol{q_i}|\boldsymbol{q_i} \qquad \text{for } i = 24 \tag{4.61}$$
$$\tilde{f}_i = \Delta p_{WT} \qquad \text{for } i = 25 \tag{4.62}$$

The following hydraulic network model shows an overall model along with the above-mentioned considerations.

$$\boldsymbol{\Delta p_1} = \boldsymbol{J}\dot{\boldsymbol{q}}_{\boldsymbol{1}} + \tilde{f}(\boldsymbol{q_1}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.63}$$

In *Equation: (4.63)* the hydraulic network model is described in terms of the flow through all the nodes and derived from the inertia model in Section *4.1.1: Pipe model.* In order to reduce the order of the model and hence, the amount of unknowns, the chord flows according to *Equation: (4.55)* are applied.

$$\Delta \boldsymbol{p_1} = \boldsymbol{J}\boldsymbol{B_1}^T\dot{\boldsymbol{z}} + f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.64}$$

Making use of the identity shown in *Equation: (4.50)*, the following is obtained

$$0 = \boldsymbol{B_1}\boldsymbol{\Delta p_1} = \boldsymbol{B_1}[\boldsymbol{J}\boldsymbol{B_1}^T\dot{\boldsymbol{z}} + f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v})] \tag{4.65}$$

Isolating the inertia matrix to the left side

$$-\boldsymbol{B_1}\boldsymbol{J}\boldsymbol{B_1}^T\dot{\boldsymbol{z}} = \boldsymbol{B_1}f(\boldsymbol{B_1}^T\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.66}$$

It is desired to know the value of the flow through the chords, hence the equation above is solved for $\dot{\boldsymbol{z}}$. In order to invert $(\boldsymbol{B_1}\boldsymbol{J}\boldsymbol{B_1}^T)$ it has to be non-singular i.e. invertible.

Setting $\boldsymbol{\mathcal{J}} = \boldsymbol{B_1}\boldsymbol{J}\boldsymbol{B_1}^T$, then for the term $\boldsymbol{\mathcal{J}}$ to be positive-definite it has to be a square matrix and its determinant has to be non-zero. Note that $\boldsymbol{\mathcal{J}}$ is

$$\boldsymbol{\mathcal{J}} = (\boldsymbol{I} \quad \boldsymbol{B_f})\begin{pmatrix}\boldsymbol{J_c} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{J_f}\end{pmatrix}\begin{pmatrix}\boldsymbol{I}\\ \boldsymbol{B_f}^T\end{pmatrix} = \boldsymbol{J_c} + \boldsymbol{B_f}\boldsymbol{J_f}\boldsymbol{B_f}^T \tag{4.67}$$

Where

$\boldsymbol{J_c} \in \mathbb{R}^{l\times l}$	is the inertia in the chord components,
$\boldsymbol{J_f} \in \mathbb{R}^{f\times f}$	is the inertia in the components of the spanning tree.

$\boldsymbol{J_c}$ is a diagonal inertia matrix containing the chord elements. Since all the components corresponding to a chord in $\boldsymbol{\mathcal{G}}$ are pipes, all the diagonal terms are positive. Thus, $\boldsymbol{J_c} > 0$.

Nevertheless, if there is a chord corresponding to a non-pipe element, *Equation: (4.67)* would still be positive-definite as long as it is possible to create a spanning tree containing all chords as pipe elements from $\boldsymbol{\mathcal{G}}$ [23].

For the remaining term $\boldsymbol{B_f J_f B_f}^T$, $\boldsymbol{J_f}$ is a non-negative matrix as all its elements are zero or describe the inertia of a pipe. Multiplying $\boldsymbol{B_f J_f B_f}^T$ by a non-zero vector column $\mathbf{x}$ and its transpose $\mathbf{x}^T$

$$\boldsymbol{x}^T \boldsymbol{B_f J_f B_f}^T \boldsymbol{x} \tag{4.68}$$

Creating a new variable $\boldsymbol{y} = \boldsymbol{B_f}^T \mathbf{x}$ and applying the definition of positive semi-definiteness [24]

$$\boldsymbol{y}^T \boldsymbol{J_f} \boldsymbol{y} \geqslant 0 \tag{4.69}$$

Thus, *Equation: (4.67)* is positive definite and it provides a sufficient condition for $\boldsymbol{\mathcal{J}}$ being invertible.

Therefore, the system can be described as follows

$$\dot{\boldsymbol{z}} = -(\boldsymbol{B_1 J B_1}^T)^{-1} \boldsymbol{B_1} f(\boldsymbol{B_1}^T \boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.70}$$

4.3.2 Pressure drop across the nodes

Equation: (4.63) describes the system by the pressure across each element except the part including the water tower. The dynamics are determined by the inertia of the pipes while the pressure relations are described by the vectorfield f. The same equation can be expressed with a reduced set of equation system with the help of the chord flows:

$$\boldsymbol{\Delta p_1} = \boldsymbol{J B_1}^T \dot{\boldsymbol{z}} + f(\boldsymbol{B_1}^T \boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.71}$$

The flow rate through the chords is found in *Equation: (4.70)*, thus the expression for $\boldsymbol{\Delta p_1}$ can be rewritten as

$$\boldsymbol{\Delta p_1} = \boldsymbol{J B_1}^T [-(\boldsymbol{B_1 J B_1}^T)^{-1} \boldsymbol{B_1} f(\boldsymbol{B_1}^T \boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v})] + f(\boldsymbol{B_1}^T \boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.72}$$

Writing in short form:

$$\boldsymbol{\Delta p_1} = (-\boldsymbol{J B_1}^T (\boldsymbol{B_1 J B_1}^T)^{-1} \boldsymbol{B_1} + \boldsymbol{\mathcal{I}}) f(\boldsymbol{B_1}^T \boldsymbol{z}, \boldsymbol{w}, \boldsymbol{k_v}) \tag{4.73}$$

a general form of the network without the part corresponding to the WT ($\boldsymbol{p_0}$ and $\boldsymbol{q_0}$) is obtained. It should be noted that the same structure applies for the complete network, which is extended with the WT. In the following sections this general model is used in a slightly different form, which is suitable for the different estimation methods.

4.4 Nonlinear Parameter identification

The behavior of the complete water distribution system is governed by the previously derived model, however certain parameters of the system are either unknown or can vary significantly from the assumed design values. Furthermore, the obtained model of the system gives non-linear relations between flows and pressures in each individual components.
In case of the valves, the conductivity function is dependent on the OD, therefore the parameter of these elements are considered to be known. The centrifugal pumps are fully described by their models and by the coefficients provided by the manufacturer. The hydraulic capacity is also considered as known in case of the WT. However, certain parameters in the model of the pipes are uncertain. Even though the necessary friction parameters can be found in the data sheet provided by the manufacturer, these values are only acceptable for new pipes, as over time material can build up on the inside of the pipes, since the laboratory setup to a large extent is built from PEX/PEM (plastic) pipes. On the other hand, the physical parameters of the pipe volumes are assumed to be known to an accuracy where there is not any benefits from estimating it. Therefore the inertia matrix is known.
Consequently, the aim of the system identification in case of the water distribution system is to estimate the missing parameters which describe the frictions and form losses in the pipes, therefore define the additional pressure losses. Due to these considerations, the importance of obtaining accurate parameters is essential in order to setup a simulation that represents the real test setup.

The block diagram of a general parameter identification method is described in *Figure 4.5* below:

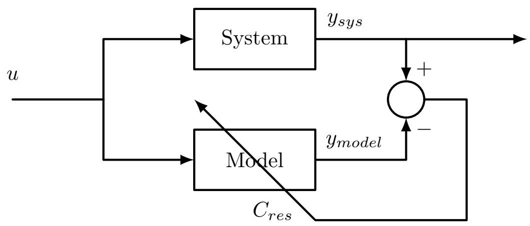


Figure 4.5. Parameter identification block diagram.

As it is shown in the figure, the measurements from the real life system are compared to the output of the simulation introducing the same input for both systems.

In order to obtain measurements from the real life test setup, the system has to be excited by various input signals. The inputs to the system are the input signals to the pumps, however in case of the parameter estimation the OD of the valves is also considered as an input. Therefore it is reasonable to reformulate the equation describing the network in *Equation: (4.70)* into such a form where the expressions for the inputs and states are isolated. Such an expression can be obtained for the whole network from the general model as follows:

$$\boldsymbol{B}\boldsymbol{J}\boldsymbol{B}^T\dot{\boldsymbol{z}} = \boldsymbol{B}g(\boldsymbol{B}^T\boldsymbol{z}) + \boldsymbol{B}u(\boldsymbol{\omega}, \boldsymbol{k_v}, \boldsymbol{B}^T\boldsymbol{z}) \tag{4.74}$$

where the vectorfield $u(.)$ contains all the functions for the elements which are dependent on the inputs and the vectorfield $g(.)$ describes the rest of the resistance terms which are responsible for the pressure drops in any part of the network. Although $u(.)$ is a function of ω and k_v, in the simulation the inputs to the pumps are specified as pressure differences, dp, because the value of these pressures are available on the test setup.

In the system, outputs are defined as differential pressures according to the available sensors on the test setup. From the system setup 8 different relative pressures can be measured. Following the notation of *Figure C.2*, sensors are placed in: n_2 n_4 n_5 n_7 n_{10} n_{11} n_{15} n_{16}. During the parameter identification, these measurements are compared to the output from the simulation and the parameters are varied until they fit.

It is important to point out that the estimation is applied for steady-state which is reasonable, since the unknown parameters are the resistances and form losses and the inertia and the capacitance only affect the dynamics, therefore have no influence on the steady-state. The inertia of the pipes, and the capacitance of the WT are considered as known parameters.

Taking the steady-state into account, the system equation for the nonlinear parameter estimation can be rewritten as:

$$0 = \boldsymbol{B}g(\boldsymbol{B}^T\boldsymbol{z}) + \boldsymbol{B}u(\boldsymbol{\omega}, \boldsymbol{k_v}, \boldsymbol{B}^T\boldsymbol{z}) \tag{4.75}$$

The aim of the parameter identification is therefore to obtain a minimum difference between the outputs by adjusting the parameters of the model. The general parameter estimation problem is the problem of the minimization of the following objective function over the variable $\boldsymbol{z}$:

$$\min_{z} \sum_{i=1}^{n} \Big(g(z_i) + B_i^T u_i\Big)^T \Big(g(z_i) + B_i^T u_i\Big) + \left(y_{sys;i} - y(z_i)\right)^T \left(y_{sys;i} - y(z_i)\right) \tag{4.76}$$

Where

n	is the number of edges in the graph-based network,	[$\cdot$]
$g(z_i)$	is the vectorfield with all resistance terms(parameters),	[bar]
u	is the vectorfield with the inputs,	[bar]
$y_{sys;i}$	is the pressure measurement over the i^{th} edge on the system,	[bar]
$y_{sys}(z_i)$	is the i^{th} pressure in the output vector in the model .	[bar]

4.4.1 Measurements on the test setup

From the system setup 8 different relative pressures can be measured as explained in the previous section. The measurements obtained from the pressure sensors placed in these nodes are relative to the atmospheric pressure. In order to compare the measurements from the system setup and the data obtained from the simulation in Matlab, an atmospheric pressure node, n_1, is set as reference point. Thus, the relation between the measured outputs and the reference point can be set, resulting in:

<u>Node 2</u>

$$y_1 = DpC2 \tag{4.77}$$

<u>Node 7</u>

$$y_2 = DpC16 \tag{4.78}$$

<u>Node 4</u>

$$y_3 = DpC18 + DpC19 + DpC23 + DpC24 \tag{4.79}$$

Node 5

$$y_4 = DpC25 + DpC26 + DpC30 + DpC31 \tag{4.80}$$

Node 10

$$y_5 = DpC20 + DpC21 \tag{4.81}$$

Node 11

$$y_6 = DpC24 \tag{4.82}$$

Node 15

$$y_7 = DpC28 + DpC27 \tag{4.83}$$

Node 16

$$y_8 = DpC31 \tag{4.84}$$

4.4.2 Estimation method

In order to carry out the parameter estimation of the water distribution, Matlab NonLinear Grey Box toolbox is used [25]. This toolbox estimates previously defined coefficients of nonlinear differential equations, to fit with the desired data. Thereby, a nonlinear model has to be designed to complete the simulation.

The comparison between the test setup measurements and the estimated data is done with Matlab function *compare*. Together with the comparison plot, the normalized root mean square (NRMSE) measure is also added which measures the goodness of the fit. This fit is calculated as a percentage [26] using:

$$fit = 100\left(1 - \frac{||y - \hat{y}||}{||y - mean(y)||}\right) \tag{4.85}$$

Where

y	is the validation data,	[Bar]
$\hat{y}$	is the output of the model	[Bar]

4.4.3 Nonlinear Estimation Outcomes

The results of the nonlinear parameter estimation are shown in *Appendix: E*. It is obvious that the estimation has been unsuccessful, seeing that the model and the measured data follow different behaviors. This might be due to the incapability to excite the system sufficiently in order to estimated the pipe parameters correctly due to limited information in the measured data. Moreover, the dynamics of the valves are slower in comparison with dynamics of the pipes, resulting in the information of the pipes being hidden into the dynamics of the valves. Thus preventing the estimation process from obtaining the correct values for the resistance as only the valve dynamic are present in the output data.

An alternative could be to only use the pumps as inputs without affecting the valves dynamics. However in order to obtain accurate values for the parameters, different scenarios have to be simulated which include varying the water consumption of the end-users as the pumps cannot excite the system sufficiently on there own.

Therefore, after seeing that the nonlinear approach is not suitable to estimate the unknown parameters for this test setup, the model will be linearized in order to preform a linear parameter estimation.

4.5 Linearization of the model

As it is shown in *Equation: (4.94)*, both $g(.)$ and $u(.)$ are vector-valued non-linear functions of the flow. Since the flows, the ODs and the differential pressure inputs, dp, are all functions of time, it can be stated that the differential equation describing the system is a first-order non-linear systems of differential equations. The number of equations are defined by the number of free variables, therefore the number of states.
In this section it is shown how the model describing the network is linearized with the help of Taylor-series. [source]

4.5.1 Taylor expansion on a simple example

The method of linearization is introduced on a simple one-state, one-input variable system. The consideration behind the example is analogous to the method applied for the water distribution system. In *Equation: (4.86)* the system with one state variable and one input can be seen:

$$\frac{d}{dt}x = f(x, u) \tag{4.86}$$

$f(x, u)$ can be written up with Taylor-series with the assumption that it is continuously differentiable, therefore the partial derivatives exist in the operating point:

$$f(x, u) = f(\bar{x}, \bar{u}) + \frac{\partial f}{\partial x}_{|\bar{x},\bar{u}} \hat{x} + \frac{\partial f}{\partial u}_{|\bar{x},\bar{u}} \hat{u} + higher\ order\ terms \tag{4.87}$$

Where

$\bar{x}$ and $\bar{u}$	is the operating point ,
$\hat{x}$ and $\hat{u}$	is the deviation from the operating point.

The aim of the linearization is to describe the function $f(x, u)$ around an operating point as a linear function. However, it should be noted that the approximation around this point is only valid for cases when the deviation from this point is small. Therefore the linearized version of a dynamic model is often called the small-signal model of the system. In *Equation: (4.87)*, the linearized term of $f(x, u)$ can be expressed. The operating point is chosen such that $f(\bar{x}, \bar{u}) = 0$, hence an equilibrium for the system is given with input $\bar{u}$. The higher order terms are not taken into account in the approximation. Since the model is described by small-signals, quadratic and higher order terms result in very small values, therefore they are negligible.
The following expression in *Equation: (4.88)* gives the approximation of the function:

$$\frac{d}{dt}x = f(x, u) \approx \frac{\partial f}{\partial x}_{|\bar{x},\bar{u}}(x - \bar{x}) + \frac{\partial f}{\partial u}_{|\bar{x},\bar{u}}(u - \bar{u}) \tag{4.88}$$

In case of a pipe or a valve component, the pressure drop across the element is described by a quadratic function of the flow if steady-state is considered and the dynamics are neglected. For the sake of illustration, *Figure 4.6* describes a non-linear function, $f(q)$, and its linearized interpretation for the operating values of pressure and flow.

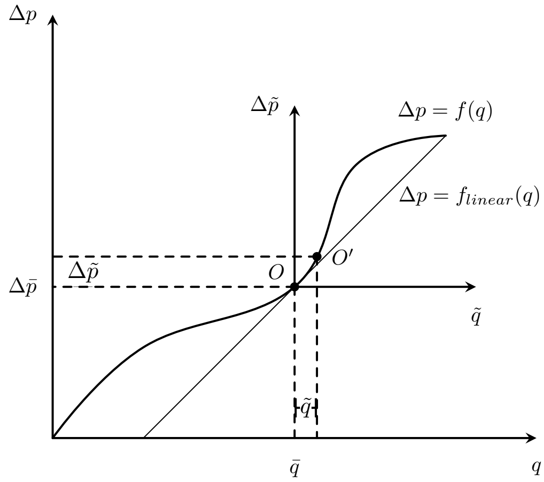


Figure 4.6. Linearization of a non-linear function $f(q)$.

As can be seen, the line inserted in the operating point, O, describes the model accurately only if the deviation is very small from this point, for example O'. Therefore the linearized model describes the system behavior in the new coordinate system $(\bar{q}, \Delta\bar{p})$. It is important to mention that in *Figure 4.6* the function $f(q)$ is an illustration of a non-linear function and not the exact same as for a pipe element.

4.5.2 Linear system model

As it is shown in *Equation: (4.56)*, the vectorfield describing the pressure in the network consists of the pressure drops of each different elements, such as pipes($\lambda(q) + \zeta + J\dot{q}$), valves($\mu(q, OD)$), pumps($\alpha(\omega, q)$), the WT ($\Delta p_{WT}$) and the WT connection ($\gamma(q)$).
Among these functions, the pipes, valves, the pumps and the edge describing the WT connection are non-linear functions, therefore they need to be linearized. The linearization is carried out according to Taylor-expansion [source].
The expression describing the pipes consists of three terms, one responsible for the resistances and form losses, one for the dynamics, and the last one for the elevation if there is any present.
The expression describing the pipes can be approximated by its linear model as follows:

$$\boldsymbol{B_1}\lambda(\boldsymbol{B_1^T z}) \approx \boldsymbol{B_1}\lambda(\boldsymbol{B_1^T \bar{z}}) + \boldsymbol{B_1}\left[\frac{\partial\lambda(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}} \boldsymbol{B_1^T \hat{z}} \tag{4.89}$$

where the partial derivative is a of all first-order partial derivatives of the vectorfield, λ, in the operating point $\bar{z}$. Since the derivation is according to $\boldsymbol{B_1^T z}$, due to the chain rule the derivative is multiplied by $\boldsymbol{B_1^T}$. The reason for only $\lambda(\boldsymbol{B_1^T z})$ being expressed is because the elevation is constant and the inertia term is a linear function of the flows.

In case of the valves, $\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})$ is not only the function of the independent flows, but also the opening degree. The conductivity function, k_v is the function of OD, which can vary in time. Therefore the linearization has to be done according to the flow and the OD:

$$\begin{aligned}\boldsymbol{B_1}\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD}) \approx \boldsymbol{B_1}\mu(\boldsymbol{B_1^T \bar{z}}, \bar{\boldsymbol{OD}}) &+ \boldsymbol{B_1}\left[\frac{\partial\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})}{\partial \boldsymbol{B_1^T z}}\right]_{(\bar{z}, \bar{OD})} \boldsymbol{B_1^T \hat{z}} \\ &+ \boldsymbol{B_1}\left[\frac{\partial\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})}{\partial \boldsymbol{OD}}\right]_{(\bar{z}, \bar{OD})} \hat{\boldsymbol{OD}}\end{aligned} \tag{4.90}$$

The Taylor-expansion is carried out in the same manner as in *Equation: (4.89)*, however the linearized valve model is in the function of two small-signal variables, the flows and the OD. Therefore the partial derivatives are calculated in the operating point defined by the operating value of $\boldsymbol{z}$ and $\boldsymbol{OD}$.
For the water tower connection, the same can be concluded as for the pipe model.

$$\boldsymbol{B_1}\gamma(\boldsymbol{B_1^T z}) \approx \boldsymbol{B_1}\gamma(\boldsymbol{B_1^T \bar{z}}) + \boldsymbol{B_1}\left[\frac{\partial\gamma(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}} \boldsymbol{B_1^T \hat{z}} \tag{4.91}$$

The pumps are operating according to the model described in *Equation: (4.43)*, where the valves around each pump are taken into account. Although this model is both dependant on the OD of the valves and the flow through the pumps, it is unnecessary to linearize it for the following reason, explained with the help of *Figure 4.7* below:

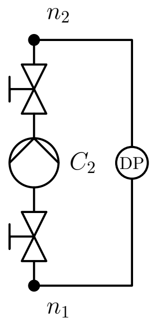


Figure 4.7. Block representing the extended pump model.

As it is shown, there is a differential pressure sensor around every pump in the system. Angular velocity is therefore not used directly as input to the system, rather the differential pressure is set. It is shown in a later chapter, Chapter *II: Control Design*, that around the pumps cascade control is designed. The pumps are controlled by PI controllers in an inner loop with control variable chosen as the differential pressure. This inner control loop linearizes the pumps and therefore the differential pressure becomes the control input.
The input, in case of the parameter estimation, is defined by the four pumps and four valves. Therefore it is convenient to define an input vector which consists of the four opening degrees controlling the valves and the four differential pressures controlling the pumps:

$$\boldsymbol{u} = \begin{bmatrix} OD_{e13} \\ OD_{e15} \\ OD_{e20} \\ OD_{e22} \\ dP_{e01} \\ dP_{e08} \\ dP_{e09} \\ dP_{e16} \end{bmatrix} \tag{4.92}$$

It should be noted here, however, that this control input representation is valid only for the parameter estimation. It is shown in a later chapter, Chapter *II: Control Design*, that for the control, the structure of the model being described here and the input vector is structured differently. However, in this case it is convenient to handle all pumps and valves as input components.
Considering the input vector, the model of the pumps can be written up in the same manner as the linearized model of pipes and valves:

$$\boldsymbol{B_1}\alpha(DP) = \boldsymbol{B_1 G_p u} \tag{4.93}$$

Where

$\boldsymbol{G_p} \in \mathbb{R}^{(e \times u)}$ is a matrix representing a linear mapping where the dimension u is the number of inputs and e is the number of edges without the WT.

$\boldsymbol{G_p}$ is an extended matrix for the eight inputs which means that the first four columns consist of zeros, since the first four elements of the input vector are the valve opening degrees. $\boldsymbol{G_p}$ can be found in *Appendix: C.7*.

G_p in the appendix should be corrected

4.5.3 State space model for linear parameter estimation

For the sake of clearance, the model describing the water distribution system in *Equation: (4.94)* is shown again:

$$\boldsymbol{BJB}^T\dot{\boldsymbol{z}} = \boldsymbol{B}g(\boldsymbol{B}^T\boldsymbol{z}) + \boldsymbol{B}u(\boldsymbol{\omega}, \boldsymbol{k_v}, \boldsymbol{B}^T\boldsymbol{z}) \tag{4.94}$$

The dynamics of the WT is described by the equation below:

$$\Delta\dot{p}_{WT} = \frac{1}{C_H} q_0 \tag{4.95}$$

These two differential equation systems give a full description of the pressures in the whole network, and describe the effect of the WT on the system. However, due to the linearization, the linear system is desired to formulate into the general state-space representation with inputs, outputs and states separated.
Before setting up the state-space form of the system, the following should be considered:

$$\boldsymbol{H_1 q_1} + \boldsymbol{H_0} q_0 = 0 \tag{4.96}$$

In *Equation: (4.96)*, the current law is shown for the WT and for the rest of the system. The two current laws sum up to zero taking into account the whole system. Expressing the flow in the WT yields:

$$q_0 = -\boldsymbol{H_0^\dagger H_1 q_1} \tag{4.97}$$

Inserting *Equation: (4.97)* into *Equation: (4.95)*, the original model of the WT, the following yields:

two letters are not the same

$$\Delta\dot{p}_{WT} = -\frac{1}{C_H}\boldsymbol{H_0^\dagger H_1 q_1} = -\underbrace{\frac{1}{C_H}\boldsymbol{H_0^\dagger H_1 B_1^T}}_{\mathbf{S}}\boldsymbol{z} \tag{4.98}$$

Therefore the dynamics of the WT can be expressed with the incidence matrix for the whole system in terms of the independent chord flow variables as follows:

$$\Delta\dot{p}_{WT} = -\boldsymbol{Sz} \tag{4.99}$$

Getting back to the original system model described in *Equation: (4.94)*, in order to formulate a state space representation, the linearized terms in vectorfields $g(.)$ and $u(.)$ should be separated according to the small signal values of the flows, the inputs and the pressure contribution from the WT. The representation is shown in *Equation: (4.100)* below:

$$\boldsymbol{B}\boldsymbol{J}\boldsymbol{B}^T\dot{\hat{\boldsymbol{z}}} = \boldsymbol{M}\hat{\boldsymbol{z}} + \boldsymbol{N}\hat{\boldsymbol{u}} + \boldsymbol{B_o}\Delta\hat{p}_{WT} \tag{4.100}$$

In *Equation: (4.100)*, the $\boldsymbol{M}$ matrix consists of the following terms:

$$\boldsymbol{M} = \boldsymbol{B_1}\left[\frac{\partial\lambda(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}}\boldsymbol{B_1^T} + \boldsymbol{B_1}\left[\frac{\partial\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})}{\partial \boldsymbol{B_1^T z}}\right]_{(\bar{z},\bar{OD})}\boldsymbol{B_1^T} + \boldsymbol{B_1}\left[\frac{\partial\gamma(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}}\boldsymbol{B_1^T} \tag{4.101}$$

And the $\boldsymbol{N}$ matrix consists of the following terms:

$$\boldsymbol{N} = \boldsymbol{B_1}\left[\frac{\partial\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})}{\partial \boldsymbol{u}}\right]_{(\bar{z},\bar{u})} + \boldsymbol{B_1 G_p} \tag{4.102}$$

$\boldsymbol{B_o}$ is the cycle matrix belonging to the WT. As can be seen, the linearized terms which are represented in the element-wise model description in *Equation: (4.56)*, are separated based on if they are multiplied by the small signal values of flow(state) or input.

In order to find a good state space representation for the system extended with the WT, first the dynamics has to be considered. As it is discussed in a previous section, two kind of elements have dynamics, the pipes and the WT. The pipes, compared to the WT, assumed to have very fast response time, which means that their time constants are small, therefore the decay time is short. Considering the control and also the parameter estimation, it means that they reach steady-state condition very quick compared to the WT. According to [source], in cases like this, the dynamics with the small time constant does not take part in the dynamics effectively, therefore they can be neglected. Due to this consideration, *Equation: (4.100)* is rewritten in steady-state form, where the derivative of the states are set to zero:

$$0 = \boldsymbol{M}\hat{\boldsymbol{z}} + \boldsymbol{N}\hat{\boldsymbol{u}} + \boldsymbol{B_o}\Delta\hat{p}_{WT} \tag{4.103}$$

The small signal value of the state vector can be expressed on the left side of the equation only if $\boldsymbol{M}$ is invertible. In *Equation: (4.101)*, the equation can be rewritten as follows:

$$\boldsymbol{M} = \boldsymbol{B_1}\left[\left[\frac{\partial\lambda(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}} + \left[\frac{\partial\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})}{\partial \boldsymbol{B_1^T z}}\right]_{(\bar{z},\bar{OD})} + \left[\frac{\partial\gamma(\boldsymbol{B_1^T z})}{\partial \boldsymbol{B_1^T z}}\right]_{\bar{z}}\right]\boldsymbol{B_1^T} \tag{4.104}$$

In *Equation: (4.104)*, the $\boldsymbol{M}$ matrix is invertible for the same reason as it is described in Section *4.3.1: Network model*, for the inertia matrix. The proof of this statement can be found in that section.
Expressing the the state vector the following yields:

$$\hat{\boldsymbol{z}} = -(\boldsymbol{M^{-1}N})\hat{\boldsymbol{u}} - (\boldsymbol{M^{-1}B_o})\Delta\hat{p}_{WT} \tag{4.105}$$

Having the independent states expressed, *Equation: (4.105)* can be inserted in the previously-derived WT model with the $\boldsymbol{S}$ matrix in *Equation: (4.99)*.

$$\Delta\dot{\hat{p}}_{WT} = (\boldsymbol{SM^{-1}B_o})\Delta\hat{p}_{WT} + (\boldsymbol{SM^{-1}N})\hat{\boldsymbol{u}} \tag{4.106}$$

Equation: (4.106) represents the linear system with the pressure drop across the water tank as a state and the input vector consisting of differential pressures from the pumps and OD values from the end-user valves. The general formulation of the state equation can be written as follows:

$$\Delta\dot{p}_{WT} = A_p\Delta\hat{p}_{WT} + \boldsymbol{B_p}\boldsymbol{\hat{u}} \tag{4.107}$$

Where

$A_p \in \mathbb{R}^{(1\times1)}$ is the system matrix for the parameter estimation, which in this case is a scalar,

$\boldsymbol{B_p} \in \mathbb{R}^{(1\times g)}$ is the input matrix for the parameter estimation.

An output equation is defined, which represents the pressure difference known from the system setup. In this way, the output equation can be compared to the data measured in the setup and proceed to estimate the unknown parameters.

$$\boldsymbol{\hat{y}} = \boldsymbol{C_1}\hat{z} + \boldsymbol{C_2}\boldsymbol{\hat{u}} \tag{4.108}$$

As in *Equation: (4.106)*, substituting the state vector, $\boldsymbol{\hat{z}}$, by the expression obtained in *Equation: (4.105)*, the equation above results in

$$\boldsymbol{\hat{y}} = \boldsymbol{C_1}(-(\boldsymbol{M^{-1}N})\boldsymbol{\hat{u}} - (\boldsymbol{M^{-1}B_o})\Delta\hat{p}_{WT}) + \boldsymbol{C_2}\boldsymbol{\hat{u}} \tag{4.109}$$

Reorganizing the terms

$$\boldsymbol{\hat{y}} = \boldsymbol{C_1}(-(\boldsymbol{M^{-1}B_o}))\Delta\hat{p}_{WT} + (\boldsymbol{C_1}(-(\boldsymbol{M^{-1}N})) + \boldsymbol{C_2})\boldsymbol{\hat{u}} \tag{4.110}$$

$$\boldsymbol{\hat{y}} = \boldsymbol{C_p}\Delta\hat{p}_{WT} + \boldsymbol{D_p}\boldsymbol{\hat{u}} \tag{4.111}$$

The equation above shows how the output equation includes a feedforward matrix, due to the outputs being affected directly by the inputs.

4.6 Model Parameters

In order to obtain a complete model of the physical setup, all the parameters describing the components have to be defined. In Section *4.4: Nonlinear Parameter identification* a detailed description of the known parameters of the system has been done. Nevertheless, in the linearized state-space model more parameters have to be indentified due to the introduction of the operating points values. Hence, in the current chapter a detailed compilation of the unknown parameters of the physical water distribution setup is carried out.

4.6.1 Unknown Parameters

The unknown parameters are the ones related with the form losses, k_f, and form friction ,f, of the pipes. Despite they are provided by the manufactures they need to be estimated. On the one hand, the form losses depend on the fittings and bends of the pipes which they are not always known. On the other hand, the friction losses dependent on the inside average roughness of pipes, ϵ, which can change its value due to passage of time and the rust generated inside pipes.

Furthermore, the operating points of the flow through the chords, $\bar{\boldsymbol{z}}$, is also unknown. These values, which correspond to the 8 flow chords, are introduced in the linearized expression of both pipes and valves, see *Equation: (4.89)* and *Equation: (4.90)*. Thus, not only pipe parameters introduce uncertainties into the system model but also the lack of knowledge of the chord operating points.

Consequently, it has been decided to estimate the total expression for the pressure across the pipes and valves in order to reduce the amount of unknowns in the system.

The system has 15 pipes in total, from *Equation: (4.89)* it can be seen that either tunning for k_f, f or $\bar{\boldsymbol{z}}$ it will have the same result for the total value of the pressure across the pipes, $\lambda(\boldsymbol{B}_1^T \boldsymbol{z})$. For this reason the pressure across the 15 pipes is estimated.

Valve linearized expression, see *Equation: (4.90)*, consists on the term depending on the chord flows and the one depending on the OD. Both terms include the operating point of the chord flows inside them, thus, the pressure difference given by both terms has to be estimated. In the system 4 valves take part, resulting in 8 unknowns in total.

The WT connection edge, see *Equation: (4.93)*, is conformed by two valves and one pump. Although the parameters corresponding to the pump are considered as known, the ones corresponding to the valves have to be estimated. Resulting in two more unknowns for the system.

All in all, the system has 24 unknown terms which will be calculated following the estimation process described in the next section.

4.7 Linear parameter estimation

The method that describes the linear parameter estimation is shown in *Figure 4.8* below:

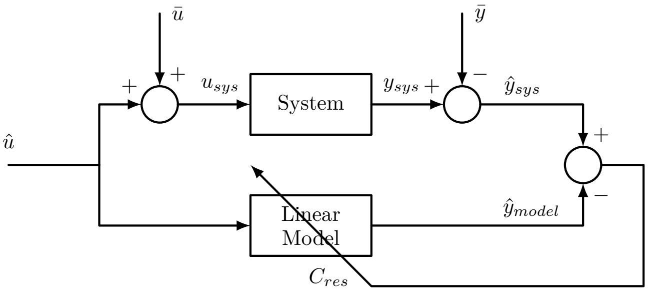


Figure 4.8. Parameter identification block diagram for the linear system.

In this case small-signal inputs are applied to both the test setup and the model. Now the linearized model is compared to the real system, therefore the operating point is taken into account. Since the linear model is only valid for small deviations around the operating point, the real system has to be excited around the same operating point to obtain identical behavior. In order to achieve it, the operating values are added to both the input and the output such as:

$$u_{sys} = \bar{u} + \hat{u} \tag{4.112}$$

Where

$\hat{u}$	is the small-signal input,	[bar]
$\bar{u}$	is the operating value of the input,	[bar]
u_{sys}	is the input to the real-life system.	[bar]

$$y_{sys} = \bar{y} + \hat{y}_{sys} \tag{4.113}$$

Where

$\hat{y}_{sys}$	is the small-signal output from the real-life system,	[bar]
$\bar{u}_{sys}$	is the operating value of the output,	[bar]
y_{sys}	is the output from the real-life system.	[bar]

During the linear parameter estimation, the same problem is solved as it is shown in *Equation: (4.76)*. In this case, however, the parameters are varied according to the comparison of the small-signal outputs.

4.7.1 Model Parameters

In order to obtain a complete model of the physical setup, all the parameters describing the components have to be defined. In Section *4.4: Nonlinear Parameter identification* a detailed description of the known parameters of the system has been done. Nevertheless, in the linearized state-space model more parameters have to be identified due to the introduction of the operating points values. Hence, in the current chapter a detailed compilation of the unknown parameters of the physical water distribution setup is carried out.

Unknown Parameters The unknown parameters are the ones related with the form losses, k_f, and form friction ,f, of the pipes. Despite they are provided by the manufactures they need to be estimated. On one hand, the form losses depend on the fittings and bends of the pipes which are not always known. On the other hand, the friction losses depend on the inside average roughness of the pipes, ϵ, which can change its value due to passage of time and rust generated inside pipes or fittings.

Furthermore, the operating points of the flow through the chords, $\bar{\boldsymbol{z}}$, is also unknown. These values, which correspond to the 8 flow chords, are introduced in the linearized expression of both pipes and valves, see *Equation: (4.89)* and *Equation: (4.90)*. Thus, not only pipe parameters introduce uncertainties into the system model but also the lack of knowledge of the chord operating points.

Consequently, it has been decided to estimate the total expression for the pressure across the pipes and valves in order to reduce the amount of unknowns in the system.

The system has 15 pipes in total, from *Equation: (4.89)* it can be seen that either tunning for k_f, f or $\bar{\boldsymbol{z}}$ it will have the same result for the total value of the pressure across the pipes, $\lambda(\boldsymbol{B_1^T z})$. For this reason the pressure across the 15 pipes is estimated.

The linearized valve expression, see *Equation: (4.90)*, consists of the term depending on the chord flows and the one depending on the OD. Both terms include the operating point of the chord flows inside them, thus, the pressure difference given by both terms has to be estimated. In the system 4 valves take part, resulting in 8 unknowns in total.

The WT connection edge, see *Equation: (4.93)*, is conformed by two valves and one pump. Although the parameters corresponding to the pump are considered as known, the ones corresponding to the valves have to be estimated. Resulting in two more unknowns for the system.

All in all, the system has 24 unknown terms which will be calculated following the estimation process described in the next section.

4.7.2 Measurements on the test setup

In order to verify the state-space model derived in Chapter *4.7: Linear parameter estimation* with the physical setup, an estimation for the parameters defined in Section *4.7.1: Model Parameters* is carried out.

From the system setup, 9 different relative pressures can be measured, following *Figure C.2* notation the sensors are placed in: n_2 n_4 n_5 n_7 n_{10} n_{11} n_{15} n_{16} n_{18}. However, the estimation will be done only regarding the four PMA relative pressure sensors across the end-users and the pressure in the WT, since those are the outputs that will be controlled in Section *4.8: State space model for control.* Thus, the estimation will only focus on obtaining the best fit for those sensor outputs relevant in the control part.

The relationship between pressures, where DpCXX describes the pressure difference for the XX component, is obtained in the same way as Section *4.4: Nonlinear Parameter identification* and can be defined as:

<u>Node 10</u>

$$y_1 = DpC20 + DpC21 \tag{4.114}$$

<u>Node 11</u>

$$y_2 = DpC24 \tag{4.115}$$

<u>Node 15</u>

$$y_3 = DpC28 + DpC20 \tag{4.116}$$

<u>Node 15</u>

$$y_4 = DpC31 \tag{4.117}$$

There is no need to define a relation with a referent point for the WT node, since the pressure across the WT is the state of the state-space model.

4.7.3 Linear Estimation Outcomes

In *Appendix: F* the results of the estimation process are shown, where the 24 terms defining the pressure across the unknowns edges are estimated. From the tests it can be seen a slightly different behavior between the model and the data from the setup, especially at time $t = 350s$. This dissimilar behavior showing up at some time samples could make the model unsuitable for the real setup, thus making it incorrect to be used in a model based control scheme.

In light of the above considerations, a different approach for the parameter estimation is attempted. In Section *4.7.1: Model Parameters* it has been described how the unknown pressures across the edges are estimated in order to build up the state-space matrices of the system. Nonetheless, as the estimation did not succeed it has been decided to estimate the final values of the state-space matrices stated in *Equation: (4.100)*, where M is a symmetric matrix, and *Equation: (4.108).*

Altogether these matrices sum up to 28 unknown parameters, which are the ones to be estimated. The outputs to be compared with the test setup remain the same as in Section *4.7.2: Measurements on the test setup*, as well as the toolbox used for estimating.

Estimation Data

As the parameter estimation is based on a linearized model an operating point for the system is chosen. This point is based on the WT being approximately half way full which allows for an equal amount of deviation in both directions. For the chosen operating point, data is gathered from the system while small steps are individually applied to the two main pumps and the opening degree of the PMA valves. In order to use the data for parameter estimation the operating point is subtracted, leaving only small signal values.

The operating point of the PMA valves is placed at 70% opening corresponding to 63° and the small signal values for the estimation can be seen in *Figure 4.9*.

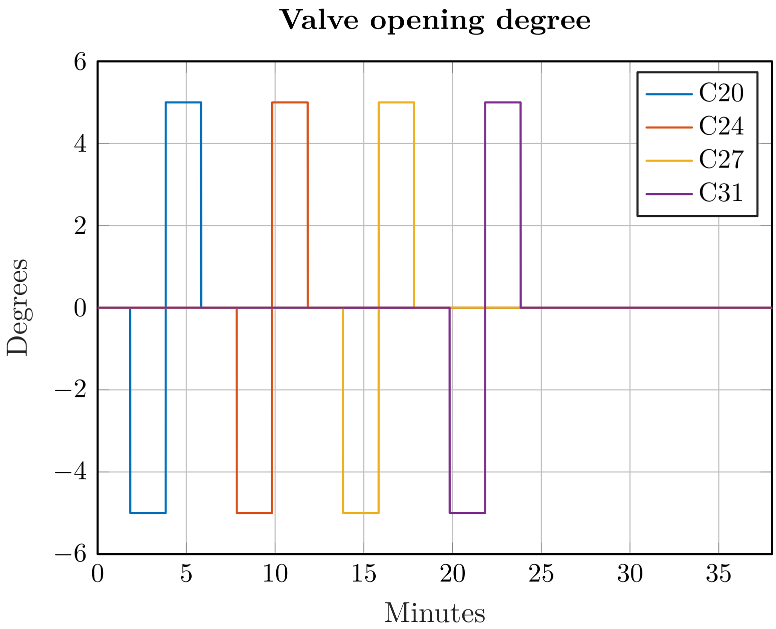


Figure 4.9. Small signal values of the opening degrees of the pma valves.

To acheive a 50% fill level of the WT in steady state combined with the chosen operating point of the valves, the operating point of the two inlet pumps C2 and C16 has to be set at a differential pressure of $\Delta p = 0.2Bar$. The required operating point is found by experimental tests made on the setup, the small signal values used in the estimation are shown in *Figure 4.10*.

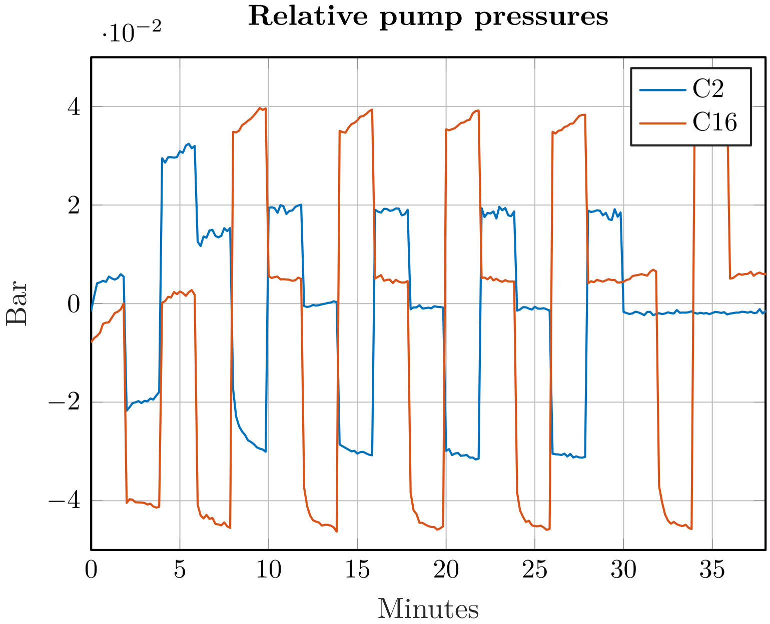


Figure 4.10. Small signal values of the differential pressure of the two main pumps.

Estimation Result

The following figures show the comparison between the data obtained from the lab and the outputs of the model with the estimated parameters.

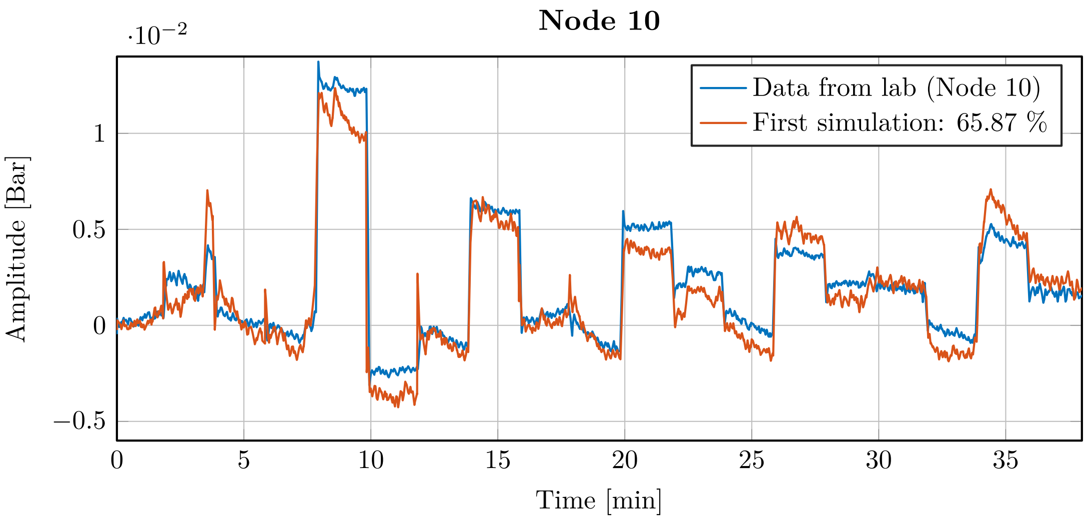


Figure 4.11. Estimation comparison for node 10.

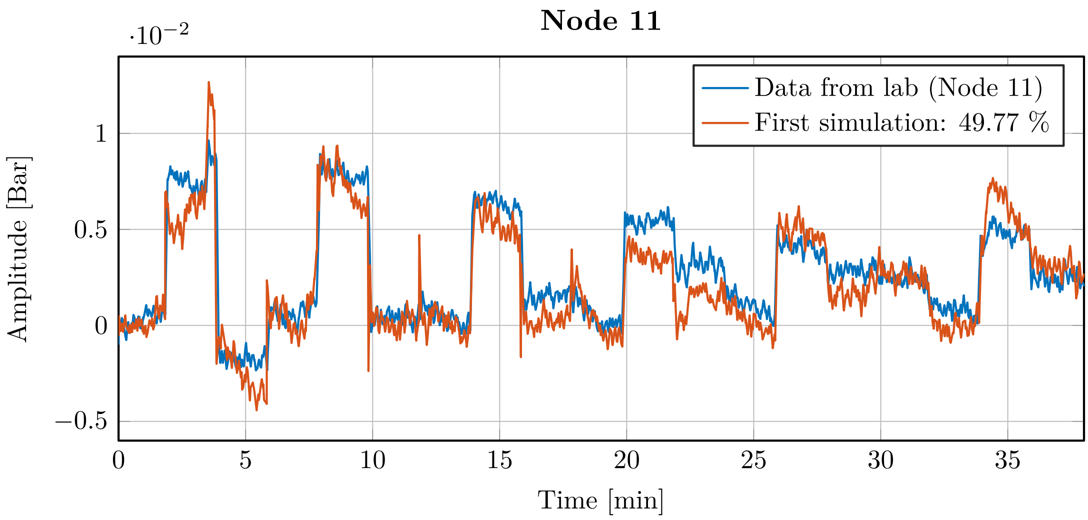


Figure 4.12. Estimation comparison for node 11.

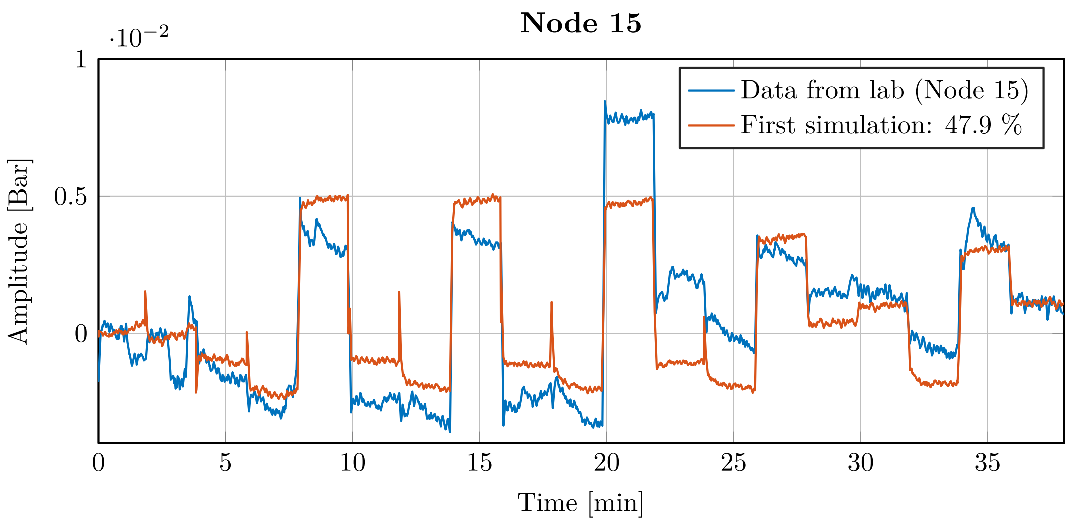


Figure 4.13. Estimation comparison for node 15.

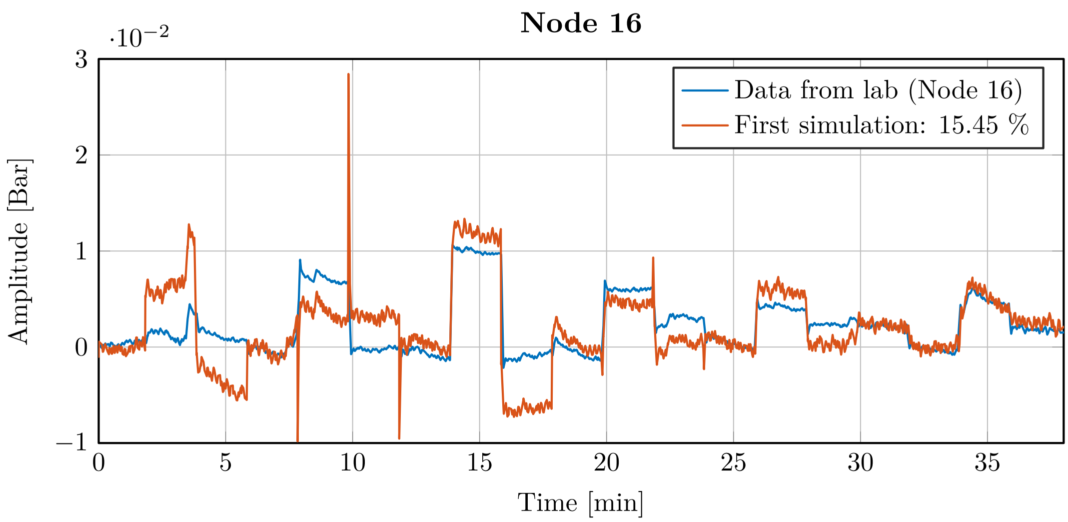


Figure 4.14. Estimation comparison for node 16.

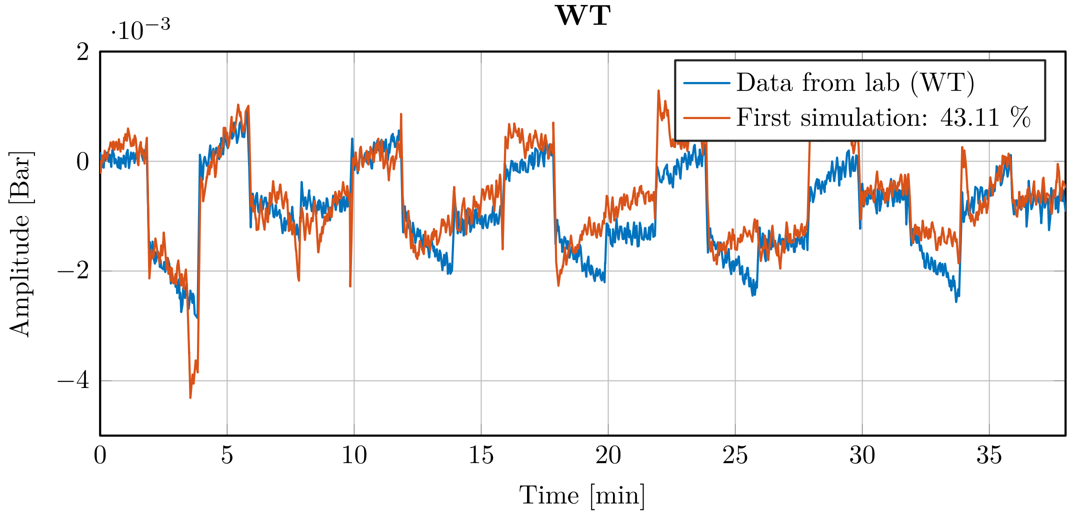


Figure 4.15. Estimation comparison for WT.

With the estimated parameters it is seen that the model follows the behavior of the measured data and that the fit percentage is within a decent margin. Based on these results can the system model and the parameters contained inside now be used for control purposes, as a description approximating the system is now obtained.

4.8 State space model for control

For the sake of clarity the steady-state, state-space representation for the parameter estimation for small-signal values is restated here:

$$0 = \boldsymbol{M}\hat{\boldsymbol{z}} + \boldsymbol{N}\hat{\boldsymbol{u}} + \boldsymbol{B_o}\Delta\hat{p}_{WT} \tag{4.118}$$

The equation for the outputs:

$$\hat{\boldsymbol{y}} = \boldsymbol{C_p}\Delta\hat{p}_{WT} + \boldsymbol{D_p}\hat{\boldsymbol{u}} \tag{4.119}$$

And the dynamic model of the WT pressure:

$$\Delta\dot{p}_{WT} = \frac{1}{C_H}q_0 \tag{4.120}$$

Although *Equation: (4.118)*, *Equation: (4.119)* and *Equation: (4.120)* include all the linearized terms that are necessary to describe the system, for the control it is being restructured. While the input vector for the parameter estimation consists of all the four pump differential pressures along with the four valve opening degrees, in case of the control, only the two main pumps are considered as inputs. The control input for the system is therefore defined as $\hat{\boldsymbol{u}} \in \mathbb{R}^{(2\times 1)}$ as follows:

$$\hat{\boldsymbol{u}} = \begin{bmatrix} dP_{e01} \\ dP_{e08} \end{bmatrix} \tag{4.121}$$

This distinction between the parameter estimation and the control is being made because of the different purposes. In case of the parameter estimation the inputs are set to excite the system in order to make different pressure scenarios in the network. Therefore the valve opening degrees and all pumps are controlled manually to create appropriate output measurements. These measurements are then used to achieve a fit with the output of the simulation by changing its parameters. However in the control this is not desired. Here the inputs are chosen for the control and therefore neither the valve opening degrees, nor the pump signals are varied manually.

The mapping for the two main pump pressures yields:

$$\boldsymbol{B_1}\alpha(DP) = \boldsymbol{B_1}\boldsymbol{G_{c;1}}\boldsymbol{\hat{u}} \tag{4.122}$$

minipage fix

Where

$\boldsymbol{G_{c;1}} \in \mathbb{R}^{(e\times u)}$ is a matrix representing a linear mapping between the vectorfield $\alpha(dP)$, describing the pressure contribution of the pumps, and between the input vector defined in *Equation: (4.121)* where the dimension u is the number of inputs and e is the number of edges without the WT.

$\boldsymbol{G_{c;1}}$ is now a mapping matrix for only the two inputs for the main pumps. $\boldsymbol{G_{c;1}}$ can be found in [appendix reference].

The input matrix for the control system can be written in the form:

$$\boldsymbol{N_c} = \boldsymbol{B_1}\boldsymbol{G_{c;1}} \tag{4.123}$$

Compared to the input matrix in the parameter estimation in *Equation: (4.102)*, it can be seen that now the linearized terms belonging to the valves are not part of the matrix since valves are not considered as control inputs.
However, the end-user valves are considered as measurable disturbances in the control system. Therefore the disturbance is defined as $\boldsymbol{d} \in \mathbb{R}^{(4\times 1)}$ as follows:

$$\boldsymbol{d} = \begin{bmatrix} OD_{e13} \\ OD_{e15} \\ OD_{e20} \\ OD_{e22} \end{bmatrix} \tag{4.124}$$

As can be seen, the PMA pumps are neither considered as inputs, nor as disturbances. The input for the PMA pumps is excluded in the linearized model for the reason that the system is described by deviation variables. It means that if the PMA pumps were included as disturbances, they would not have an effect on the linearized system because they are kept constant all the time.
The disturbance matrix for the state space model consists of the linearized terms of the end-user valves. The matrix can be formulated as follows:

$$\boldsymbol{Q_c} = \boldsymbol{B_1}\left[\frac{\partial\mu(\boldsymbol{B_1^T z}, \boldsymbol{OD})}{\partial \boldsymbol{d}}\right]_{(\bar{z},\bar{u})} \tag{4.125}$$

Taking the same considerations into account as for the parameter estimation, the steady-state, state equation for small signals can be formulated as:

$$0 = \boldsymbol{M_c}\boldsymbol{\hat{z}} + \boldsymbol{N_c}\boldsymbol{\hat{u}} + \boldsymbol{Q_c}\boldsymbol{\hat{d}} + \boldsymbol{B_o}\Delta\hat{p}_{WT} \tag{4.126}$$

where the system matrix, $\boldsymbol{M_c}$ is the same as the system matrix, $\boldsymbol{M_p}$, in case the parameter estimation.
$\boldsymbol{M_c}$ is invertible for the same reason as described in *Equation: (4.3.1)*, thus:

$$\hat{\boldsymbol{z}} = -(\boldsymbol{M_c^{-1}N_c})\hat{\boldsymbol{u}} - (\boldsymbol{M_c^{-1}Q_c})\hat{\boldsymbol{d}} - (\boldsymbol{M_c^{-1}B_o})\Delta\hat{p}_{WT} \tag{4.127}$$

Inserting the states into *Equation: (4.98)*:

$$\Delta\dot{\hat{p}}_{WT} = (\boldsymbol{SM_c^{-1}B_o})\Delta\hat{p}_{WT} + (\boldsymbol{SM_c^{-1}N_c})\hat{\boldsymbol{u}} + (\boldsymbol{SM_c^{-1}Q_c})\hat{\boldsymbol{d}} \tag{4.128}$$

Which in standard state-space form can be written as:

$$\Delta\dot{\hat{p}}_{WT} = A_c\Delta\hat{p}_{WT} + \boldsymbol{B_c}\hat{\boldsymbol{u}} + \boldsymbol{E_c}\hat{\boldsymbol{d}} \tag{4.129}$$

Where

$A_c = \boldsymbol{SM_c^{-1}B_o}$	is the system matrix for the control,
$\boldsymbol{B_c} = \boldsymbol{SM_c^{-1}N_c}$	is the input matrix for the control,
$\boldsymbol{E_c} = \boldsymbol{SM_c^{-1}Q_c}$	is the disturbance matrix for the control.

The output of the control model is defined as two measurement points around the valves in the two different PMAs. Therefore the output vector is defined as:

$$\boldsymbol{y} = \begin{bmatrix} dP_{e15} \\ dP_{e22} \end{bmatrix} \tag{4.130}$$

Since the outputs are pressures around two end-user valves, the output expression should be written in the form:

$$\hat{\boldsymbol{y}} = \boldsymbol{C_1}\hat{\boldsymbol{z}} + \boldsymbol{C_2}\hat{\boldsymbol{d}} \tag{4.131}$$

Where

$\boldsymbol{C_1}$	is the matrix consisting of the mapping between the vectorfield,μ, and the output vector. Furthermore it includes the partial derivative matrix of the vectorfield according to the independent states,
$\boldsymbol{C_2}$	is the matrix consisting of the mapping between the vectorfield, μ, and the output vector. Furthermore it includes the partial derivative matrix of the vectorfield according to the ODs.

As it is shown, the output equation includes feedforward from the measured disturbances. Expressing the independent flow variables with *Equation: (4.127)*, the following yields:

$$\hat{\boldsymbol{y}} = \boldsymbol{C_1}\big[-(\boldsymbol{M_c^{-1}N_c})\hat{\boldsymbol{u}} - (\boldsymbol{M_c^{-1}Q_c})\hat{\boldsymbol{d}} - (\boldsymbol{M_c^{-1}B_o})\Delta\hat{p}_{WT}\big] + \boldsymbol{C_2}\hat{\boldsymbol{d}} \tag{4.132}$$

And the output equation in standard state-space form:

$$\hat{\boldsymbol{y}} = \boldsymbol{C_c}\Delta\hat{p}_{WT} + \boldsymbol{D_c}\hat{\boldsymbol{u}} + \boldsymbol{K_c}\hat{\boldsymbol{d}} \tag{4.133}$$

Where

$C_c = -\boldsymbol{C_1M_c^{-1}B_o}$	is the output matrix for the control,
$\boldsymbol{D_c} = -\boldsymbol{C_1M_c^{-1}N_c}$	is the feed-forward matrix for the control,
$\boldsymbol{K_c} = \boldsymbol{C_2} - \boldsymbol{C_1M_c^{-1}Q_c}$	is the disturbance matrix affecting the output.

The continuous state space representation thus given by *Equation: (4.129)* and *Equation: (4.133)*.

4.8.1 Discretization of state space model

The dynamics of the water distribution system are now described. To use this linear continuous model to be subjected to MPC, the model needs to be discretized by assuming zero-order-hold(ZOH) of the variables at specified sampling points. This means that the variables are constant between these points. The aim is to have a linear discrete-time state-space model with piecewise constant $\Delta\hat{p}_{WT}[k]$, $\hat{\boldsymbol{u}}[k]$ and $\hat{\boldsymbol{d}}[k]$. The method is chosen as forward Euler-method and the detailed presentation can be found in *Appendix: J*. The final discretized state-space model is stated here with respect to the sampling time T_s:

$$\Delta\hat{p}_{WT}[k+1] = A_d\Delta\hat{p}_{WT}[k] + \boldsymbol{B_d}\hat{\boldsymbol{u}}[k] + \boldsymbol{E_d}\hat{\boldsymbol{d}}[k] \tag{4.134}$$

and the output equation:

$$\hat{\boldsymbol{y}}[k] = \boldsymbol{C_d}\Delta\hat{p}_{WT}[k] + \boldsymbol{D_d}\hat{\boldsymbol{u}}[k] + \boldsymbol{K_d}\hat{\boldsymbol{d}}[k] \tag{4.135}$$

Where

$A_d \in \mathbb{R}^{(1\times1)}$	is the discrete state matrix,
$\boldsymbol{B_d} \in \mathbb{R}^{(1\times2)}$	is the discrete input matrix,
$\boldsymbol{E_d} \in \mathbb{R}^{(1\times4)}$	is the discrete disturbance matrix,
$\boldsymbol{C_d} \in \mathbb{R}^{(2\times1)}$	is the discrete output matrix, which is the same as in continuous case
$\boldsymbol{D_d} \in \mathbb{R}^{(2\times2)}$	is the discrete feed-forward matrix, which is the same as in continuous case
$\boldsymbol{K_d} \in \mathbb{R}^{(2\times4)}$	is the discrete disturbance matrix affecting the output, which is the same as in continuous case

Part II

Control Design

Controller 5

In this chapter the design of the controller is explained including the control problem, optimality and structure. The chosen control approach is then described and leads up to a section where the implementation is discussed.

5.1 Control Problem

The water distribution system described in Section *2.1: System overview* is to be controlled with respect to the requirements and constraints stated in Section *3: Requirements and Constraints* and the dynamics of the system described in Section *4.8: State space model for control*. For a better clearance, the requirements and constraints are listed again below:

- Consumer pressure requirements, $0.08 < \boldsymbol{y} < 0.14$ [bar]
- Minimizing the total running costs
- Performance constraints on the pumps

The system consists of four pumps and is fully controlled by the two main pumps. The two pumps in the PMAs influence only the operating point with a fixed pressure lift. The project deals with a control that optimizes the WT such that the pressure at the end-users is maintained and the cost of pumping effort in the main pumps is minimised. In other words, the use of the main pumps and the WT is controlled according to the constraints and requirements stated above.

In order to achieve such an optimal behavior, Model Predictive Control, MPC, is applied. With MPC, the dynamics of the model is used to predict the system behavior, therefore taking the future electrical prices into account.

Apart from the MPC control, all pumps are controlled by PI controllers in order to deliver the required minimum pressure to the end-users. The MPC could be used to control the main pumps directly, but to mimic a real world scenario, without PMA pumps, it is chosen to let the MPC set differential pressure references to the main pumps. As a consequence of the two control method, the following cascade structure can be formulated for the two main pumps as it is shown in *Figure 5.1*:

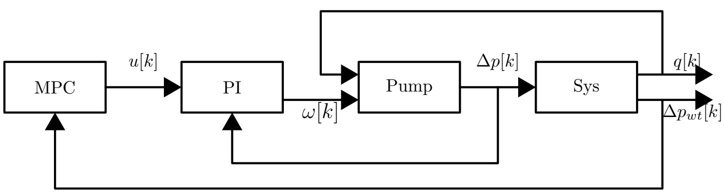


Figure 5.1. Cascade control structure with MPC and PI controllers.

As it is shown, the PI controllers are responsible for setting the control inputs to the main pumps, using the feedback from the differential pressure output. It should be noted however, that typically the flow out of the pumping station is controlled instead of the pressure. Here the use of pressure feedback has the advantage that in case of system failure the pressure is important. The flow would typically either lead to too high or too low pressure or would make the WT overrun. On the other hand, the reference to the PI controllers is set by the MPC algorithm which takes the measurements from the output into account. These references are set as differential pressures. One of the advantage of such a cascade control is that the pump controllers are present for each pump, also including the PMA pumps. At the event of implementation or communication failure between the MPC and the pump controllers, the pumping remains operational.

In order to define an objective function for the MPC problem, the cost of running the system has to be taken into account along with the hydraulic power consumption of the pumps. Such an objective function can be written in the form:

$$\Upsilon(\boldsymbol{u}[k], \boldsymbol{q_p}[k], c_p[k]) = \frac{1}{\eta} \sum_{i=1}^{H_p-1} \Big(\boldsymbol{u^T}[k+i|k] \cdot \boldsymbol{q_p}[k+i|k]\Big) \cdot c_p[k+i|k] \tag{5.1}$$

Where

Hp	is the prediction horizon,	[·]
$\boldsymbol{u}$	is the differential pressure input to the main pumps,	[bar]
$\boldsymbol{q_p}$	is the flow through the main pumps,	[m^3/h]
η_i	is the efficiency of the main pumps,	[·]
$c_p[k]$	is the electricity price cost sequence.	[DKK/MWh]

It should be pointed out that the input and flow variables of the objective function are not small signal values since the energy is to be optimized. Therefore only the deviation from the operating point would not result in the total amount of energy consumed up by the pumps.
The efficiency of the two main pumps is assumed to be the same for the reason that the pumps are of the same type. Furthermore it is considered as constant since the operating point is the same for the main pumps and it is assumed that for small deviations in flow and pressure the efficiency does not change significantly.
The control problem can be formulated as a minimization problem of the form:

$$\underset{\boldsymbol{u}}{min}\, \Upsilon(\cdot) = \underset{\boldsymbol{u}}{min}\, \frac{1}{\eta} \sum_{i=0}^{H_p-1} \Big(\boldsymbol{u^T}[k+i|k] \cdot \boldsymbol{q_p}[k+i|k]\Big) \cdot c[k+i|k] \tag{5.2}$$

$$s.t. \quad \Delta\hat{p}_{wt}[k+i+1|k] = A_d \Delta\hat{p}_{wt}[k+i|k] + \boldsymbol{B_d}\hat{\boldsymbol{u}}[k+i|k] + \boldsymbol{E_d}\hat{\boldsymbol{d}}[k+i|k] \tag{5.3}$$

$$\hat{\boldsymbol{y}}[k+i|k] = \boldsymbol{C_d}\Delta\hat{p}_{wt}[k+i|k] + \boldsymbol{D_d}\hat{\boldsymbol{u}}[k+i|k] + \boldsymbol{K_d}\hat{\boldsymbol{d}}[k+i|k] \tag{5.4}$$

$$\underline{y} \leq \boldsymbol{y} \leq \overline{y} \tag{5.5}$$

$$\underline{\Delta p_{wt}} \leq \Delta p_{wt} \leq \overline{\Delta p_{wt}} \tag{5.6}$$

$$\boldsymbol{u} \leq \overline{u} \tag{5.7}$$

Where

$\boldsymbol{y}$	is the output vector with the PMA pressures,	[bar]
Δp_{wt}	is the state which is the pressure in the WT.	[bar]

As it is shown in *Equation: (5.2)* the optimal input signal is obtained such that the cost of running the pumps is minimized. Therefore the objective function gives a price for all the consumed power over the control horizon, which is a specific future time interval. Furthermore, it should be noted that both the objective function and the constraints are linear, therefore the problem is a linear programming formulation. The minimization is

subject to the dynamics of the water distribution network, and the constraints. As can be seen, there is a constraint on the output of the system, which is considered as the pressure demand for the end-users. There is a constraint on the pressure in the WT which represents the states in the dynamics and furthermore, there is a constraint on the input signal to the pumps.
The dynamic model of the system is explained in Chapter *4: Modelling* and linearized in Section *4.7: Linear parameter estimation.* The electrical price, $c_p[k]$, is described in *Appendix: G.*

In the following sections the design of the control system is explained.

5.2 Model predictive control

Model predictive control (MPC) is an advanced control method that depends on the dynamic model of the plant. MPC allows to calculate an optimal control signal taking the future costs of some matter into account. The control structure of MPC is in general:

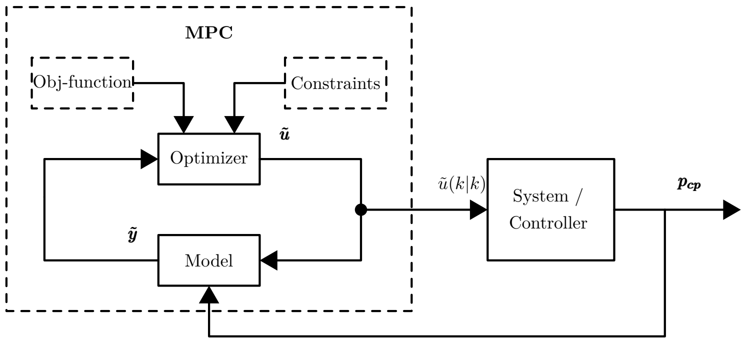


Figure 5.2. The block of MPC algorithm [27].

Figure 5.2 is of the same structure as *Figure 5.1*, however here the MPC block is specified. A MPC is an iterative process that can be summarized as follows:

1: A model predict the future outputs, $\tilde{\boldsymbol{y}}$.
2: The future outputs are feed back to the optimizer that calculate the future inputs $\tilde{\boldsymbol{u}}$.
3: $\tilde{u}(k|k)$, the optimal control signal to the current time k, is sent as the control signal.
4: $\tilde{\boldsymbol{u}}$ are applied to the model, together with the old output.
5: Go back to step 1

The future outputs $\tilde{\boldsymbol{y}}$ and the future inputs $\tilde{\boldsymbol{u}}$ are defined by:

$$\tilde{\boldsymbol{y}} = \begin{bmatrix} \tilde{y}(k|k) \\ \vdots \\ \tilde{y}(k+H_p|k) \end{bmatrix} \tag{5.8}$$

$$\tilde{\boldsymbol{u}} = \begin{bmatrix} \tilde{u}(k|k) \\ \vdots \\ \tilde{u}(k+H_p|k) \end{bmatrix} \tag{5.9}$$

Where

	$\tilde{y}(k|k)$	is the predicted output to time k based on the current output,
	H_p	is the prediction horizon,
and	$\tilde{u}(k|k)$	is the predicted optimal control signal to the current time k.

To calculate the constraint matrices H_p need to be determined. In Section *G: Electrical price* it can be seen that the price fluctuates a lot but some periodicity can be seen every 24 hours. Furthermore water consumption can be seen as periodic, with a periodicity of 24 hours, due to the daily rhythm of the population. Therefore H_p is chosen to 24.

5.3 Reformulation of the objective function

The minimization problem formulated in Section *5.1: Control Problem* describes the optimization subject to the WT dynamics and constraints. In order to make this problem solvable, the objective function has to match the state-space dynamics, therefore it has to be reformulated. As it is shown in *Equation: (5.1)*, the hydraulic power is expressed with the flow through the pumps, however the dynamics include the WT pressure as a state.

In order to help the reader, the equation used for the reformulation of the pump flows is recalled:

$$\hat{\boldsymbol{z}} = -(\boldsymbol{M_c^{-1}N_c})\hat{\boldsymbol{u}} - (\boldsymbol{M_c^{-1}Q_c})\hat{\boldsymbol{d}} - (\boldsymbol{M_c^{-1}B_o})\Delta\hat{p}_{WT} \tag{5.10}$$

Equation: (5.10) explains how the independent flows of the network can be calculated with the system matrices. Using this equation and plugging this into the objective function, the flows through the pumps can be obtained. Recalling that all flows can be obtained from the independent variables, the flow through the pumps can be given with a linear mapping such that:

$$\boldsymbol{q_p}[k] = \boldsymbol{G_2 B_1^T z}[k] \tag{5.11}$$

Where

$\boldsymbol{G_2} \in \mathbb{R}^{(2\times e)}$ is a matrix representing a linear mapping between the flow through the two main pumps and the edge flows in the system. The dimension e is the number of edges without the WT while the number of rows is the number of main pumps.

The system dynamics are only valid for small-signal values of the chord flows, and therefore for the small-signal values of the pump flows. As a consequence of this, $\boldsymbol{q_p}[k]$ can be written up as a sum of the operating point and the deviation from it:

$$\boldsymbol{q_p}[k] = \bar{\boldsymbol{q}}_{\boldsymbol{p}} + \hat{\boldsymbol{q}}_{\boldsymbol{p}}[k] = \bar{\boldsymbol{q}}_{\boldsymbol{p}} + \boldsymbol{G_2 B_1^T} \hat{\boldsymbol{z}}[k] \tag{5.12}$$

Where

$\bar{\boldsymbol{q}}_{\boldsymbol{p}}$	is the operating value of the flow through the main pumps,	$[\mathrm{m^3/h}]$
$\hat{\boldsymbol{q}}_{\boldsymbol{p}}[k]$	is the small-signal value of the flows through the main pumps.	$[\mathrm{m^3/h}]$

The operating value for the pump flows can be determined by the non-linear model of the pumps with the available pressure measurements. This is described in detail in *Appendix:* **??**.

Plugging the expression for the small-signal chord flows given in *Equation: (5.10)* into *Equation: (5.12)* and then into the objective function, the following yields:

$$\hat{\boldsymbol{q}}_{\boldsymbol{p}}[k] = \boldsymbol{G_2 B_1^T}(-\boldsymbol{M_c^{-1}N_c} \cdot \hat{\boldsymbol{u}}[k] - \boldsymbol{M_c^{-1}Q_c} \cdot \hat{\boldsymbol{d}}[k] - \boldsymbol{M_c^{-1}B_o} \cdot \Delta\hat{p}_{wt}[k]) \tag{5.13}$$

Therefore the small-signal flow through the main pumps can be written as:

$$\hat{\boldsymbol{q}}_{\boldsymbol{p}}[k] = \boldsymbol{\Lambda_1}\hat{\boldsymbol{u}}[k] + \boldsymbol{\Lambda_2}\hat{\boldsymbol{d}}[k] + \boldsymbol{\Lambda_3}\Delta\hat{p}_{wt}[k] \tag{5.14}$$

Now define

$$\boldsymbol{\Lambda_1} = -\boldsymbol{G_2}\boldsymbol{B_1^T}\boldsymbol{M_c^{-1}}\boldsymbol{N_c} \in \mathbb{R}^{(2\times 2)}$$
$$\boldsymbol{\Lambda_2} = -\boldsymbol{G_2}\boldsymbol{B_1^T}\boldsymbol{M_c^{-1}}\boldsymbol{Q_c} \in \mathbb{R}^{(2\times 4)}$$
$$\boldsymbol{\Lambda_3} = -\boldsymbol{G_2}\boldsymbol{B_1^T}\boldsymbol{M_c^{-1}}\boldsymbol{B_o} \in \mathbb{R}^{(2\times 1)}$$

Hence the objective function:

$$\Upsilon(\cdot) = \frac{1}{\eta}\sum_{i=0}^{H_p-1}\Big[\boldsymbol{u^T}[k+i|k]\cdot\Big(\bar{\boldsymbol{q}}_{\boldsymbol{p}} + \boldsymbol{\Lambda_1}\hat{\boldsymbol{u}}[k+i|k] + \boldsymbol{\Lambda_2}\hat{\boldsymbol{d}}[k+i|k] + \boldsymbol{\Lambda_3}\Delta\hat{p}_{wt}[k+i|k]\Big)\Big]\cdot c_p[k+i|k] \tag{5.15}$$

As can be seen in *Equation: (5.15)*, the objective function now includes both the full- and small-signal inputs, the small-signal disturbances and the small-signal pressure in the WT. It is important to point out that the system dynamics are described by small-signals, however as it is mentioned in the problem formulation, the optimization for the cost has to be according to full-signals. Since the dynamics are meant to be plugged into the objective function, the following has to be considered:

$$\boldsymbol{u} = \bar{\boldsymbol{u}} + \hat{\boldsymbol{u}} \tag{5.16}$$

and

$$\boldsymbol{d} = \bar{\boldsymbol{d}} + \hat{\boldsymbol{d}} \tag{5.17}$$

which shows that the signals are decomposed to their constant values in the operating point and the small- signal deviations.

As it was shown before, the WT pressure is described as a state in the dynamics governing the water distribution system. Therefore the state-space model in *Equation: (5.3)* can be used to substitute this pressure term in *Equation: (5.15)*.

However, before substituting the dynamics into the objective function, it is important to point out that an initial measurement of the states has to be available. In other words, an initial small-signal pressure value is required in the WT. The deviation from the operating point can be determined by subtracting the operating point value from the measurement. Due to the available sensors it is possible to obtain the initial WT pressure at $i = 0$.

It is important to emphasize furthermore that the cost function is formulated in such a way that from the current time step,k , the future values are iterated from $i = 0$ to $i = H_p - 1$ when calculating the price of energy usage for the whole interval. In order to replace this iterative summation with vector and matrix products, all signals and matrices are written up for the complete prediction horizon. Therefore the elements of the input vector now represent input configurations at different time steps moving towards the end of the interval, such that:

$$\boldsymbol{u_{Hp}} = \begin{bmatrix} \boldsymbol{u}[k|k] \\ \vdots \\ \boldsymbol{u}[k+H_p-1|k] \end{bmatrix} \in \mathbb{R}^{(2H_p\times 1)} \tag{5.18}$$

and the disturbance vector therefore formulated as:

$$\boldsymbol{d_{Hp}} = \begin{bmatrix} \boldsymbol{d}[k|k] \\ \vdots \\ \boldsymbol{d}[k+H_p-1|k] \end{bmatrix} \in \mathbb{R}^{(4H_p \times 1)} \tag{5.19}$$

Thus at time step k, these signals consist of the present and future input and disturbance vectors until the end of the horizon, which is $H_p - 1$ since i is iterated from zero. In the further discussion the same notation is used for all signals as in *Equation: (5.18)* and *Equation: (5.19)*.

In order to substitute the WT dynamics into the objective function, it has to be written up for the whole prediction horizon, with extended matrices and signal vectors that represent the above-mentioned predicted values of the inputs, states and disturbances. For $i = 1$, the iteration of the state equation gives:

$$\Delta\hat{p}_{wt}[1] = A_d\Delta\hat{p}_{wt}[0] + \boldsymbol{B_d}\boldsymbol{\hat{u}}[0] + \boldsymbol{E_d}\boldsymbol{\hat{d}}[0] \tag{5.20}$$

Now $\Delta\hat{p}_{wt}[2]$ is calculated in the same way, but with $\Delta\hat{p_{wt}}[1]$ substituted:

$$\Delta\hat{p}_{wt}[2] = A_d[A_d\Delta\hat{p}_{wt}[0] + \boldsymbol{B_d}\boldsymbol{\hat{u}}[0] + \boldsymbol{E_d}\boldsymbol{\hat{d}}[0]] + \boldsymbol{B_d}\boldsymbol{\hat{u}}[1] + \boldsymbol{E_d}\boldsymbol{\hat{d}}[1] \tag{5.21}$$

As can be seen, moving further in time steps, the predicted states only depend on the past values of the input, disturbance and the initial state measurement. Writing up the matrix equation until the $i = H_p - 1$ time steps, thus until the end of the prediction horizon, the following state dynamic matrix equation yields:

$$\begin{aligned}
\underbrace{\begin{bmatrix} \Delta\hat{p}_{wt}[k+1|k] \\ \Delta\hat{p}_{wt}[k+2|k] \\ \vdots \\ \Delta\hat{p}_{wt}[k+H_p] \end{bmatrix}}_{\boldsymbol{\Delta\hat{p}}_{wt,Hp}} = \underbrace{\begin{bmatrix} A_d \\ A_d^2 \\ \vdots \\ A_d^{H_p} \end{bmatrix}}_{\boldsymbol{\Phi}} \Delta\hat{p}_{wt}[0] + \underbrace{\begin{bmatrix} \boldsymbol{B_d} & \boldsymbol{0} & \dots & \boldsymbol{0} \\ A_d\boldsymbol{B_d} & \boldsymbol{B_d} & \dots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ A_d^{H_p}\boldsymbol{B_d} & A_d^{H_p-1}\boldsymbol{B_d} & \dots & \boldsymbol{B_d} \end{bmatrix}}_{\boldsymbol{\Gamma}} \underbrace{\begin{bmatrix} \boldsymbol{\hat{u}}[k|k] \\ \boldsymbol{\hat{u}}[k+1|k] \\ \vdots \\ \boldsymbol{\hat{u}}[k+H_p-1|k] \end{bmatrix}}_{\boldsymbol{\hat{u}}_{Hp}} \\
+ \underbrace{\begin{bmatrix} \boldsymbol{E_d} & 0 & \dots & 0 \\ A_d\boldsymbol{E_d} & \boldsymbol{E_d} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_d^{H_p}\boldsymbol{E_d} & A_d^{H_p-1}\boldsymbol{E_d} & \dots & \boldsymbol{E_d} \end{bmatrix}}_{\boldsymbol{\Psi}} \underbrace{\begin{bmatrix} \boldsymbol{\hat{d}}[k|k] \\ \boldsymbol{\hat{d}}[k+1|k] \\ \vdots \\ \boldsymbol{\hat{d}}[k+H_p-1|k] \end{bmatrix}}_{\boldsymbol{\hat{d}}_{Hp}}
\end{aligned} \tag{5.22}$$

Which in short form, expressed with the newly introduced vectors and matrices can be written as:

$$\Delta\hat{p}_{wt,H_p} = \boldsymbol{\Phi}\Delta\hat{p}_{wt}[0] + \boldsymbol{\Gamma}\boldsymbol{\hat{u}_{Hp}} + \boldsymbol{\Psi}\boldsymbol{\hat{d}_{Hp}} \tag{5.23}$$

Where

$\boldsymbol{\Delta\hat{p}}_{wt,H_p}$	$\in \mathbb{R}^{(H_p\times 1)}$	is the predicted state vector calculated for the whole prediction horizon,
$\Delta\hat{p}_{\boldsymbol{wt}}[0]$	$\in \mathbb{R}^{(1\times 1)}$	is the initial state describing the whole prediction horizon,
$\boldsymbol{\hat{u}_{Hp}}$	$\in \mathbb{R}^{(2H_p\times 1)}$	is the predicted input vector consisting of all the predicted values from the current time step until $k = H_p - 1$,
$\boldsymbol{\hat{d}_{Hp}}$	$\in \mathbb{R}^{(4H_p\times 1)}$	is the disturbance vector consisting of all the future values from the current time step until $k = H_p - 1$,
$\boldsymbol{\Phi}$	$\in \mathbb{R}^{(H_p\times 1)}$	is the state matrix along the prediction horizon, taking A_d into account at each time step,
$\boldsymbol{\Gamma}$	$\in \mathbb{R}^{(H_p\times 2H_p)}$	is the input matrix along the prediction horizon, taking A_d and $\boldsymbol{B_d}$ matrix into account at each time step,
and $\boldsymbol{\Psi}$	$\in \mathbb{R}^{(H_p\times 4H_p)}$	is the disturbance matrix along the prediction horizon, taking the A_d and $\boldsymbol{E_d}$ matrix into account at each time step .

It is shown therefore that the state, input and disturbance matrices, described above, may be calculated prior to solving the MPC optimization. *Equation: (5.23)* defines the future state trajectories for each time step and for the whole prediction horizon. Furthermore, it is noted here that these matrices now describe the variables for the whole prediction horizon, and therefore the signals are subscripted with H_p, as mentioned above.

The objective function formulated in *Equation: (5.15)* consists of the product of the $\boldsymbol{\Lambda_{1,2,3}}$ matrices and the signals. In order to express the objective function in vectorial form, the cost function is multiplied with these $\boldsymbol{\Lambda_{1,2,3}}$ matrices such that:

$$\boldsymbol{\Lambda_{1,Hp}} = \begin{bmatrix} \boldsymbol{\Lambda_1} c_p[k|k] & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \boldsymbol{\Lambda_1} c_p[k+H_p-1|k] \end{bmatrix} \in \mathbb{R}^{(2H_p\times 2H_p)} \tag{5.24}$$

$$\boldsymbol{\Lambda_{2,Hp}} = \begin{bmatrix} \boldsymbol{\Lambda_2} c_p[k|k] & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \boldsymbol{\Lambda_2} c_p[k+H_p-1|k] \end{bmatrix} \in \mathbb{R}^{(2H_p\times 4H_p)} \tag{5.25}$$

$$\boldsymbol{\Lambda_{3,Hp}} = \begin{bmatrix} \boldsymbol{\Lambda_3} c_p[k|k] & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \boldsymbol{\Lambda_3} c_p[k+H_p-1|k] \end{bmatrix} \in \mathbb{R}^{(2H_p\times H_p)} \tag{5.26}$$

Now, that all the matrices and signals are represented in vectorial form along with their predicted values, the objective function can be written up such that:

$$\Upsilon(\cdot) = \frac{1}{\eta}(\boldsymbol{\bar{u}_{Hp}} + \boldsymbol{\hat{u}_{Hp}})^T \left[\boldsymbol{\bar{q}_{p,Hp}} + \boldsymbol{\Lambda_{1,Hp}}\boldsymbol{\hat{u}_{Hp}} + \boldsymbol{\Lambda_{2,Hp}}\boldsymbol{\hat{d}_{Hp}} + \boldsymbol{\Lambda_{3,Hp}}\boldsymbol{\Delta\hat{p}_{wt,Hp}} \right] \tag{5.27}$$

And then the dynamics of the system can be plugged into *Equation: (5.27)*:

$$\begin{aligned} \Upsilon(\cdot) = \frac{1}{\eta}(\boldsymbol{\bar{u}_{Hp}} + \boldsymbol{\hat{u}_{Hp}})^T \Big[& \boldsymbol{\bar{q}_{p,Hp}} + \boldsymbol{\Lambda_{1,Hp}}\boldsymbol{\hat{u}_{Hp}} + \boldsymbol{\Lambda_{2,Hp}}\boldsymbol{\hat{d}_{Hp}} \\ & + \boldsymbol{\Lambda_{3,Hp}}\Big(\boldsymbol{\Phi}\Delta\hat{p}_{wt,Hp}[0] + \boldsymbol{\Gamma}\boldsymbol{\hat{u}_{Hp}} + \boldsymbol{\Psi}\boldsymbol{\hat{d}_{Hp}}\Big)\Big] \end{aligned} \tag{5.28}$$

By expressing the terms in *Equation: (5.28)*, the following yields:

$$\Upsilon(\cdot) = \frac{1}{\eta}\Big[\underbrace{(\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \bar{\boldsymbol{q}}_{\boldsymbol{p,Hp}}}_{\text{I}} + \underbrace{(\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \boldsymbol{\Lambda}_{\boldsymbol{1,Hp}} \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}}_{\text{II}} + \underbrace{(\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \boldsymbol{\Lambda}_{\boldsymbol{2,Hp}} \hat{\boldsymbol{d}}_{\boldsymbol{Hp}}}_{\text{III}}$$
$$+ \underbrace{(\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Phi} \Delta\hat{p}_{wt,Hp}[0]}_{\text{IV}} + \underbrace{(\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Gamma} \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}}_{\text{V}} + \underbrace{(\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Psi} \hat{\boldsymbol{d}}_{\boldsymbol{Hp}}}_{\text{VI}}\Big] \tag{5.29}$$

The product of the different terms with the operating and deviation values results in the following:

$$\text{I)}\quad (\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \bar{\boldsymbol{q}}_{\boldsymbol{p,Hp}} = \bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \bar{\boldsymbol{q}}_{\boldsymbol{p,Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \bar{\boldsymbol{q}}_{\boldsymbol{p,Hp}} \tag{5.30}$$
$$\text{II)}\quad (\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \boldsymbol{\Lambda}_{\boldsymbol{1,Hp}} \hat{\boldsymbol{u}}_{\boldsymbol{Hp}} = \bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{1,Hp}} \hat{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{1,Hp}} \hat{\boldsymbol{u}}_{\boldsymbol{Hp}} \tag{5.31}$$
$$\text{III)}\quad (\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \boldsymbol{\Lambda}_{\boldsymbol{2,Hp}} \hat{\boldsymbol{d}}_{\boldsymbol{Hp}} = \bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{2,Hp}} \hat{\boldsymbol{d}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{2,Hp}} \hat{\boldsymbol{d}}_{\boldsymbol{Hp}} \tag{5.32}$$
$$\begin{aligned}\text{IV)}\quad & (\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Phi} \Delta\hat{p}_{wt,Hp}[0] = \bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Phi} \Delta\hat{p}_{wt,Hp}[0] \\ & + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Phi} \Delta\hat{P}_{wt,Hp}[0]\end{aligned} \tag{5.33}$$
$$\text{V)}\quad (\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Gamma} \hat{\boldsymbol{u}}_{\boldsymbol{Hp}} = \bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Gamma} \hat{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Gamma} \hat{\boldsymbol{u}}_{\boldsymbol{Hp}} \tag{5.34}$$
$$\text{VI)}\quad (\bar{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}})^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Psi} \hat{\boldsymbol{d}}_{\boldsymbol{Hp}} = \bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Psi} \hat{\boldsymbol{d}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Psi} \hat{\boldsymbol{d}}_{\boldsymbol{Hp}} \tag{5.35}$$

As can be seen, now the objective functions is expressed with the vector of small-signal inputs. The operating values of the inputs are the operating pressures of the two main pumps. Both the small-signal and operating point values of the disturbances are present. However, these disturbances are the ODs of the end-user valves, therefore they describe some kind of characteristics of water usage. From the expressions derived above it can be clearly seen that predictions are necessary for this water usage. The operating point values are known and are constant in the whole sequence. The small signal values are the deviations from this constant OD and also known, since the full-signal value of the whole disturbance sequence is known.
As it is shown, all matrices in the objective function are expressed using the predicted value of the energy cost which is changing in time. Therefore the matrices need to be updated at every time step.
After rearranging the terms, it is shown in *Equation: (5.35)* that the objective function consists of quadratic and linear terms of the vector $\hat{\boldsymbol{u}}_{\boldsymbol{Hp}}$. Furthermore, there are constants due to the operating values of the disturbances and inputs.

Hence the quadratic term results in:

$$\hat{\boldsymbol{u}}_{Hp}^T(\boldsymbol{\Lambda}_{1,Hp} + \boldsymbol{\Lambda}_{3,Hp}\boldsymbol{\Gamma})\hat{\boldsymbol{u}}_{Hp} \tag{5.36}$$

The linear term is given by:

$$\begin{aligned}&\bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T(\boldsymbol{\Lambda}_{\boldsymbol{1,Hp}} + \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}}\boldsymbol{\Gamma})\hat{\boldsymbol{u}}_{\boldsymbol{Hp}} + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}^T(\boldsymbol{\Lambda}_{\boldsymbol{2,Hp}} + \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}}\boldsymbol{\Psi})\hat{\boldsymbol{d}}_{\boldsymbol{Hp}} \\ &+ \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Phi} \Delta\hat{p}_{wt,Hp}[0] + \hat{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \bar{\boldsymbol{q}}_{\boldsymbol{p,Hp}}\end{aligned} \tag{5.37}$$

And the constants are:

$$\bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \bar{\boldsymbol{q}}_{\boldsymbol{p,Hp}} + \bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{2,Hp}} \hat{\boldsymbol{d}}_{\boldsymbol{Hp}} + \bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Phi} \Delta\hat{p}_{wt,Hp}[0] + \bar{\boldsymbol{u}}_{\boldsymbol{Hp}}^T \boldsymbol{\Lambda}_{\boldsymbol{3,Hp}} \boldsymbol{\Psi} \hat{\boldsymbol{d}}_{\boldsymbol{Hp}} \tag{5.38}$$

Therefore it is shown that this optimization simplifies to a quadratic problem that can be written up in such a form as follows:

$$\Upsilon(\cdot) = \frac{1}{\eta}\Big(\frac{1}{2}\boldsymbol{\hat{u}^T_{Hp}R\hat{u}_{Hp}} + \boldsymbol{b\hat{u}_{Hp}} + c\Big) \tag{5.39}$$

Where
$\boldsymbol{R} \in \mathbb{R}^{(2H_p \times 2H_p)}$
$\boldsymbol{b} \in \mathbb{R}^{(1 \times H_p)}$
$c \in \mathbb{R}^{(1 \times 1)}$

Therefore the $\boldsymbol{R}$ matrix in the quadratic problem can be given as:

$$\boldsymbol{R} = 2\Big(\boldsymbol{\Lambda_{1,Hp}} + \boldsymbol{\Lambda_{3,Hp}\Gamma}\Big) \tag{5.40}$$

The vector $\boldsymbol{b}$ can be given as:

$$\boldsymbol{b} = \boldsymbol{\bar{u}^T_{Hp}}(\boldsymbol{\Lambda_{1,Hp}}+\boldsymbol{\Lambda_{3,Hp}\Gamma})+\boldsymbol{\hat{d}^T_{Hp}}(\boldsymbol{\Lambda_{2,Hp}}+\boldsymbol{\Lambda_{3,Hp}\Psi})^T + \Delta\hat{p}^T_{wt,Hp}[0](\boldsymbol{\Lambda_{3,Hp}\Phi})^T + \boldsymbol{\bar{q}^T_{p,Hp}} \tag{5.41}$$

and c can be given as:

$$c = \boldsymbol{\bar{u}^T_{Hp}\bar{q}_{p,Hp}} + \boldsymbol{\bar{u}^T_{Hp}\Lambda_{2,Hp}\hat{d}_{Hp}} + \boldsymbol{\bar{u}^T_{Hp}}(\boldsymbol{\Lambda_{3,Hp}\Phi})\Delta\hat{p}_{wt,Hp}[0] + \boldsymbol{\bar{u}^T_{Hp}}(\boldsymbol{\Lambda_{3,Hp}\Psi})\boldsymbol{\hat{d}_{Hp}} \tag{5.42}$$

Hence the optimization is given by: the following minimization problem

$$\min_{\boldsymbol{\hat{u}_{Hp}}} \Upsilon(\cdot) = \min_{\boldsymbol{\hat{u}_{Hp}}} \frac{1}{\eta}\Bigg(\frac{1}{2}\boldsymbol{\hat{u}^T_{Hp}R\hat{u}_{Hp}} + \boldsymbol{b\hat{u}_{Hp}} + c\Bigg) \tag{5.43}$$

5.4 Constraints

The reformulation of the objective function, as it was shown, results in a constrained quadratic problem. This problem is the process of optimizing the objective function described in *Equation: (5.43)* with respect to the input signal in the presence of constraints on the input. As it was shown in the control problem, constraints are defined as inequalities on the output, state and input signals respectively. Since all constraints should be set up as a constraint on the controller input, $\boldsymbol{u_{Hp}}$, these inequalities must be reformulated.

First, let's consider the constraints on the output signal which are the upper and lower bound constraints for the end-user pressures in the two PMAs. Recalling the output equation constructed for the control and now extending it for the prediction horizon, similarly as it was done for the state equation, yields:

$$\boldsymbol{\hat{y}_{Hp}} = \boldsymbol{\Theta\Delta}\hat{p}_{wt}[0] + \boldsymbol{\Omega\hat{u}_{Hp}} + \Pi\boldsymbol{\hat{d}_{Hp}} \tag{5.44}$$

Where the output, feedforward and disturbance matrices are respectively:

$$\boldsymbol{\Theta} = \begin{bmatrix} \boldsymbol{C_d} & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{C_d} \end{bmatrix} \in \mathbb{R}^{(2H_p \times H_p)} \tag{5.45}$$

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{D_d} & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{D_d} \end{bmatrix} \in \mathbb{R}^{(2H_p \times 2H_p)} \tag{5.46}$$

$$\boldsymbol{\Pi} = \begin{bmatrix} \boldsymbol{K_d} & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{K_d} \end{bmatrix} \in \mathbb{R}^{(2H_p \times 4H_p)} \tag{5.47}$$

Now inserting the extended state equation back into the output equation the following yields:

$$\boldsymbol{\hat{y}_{Hp}} = \boldsymbol{\Theta}[\boldsymbol{\Phi}\Delta\hat{p}_{wt}[0] + \boldsymbol{\Gamma}\boldsymbol{\hat{u}_{Hp}} + \boldsymbol{\Psi}\boldsymbol{\hat{d}_{Hp}}] + \boldsymbol{\Omega}\boldsymbol{\hat{u}_{Hp}} + \boldsymbol{\Pi}\boldsymbol{\hat{d}_{Hp}} \tag{5.48}$$

And summing up the matrix multiplications, all signals can be expressed such that:

$$\boldsymbol{\hat{y}_{Hp}} = \boldsymbol{\Theta\Phi}\Delta\hat{p}_{wt}[0] + (\boldsymbol{\Theta\Gamma} + \boldsymbol{\Omega})\boldsymbol{\hat{u}_{Hp}} + (\boldsymbol{\Theta\Psi} + \boldsymbol{\Pi})\boldsymbol{\hat{d}_{Hp}} \tag{5.49}$$

Equation: (5.49) defines the equality between the small-signal output and small-signal inputs. Inserting the upper-and lower-bound values of the small-signal outputs in this equation, the inequality constraint is obtained for the outputs as an affine function of the small-signal input signals. It has to be considered however, that the constraints are originally set for full-signal values, therefore in this case the upper and lower values are the maximum allowed and minimum required pressure values delivered to the end-users. As a consequence of this, when inserting the bounds back into *Equation: (5.49)*, the operating point has to be subtracted at both sides to transform each term to small-signals. The constraint therefore is given as:

$$\underline{\boldsymbol{y_{Hp}}} - \boldsymbol{\bar{y}_{Hp}} \leq \boldsymbol{\Theta\Phi}\Delta\hat{p}_{wt}[0] + (\boldsymbol{\Theta\Gamma} + \boldsymbol{\Omega})\boldsymbol{\hat{u}_{Hp}} + (\boldsymbol{\Theta\Psi} + \boldsymbol{\Pi})\boldsymbol{\hat{d}_{Hp}} \leq \overline{\boldsymbol{y_{Hp}}} - \boldsymbol{\bar{y}_{Hp}} \tag{5.50}$$

Where

$\overline{\boldsymbol{y_{Hp}}}$	is the upper-bound of the end-user pressure,	[bar]
$\underline{\boldsymbol{y_{Hp}}}$	is the lower-bound of the end-user pressure.	[bar]

The constraint on the states can be reformulated using the same idea as for the output, except that it is sufficient to use the extended state dynamic equation such that:

$$\underline{\boldsymbol{\Delta p_{wt,Hp}}} - \boldsymbol{\Delta\bar{p}_{wt,Hp}} \leq \boldsymbol{\Theta\Phi}\Delta\hat{p}_{wt}[0] + (\boldsymbol{\Theta\Gamma} + \boldsymbol{\Omega})\boldsymbol{\hat{u}_{Hp}} + (\boldsymbol{\Theta\Psi} + \boldsymbol{\Pi})\boldsymbol{\hat{d}_{Hp}} \leq \overline{\boldsymbol{\Delta p_{wt,Hp}}} - \boldsymbol{\Delta\bar{p}_{wt,Hp}} \tag{5.51}$$

Where

$\overline{\boldsymbol{\Delta p_{wt,Hp}}}$	is the upper-bound of the WT pressure,	[bar]
$\underline{\boldsymbol{\Delta p_{wt,Hp}}}$	is the lower-bound of the WT pressure.	[bar]

And the constraint for the input signals are given by:

$$\underline{\boldsymbol{u}} - \boldsymbol{\bar{u}} \leq \boldsymbol{\hat{u}_{Hp}} \leq \overline{\boldsymbol{u}} - \boldsymbol{\bar{u}} \tag{5.52}$$

Where

$\overline{\boldsymbol{u}}$	is the upper-bound of the control input,	[bar]
$\underline{\boldsymbol{u}}$	is the lower-bound of the control input.	[bar]

In *Equation: (5.52)*, the full-signal lower-bound constraint equals to zero and from now on in the further discussion, zero is used.

As it is shown, the constraints are all set up as constraints on the input signal in *Equation: (5.50)*, *Equation: (5.52)* and in *Equation: (5.51)*. In order to formulate all the constraints as one affine inequality system, the three constrain inequalities are divided into six inequalities such that the lower-bound for the input is given by:

$$\underbrace{\mathbf{0}}_{\hat{\boldsymbol{u}}_1} \leq \hat{\boldsymbol{u}}_{\boldsymbol{H_p}} \tag{5.53}$$

The upper bound of the input is given by:

$$\underbrace{\overline{\boldsymbol{u}} - \bar{\boldsymbol{u}}_{\boldsymbol{Hp}}}_{\hat{\boldsymbol{u}}_2} \geq \hat{\boldsymbol{u}}_{\boldsymbol{Hp}} \tag{5.54}$$

The lower-bound of the output is given by:

$$\underbrace{\underline{\boldsymbol{y}_{\boldsymbol{Hp}}} - \bar{\boldsymbol{y}}_{\boldsymbol{Hp}} - \boldsymbol{\Theta\Phi}\Delta\hat{p}_{wt}[0] - (\boldsymbol{\Theta\Psi} + \boldsymbol{\Pi})\hat{\boldsymbol{d}}_{\boldsymbol{Hp}}}_{\hat{\boldsymbol{y}}_1} \leq \underbrace{(\boldsymbol{\Theta\Gamma} + \boldsymbol{\Omega})}_{\boldsymbol{L_y}}\hat{\boldsymbol{u}}_{\boldsymbol{Hp}} \tag{5.55}$$

The upper-bound of the output is given by:

$$\underbrace{\overline{\boldsymbol{y}_{\boldsymbol{Hp}}} - \bar{\boldsymbol{y}}_{\boldsymbol{Hp}} - \boldsymbol{\Theta\Phi}\Delta\hat{p}_{wt}[0] - (\boldsymbol{\Theta\Psi} + \boldsymbol{\Pi})\hat{\boldsymbol{d}}_{\boldsymbol{Hp}}}_{\hat{\boldsymbol{y}}_2} \geq \underbrace{(\boldsymbol{\Theta\Gamma} + \boldsymbol{\Omega})}_{\boldsymbol{L_y}}\hat{\boldsymbol{u}}_{\boldsymbol{Hp}} \tag{5.56}$$

The lower-bound of the state is given by:

$$\underbrace{\underline{\boldsymbol{\Delta p}_{\boldsymbol{wt,Hp}}} - \boldsymbol{\Delta}\bar{\boldsymbol{p}}_{\boldsymbol{wt,Hp}} - \boldsymbol{\Phi}\Delta\hat{p}_{wt}[0] - \boldsymbol{\Psi}\hat{\boldsymbol{d}}_{\boldsymbol{Hp}}}_{\boldsymbol{\Delta}\hat{\boldsymbol{p}}_{\boldsymbol{wt,1}}} \leq \boldsymbol{\Gamma}\hat{\boldsymbol{u}}_{\boldsymbol{Hp}} \tag{5.57}$$

The upper-bound of the state is given by:

$$\underbrace{\overline{\boldsymbol{\Delta p}_{\boldsymbol{wt,Hp}}} - \boldsymbol{\Delta}\bar{\boldsymbol{p}}_{\boldsymbol{wt,Hp}} - \boldsymbol{\Phi}\Delta\hat{p}_{wt}[0] - \boldsymbol{\Psi}\hat{\boldsymbol{d}}_{\boldsymbol{Hp}}}_{\boldsymbol{\Delta}\hat{\boldsymbol{p}}_{\boldsymbol{wt,2}}} \geq \boldsymbol{\Gamma}\hat{\boldsymbol{u}}_{\boldsymbol{Hp}} \tag{5.58}$$

Collecting all the constraints, the following affine constraint inequality is obtained:

$$\begin{bmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \\ \boldsymbol{L_y} \\ -\boldsymbol{L_y} \\ \boldsymbol{\Gamma} \\ -\boldsymbol{\Gamma} \end{bmatrix} \hat{\boldsymbol{u}}_{\boldsymbol{Hp}} \geq \begin{bmatrix} \mathbf{0} \\ \hat{\boldsymbol{u}}_2 \\ \hat{\boldsymbol{y}}_1 \\ \hat{\boldsymbol{y}}_2 \\ \boldsymbol{\Delta}\hat{\boldsymbol{p}}_{\boldsymbol{wt,1}} \\ \boldsymbol{\Delta}\hat{\boldsymbol{p}}_{\boldsymbol{wt,2}} \end{bmatrix} \tag{5.59}$$

5.5 Convexity

Due to the reformulation of the objective function and constraints, the MPC optimization problem becomes the following:

$$\underset{\hat{\boldsymbol{u}}_{\boldsymbol{Hp}}}{min}\ \Upsilon(\cdot) = \underset{\hat{\boldsymbol{u}}_{\boldsymbol{Hp}}}{min}\ \frac{1}{\eta}\left(\frac{1}{2}\hat{\boldsymbol{u}}_{\boldsymbol{Hp}}^T\boldsymbol{R}\hat{\boldsymbol{u}}_{\boldsymbol{Hp}} + \boldsymbol{b}\hat{\boldsymbol{u}}_{\boldsymbol{Hp}} + c\right) \tag{5.60}$$

$$s.t. \quad \begin{bmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \\ \boldsymbol{L_y} \\ -\boldsymbol{L_y} \\ \boldsymbol{\Gamma} \\ -\boldsymbol{\Gamma} \end{bmatrix} \boldsymbol{\hat{u}_{Hp}} \geq \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{\hat{u}_2} \\ \boldsymbol{\hat{y}_1} \\ \boldsymbol{\hat{y}_2} \\ \boldsymbol{\Delta\hat{p}_{wt,1}} \\ \boldsymbol{\Delta\hat{p}_{wt,2}} \end{bmatrix} \tag{5.61}$$

As can be seen, the objective function and the constraints are all written up according to the small-signal value of the control signal.

In this section:
-show how the quadratic problem can be solved -prove positive semi-definiteness

5.6 PI controller

As described in Section *5.1: Control Problem* the two PMA pumps should generate a constant differential pressure. Furthermore Section *5.1: Control Problem* conclude that a simple PI controller, reacting to a reference calculated by the MPC, should be used to control the pressure generated by the main pumps.

In *Appendix:* **??** a linearized transfer function is derived. Whit the operating point of the four pumps and their respective parameters shown in *Appendix: C*, four different models are derived as

include the models

The controllers are design through the Matlab toolbox *Control System Designer App*[28], with the approach of the same control characteristics. Since the time constant of the water system is large, the settling time of the PI controllers has been chosen to 5 seconds.

With the settling time in mind the following controllers has been designed

include the PI controllers

6 Implementation of controller

This chapter will explain how the controller designed in Chapter *5: Controller* is implemented in MATLAB simulink.

Part III

Conclusion and verification

7 Accepttest

Discussion 8

Conclusion 9

Part IV

Appendices

Unit Conversion A

Due to the large difference between the SI-units of flow, [m^3/s], and pressure, [Pa], a conversion from seconds to hours and pascal to Bar is made.

The final pipe model from *Equation: (4.21)*, is shown below.

$$\begin{aligned} \frac{L\rho}{A}\frac{dq}{dt} &= \Delta p - \frac{8fL}{\pi^2 g D^5}\rho g|q|q - k_f\frac{8}{\pi^2 g D^4}\rho g|q|q - \Delta z\rho g \\ &= \Delta p - (\frac{8fL}{\pi^2 g D^5} + k_f\frac{8}{\pi^2 g D^4})\rho g|q|q - \Delta z\rho g \end{aligned} \tag{A.1}$$

$1[\text{bar}] = 10^5[\text{Pa}]$. Therefore we can rewrite *Equation: (A.1)* to:

$$\begin{aligned} \frac{L\rho}{A\cdot 10^5}\frac{dq}{dt} &= \Delta\frac{p}{10^5} - (\frac{8fL}{\pi^2 g D^5\cdot 10^5} + k_f\frac{8}{\pi^2 g D^4\cdot 10^5})\rho g|q|q - \frac{\Delta z\rho g}{10^5} \\ \frac{L\rho}{A\cdot 10^5}\frac{dq}{dt} &= \Delta p_{bar} - (\frac{8fL}{\pi^2 g D^5\cdot 10^5} + k_f\frac{8}{\pi^2 g D^4\cdot 10^5})\rho g|q|q - \frac{\Delta z\rho g}{10^5} \end{aligned} \tag{A.2}$$

The conversion from $[\frac{m^3}{s}]$ to $[\frac{m^3}{h}]$ is $\frac{m^3}{s}3600 = \frac{m^3}{s}$. *Equation: (A.1)* can be written as:

$$\frac{L\rho}{A\cdot 10^5}\frac{d}{dt}\frac{q}{3600} = \Delta\frac{p}{10^5} - (\frac{8fL}{\pi^2 g D^5\cdot 10^5} + k_f\frac{8}{\pi^2 g D^4\cdot 10^5})\rho g\frac{|q|}{3600}\frac{q}{3600} - \frac{\Delta z\rho g}{10^5} \tag{A.3}$$

There is no need to apply the unit conversion to the final valve model from *Equation: (4.29)*, due to the parameter k_v being designed for the water flow in m^3 through the valve in one hour and at a pressure drop across the valve of 1 Bar.

In the pump final model *Equation: (4.31)* the constants are scaled so the pump equation has the units in Bar and the flow has the units in m^3/h.

Assumption List B

Number	Assumptions	Section reference
1	The fluid in the network is water.	Section *4.1.1: Pipe model*
2	All pipes in the system are filled up fully with water at all time.	Section *4.1.1: Pipe model*
3	The pipes have a cylindrical structure and the cross section, $A(x)$, is constant for every $x \in [0, L]$.	Section *4.1.1: Pipe model*
4	The flow of water is uniformly distributed along the cross sectional area of the pipe and the flow is turbulent.	Section *4.1.1: Pipe model*
5	Δz, the change in elevation only occurs in pipes.	Section *4.1.2: Valve model*
6	The pumps in the network are centrifugal pumps.	Section *4.1.3: Pump model*
7	The storage of the WT has a constant diameter. In other words, the walls of the WT are vertical.	Section *4.1.4: Water Tower*
8	Valves in the water distribution system are modelled according to the assumption that the length,L, is zero.	Section *4.1.2: Valve model*
9	$\mathcal{G}$ is a connected graph.	Section *4.3: Graph representation*
10	The pipe volume is assumed to be known to an accuracy where there is not any benefit from estimating it. Thereby the estimation problem is simplified.	Section *4.4: Nonlinear Parameter identification*
11	Functions describing the pressure drops across the elements of the system are continuously differentiable. Therefore can be approximated with their Taylor-series.	Section *4.5.1: Taylor expansion on a simple example*
12	The operating point for the system is chosen such that $f(\bar{x}, \bar{u}) = 0$.	Section *4.5.1: Taylor expansion on a simple example*
13	The efficiency of the main pumps are considered constant for small deviations around the same operating point.	Section *5.1: Control Problem*

Table B.1. List of assumptions

System Description C

C.1 Components of the System

Here all necessary data (from the datasheets) and notations should be listed about the components (pipes, pumps, valves, etc)

Part	Component	Length	Diameter	Material	ϵ	Δz	Fittings	Σk_f
Ring	C_4	5m+0.3m	20 mm	25 mm PEM	0.01 mm	0 m	b,c,c,a,a	4.42
	C_8	10 m	20 mm	25 mm PEM	0.01 mm	0 m	c,b,a,c,b	3.92
	C_9	10 m	20 mm	25 mm PEM	0.01 mm	0 m	c	0.51
	C_{10}	10 m	20 mm	25 mm PEM	0.01 mm	0 m	c,a,a	3.11
	C_{11}	10 m	20 mm	25 mm PEM	0.01 mm	0 m	c,a	1.81
	C_{12}	10 m	20 mm	25 mm PEM	0.01 mm	0 m	c,c,a,c,b	3.63
	C_{13}	10 m	20 mm	25 mm PEM	0.01 mm	0 m	c	0.51
	C_{14}	5m+4m	20 mm	25 mm PEM	0.01 mm	0 m	a,c	1.81
PMA1	C_{19}	2 m	10 mm	15 mm PEX	0.007 mm	0 m	b,c,d,c,e,a	3.57
	C_{21}	1 m	10 mm	15 mm PEX	0.007 mm	0 m	c,d,b	1.46
	C_{22}	1 m	10 mm	15 mm PEX	0.007 mm	0 m	c,d,b,e,b	7.68
	C_{23}	2 m	10 mm	15 mm PEX	0.007 mm	0 m	a,b,d,e	2.55
PMA2	C_{23}	3 m	10 mm	15 mm PEX	0.007 mm	0.5 m	d,c,a,c,e	2.77
	C_{23}	1 m	10 mm	15 mm PEX	0.007 mm	0 m	c,e	0.81
	C_{23}	1 m	10 mm	15 mm PEX	0.007 mm	0 m	b,d,c,b	2.26
	C_{23}	2 m	10 mm	15 mm PEX	0.007 mm	0 m	b,a	2.10
-	C_{42}	2 m	10 mm	15 mm PEX	0.007 mm	0.5 m	c,c,a,d,e	2.77
-								
-								

Table C.1. Table with details about the pipes in the water system, shown on *Figure C.2*. Note that Σk_f is an initial guess for the parameter estimation in Section *4.4: Nonlinear Parameter identification*.

Fitting	Symbol	k_f
Tee - Over all loss	$k_{f,a}$	1.3
Tee - Straigh through	$k_{f,b}$	0.8
90° bend - Diameter/radious ration 1:1	$k_{f,c}$	0.51
Sudden enlarger - Diameter ratio 1:2	$k_{f,d}$	0.15
Sudden contractor - Diameter ratio 1:2	$k_{f,e}$	0.3

Table C.2. Table with details about the fittings in the water system. The finttings are not shown in *Figure C.2.* The values are found in [29, 30].

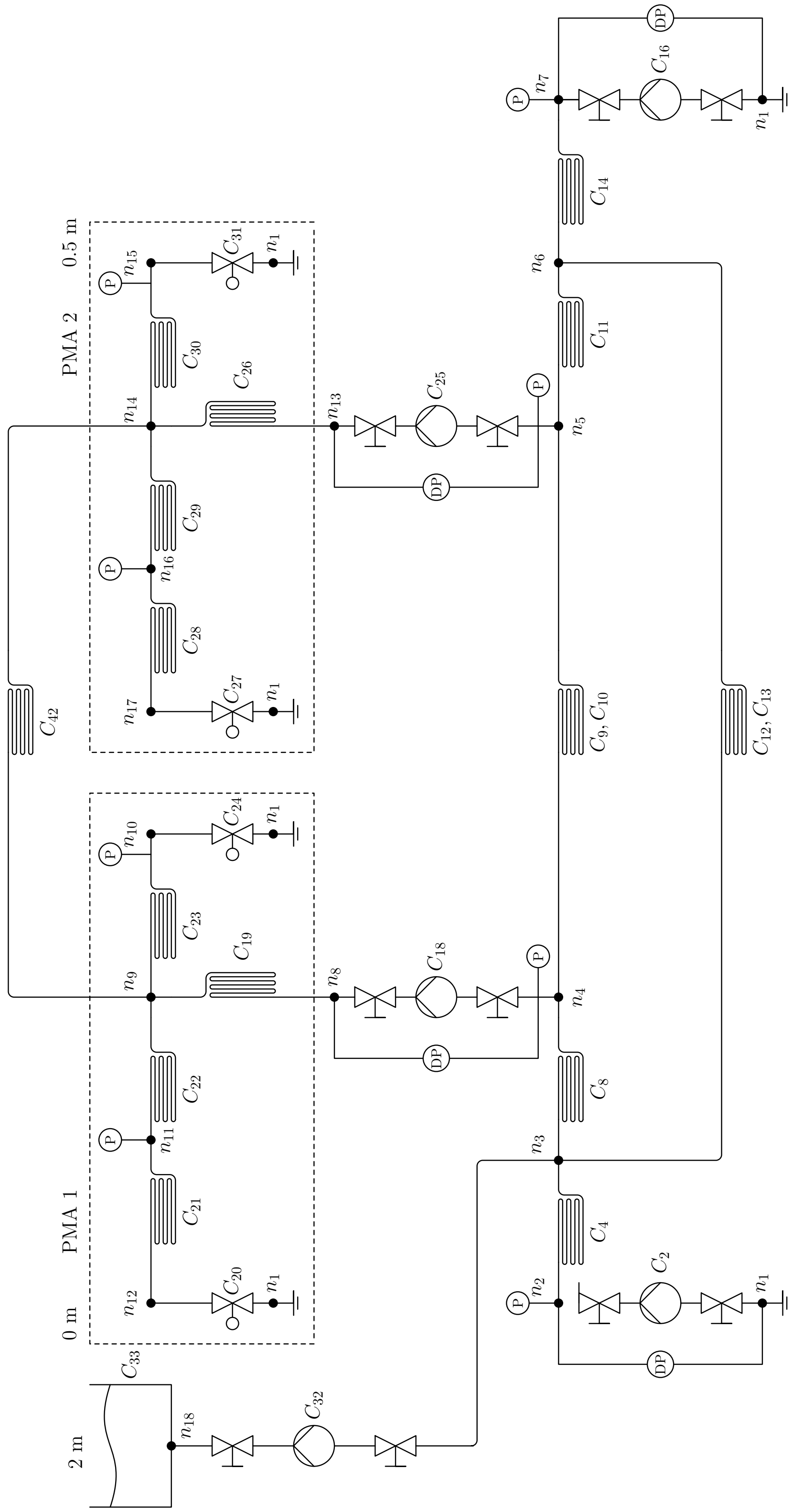
Part	Component	Valve fitting	k_{vs}	n_{gl}	θ_{off}	θ_{max}	Valve motor
PMA1	C_{20}, C_{24}	Belimo R2015-1-S1	1	3.2	15°	90°	Belimo LRQ24A-SR
PMA2	C_{27}, C_{31}	Belimo R2015-1-S1	1	3.2	15°	90°	Belimo LRQ24A-SR

Table C.3. Table with details about the valves in the water system, shown on *Figure C.2.* The parameters are found in [31, 32]

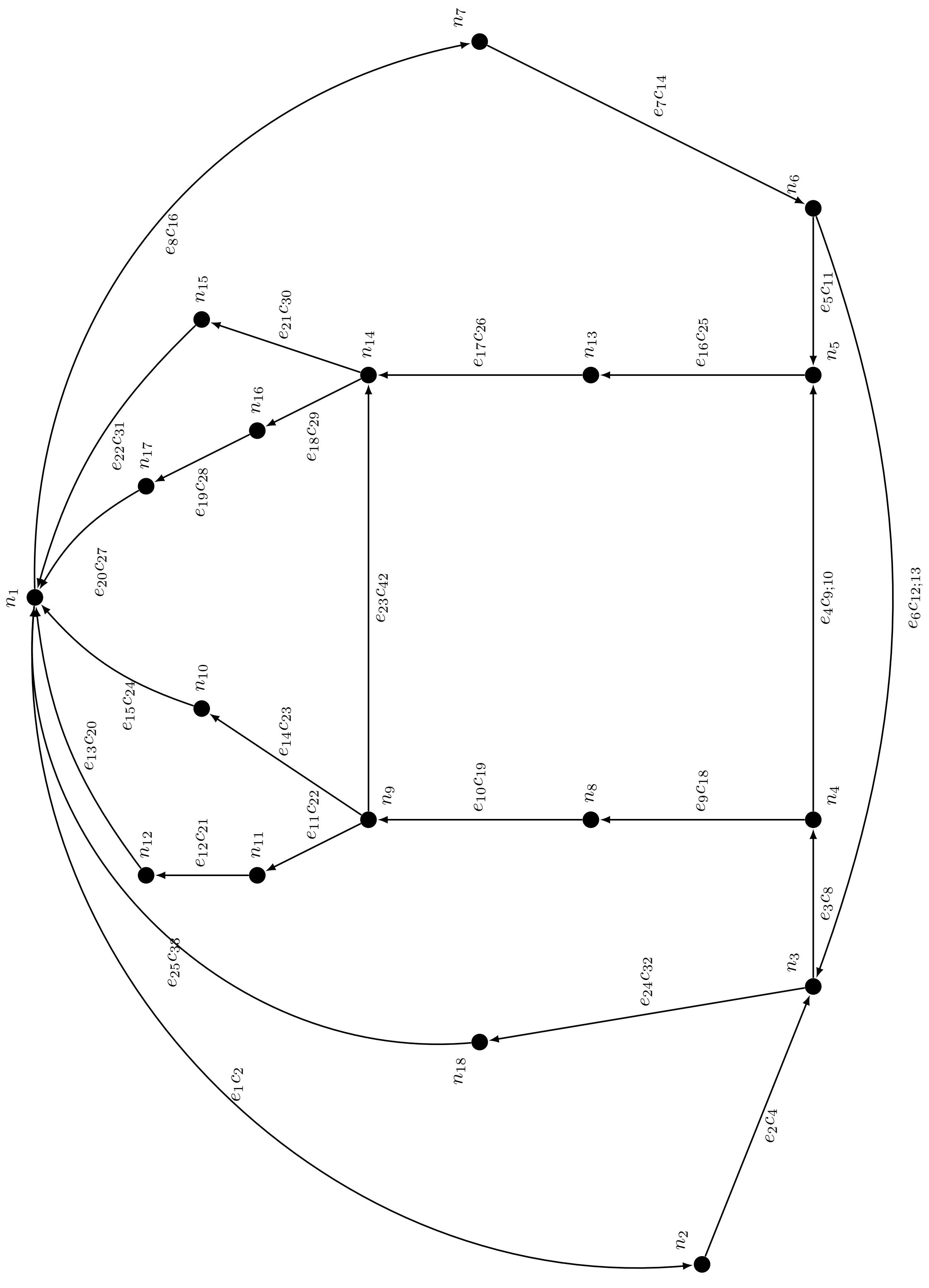
Part	Component	Pump type	Constants
Ring	C_2, C_{16}	Grundfors UPMXL GEO 25-125 180	a_{h0} = 1.2024
			a_{h1} = 0.0098
			a_{h2} = 0.0147
			B_0 = 9.8924
PMA(1,2)	C_{18}, C_{25}, C_{32}	Grundfors UPM2 25-60 180	a_{h0} = 0.6921
			a_{h1} = -0.0177
			a_{h2} = 0.0179
			B_0 = 0.0698

Table C.4. Table with details about the pumps in the water system, shown on *Figure C.2.* The parameters are provided by Grundfos.

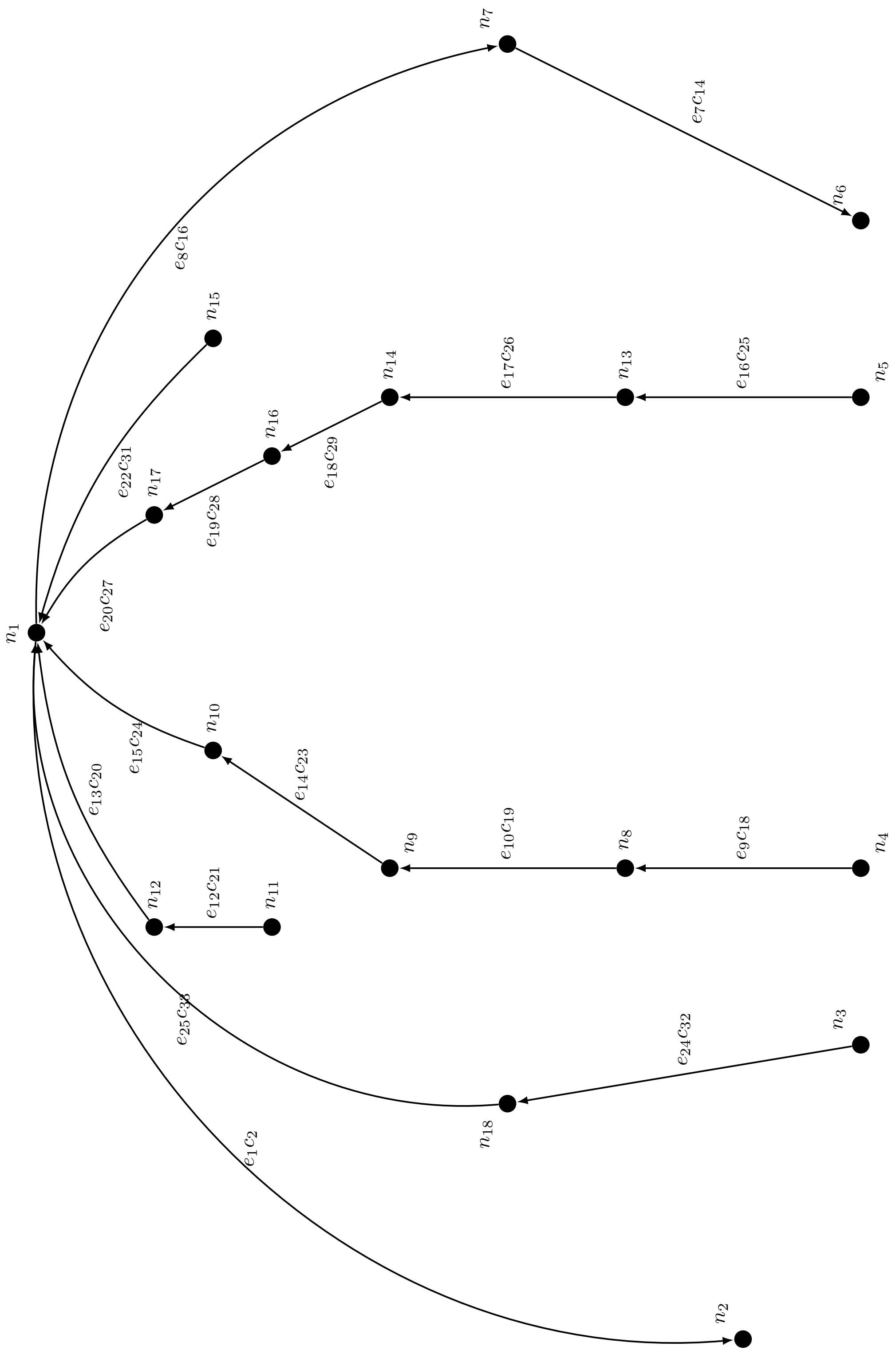
C.2 System Topology



C.3 System Graph



C.4 Spanning Tree



C.5 Incidence Matrix

Vinkel! this matrix appears in a different page than the section title, Also with the cycle matrix

C.6 Cycle Matrix

C.7 G mapping matrix

$$G = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{C.3}$$

$$
H = \begin{array}{cc}
\text{18x24} & \begin{array}{ccccccccccccccccccccccccc} e2 & e3 & e4 & e5 & e6 & e11 & e21 & e23 & e1 & e7 & e8 & e9 & e10 & e12 & e13 & e14 & e15 & e16 & e17 & e18 & e19 & e20 & e22 & e24 & e25 \end{array} \\
\begin{array}{c} n1 \\ n2 \\ n3 \\ n4 \\ n5 \\ n6 \\ n7 \\ n8 \\ n9 \\ n10 \\ n11 \\ n12 \\ n13 \\ n14 \\ n15 \\ n16 \\ n17 \\ n18 \end{array} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix}
\end{array}
\tag{C.1}
$$

$$B = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0
\end{bmatrix} \tag{C.2}$$

Linearization D

Valve equation

The Kv value as a function of the opening degree is for a valve in the system given by:

$$Kv(OD) = kv_{100}e^{(n_{gl}\gamma)} \tag{D.1}$$

The pressure across the valve as a function of the flow is given by:

$$\mu(q) = \frac{1}{(Kv)^2}q|q| \tag{D.2}$$

Gathering the two previous equations allows to describe the pressure pressure across the valve as a combined function of both the flow and the opening degree.

$$M(q, OD) = \frac{1}{(kv_{100}e^{(n_{gl}\gamma)})^2}q|q| \tag{D.3}$$

The linerization of the function $M(q, OD)$ by multi variable Taylor expansion in the operating points $\bar{q}$ and $\bar{OD}$ is given by the form

$$\begin{aligned} M(q, OD) &\approx M(a, b) + \frac{\partial}{\partial x}(M(a, b))(x - a) + \frac{\partial}{\partial y}(M(a, b))(y - b) \\ &\approx e^{\frac{2(\theta_{off}-\bar{OD})n_{gl}}{\theta_{max}-\theta_{off}}+2}\bar{q}|\bar{q}| - 2\frac{e^{\frac{2(\theta_{off}-\bar{OD})n_{gl}}{\theta_{max}-\theta_{off}}+2}n_{gl}\hat{OD}\bar{q}|\bar{q}|}{\theta_{max}-\theta_{off}} + 2e^{\frac{2(\theta_{off}-\bar{OD})n_{gl}}{\theta_{max}-\theta_{off}}+2}\hat{q}|\bar{q}| \end{aligned} \tag{D.4}$$

Where

	$a = \bar{q}$	$\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$
	$x = \bar{q} + \hat{q}$	$\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$
	$b = \bar{OD}$	$[°]$
and	$y = \bar{OD} + \hat{OD}$	$[°]$

Pipe equation

The pressure across a pipe as a function of the flow is given by:

$$\mu(q) = C_p q|q| \tag{D.5}$$

The first order linear Taylor expansion in the operating point $\bar{q}$ is given as:

$$\begin{aligned} \mu(x) &\approx \mu(a) + \frac{\partial}{\partial x}\mu(a)(x - a) \\ &\approx C_p\bar{q}|\bar{q}| + 2C_p\bar{q}\hat{q} \end{aligned} \tag{D.6}$$

Pump equation

Concerning the pump that connects the WT with the remaining system the rotational speed is zero. Therefore will the pumps influence be described by a resistive term which is gives a differential pressure drop as a function of the flow.

$$\Delta p = (\frac{2}{kv_{100}^2} - a_{h2})q|q| \tag{D.7}$$

The first order linear Taylor expansion in the operating point $\bar{q}$ is given as:

$$\begin{aligned}\Delta p(x) &\approx \Delta p(a) + \frac{\partial}{\partial x}\Delta p(a)(x-a) \\ &\approx (\frac{2}{kv_{100}^2} - a_{h2})\bar{q}|\bar{q}| + 2(\frac{2}{kv_{100}^2} - a_{h2})\bar{q}\hat{q}\end{aligned} \tag{D.8}$$

Nonlinear Estimation E

Estimation data In order to estimate accurately the unknown parameters of the system, adequate inputs signals have to be applied to the model. In this way, the system will work on different scenarios regarding the different combination of inputs signals.

Figure E shows the different combination between the OD of the PMA valves, and the steps applied to the main pumps.

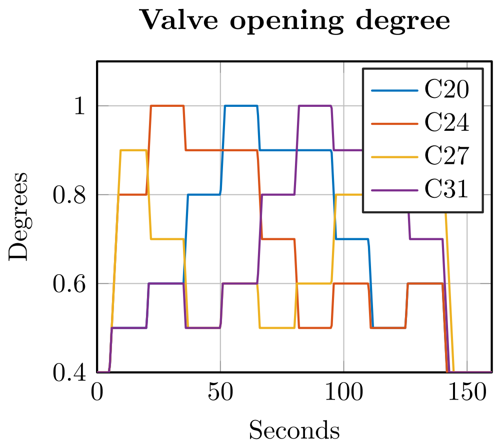


Figure E.1. Inputs to the parameter identification

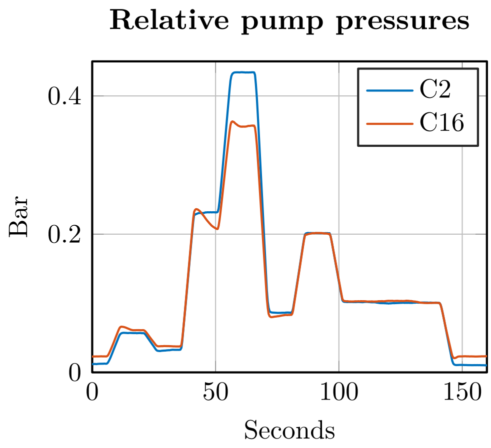


Figure E.2. Output pressure measurements

Together with the inputs from the lab and the nonlinear differential model the nonlinear parameter estimation is carried out.

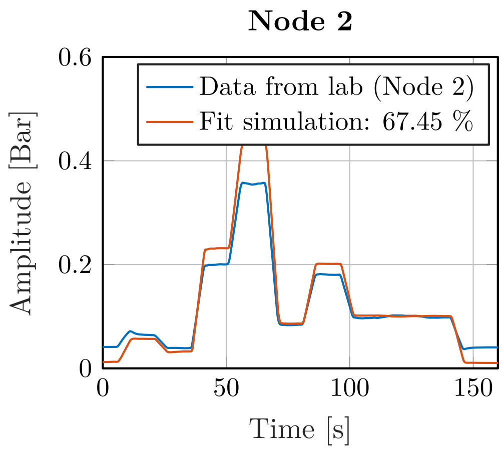


Figure E.3. Estimation comparison for node 2.

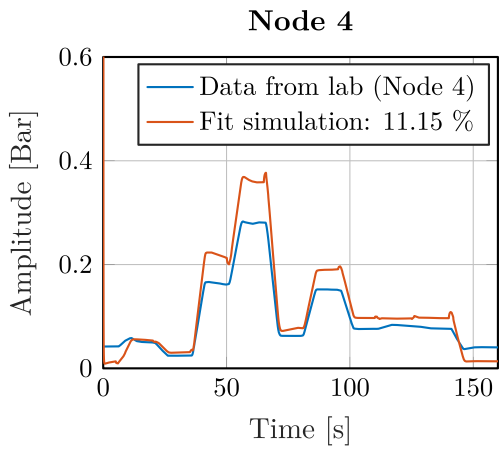


Figure E.4. Estimation comparison for node 4.

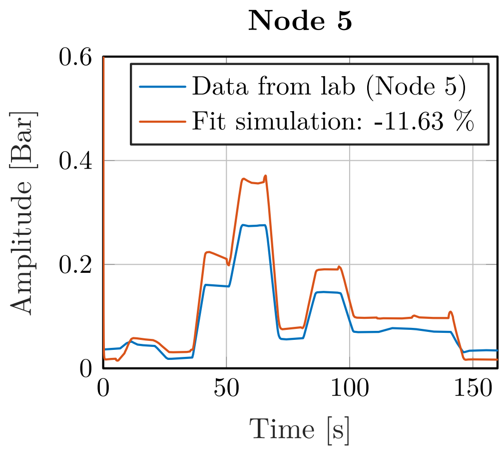


Figure E.5. Estimation comparison for node 5.

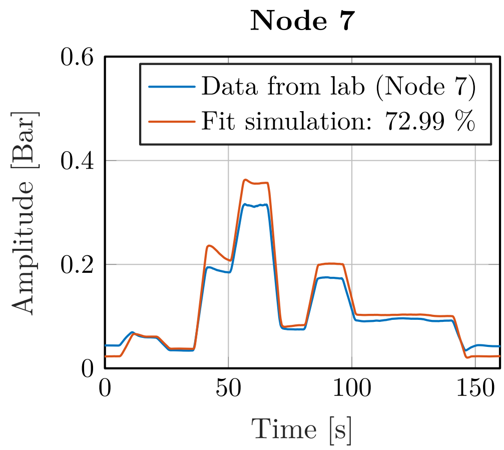


Figure E.6. Estimation comparison for node 7.

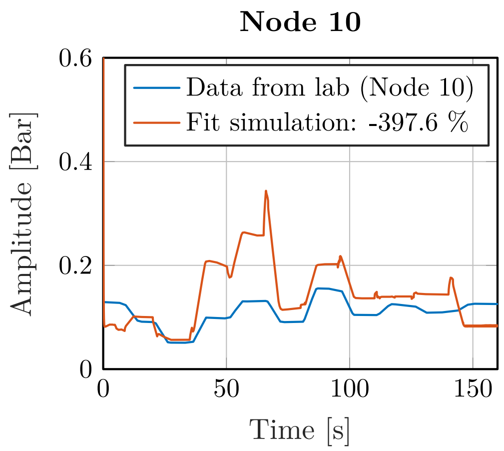


Figure E.7. Estimation comparison for node 10.

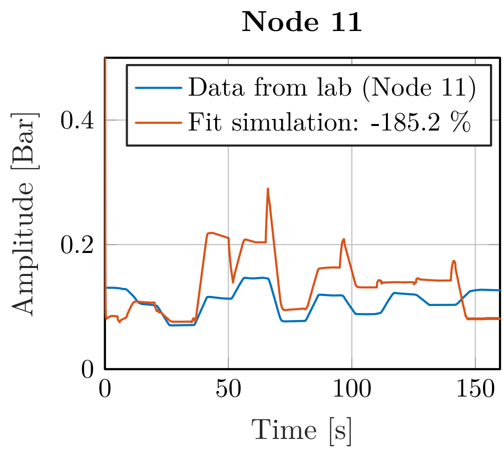


Figure E.8. Estimation comparison for node 11.

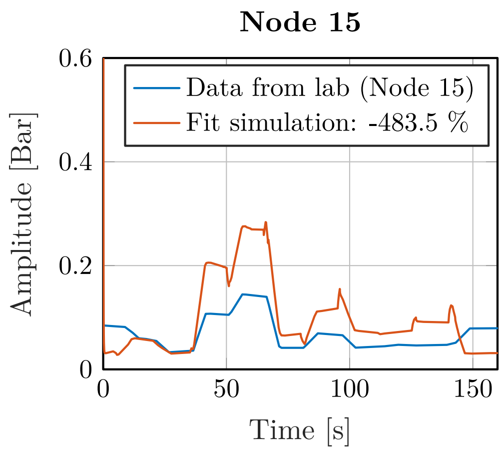


Figure E.9. Estimation comparison for node 15.

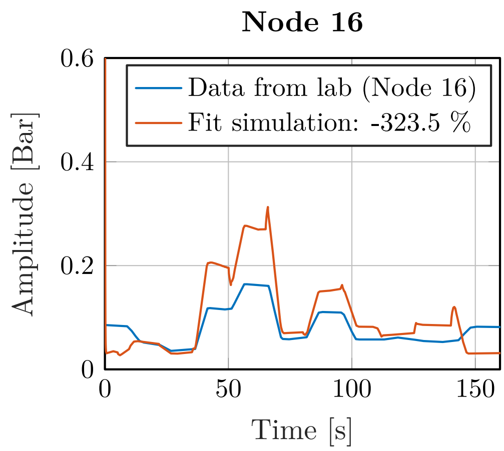


Figure E.10. Estimation comparison for node 16.

Linearization Results F

Estimation Data

The input signals used for the linear parameter estimation are presented in this chapter. The inputs to the end user vales are shown in *Figure F.1* and the inputs to the pumps are shown in *Figure F.2*.

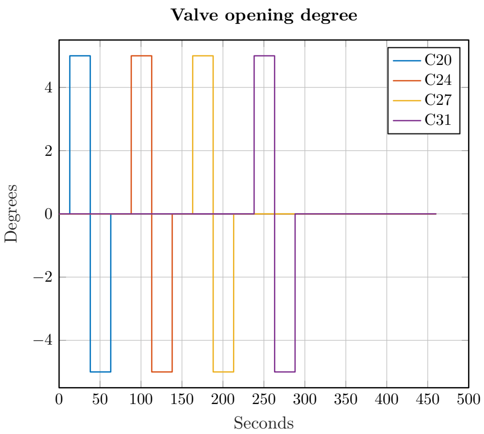


Figure F.1. Small signal values of the opening degrees of the pma valves.

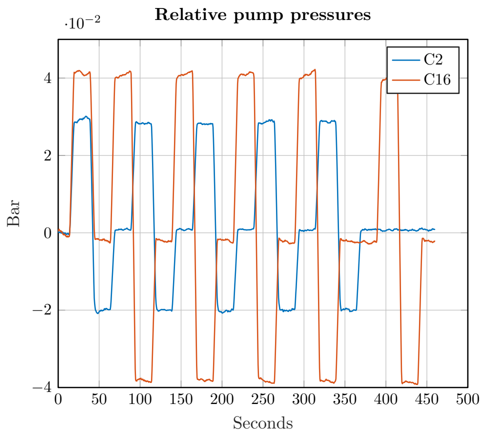


Figure F.2. Small signal values of the angular velocity of the two main pumps.

Estimation Method

The estimation of the unknown parameters is carried through Matlab Linear Grey-Box model estimation toolbox. This toolbox allows to estimate continuos-time grey-box models for differential equations using multiple input/output time-domain data [33]. The numerical search method used for the estimation of the unknown parameters is the *Subspace Gauss-Newton least squares* search.

This method is automatically set by the parameter estimation process, thus any further understanding regarding the estimation method is considered out of scope of this project.

Estimation Result

The following figures show the comparison between the data obtained from the lab and the estimated outputs of the model.

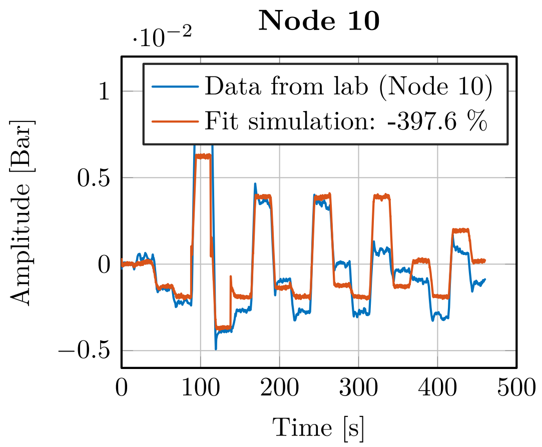


Figure F.3. Estimation comparison for node 10.

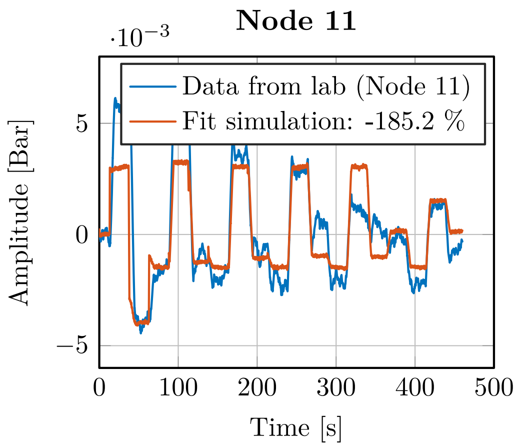


Figure F.4. Estimation comparison for node 11.

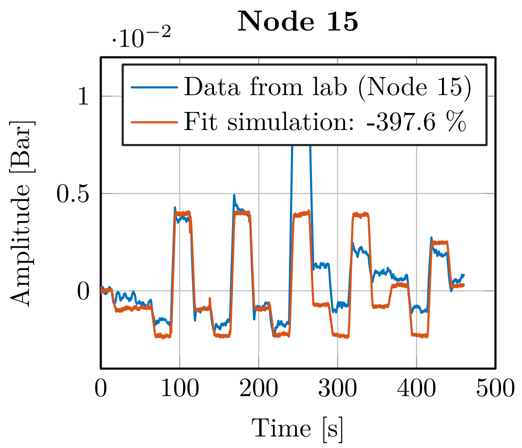


Figure F.5. Estimation comparison for node 15.

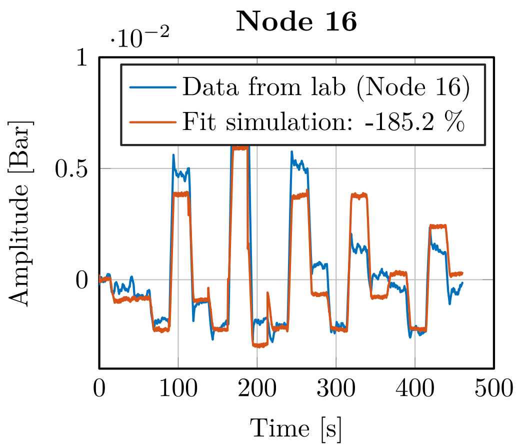


Figure F.6. Estimation comparison for node 16.

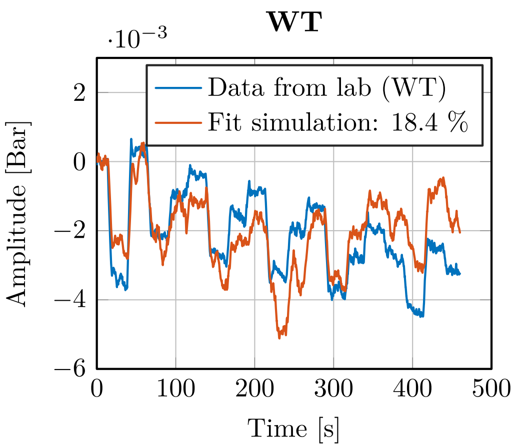


Figure F.7. Estimation comparison for WT.

Electrical price G

To minimize the running cost of the system the power consumption of the pumps, P_e Cf. Section *4.1.3: Pump model*, and the electrical price, $c[k]$, is needed. Predicting future prices is an extensive task that depends on many factors e.g user consumption and weather conditions. Due to the fact that the learning goals of this project is not to derive a high precision predictive model that describe future electrical prices, data gathered from [34] is used instead. The pricing can be seen on *Figure G.1*.

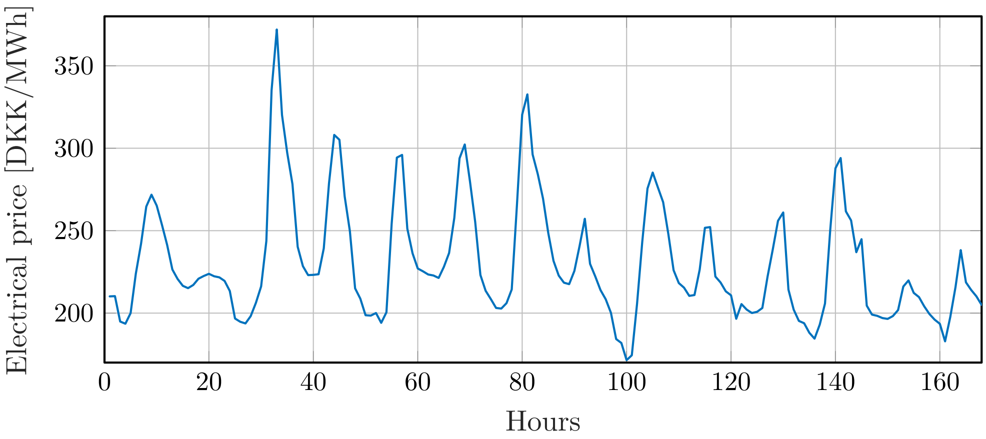


Figure G.1. $c[k]$, describing the electricity prices in Denmark from the 27-03-2017 to 02-04-2017.

As this is real data from a given period, it will most likely not fit the pricing in any other given week, as the pricing is fluctuating a lot from day to day. However the data indicates the pricing is higher in the morning and evening which is applicable for any given week and thereby a general property of the time dependent pricing. This behavior can be seen as the periodicity of the data with two peaks a day. The chosen data will thus give a realistic idea of the improvement the controller can achieved in a real world scenario based on the week the data is recorded.

H Pump linearization and PI controller - V2

The control structure chosen in Chapter *5.1: Control Problem* and *Figure 5.1* includes a PI controller, which is to designed. The PI controllers purpose is to use the optimized control output from the MPC as a control reference.

The control focus of this project has been on the model predictive control, see Section *5.2: Model predictive control*. Therefore a simple PI controller has been design, where the control structure can be seen on *Figure I.1*.

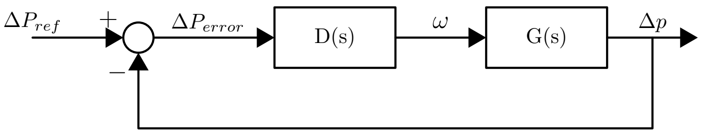


Figure H.1. The structure of the PI controller.

The output from the MPC is a differential pressure, which is to be controlled through the rotational speed of the pumps. The model for the pumps is explained in Section *4.1.3: Pump model* and defined as

$$\Delta p = -a_{h2}{q_i}^2 + a_{h1}\omega_r q_i + a_{h0}{\omega_r}^2$$

which is a nonlinear model. By the assumption that the flow is constant, the expression can through a Taylor expansion be linearized, with respect to ω, to a small signal model.

For simplification the pump model is separated into smaller chunks, see *Equation: (H.1)*, then Taylor approximated, see *Equation: (H.2)*, where $\omega = \hat{\omega} + \bar{\omega}$.

$$\begin{aligned} f_1(\bar{\omega}) &= a_{h1}\bar{q}\bar{\omega} \\ f_2(\bar{\omega}) &= a_{h0}\bar{\omega}^2 \end{aligned} \tag{H.1}$$

$$\begin{aligned} f_{t1}(\omega) &= f_1(\bar{\omega}) + f_1'(\bar{\omega}) \cdot (\omega - \hat{\omega}) \\ f_{t2}(\omega) &= f_2(\bar{\omega}) + f_2'(\bar{\omega}) \cdot (\omega - \hat{\omega}) \end{aligned} \tag{H.2}$$

From *Equation: (H.1)* an expression for the workspace of, $\Delta\bar{P}$, can be made.

$$\Delta\bar{P} = f_1(\bar{\omega}) + f_2(\bar{\omega}) + c \tag{H.3}$$

The linear pump model can then be expressed as seen in *Equation: (H.4)*

$$\begin{aligned} 0 &= -(\Delta\bar{P} + \Delta\hat{P}) + f_{t1}(\bar{\omega}) + f_{t2}(\bar{\omega}) + c \\ &= -(f_1(\bar{\omega}) + f_2(\bar{\omega}) + c + \Delta\hat{P}) + f_{t1}(\bar{\omega}) + f_{t2}(\bar{\omega}) + c \\ &= -\Delta\hat{P} + f_1'(\bar{\omega}) \cdot \hat{\omega} + f_2'(\bar{\omega}) \cdot \hat{\omega} \end{aligned} \tag{H.4}$$

Equation: (H.4) is then Laplace transformed and solved for the input output relationship as seen on *Figure I.1*

$$G(s) = \frac{\Delta \hat{P}}{\hat{\omega}} = f_1'(\bar{\omega}) + f_2'(\bar{\omega}) = a_{h1}\bar{q} + 2a_{h0}\bar{\omega} \tag{H.5}$$

With the following operating points set for the pumps, the differential pressure over the pumps and their respective valves are measured. The data can be found on the attached storage under the path: `CD:/Data/Steadystate`.

$$\begin{aligned} \bar{\omega}_{C18} &= 0.16, & \Delta P_{C18} &= 0.0877 \\ \bar{\omega}_{C25} &= 0.4, & \Delta P_{C25} &= 0.2598 \\ \bar{\omega}_{C2} &= 0.4, & \Delta P_{C2} &= 0.2008 \\ \bar{\omega}_{C16} &= 0.4, & \Delta P_{C16} &= 0.1960 \end{aligned} \tag{H.6}$$

From this the flow through the pumps can be estimated by *Equation: (4.43)* as the respective pump parameters are found in *Appendix: C* and thus q is the only unknown variable. Solving for q yields two results due to the second order equation, thus only the positive results is used as negative flow trough the pump is infeasible combined with positive differential pressure.

$$q_{C18} = 0.229 \qquad q_{C25} = 0.430 \qquad q_{C2} = 0.442 \qquad q_{C16} = 0.440 \tag{H.7}$$

By inserting the values for the linearized model in *Equation: (H.5)*, the final models ends up as gains.

$$G_{C18}(s) = 0.217 \qquad G_{C25}(s) = 0.546 \qquad G_{C2}(s) = 0.966 \qquad G_{C16}(s) = 0.976 \tag{H.8}$$

As it can be seen in *Appendix: L.1* the dynamics of the system are slow. Therefore will a controller that can settle within 3-5 seconds be sufficiently fast for the system. By using the Matlab design tool box for dynamical systems and having the settling time of 3 seconds in mind, the gains for the PI controller can be found as shown in *Equation: (H.9)*. Due to high similarities between these controllers only one step response in shown on *Figure H.3*.

$$K_{C18} = 8 \qquad K_{C25} = 3 \qquad K_{C2} = 2 \qquad K_{C16} = 2 \tag{H.9}$$

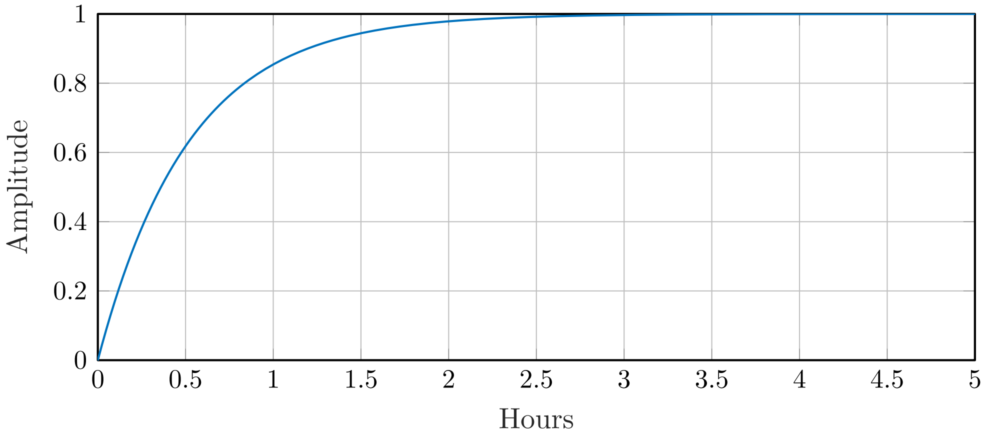


Figure H.2. Step response of one pump PI system.

PI test

The PI controller was implemented on the water system described in Chapter *2.1: System overview*. From the test it where concluded that some tuning of the gains had to be done, due to large overshoots and marginal stabilities on some of the pumps. The final values where found by trail and errors and can be seen in *Equation: (H.10)*.

$$K_{C18} = 6 \qquad K_{C25} = 4 \qquad K_{C2} = 1.4 \qquad K_{C16} = 1.4 \tag{H.10}$$

With these gains the systems gets a slower response than the settling time of 4 seconds. However this is still deemed sufficient to this project, since the dynamics of the WT are much slower.

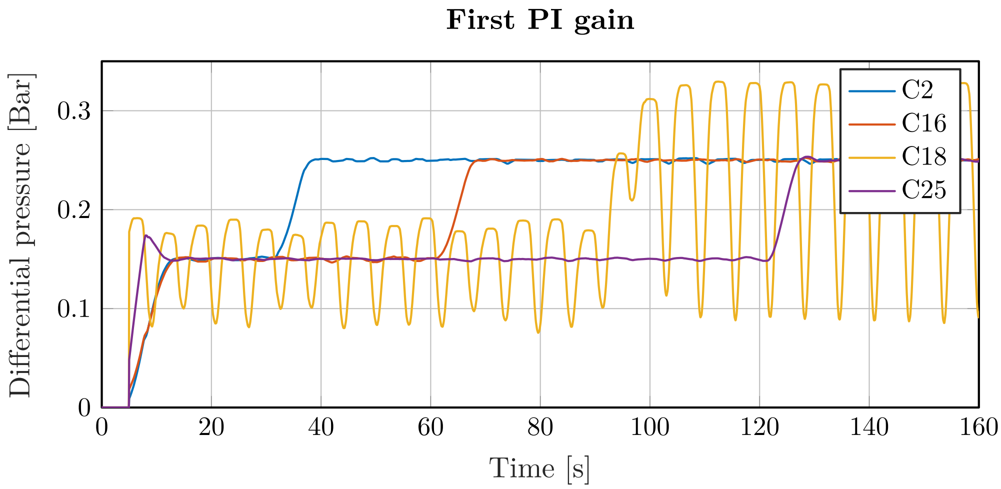


Figure H.3. Step response of one pump PI system.

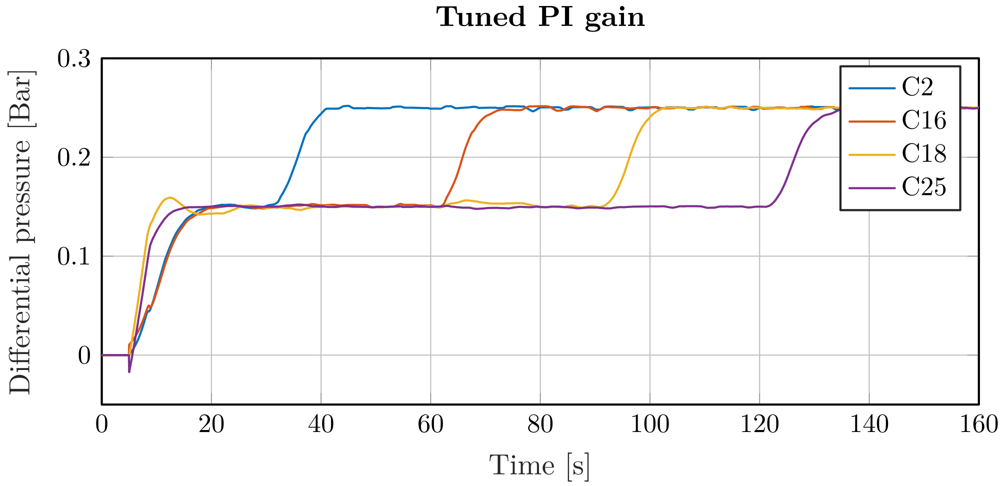


Figure H.4. Step response of one pump PI system.

Based on the step response shown in *Figure H.4*, it can be concluded that a pressure controller that satisfies the requirements has been found for each pump.

Pump linearization and PI controller - V2

I

The control structure chosen in Chapter *5.1: Control Problem* and *Figure 5.1* includes a PI controller, which is to designed. The PI controllers purpose is to use the optimized control output from the MPC as a control reference.

The control focus of this project has been on the model predictive control, see Section *5.2: Model predictive control*. Therefore a simple PI controller has been design, where the control structure can be seen on *Figure I.1*. The steady state error is desired to be zero, which is possible by an integrator pole in the controller. The gain of the controller is then to be found.

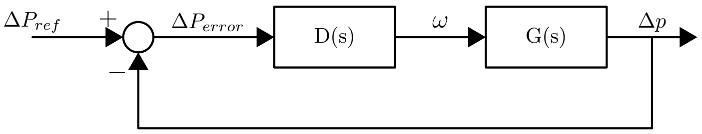


Figure I.1. The structure of the PI controller.

The output from the MPC is a differential pressure, which is to be controlled through the rotational speed of the pumps. However the pump/valve model in Section *4.2: Simplification and electrical analogy* and seen beneath, is to be used, since the feedback of ΔP is measured over two valves and a pump, see *Figure 4.4* in Section *4.2: Simplification and electrical analogy*.

$$\Delta p = \left(\frac{2}{k_{v100}^2} - a_{h2}\right)|q|q + a_{h1}\omega_r q + a_{h0}{\omega_r}^2$$

The two terms ω and q are time dependent variables, which is to be linearized around the operating point. Through a first order Taylor approximation the pump expression end up as see in *Equation: (I.1)*

$$\hat{\Delta P} = (4\bar{q} - 4\bar{q}a_2 + a_1\bar{\omega})\hat{q} + (a_1\bar{q} + 2a_0\bar{\omega})\hat{\omega} \tag{I.1}$$

$$\frac{\hat{\Delta P}}{\hat{\omega}} = ?? \tag{I.2}$$

J Discretization of state space model

Here the discretization of the continuous SS model is explained.

Nonlinear differential script K

The nonlinear differential script contains all the information about the system and how it is constructed. The model needs 8 different inputs(u): the opening degree of the 4 PMA valves and the differential pressure across the main and PMA pumps. This inputs are obtained from the lab and are inserted directly into the model. Furthermore, it has been decided to include the dynamics of the system so the model remains as a nonlinear differential system.

First, the states of the system are set. Previously in the report, the states have been defined as the chord flows and denoted as $z = [z_1 z_2 z_3 z_4 z_5 z_6 z_7]$. The value of the states will be dependent on time, which will be specified by the data obtained from the Water Wall Setup in the lab. In this way, using the relation stated in *Equation: (4.55)* the flow through the edges is obtained.

The pressure across the edges has been defined in *Equation: (4.57)*, and in the same way has been done in Matlab.

- λ and ζ denote the pressure in the pipes due to the friction and the elevation respectively.
- ν represents the pressure across the valves.
- α represent the pressure across the pumps, which is obtained from the inputs to the model.

In the system in total there are 15 pipes, the pressure across them is obtained as:

$$\lambda = (C_p * abs(q) * q/(10^5 * 3600^2)) + ((Z) * g * 1000/10^5) \tag{K.1}$$

Where, C_p denotes the friction coefficient to be estimated by the NonLinear Grey Box and Z is the height of the pipes. An initial guess is done for the value of C_p to minimize the error as possible. Moreover, the unit conversion from Bar to Pas and seconds to hours is introduced.

$$Cp = f * ((8 * L * rho)/(pi^2 * D^5)) + k_f * ((8 * rho)/(pi^2 * D^4)) \tag{K.2}$$

The f and k_f values are obtained as described in *Equation: (4.15)* and *Equation: (4.18)* with the range of values considered in Section *4.1.1: Pipe model*.

In case of the valves the pressure across them is obtained as following

$$mu = (1/C_v^2) * abs(q) * q/3600^2; \tag{K.3}$$

The value of C_v is obtained from the input data. When the valve is closed μ is set to 0.

For the pumps, the pressure across them is obtained from the lab and is set as α.

As pointed out earlier, the inertia of the pipes is taken into account in the model and is calculated as

$$J = diag((4 * L * rho)/(D^2 * pi * 10^5 * 3600)); \tag{K.4}$$

Where, L and D are the pipes length and diameters respectively. The unit conversion is also added.

A new matrix variable is defined containing the pressure across the edges.

$$F = \lambda + \mu - \alpha \tag{K.5}$$

The derivative of the chords can be obtained isolating $\dot{z}$ in *Equation: (4.66)*

$$\dot{z} = -(B_1 J B_1{}^T)^{-1} B_1 F \tag{K.6}$$

Where, B_1 is the cycle matrix defined in *Appendix: C.6.*

Known the derivative of the chords, the derivative of the flow through the edges is obtained

$$\dot{q} = (B_1)^T * \dot{z} \tag{K.7}$$

Equation: (4.64) is now applied substituting $\dot{q}$ with the relations set in *Equation: (K.7)* and *Equation: (K.6)*, resulting in

$$\Delta P = [(J * (B_1)^T * (B_1 J B_1{}^T)^{-1} B_1(-\mu - \lambda + \alpha)] + \mu + \lambda - \alpha \tag{K.8}$$

Now the pressure across the edges are calculated, nevertheless, from the data of the lab the relative pressure at the nodes are obtained. This implies to have to calculate the pressure in the nodes so the comparison is done correctly.

The pressure at each node can be found by applying

$$\Delta P = (H_1)^T p \tag{K.9}$$

ΔP is known from *Equation: (K.8)*, the vector of pressure in the nodes is split up as $p = [p_o \quad p_r]$ where p_o is the atmospheric pressure at node 1 and p_r the pressure in the remaining nodes. Splitting H_1 matrix also

$$\Delta P = [H_o{}^T \quad H_r{}^T] \begin{bmatrix} p_o \\ p_r \end{bmatrix} \tag{K.10}$$

Isolating p_r from the equation the pressure at each node is obtained

$$p_r = H_r^{\dagger}(\Delta p - (H_o)^T p_o) \tag{K.11}$$

$H_r^{\dagger}$ being the pseude-inverse of matrix $H_r{}^T$.

The outputs are set as node 2, node 4, node 5, node 7, node 10, node 11, node 15 and node 16 (see *Appendix: C.2* for the notation). Which will be the ones that Matlab will try to fit to the data obtained from the setup.

Measurements L

L.1 Water tower time constant

Purpose:

The purpose of this test is to determine the time constant and settling time of WT.

Test equipment:

- The water distribution system at AAU.

Procedure:

The following procedure was made for finding the time constant:

1. Wait for the system to get into a steady state position with the following system setup: valve opening at 0.7% for all consumer valves and differential pressure over pumps at $C2 = C16 = 0.2Bar$, $C18 = 0.1Bar$, $C25 = 0.25Bar$.
2. Increase the differential pressure over C2 with 0.1 Bar.
3. Wait 1.5 hour.

Measuring data:

The measurements data can be found on the attached storage under the path: `CD: /Data/WTtimeconstant`, a plot of the data is shown in *Figure L.1*.

Results:

The time constant of the WT can be found through the linear differential equation shown in *Equation: (4.107)*. By Laplace transform and solving for the input output relation, the standard form of the transfer function for the WT can be derived as see in *Equation: (L.1)*

$$\begin{aligned}
&\Delta\dot{p}_{WT} = A_p \Delta\hat{p}_{WT} + \boldsymbol{B_p}\boldsymbol{\hat{u}} \\
&s\Delta p_{WT}(s) = A_p \Delta\hat{p}_{WT}(s) + \boldsymbol{B_p}\boldsymbol{\hat{u}(s)} \\
&\frac{\Delta p_{WT}(s)}{u(s)} = \frac{B}{s-A} = \frac{\frac{B}{A}}{\frac{1}{A}s+1}
\end{aligned} \tag{L.1}$$

From the denominator, the time constant of the WT can be directly read as seen in *Equation: (L.2)*

$$\tau s + 1 \tag{L.2}$$

Since A in *Equation: (4.107)* is a constant, due to the tank only having one state, the WT has one time constant. However the unit of A is in $[\frac{m^3}{h}]$ and needs to be convert to seconds.

Convert from

$m3^3/h$

and see how much it fits compared to the test below

As the dynamics of the WT are described by a first order system, the time constant can be found as the time the system uses to reach 63.2% of the steadystate pressure. This pressure at 63.2% of the steadystate values is based on the minimum and maximum pressure values during the step and determined to:

$$(0.137 - 0.127) * 63.2\% = 0.0063 \rightarrow 0.127 + 0.0063 = 0.133 Bar \tag{L.3}$$

Based on the data is it found that at a pressure of 0.133 bar the time is passed is 1155 seconds corresponding to 19,25 minutes.

On *Figure L.1* the measurement data used to determine the time constant of the WT is shown. A small red dot indicates the time constant for the tank.

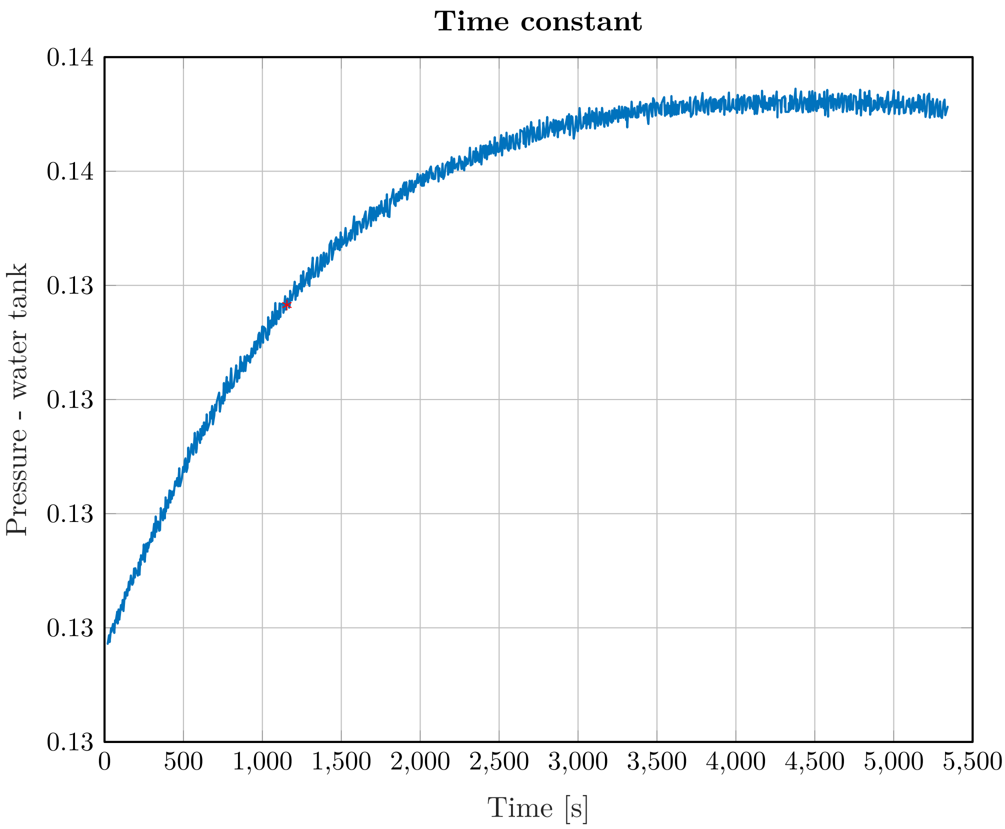


Figure L.1. The WT pressure during a step and a red dot indicating the time constant.

Based on the determined time constant and the first order model, the settling time of the tank can be found as $1155 \cdot 5 = 5775$ seconds equal to 1.6 hours.

Uncertainties of measurement:

- The settling time for the initial state was not reached after 1.5 hours.

Conclusion:

From this test both the time constant and the settling time of the WT are determined. The results are based on the fact that the WT dynamics can be described by a first order system, thereby is the time constant found by applying a step to the system and determined to be 19,25 minutes. Furthermore is the settling determined to be 1.6 hours. It can thus be concluded that the purpose of this test is fulfilled and the dynamics of the WT are found.

L.2 Small signal pump measurements

In this section the linear small signal deviation of the linearized pump model is measured and calculated.

Test equipment:

- The water distribution system at AAU.

Procedure:

The following procedure was made for finding the small signal deviations of the pumps

1. Wait for the system to reach the steady state operating point.
2. Add 0.05 to the rotational speed for one of the pumps and wait 30 minutes for steady state.
3. Note differential pressure over the pump, set the rotational speed to the operating point and 30 minutes to reach operating steady state.

Point 2-3 is done for each pump where the pressure over the pump is noted

Measuring data:

The data from the measurements can be seen on

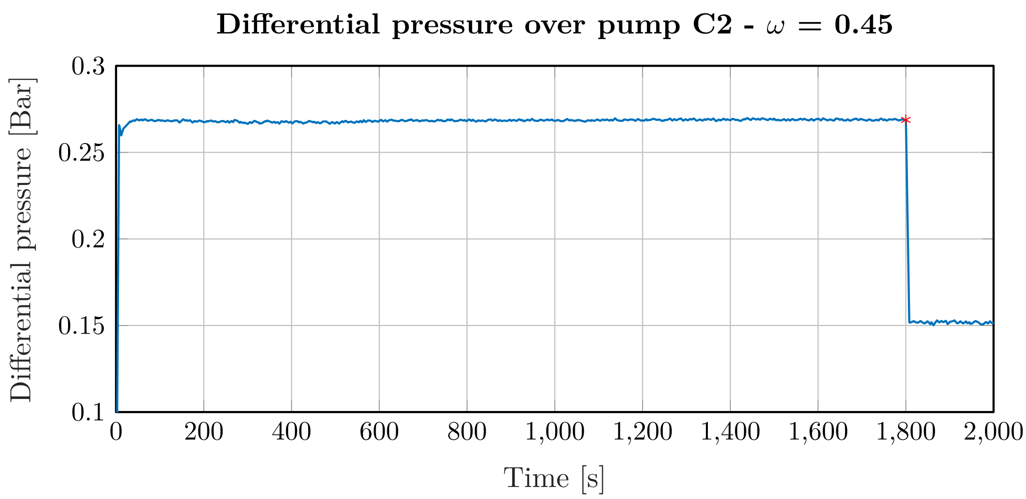


Figure L.2. Small signal measurements for pump C2.

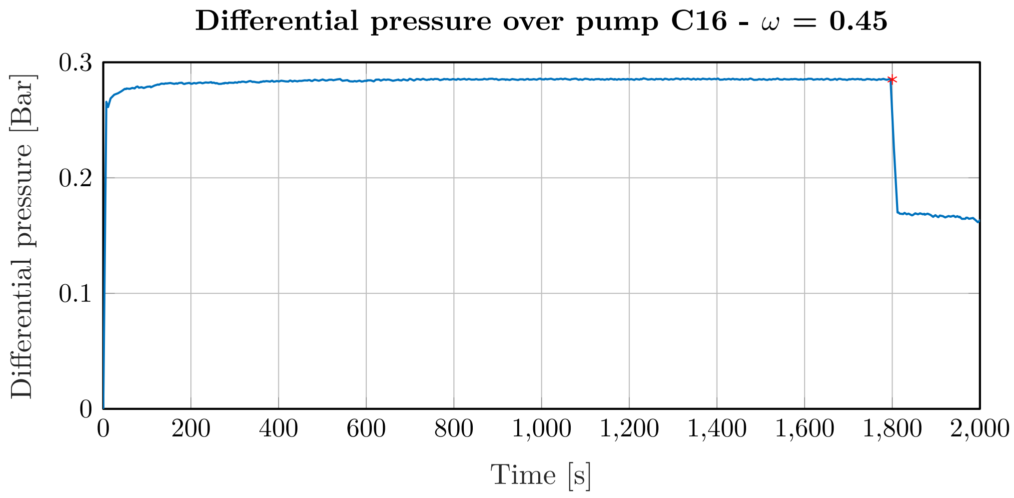


Figure L.3. Small signal measurements for pump C16.

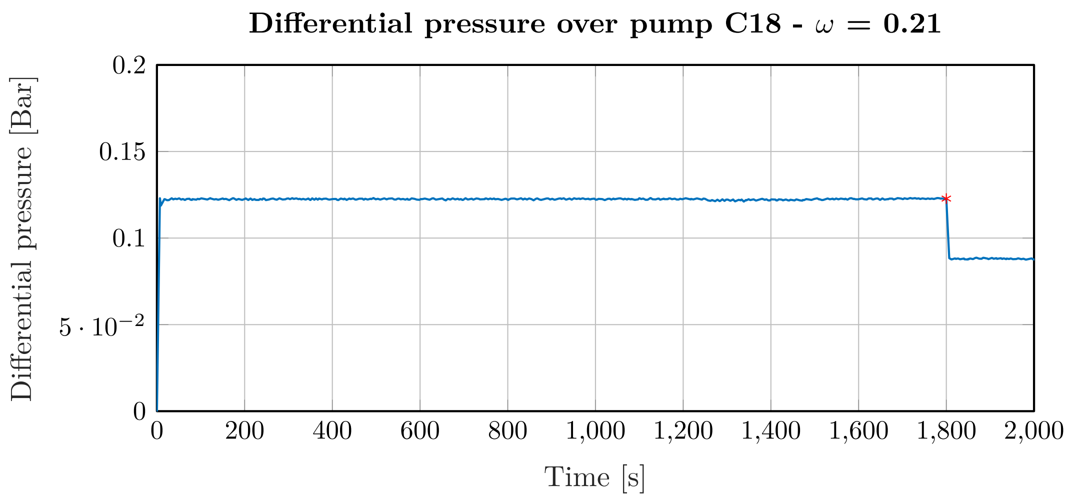


Figure L.4. Small signal measurements for pump C18.

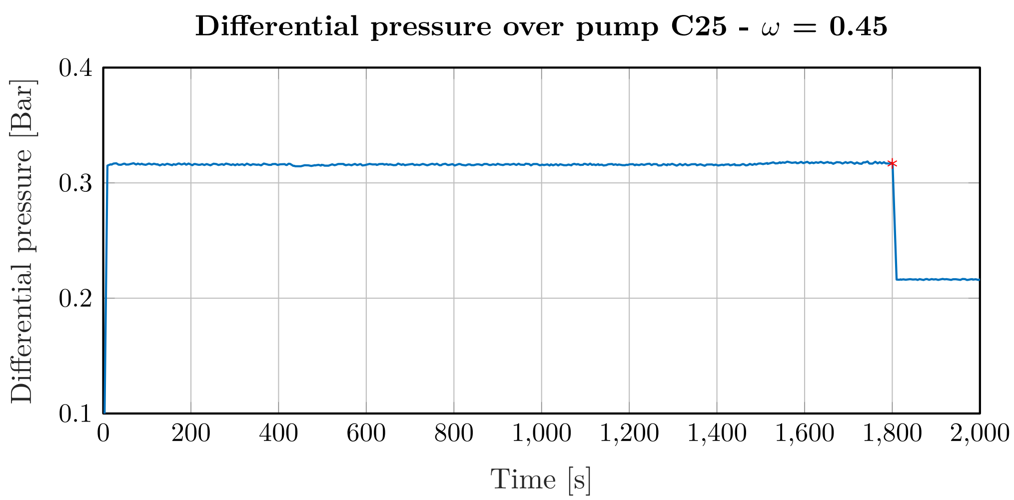


Figure L.5. Small signal measurements for pump C25.

Results:

The steady state differential pressure for time 1800 is noted and used for the small signal deviation. These values has been picked as this is the time when the signal to one of the pumps are change and where the differential pressure over the pump should have reached a reasonable steady state. The differential pressure shown in *Equation: (L.4)*, is show with respect to $\omega = \bar{\omega} + \hat{\omega}$ and $\Delta P = \bar{\Delta P} + \hat{\Delta p}$.

$$
\begin{aligned}
\Delta P_{C_2} &= 0.2688 \\
\Delta P_{C_{16}} &= 0.2852 \\
\Delta P_{C_{18}} &= 0.1228 \\
\Delta P_{C_{25}} &= 0.3169
\end{aligned}
\tag{L.4}
$$

From these results, the small signal pressure can be derived as see in *Equation: (L.5)* to *Equation: (L.8)*.

$$
\frac{\Delta \hat{P}_{C_2}}{\hat{\omega_{C_2}}} = \frac{0.2 - 0.2688}{0.4 - 0.45} = 1.376 \tag{L.5}
$$

$$
\frac{\Delta \hat{P}_{C_16}}{\hat{\omega_{C_2}}} = \frac{0.25 - 0.2852}{0.4 - 0.45} = 0.704 \tag{L.6}
$$

$$
\frac{\Delta \hat{P}_{C_18}}{\hat{\omega_{C_2}}} = \frac{0.0807 - 0.1228}{0.16 - 1.21} = 1.473 \tag{L.7}
$$

$$
\frac{\Delta \hat{P}_{C_25}}{\hat{\omega_{C_2}}} = \frac{0.25 - 0.3169}{0.4 - 0.45} = 1.338 \tag{L.8}
$$

Uncertainties of measurement:

- The settling time for the initial state was not reached after 0.5 hours.
- Corrupt pressure measurements or noise.

Conclusion:

From these measurements the small signal deviations have been found and will be used in the linearized small signal model for the pumps.

L.3 Test of model predictive control

In this section, the model predictive controller obtained in Chapter *II: Control Design* is implemented and tested on the water system available at Aalborg university.

Purpose:

To conclude if the controller obtained meets the requirements stated in Chapter *3: Requirements and Constraints* and furthermore check if it lower the running cost of the system.

Test equipment:

- The water system at Aalborg university [AAU: 100911]

Procedure:

Due to time limitations the tests has been scaled to minimize the time spend on the measurements. The scaling factor have been chosen with the dynamics of the WT in mind. The time constant of the WT has been found as 1155 seconds, where the scaling have been chosen in such a way that a one hour optimization period should take half a time constant of the WT. This value have been rounded up to 10 minutes. Since the MPC optimize over a horizon of 24 hour, the test will in total take four hours.

1. ...

Measuring data:

Results:

Uncertainties of measurement:

- ...

Conclusion:

Bibliography

[1] N. R. Council *et al.*, *Public Water Supply Distribution Systems: Assessing and Reducing Risks–First Report.* National Academies Press, 2005.

[2] B. Appelbaum, "Water & sustainability (volume 4): Us electricity consumption for water supply & treatment—the next half century," 2002.

[3] Y. X. J.N. Tsitsiklis, "Pricing of fluctuations in electricity markets," *European Journal of Operational Research*, 2015.

[4] M. Feldman, "Aspects of energy efficiency in water supply systems," in *Proceedings of the 5th IWA Water Loss Reduction Specialist Conference, South Africa*, pp. 85–89, 2009.

[5] Watts, "Water pressure regulators frequently asked questions." Last visited 14-02-2017, http://www.watts.com/pages/faq.asp?catId=64&faqId=7.

[6] Lenntech, "Why oxygen dissolved in water is important." Last visited 14-02-2017, http://www.lenntech.com/why_the_oxygen_dissolved_is_important.htm.

[7] S. Murphy, "General information on dissolved oxygen." Last visited 14-02-2017, http://bcn.boulder.co.us/basin/data/NEW/info/DO.html.

[8] Grundfos, "Upml geo and upmxl geo oem." Last visited 02-03-2017, http://www.grundfos.com/products/find-product/upml-geo-upmxl-geo-oem.html#overview.

[9] Grundfos, "Upm2, upm geo, upm2k." Last visited 02-03-2017, net.grundfos.com/Appl/WebCAPS/Grundfosliterature-4927111.pdf.

[10] PE100, "What is the meaning of the designations pe80 and pe100." Last visited 02-03-2017, https://www.pe100plus.com/PE-Pipes/PE-technical-guidance/model/Materials/mrs/What-is-the-meaning-of-the-designations-PE80-and-PE100-i254.html.

[11] M. MØlgaard and B. G. Pétursson, *Energy Optimization of Water Distribution Networks.* 2015. Master thesis, Aalborg University.

[12] C. Kallesøe and C. Persis, "Pressure regulation in nonlinear hydraulic networks by positive controls," *Proceedings of the European Control Conference Budapest, Hungary*, 2009.

[13] B. Hunt, *Fluid Mechanics for Civil Engineers.* University of Canterbury, 1995.

[14] A. K. S. Prabhata K. Swamee, *Design of Water Supply Pipi Networks.* John Wiley & Sons, 2008.

[15] Y. Nakayama, *Introduction to Fluid Mechanics.* Butterworth Heinemann, 1999.

[16] N. Trifunovic, *Introduction to uUrban Water Distribution.* Balkema, 2006.

[17] H. Boysen, k_v*:what,why,how,whence?* Danfoss A/S, 2011.

[18] U. Keller, *Method for the Hydraulic Compensation and Control of a Heating or Cooling System and Compensation and Control Valve Therefor*. BELIMO Holding AG, Hinwil(CH), 2009.

[19] C. KallesØe, *Fault Detetection and Isolation in Centrifugal Pumps*. 2005. Ph.D. thesis, Aalborg University.

[20] W. Borutzky, *Bond Graph Methodology*. Springer London, 2010.

[21] C. D. Persis and C. S. Kallesøe, "Pressure regulation in nonlinear hydraulic networks by positive and quantized controls," *IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY*, vol. 19, November 2011.

[22] N. Deo, *Graph Theory with Applications to Engineering and Computer Science*. 2004.

[23] C. K. J. L. Maryamsadat Tahavori, Tom Nørgaard Jensen and R. Wisniewski, "Toward model-based control of non-linear hydraulic networks," July 2012.

[24] M. S. P. Kaare Brandt Petersen, *The Matrix Cookbook*. 2012.

[25] PE100, "Nonlinear grey-box models." Last visited 07-04-2017, https://se.mathworks.com/help/ident/nonlinear-grey-box-models.html.

[26] MathWorks, "Compare." Last visited 07-05-2017, https://se.mathworks.com/help/ident/ref/compare.html.

[27] E. F. Camacho and C. Bordons, *Generalized Predictive Control*, pp. 47–79. London: Springer London, 2007.

[28] MathWorks, "Getting started with the control system designer." Last visited 07-05-2017, https://se.mathworks.com/help/control/examples/getting-started-with-the-control-system-designer.html?prodcode=ML.

[29] Polypipe, "Effast pvcu and abs - high performance pvcu and abs pressure pipe systems. 2008." Last visited 07-05-2017, http://www.sbsdownloads.co.uk/Supplier/Polypipe%20Terrain/Product%20&%20Installation%20Guides/Installation%20Guides/Effast%20PVCu%20and%20ABS%20Technical%20Installation%20Guide.pdf.

[30] G. Fischer, "For pe piping systems in utilities." Last visited 07-05-2017, http://www.gfps.com/content/dam/gfps/master%20Content/Documents/EN/Planning%20Fundamentals/PL%20VS%20en.pdf.

[31] Belimo, "Technical databook." Last visited 07-05-2017, http://www.belimo.com.cn/pdf/CCV_databook%5B15.04.2011%5D.pdf.

[32] Belimo, "2-way and 3-way characterised control valves." Last visited 07-05-2017, http://www.belimo.ch/pdf/e/project_2-way_3-way_Characterised%20control%20valves.pdf.

[33] MathWorks, "Estimate linear grey-box models." Last visited 07-05-2017, https://se.mathworks.com/help/ident/ug/estimating-linear-grey-box-models.html.

[34] "Elspot prices." Last visited 19-04-2017, http://www.nordpoolspot.com/Market-data1/Elspot/Area-Prices/DK/Hourly/?dd=DK2&view=chart.

Rettelser

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