Aalborg University

Optimal Control for Water Distribution

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Group:
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Student report

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STUDENT REPORT

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Preface

This project comprises of implementing a functional controller system for

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Explanation of notation

Acronyms

PMA	Pressure Management Area
CP	Critical Point
WT	Water Tower
MP	Minimization Problem

Symbols

Symbol	Description	Unit
C_k	The k^{th} component of the distribution network	$[\cdot]$
n_i	The i^{th} node of the distribution network	$[\cdot]$
q_k	Flow through the k^{th} component	$[m^3/h]$

Contents

1 Introduction

Water pressure management is a vital part of the infrastructure all over the world. It ensures that a positive water pressure is present such that the consumers are supplied with water at all time. Maintaining a minimum pressure in the network is an important task as the end user is ensured a decent water pressure and also for minimizing the risk of contamination in the water system[1].

In the U.S alone 4 % of the national energy consumption is used on moving and treating water/wastewater[2]. The current environmental situation that our world is facing has lead to an increase in the care of the planet. This concern has yielded to the rise of green energies, augmenting the amount of renewable energy sources added to the grid. Nevertheless, the intermittent behavior of renewable energies and time-dependent consumer preferences result in the available power fluctuating. This means that the price for electric power also varies [3]. For minimizing the cost of running a water distribution network, potential energy can be used to maintain a minimum pressure. When electric prices are low, water is pumped to a higher altitude, a water tower (WT), and thereby energy is stored. The potential energy stored in the WT can then be used to maintain a minimum pressure that is required at the end consumer. If the pressure however should become to low, the pumps can be activated to increase the pressure again.

Maximum allowed pressure should also be considered as the water leakage increases proportional to the pressure[4], thus increasing the water losses which leads to a higher energy consumption. In [4] it is stated that the estimated world wide water loss is at 30 %, so the energy used on cleaning the water for filth, bacteria and pressurizing it is lost. Another problem that should be highlighted to a high pressure is that a high pressure will increase the ware on the pipes in a system[5], which leads to higher maintenances as they got to be replaced more often. This is not an easy task, since the pipes usually is under ground and needs to be dug up which is an expensive part, especially in a city since it can have a negative impact on significant infrastructures.

Some constraints to this solution is still needed to be taken into account. One of them being the quality of the water. If stored for too long the quality of the water will start to decrease due to a decreasing oxygen level [6, 7], thus the water should not be stored for to long.

This leads to the following problem statement:

- *How can a water tower, implemented in a water distribution network, be controlled to minimize the cost of running a water distribution network.*

Part I

Analysis

System description 2

This section will give an introduction to the available test system, including structure and components overview.

2.1 System overview

To develop and test different control methods for a water distribution system a test setup is required. Such a setup is available at Aalborg university which is based on a real water distribution system, though as a 1:20 downscaled version.

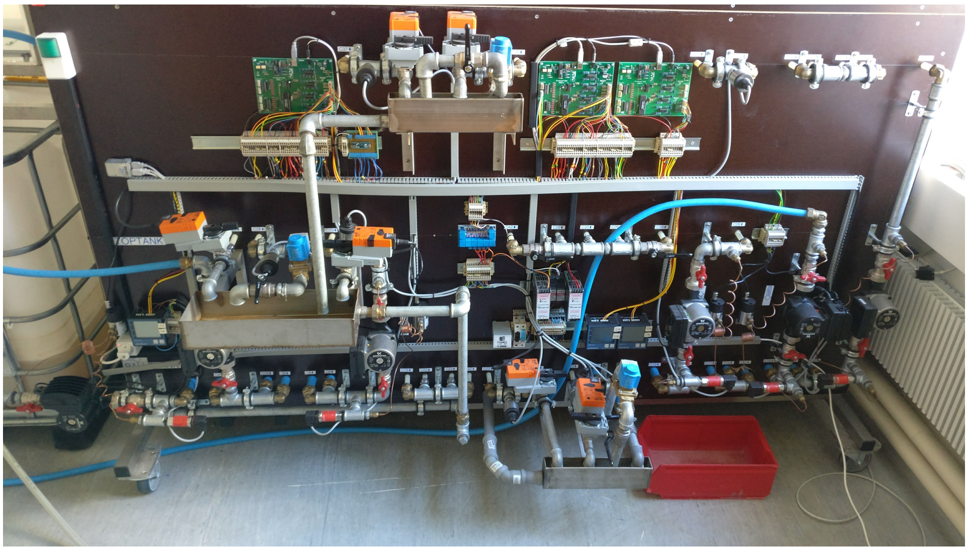


Figure 2.1. The available test setup used to represent a real water distribution system.

The test setup represents a real system, thus the same structure concerning piping, levelling and all the other components. To achieve different elevation levels between system parts, the setup is mounted on a wall. This also allows for a quick overview of the complete setup and eases access to the components. As the system is used for various test scenarios other equipment are also present in the test setup shown in *Figure 2.1*, enabling the test system to mimic a variety of different system types and scenarios. A diagram representing the part of the test setup that will be used in this project is shown in *Figure 2.2*.

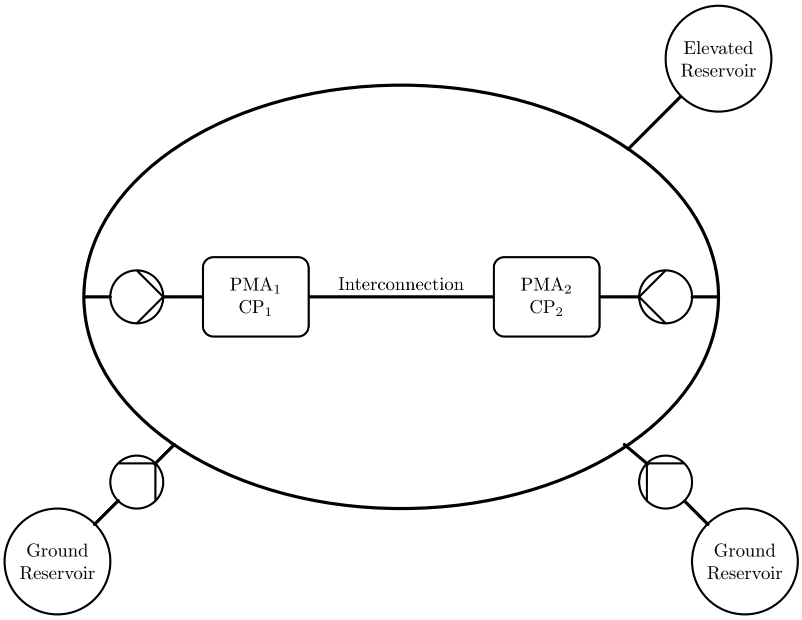


Figure 2.2. Overview of the reduced system that fulfills the scenario of this project.

Should we consider changing the name "elevated reservoir" to "water tower". We have uesed the word water tower in the introduction

The system consists of different parts, the main part being a water reservoir placed on ground level, used to supply the system. Two pumps are connected to the reservoir and they supply water to the main water ring formed around the consumers. Another water reservoir is connected to the water ring by a dedicated pump. This reservoir is elevated and can, when filled, be used to pressurize the system. From the water ring two PMA's are connected via their own pump. In each PMA a measuring point is placed and the pressure at this point shall be kept. Furthermore two consumers are placed in each PMAs.

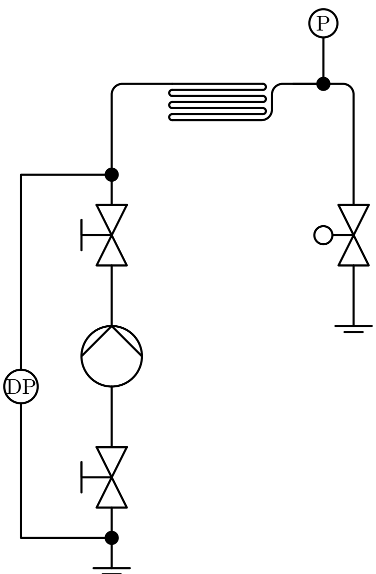


Figure 2.3. Basic water distribution network.

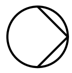
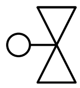
Symbol	**Name**
	Pump
	Manual valve
	Electronic valve
	Pipe segment
	Pressure sensor
	Differential pressure sensor
	Gnd

Table 2.1. Symbol and name for component in the water network.

Modelling 3

3.1 Hydraulic Model

Water distribution networks are designed to deliver water to consumers in terms of sufficient pressure and appropriate chemical composition. Distribution systems as such are generally consist of four main components: pipes, pumps, valves and reservoirs. The common property is that they are all two-terminal components, therefore they can be characterized by the dynamic relationship between the pressure drop accross the two endpoints and the flow through the element.[8] *Equation: (3.1)* shows the state vector of one component.

$$\begin{bmatrix} \Delta P \\ q \end{bmatrix} = \begin{bmatrix} P_{in} - P_{out} \\ q \end{bmatrix} \tag{3.1}$$

In the following chapter the hydraulic model analysis of the system is done by control volume approach.[9] The relationship between the two state parameters are introduced for each component in the hydraulic network.

3.1.1 Pipe Model

Pipes are major components of water distribution systems since they are used for carrying pressurized and treated fresh water. A detailed model of pipes has to be derived in a general form in order to describe the relationship of pressure and flow for each pipe component. The dynamic model of a pipe can be originated from Newton's second law. *Equation: (3.2)* describes the proportionality between the rate of change of the momentum of the fluid(water) and the force acting on it.

$$\frac{d}{dt}P = \sum F \tag{3.2}$$

Where

F	is the force acting on the water,	[N]
P	is the linear momentum of the water flow.	$\left[\frac{\text{kgm}}{\text{s}}\right]$

The dynamics of a pipe component is derived under the assumption that the flow of the fluid is uniformly distributed along the cross sectional area of the pipe and the flow is turbulent. In other words, all pipes in the system are filled up fully with water all the time and the water is assumed to be incompressible. Thus the density of water and the volume of the fluid is constant in time, as the mass of the water is.

Rewriting *Equation: (3.2)*, because of the above-mentioned consideration, the mass of the water can be taken out in front of the derivative.

$$\frac{d}{dt}P = \frac{d(Mv)}{dt} = M\frac{dv}{dt} = \sum F \tag{3.3}$$

Where

M	is the mass of the water,	[kg]
v	is the absolute value of the velocity of the water at each point of the pipe.	$\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$

The sum of the forces acting on the control volume can be seen as input forces(acting on the inlet of the pipe), output forces(acting on the outlet) and resistance forces. These forces are expressed in terms of pressure in order to obtain the model of the pressure drop in the pipes.

The diameter of the pipe is assumed to be constant and to have cylindrical structure:

$$A_{in} = A_{out} = \frac{1}{4}\pi D^2 \tag{3.4}$$

Continuity law in fluid mechanics is applied to analyse the flow in the pipe.[9] A steady and uniform mass flow is assumed in the pipe. Hence, the water flow can be described as:

$$q = A \cdot V \tag{3.5}$$

Where

A	is the cross sectional area of a pipe	$\left[\mathrm{m}^2\right]$
q	is the volumetric rate of flow.	$\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$

In *Equation: (3.6)* the forces acting on the pipe are included, the difference between F_{in} and F_{out} represents the pressure drop between two endpoints.

$$M\frac{dv}{dt} = F_{in} - F_{out} - F_{res} \tag{3.6}$$

In order to obtain an equation consisting of only pressure parameters, the relationship between forces and pressures is used.

$$AL\rho\frac{dv}{dt} = Ap_{in} - Ap_{out} - F_{res} \tag{3.7}$$

The velocity can be written in terms of volumetric water flow and cross sectional area according to the continuity law.

$$AL\rho\frac{d}{dt}\frac{q}{A} = Ap_{in} - Ap_{out} - F_{res} \tag{3.8}$$

Reducing the cross sectional area to obtain an expression for the pressure:

$$\frac{L\rho}{A}\frac{dq}{dt} = p_{in} - p_{out} - \frac{F_{res}}{A} \tag{3.9}$$

Thus the desired pressure drop between two endpoint is obtained. *Equation: (3.10)* differential equation describes the change in flow as a function of the pressure drops in the system.

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - \frac{F_{res}}{A} \tag{3.10}$$

Where

L	is the length of a pipe	[m]
F_{res}	is the resistance force	[N]
Δp	is the pressure drop across the two endpoints	[Pa]
ρ	is the density of water.	$\left[\frac{\text{kg}}{\text{m}^3}\right]$

In *Equation: (3.10)* the term F_{res} is the resistance force acting on the pipe, which consists of two parts: surface resistance(h_f), the friction loss, form resistance(h_m) and the resistance due to change of elevation, Δz.

3.1.1.1 Surface Resistance (h_f)

The flow of a liquid through a pipe suffers resistance from the turbulence occurring along the internal walls of the pipe, caused by the roughness of the surface. This surface resistance is given by the Darcy-Weisbach equation [10].

$$h_f = \frac{fLV^2}{2gD} \tag{3.11}$$

Where

f	is the Moody friction factor	[−]
g	is acceleration due to gravity	$\left[\frac{\text{m}}{\text{s}^2}\right]$
D	is the diameter of the pipe.	[m]

Applying the continuity law, the velocity can be substituted by the volumetric flow and pipe area, resulting in:

$$h_f = \frac{8fLq^2}{\pi^2 gD^5} \tag{3.12}$$

The unknown parameter in 3.12 is the Moody friction factor which is non-dimensional and is a function of the Reynold´s number. This friction factor depends on the flow if it is laminar, transient or turbulent, and the roughness of the tube.

The Reynold´s number can be used to determine the regime of the flow. When $Re < 2300$ as laminar, if $2300 < Re < 4000$ as transient and if $Re > 4000$ as turbulent [11].

$$Re = \frac{vD}{\nu} \tag{3.13}$$

Where

ν	is the kinematic viscosity	$\left[\frac{\mathrm{kg}}{\mathrm{ms}}\right]$

The kinematic viscosity in [10] is given by :

$$\nu = 1.792 \cdot 10^{-6}\left[1+\left(\frac{T}{25}\right)^{1.165}\right]^{-1} \tag{3.14}$$

Where

T	is the water temperature	[°C]

In order to estimate the range of the Reynolds number of a common water distribution, typical values of the temperature, velocity and the radius of the pipes are considered.[12].

- $v = 0.5 - 1.5 \quad \frac{m}{s}$
- $D = 50 - 1500 \quad mm$
- $T = 10 - 20 \quad {}^{\circ}C$

These values result in a Reynold's number between 19000 and 225000, which yields a turbulent fluid flow. For turbulent flow the Moody friction factor is given by [10] :

$$f = 1.325\left(ln\left(\frac{\epsilon}{3.7D}+\frac{5.74}{R^{0.9}}\right)\right)^{-2} \tag{3.15}$$

Where

ϵ	is the average roughness of the wall inside the pipe.	[−]

3.1.1.2 Form Resistance (h_m)

Form resistance losses appear at any time the flow changes direction, due to elbows, bends, or due to enlargers and reducers. It is a particular frictional resistance due to the fittings of a pipe. Form loss can be expressed as:

$$h_m = k_f\frac{V^2}{2g} \tag{3.16}$$

Applying the continuity law:

$$h_m = k_f\frac{8q^2}{\pi^2 g D^4} \tag{3.17}$$

Where

k_f	is the form-loss coefficient.	[−]

The form-loss coefficient can be split into different loss depending on the fitting of the pipes: pipe bend and elbows.
Pipe bends principally is determined by the bend angle α and bend radius R, it is given by the following expression [10]:

$$k_f = \left[0.0733 + 0.923\left(\frac{D}{R}\right)^{3.5}\right]\alpha^{0.5} \tag{3.18}$$

Pipe elbows are also used to change the direction of the flow but providing sharp turns in pipelines. The coefficient for the losses in elbows is determined by the angle of an elbow α and is given by:

$$k_f = 0.442\alpha^{2.17} \tag{3.19}$$

3.1.1.3 Complete Pipe Model

In *Equation: (3.12)* and *Equation: (3.17)*, the head loss of the friction losses are determined. These terms are introduced in *Equation: (3.10)* in terms of pressure. Thus, the friction factors are multiplied by the water density and gravity. Nevertheless, the head loss due to elevation has to be added in the model, yielding the final expression:

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - h_f\rho g - h_m\rho g - \Delta z\rho g \tag{3.20}$$

Substituting the terms h_f and h_m with their respective values:

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - \frac{8fLq^2}{\pi^2 gD^5}\rho g - k_f\frac{8q^2}{\pi^2 gD^4}\rho g - \Delta z\rho g \tag{3.21}$$

Taking out the flow q as the common factor:

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - \frac{8fLq^2}{\pi^2 gD^5}\rho g - k_f\frac{8q^2}{\pi^2 gD^4}\rho g - \Delta z\rho g \tag{3.22}$$

3.1.2 Valve Model

Valves in the water distribution system are modelled according to the same principle as pipes with the difference that the length of each valve(L) and the change in elevation(Δz) are assumed to be zero. Therefore it is assumed that the length of the valve does not influence the flow and the pressure between the endpoints considering the fact that the length of a valve is considerably smaller than the length of a pipe. Another fair assumption is that in case of a valve, elevation is not present.
In the given system, valves are considered as end-user components since they are placed only in the PMAs. These user valves have a variable opening degree(OD) which influences the pressure drop across the endpoints. Valves can be also seen as pipe fittings where the OD is constant for all times, however there are not any fittings in the system, moreover the model of pipes covers it anyway.
Due to the above-mentioned considerations, by recalling *Equation: (3.22)*, it simplifies as follows:

$$\Delta p = -k_f\frac{8q^2}{\pi^2 gD^4}\rho g \tag{3.23}$$

In *Equation: (3.23)*, the form-loss coefficient is taken into account in order to determine the pressure drop. Although k_f is a coefficient that describes the resistance of the valve, manufacturers provide another constant which indicates the valve capacity instead. This coefficient is called the k_{v100}- factor that describes the conductivity of the valve at maximum OD. According to the definition of this parameter, it sets the relationship between the capacity through the valve and the pressure drop of $\Delta p = 1[bar]$ at a fully open state of the valve. According to [13], the properties of water fulfil the requirements which allows to write up the following expression for flow and pressure:

$$q = k_{v100}\sqrt{\Delta p} \tag{3.24}$$

Equation: (3.24) can be derived in detail using the law of continuity for each endpoint of the valve, however the exact derivations can be found in the datasheet [13]. In the further description and derivations, the coefficients and all the technical considerations are based on this datasheet.

3.1.2.1 Valve conductivity function $k_v(OD)$

Instead of k_{v100}, more generally $k_v(OD)$ can be used which is a function of the opening degree, where $OD \in [0, 1]$. In case of user-operated valves, k_v does not remain constant, it ranges over a compact set of values as the opening degree varies too. [8]

All valves in the system share the same characteristics, therefore the following characteristics of k_v are valid for all valves.

Figure 3.1. Valve characteristics - Valve conductivity in the function of OD

According to [14], the following definition can be written up for the conductivity function, $k_v(OD)$:

$$k_v(OD) = \begin{cases} k_{v100}\,\frac{\theta_{OD}}{\theta_{max}}\,n_{gl}\,e^{(1-n_{gl})}, & \text{if } \frac{\theta_{OD}}{\theta_{max}} \le \frac{1}{n_{gl}}; \\ k_{v100}\,e^{(n_{gl}\,(\frac{\theta_{OD}}{\theta_{max}}-1))}, & \text{if } \frac{\theta_{OD}}{\theta_{max}} \ge \frac{1}{n_{gl}} \end{cases} \tag{3.25}$$

Where

θ_{OD}	is the opening degree	$[^\circ]$
θ_{max}	is the maximum of the opening degree	$[^\circ]$
n_{gl}	is the valve characteristic curve factor.	$[-]$

As can be seen, a new parameter, θ_{max} is introduced which describes the maximum angle where the actuator closes the valve. The same can be stated for a minimum angle. The valve is closed when the position of the actuator $\in [0°, 15°]$. As a consequence, there is an offset in the curve as it is shown in *Figure 3.1.2.1.* Introducing the following angle:

$$\gamma = \frac{\theta_{OD} - \theta_{off}}{\theta_{max} - \theta_{off}} \tag{3.26}$$

Where

θ_{off} is the minimum angle where the valve opens. [°]

Equation: (3.25) modifies to:

$$k_v(OD) = \begin{cases} k_{v100}\,\gamma\,n_{gl}\,e^{(1-n_{gl})}, & \text{if } \gamma \leq \frac{1}{n_{gl}}; \\ k_{v100}\,e^{(n_{gl}\,\gamma)}, & \text{if } \gamma \geq \frac{1}{n_{gl}} \end{cases} \tag{3.27}$$

As it is shown, the conductivity function of the valve consists of two types of functions:

$$k_v(OD) = \begin{cases} k_v(\theta_{OD}) \sim linear(), & \text{if } \gamma \leq \frac{1}{n_{gl}}; \\ k_v(\theta_{OD}) \sim exponential(), & \text{if } \gamma \geq \frac{1}{n_{gl}} \end{cases} \tag{3.28}$$

Since exponential functions never cross the zero point, it is reasonable to use linear characteristics in the lower range. The transition from linear to exponential part has to be continuously differentiable and predetermined by n_{gl}. [8] [14]

3.1.2.2 Complete valve model

Using *Equation: (3.24)* with the conductivity function $k_v(OD)$ and expressing Δp yields:

$$\Delta p = \frac{1}{k_v(OD)^2}|q|q \tag{3.29}$$

Expressing it in a compact form for the kth valve in the network yields:

$$\Delta p_k = \mu_k(q_k, k_v(OD)) \tag{3.30}$$

3.1.2.3 Unit transformation

3.1.3 Pump Model

3.1.4 Water Tank Model

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Requirements 4

In this chapter the requirements for the system operation is described.

- Minimum pressure at CP, $\rho > x$ [bar]
- Minimum flow through water tower, $q_{wt} > x$ $[\frac{m^3}{h}]$
- Minimizing the total cost of running the system

put in numbers

Part II

Control Design

5 Controller

In this chapter the design of the controller is explained. Furthermore the optimization controller is designed and this is implemented in simulink.

5.1 Control Problem

The water distribution system explained in Section *2.1: System overview* need to be controlled according the Section *4: Requirements*. The requirements can be summarized as:

- Minimum pressure at CP, $\rho > x$ [bar]
- Minimum flow through water tower, $q_{wt} > x$ $[\frac{m^3}{h}]$
- Minimizing the total cost of running the system

put in numbers

The pressure at a given CP can be controlled by both the water level in the WT and the rotational speed of the pump connected to the given PMA. To fulfill the requirement of a minimum pressure at a given CP a controller have to be develop that takes both pressure actuators into account.

The flow through the WT is controlled by a pump, . Whenever the pump, , is pumping water into the WT can be seen as a consumer. This mean that the others pumps have to do ensure pressure at all CP. Furthermore the flow rate, q_{WT}, must meet a minimum requirement to insure the water quality. This can be seen as a constrain of the operation area of the system.

name of WT pump

name of WT pump

At the same time the total cost of running the system should be minimized. Therefor a cost function is needed. This cost function purpose is to find the optimal control signal which minimize the cost of running the pumps. Thereby spending the least money on running the total system.

Considering both the cost function and the constrain, this leads to a description of the systems operate area, $C_T(\Delta p_i, q_i)$ wherein the system must operate. By considering the total cost of running, C_t this can be seen as a minimization problem:

$$\underset{u}{min} \quad C_T(\Delta p_i, q_i) \tag{5.1}$$

s.t

$$p_i \geq x$$
$$q_4 \geq x$$

Where

	Δp_i	is the pressure gain over a pump,	[Bar]
and	q_i	is the flow at a pump.	$[\mathrm{m}^3 \cdot \mathrm{s}^{-1}]$

5.2 Pressure Control

5.3 Minimization Problem

In this section the cost function for the minimization problem and the constraints is explained. Furthermore the optimization controller is designed.

Implementation of controller 6

This chapter will explain how the controller designed in Chapter *5: Controller* is implemented in MATLAB simulink.

Part III

Conclusion and verification

7 Accepttest

8 Discussion

9 Conclusion

Part IV

Appendices

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Rettelser

Todo list