Aalborg University

Optimal Control for Water Distribution

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Student report

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STUDENT REPORT

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Preface

This project comprises of implementing a functional controller system for

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Explanation of Notation

Acronyms

PMA	Pressure Management Area
CP	Critical Point
WT	Water Tower
MP	Minimization Problem
OD	Opening degree

Symbols

Symbol	**Description**	**Unit**
A	Cross sectional area	$[m^2]$
C_k	The k^{th} component of the distribution network	$[\cdot]$
C	Electric capacitance	$[F]$
C_H	Hydraulic capacitance	$[m^3/(N/m^2)]$
D	Diameter	$[m]$
f	Moody friction factor	$[-]$
F	Force	$[N]$
g	Acceleration due to gravity	$[m/s^2]$
h_f	Pressure given in head	$[m]$
h_m	Form loss	$[?]$
J_k	Water inertia of the k^{th} component	$[kg/m^4]$
k_f	Form loss coefficient	$[-]$
L	Length	$[m]$
m	Mass of body	$[kg]$
M	Linear momentum	$[kgm/s]$
n_i	The i^{th} node of the distribution network	$[\cdot]$
n_{gl}	Valve characteristic curve factor	$[-]$
p_a	Atmospheric pressure	$[Pa]$
Δp_k	The pressure drop across the i^{th} component	$[Pa]$
q_k	Flow through the k^{th} component	$[m^3/h]$
T	Temperature	$[^\circ]$
v	Velocity	$[m/s]$
V_t	Volume of the water in the water tower	$[m^3]$
$\alpha_k(.)$	The pressure boost given by the k^{th} pump	$[Pa]$
ϵ	Average roughness	$[-]$
ζ	Pressure drop from elevation difference across the k^{th} component	$[Pa]$
θ_{max}	Maximum angle of the opening degree	$[^\circ]$
θ_{off}	Minimum angle where the valve closes	$[^\circ]$
θ_{OD}	Angle of opening degree	$[^\circ]$
$\lambda_k(.)$	Function of hydraulic resistance in the k^{th} pipe	$[Pa]$
$\mu_k(.)$	Function of hydraulic resistance in the k^{th} valve	$[Pa]$
ν	Kinematic viscosity	$[kg/ms]$
ρ	Density	$[kg/m^3]$
ω_r	Impeller angular velocity	$[rad/s]$

Contents

1 Introduction

Water pressure management is a vital part of the water supply infrastructure all over the world. It ensures that a positive water pressure is present such that the consumers are supplied with water at all time. Maintaining a minimum pressure in the network is an important task as it ensures the end user a decent water pressure and also minimizes the risk of contamination in the water system[1].

In the U.S alone 4 % of the national energy consumption is used on moving and treating water/wastewater[2]. With an increasing focus on green energy, more and more renewable energy sources are added to the grid. Nevertheless, the intermittent behavior of renewable energy sources and time-dependent consumer preferences result in fluctuation in the available power. This means that the price for electric power also varies [3]. To minimize the cost of running a water distribution network, potential energy can be used to maintain a minimum pressure. When electric prices are low, water can be pumped to a higher altitude and stored in a water tower (WT), and thereby energy is stored for future use. The potential energy of the water stored in the WT can then be used to maintain a minimum pressure that is required at the end consumer. However when a WT is included in a water distribution network, the pressure in the system is defined by the water level and hight of the WT. This means that to control pressure, the water level of the WT should be controlled.

Maximum allowed pressure in water distribution networks should also be considered as the risk of water leakage increases when pressure is increased[4], thus increased water losses due to leakage will lead to a higher energy consumption. In [4] it is stated that the estimated world wide water loss is at 30 %, so the energy used on cleaning the water for filth, bacteria and pressurizing it is lost. Another problem that should be highlighted regarding high pressure is that a high pressure will increase the wear on the pipes in a system[5], this leads to higher maintenances costs as pipes and fittings have to be replaced more frequently. Additionally, maintenances is not always an easy task, since the pipes usually are placed under ground and need to be dug up. Thereby the expense of maintenance is increased, especially in a city, were the operation also can have a negative impact on significant infrastructures. Based on these facts, the maximum pressure in a water distribution network is a vital parameter of the systems profitability. In a system with a WT the maximum allowed pressure will likely be defined by the maximum allowed water level in the WT, as the WT in most situations will be able to provide a dominant pressure compared to the desired network pressure.

Some constraints regarding a solution that implements a WT are still necessary to be taken into account. One of them being the quality of the water in the tower. If stored for too long the quality of the water will start to decrease due to a decreasing oxygen level [6, 7], thus the water should not be stored for to long. The oxygen level of the water also depends on the water temperature and therefor the water should not be too warm. Furthermore it is undesirable that the water remains stagnant in the tower or pipe as it also effects the water quality.

This leads to the following problem statement:

- *How can a water tower, implemented in a water distribution network, be controlled to minimize the cost of running a water distribution network without compromising the water quality.*

Part I

Analysis

System Description 2

This section will give an introduction to the available test system, including structure and components overview.

2.1 System Overview

To develop and test different control methods for a water distribution system a test setup is required. Such a setup is available at Aalborg university which is based on a real water distribution system, though as a 1:20 downscaled version.

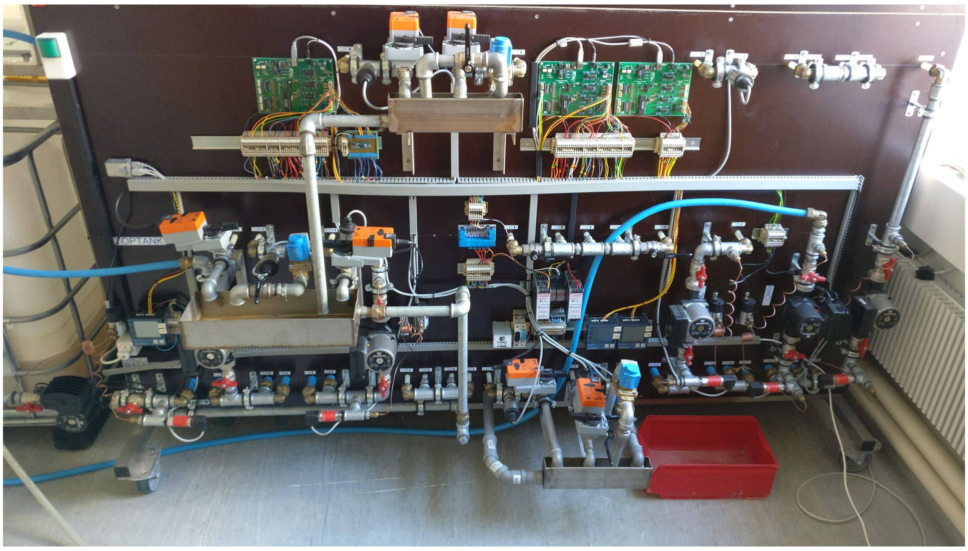


Figure 2.1. The available test setup used to represent a real water distribution system.

The test setup represents a real system, thus the same structure concerning piping, leveling and all the other components. To achieve different elevation levels between system parts, the setup is mounted on a wall. This also allows for a quick overview of the complete setup and eases access to the components. As the system is used for various test scenarios other equipment is also present in the test setup shown in *Figure 2.1*, enabling the test system to mimic a variety of different system types and scenarios. A simplified diagram representing the structure of the test setup that will be used in this project is shown in *Figure 2.2*.

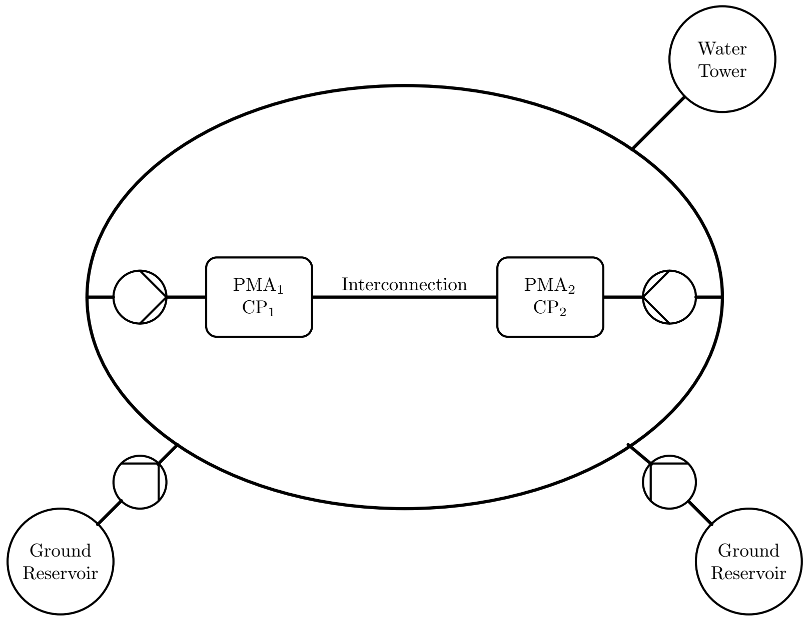


Figure 2.2. Overview of the reduced system that fulfills the scenario of this project.

The system can be split up into different parts, where the main part is a water reservoir placed at ground level, used to supply the system. Two pumps are connected to the reservoir and they supply water to a main water ring formed around the PMA's. A water tower is also connected to the main water ring, and will act both as an additional water reservoir and pressurize the ring due to the elevation of the tower. The direction of water flow with respect to the tower will depend on the pressure in the main ring and the tower can thus be filled by pressurizing the ring or be used to pressurize and supply water to the ring instead of the pumps. From the water ring two PMA's are connected, each via their own pump. In each PMA a measuring point called the critical point (CP) is placed and the pressure at this point shall be kept to accommodate supply demands of the consumers. Furthermore two consumers are placed in each PMA, these are simulated by valves with a variable opening degree where the water flows back to the main reservoir.

As the test setup consist of different components as valves, pumps and pipes, a basic water distribution network is shown in *Figure 2.3* which will be used to illustrate and explain the individual components in the system and their functionality.

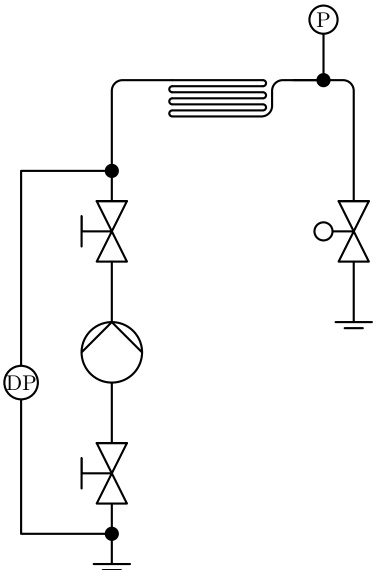


Figure 2.3. Basic water distribution network.

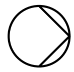
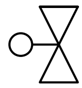
Symbol	Name
	Pump
	Manual valve
	Electronic valve
	Pipe segment
	Pressure sensor
	Differential pressure sensor
	Gnd

Table 2.1. Symbol and name for component in the water network.

In the system two different types of Grundfos pumps are used. For supplying the water ring two pumps of the type UPMXL GEO 25-125[8] are used. Whereas the pumps used in each PMA are of the type UPM2 25-60[9] which is a smaller pump and typically used at the end-user.

In order to close off parts of the system that will not be used for a specific scenario or to simulate faulty behavior, manual rotary ball valves are placed trough out the system. To simulate a consumer, an electronically controlled belimo valve is used. Thereby is it possible to vary the opening degree of the valve over time according to a specific consumer behavior.

For the pipes there are used two different material types. The pipes used in the main ring, the connections to both the reservoir and the water tower are made of polyethylene grade 80 called PE80[10]. The pipes used to connect the PMA's to the ring and the internal connections in the PMA's are made of polyethylene with cross-links called PEX. In addition to the pipes, fittings, bends, and elbows are also present and found in various metals as iron and brass.

The pressure measuring in each PMA is done with a Jumo pressure sensor. The pressure is measured relative to a reference called Gnd, for the test system Gnd is atmospheric pressure. Furthermore both the differential pressure over each pump and the absolute pressure at the pump is measured with a Grundfos direct sensor DPI v.1 and a Danfoss mbs32/33 pressure sensor, respectively.

The main reservoir has a volume of 1000 L and the WT a volume of 200 L. THe volume of the WT in this report is denoted as V_t. A system diagram of the entire test setup, including pipe dimensions, naming and so on, can be seen in *Appendix: C.2.*

3 Requirements and Constraints

Adding a WT to an existing water distribution network will introduce constrains and these need to be taken into account.

As mentioned in Section *1: Introduction*, a minimum pressure must be maintained at the end user. Furthermore the pressure can not exceed a maximum level as this will both increase the possibility of water leakage and wear on the pipes in the system. The system described in Section *2.1: System Overview*, is designed to operate at a pressure around 0.1 bar, relative to the environment [11]. For the purpose of this project the interval for which the pressure should be within, is chosen to be between $0.08 < p_{cp} < 0.14$ [Bar], where p_{cp} is the pressure at a critical point.

Another important aspect when implementing a WT is water quality. If the water is stored, in the WT, for too long the quality will decrease due to decreasing oxygen level. Because of this a requirement for water quality has to be formulated. As described in Section *2.1: System Overview*, the WT has one combined input/output connection. Therefor a requirement only for flow is hard to formulate as the direction will change dependent on the usage. This could result in a flow based constraint being fulfilled by rapidly changing flow direction without actually replacing any significant water volume in the tower. Instead, a requirement for how often the content of the WT should be exchanged per time unit is proposed. For the purpose of this project the minimum requirement to volume exchange, is chosen to 30% of the maximum volume of the WT, V_T per day. This can be written as $\bar{q}_{wt} > 0.3 \cdot V_T \left[\frac{m^3}{day}\right]$. As stated in Section *2.1: System Overview* $V_T = 200\ L$ so therefor $\bar{q}_{wt} > 0.06 \left[\frac{m^3}{day}\right]$.

This results in the following requirements:

- Pressure at CP, $0.08 < p_{cp} < 0.14$ [bar]
- Minimum water exchange , $\bar{q}_{wt} > 0.06 \left[\frac{m^3}{day}\right]$
- Minimizing the total cost of running the system

Modelling 4

4.1 Hydraulic Modeling

Water distribution networks are designed to deliver water to consumers in terms of sufficient pressure and appropriate chemical composition. Distribution systems as such are generally consisting of four main components: pipes, pumps, valves and reservoirs. The common property is that they are all two-terminal components, therefore they can be characterized by the dynamic relationship between the pressure drop across the two endpoints and the flow through the element [12]. *Equation: (C.2)* shows the dual variables which describes one component.

$$\begin{bmatrix} \Delta P \\ q \end{bmatrix} = \begin{bmatrix} P_{in} - P_{out} \\ q \end{bmatrix} \tag{4.1}$$

Where

Δp	is the pressure drop across the two endpoints	[Pa]
q	is the flow through the element.	$\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$

In the following chapter the hydraulic model of the system is derived by control volume approach [13]. The relationship between the two variables are introduced for each component in the hydraulic network.

4.1.1 Pipe Model

Pipes are important components of water distribution systems since they are used for carrying pressurized and treated fresh water. A detailed model of pipes has to be derived in order to describe the relationship of pressure and flow for each pipe component. The dynamic model of a pipe can be originated from Newton's second law. *Equation: (4.2)* describes the proportionality between the rate of change of the momentum of the water and the force acting on it.

$$\frac{d}{dt}M = \sum_i F_i \tag{4.2}$$

Where

M	is the linear momentum of the water flow	$\left[\frac{\mathrm{kgm}}{\mathrm{s}}\right]$
F_i	is the set of forces acting on the water.	[N]

The dynamic model of a pipe component is derived under the assumption that the flow of the fluid is uniformly distributed along the cross sectional area of the pipe. In other words, all pipes in the system are filled up fully with water all the time. Thus the density of water and the volume of the fluid is constant in time, as is the mass of the water.
Rewriting *Equation: (4.2)*, because of the above-mentioned asumptions, the mass of the water can be taken out in front of the derivative.

$$\frac{d}{dt}M = \frac{d(mv)}{dt} = m\frac{dv}{dt} = \sum_i F_i \tag{4.3}$$

Where

m	is the mass of the water	[kg]
v	is the value of the velocity of the water at each point of the pipe.	$\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$

The sum of the forces acting on the control volume can be seen as input forces, acting on the inlet of the pipe, output forces, acting on the outlet, resistance forces and gravitational force effect. These forces are expressed in terms of pressure in order to obtain the model of the pressure drop in the pipes. In *Figure 4.1* all forces acting on a pipe segment are shown:

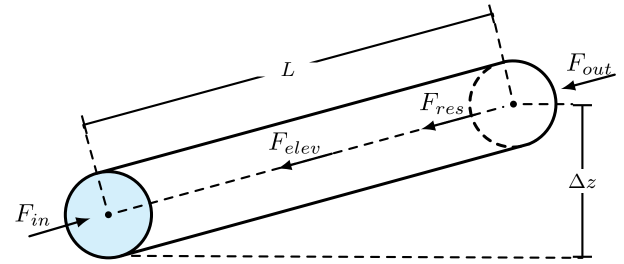


Figure 4.1. Free-body diagram describing the forces acting on a segment of a pipe.

The pipe is assumed to have a cylindrical structure. Furthermore, the cross section of the pipe, $A(x)$, is constant for every $x \in [0, L]$, where L is the length of the pipe:

$$A_{in} = A_{out} = \frac{1}{4}\pi D^2 \tag{4.4}$$

Where

A	is the cross sectional area of a pipe	$[\mathrm{m}^2]$
D	is the diameter of the pipe.	[m]

Water flow(q) can be expressed in terms of velocity(v) and cross sectional area(A), resulting in:

$$q = A \cdot v \tag{4.5}$$

In *Equation: (4.6)* the forces acting on the pipe are included, the difference between F_{in} and F_{out} which indicates the pressure drop between two endpoints, the resistance forces F_{res} and the gravitational force effect due to change in elevation F_{elev}.

$$m\frac{dv}{dt} = F_{in} - F_{out} - F_{res} - F_{elev} \tag{4.6}$$

In order to obtain an equation consisting of only pressure variables, the relationship between forces and pressures is used.

$$AL\rho\frac{dv}{dt} = Ap_{in} - Ap_{out} - F_{res} - F_{elev} \tag{4.7}$$

Where

A	is the cross sectional area of a pipe	$\left[\mathrm{m}^2\right]$
L	is the length of a pipe	$[\mathrm{m}]$
ρ	is the density of water	$\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$
p	is the pressure at a point.	$[\mathrm{Pa}]$

Rewriting the velocity in terms of volumetric water flow and cross sectional area.

$$AL\rho\frac{d}{dt}\frac{q}{A} = Ap_{in} - Ap_{out} - F_{res} - F_{elev} \tag{4.8}$$

By dividing the equation with the cross sectional area, A, it can be seen that the equation is dependant on the pressure difference between two endpoints.

$$\frac{L\rho}{A}\frac{dq}{dt} = p_{in} - p_{out} - \frac{F_{res}}{A} - \frac{F_{elev}}{A} \tag{4.9}$$

Thus the desired pressure drop between two endpoint is obtained. *Equation: (4.10)* differential equation describes the change in flow as a function of the pressure drops in the system.

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - \frac{F_{res}}{A} - \frac{F_{elev}}{A} \tag{4.10}$$

In *Equation: (4.10)* the term F_{res} is the resistance force acting on the pipe, which consists of two parts: surface resistance(h_f), the friction loss, and the form resistance(h_m) due to the fittings. F_{elev} is the force of gravity due to change in elevation, Δz.

4.1.1.1 Surface Resistance (h_f)

The flow of a liquid through a pipe suffers resistance from the turbulence occurring along the internal walls of the pipe, caused by the roughness of the surface. This surface resistance is given by the Darcy-Weisbach equation [14].

$$h_f = \frac{fLv^2}{2gD} \tag{4.11}$$

Where

f	is the Moody friction factor	$[-]$
h_f	is the pressure given in head	$[\mathrm{m}]$
g	is acceleration due to gravity	$\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$
D	is the diameter of the pipe.	$[\mathrm{m}]$

Equation: (4.11) is under the assumption that $v > 0$.Assuming that the flow is not unidirectional and substituting the velocity by the volumetric flow and pipe area:

$$h_f = \frac{8fL}{\pi^2 g D^5}|q|q \tag{4.12}$$

The unknown parameter in 4.12 is the Moody friction factor which is non-dimensional and is a function of the Reynold´s number. This friction factor depends on whether the flow is laminar, transient or turbulent, and the roughness of the pipe [15].

The Reynold´s number can be used to determine the regime of the flow [15]. When $Re < 2300$ as laminar, if $2300 < Re < 4000$ as transient and if $Re > 4000$ as turbulent.

$$Re = \frac{vD}{\nu} \tag{4.13}$$

Where

ν is the kinematic viscosity. $\left[\frac{\mathrm{kg}}{\mathrm{ms}}\right]$

The kinematic viscosity in [14] is given by :

$$\nu = 1.792 \cdot 10^{-6} \left[1 + \left(\frac{T}{25}\right)^{1.165}\right]^{-1} \tag{4.14}$$

Where

T is the water temperature [°C]

In order to estimate the range of the Reynolds number in a common water distribution, typical values for the temperature, velocity and the radius of the pipes are considered [16].

- $0.5 \leq v \leq 1.5 \quad \frac{m}{s}$
- $50 \leq D \leq 1500 \quad mm$
- $10 \leq T \leq 20 \quad °C$

These values result in a Reynold's number between 19000 and 225000, which yields to consider a turbulent fluid flow through the pipes. For turbulent flow the Moody friction factor is given by [14]:

$$f = 1.325 \left(ln \left(\frac{\epsilon}{3.7D} + \frac{5.74}{Re^{0.9}} \right) \right)^{-2} \tag{4.15}$$

Where

ϵ is the average roughness of the wall inside the pipe. [m]

4.1.1.2 Form Resistance (h_m)

Form resistance losses appear at any time the flow changes direction, due to elbows, bends, or due to enlargers and reducers. It is a particular frictional resistance due to the fittings of a pipe. Form loss can be expressed as:

$$h_m = k_f \frac{v^2}{2g} \tag{4.16}$$

Applying the definition of volumetric flow:

$$h_m = k_f \frac{8}{\pi^2 g D^4} |q| q \tag{4.17}$$

Where

k_f is the form-loss coefficient. [–]

The form-loss coefficient can be split into different losses depending on the fitting of the pipes.
Pipe bends are principally determined by the bend angle α and bend radius r, this is given by the following expression [14]:

$$k_f = \left[0.0733 + 0.923 \left(\frac{D}{r}\right)^{3.5}\right] \alpha^{0.5} \tag{4.18}$$

Pipe elbows are also used to change the direction of the flow but providing sharp turns in pipelines. The coefficient for the losses in elbows is determined by the angle of an elbow α and is given by:

$$k_f = 0.442\alpha^{2.17} \tag{4.19}$$

4.1.1.3 Complete Pipe Model

In *Equation: (4.12)* and *Equation: (4.17)*, the head loss of the friction losses are determined. These terms are introduced in *Equation: (4.10)* in terms of pressure. Thus, the friction factors are multiplied by the water density and gravity. Nevertheless, the head loss due to elevation has to be added in the model, yielding the final expression:

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - h_f\rho g - h_m\rho g - \Delta z\rho g \tag{4.20}$$

Substituting the terms h_f and h_m with their respective values:

$$\frac{L\rho}{A}\frac{dq}{dt} = \Delta p - \frac{8fL}{\pi^2 gD^5}\rho g|q|q - k_f\frac{8}{\pi^2 gD^4}\rho g|q|q - \Delta z\rho g \tag{4.21}$$

Equation: (4.21) describes the rate of flow in terms of pressure losses due to pressure change, frictions and elevation. A more compact form can be expressed for the kth component as such:

$$J_k\dot{q_k} = \Delta p_k - \lambda_k(q_k) - \zeta_k \tag{4.22}$$

Where

J_k	is an analogous parameter as inertia for the water
$\lambda_k(q_k)$	is the friction as a function of flow
ζ_k	is the pressure drop due to the elevation.

As can be seen in *Equation: (4.22)*, the flow dynamics of the kth pipe is described by J_k which is an analogous parameter as inertia in mechanical systems. Where J_k is a diagonal matrix with zeros for the diagonal elements not related to a pipe, $J = diag(J_i)$.

However, it is assumed(prior to the tests carried out on the system) that the presence of the water tower in the system has a slow effect on the flow due to slow integration behavior. In other words it means that the water tower might have a relatively big time constant compared to the time constant of the pipe. Due to this consideration it would be a fair assumption that the parameter J_k does not influence the flow significantly in the system, therefore it could be neglected. However, the parameter is kept until this assumption is not verified by tests. The complete model of a pipe yields:

$$\Delta p_k = \lambda_k(q_k) + J_k\dot{q_k} + \zeta_k \tag{4.23}$$

4.1.2 Valve Model

Valves in the water distribution system are modelled according to the assumption that the length of each valve, L, and the change in elevation, Δz, are zero. Therefore it is assumed that the length of the valve does not influence the flow and the pressure between the endpoints. This is due to the fact that the length of a valve is considerably smaller than the length of a pipe. Another fair assumption is that in case of a valve, elevation is

not present.
In the given system, valves are considered as end-user components since they are placed only in the PMAs. These user valves have a variable Opening Degree(OD) which influences the pressure drop across the endpoints.

In case of valves, manufacturers provide a parameter which indicates the valve capacity. This coefficient is called the k_{v100}- factor that describes the conductivity of the valve at maximum OD. According to the definition of this parameter, it sets the relationship between the capacity through the valve and the pressure drop of $\Delta p = 1[bar]$ at a fully open state of the valve. According to [17], the properties of water fulfil the requirements which allows to write up the following expression for flow and pressure:

$$q = k_{v100}\sqrt{\Delta p} \tag{4.24}$$

Where

k_{v100} is the valve maximum capacity factor $\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$

Equation: (4.24) can be derived in detail using the law of continuity for each endpoint of the valve, however the exact derivations can be found in the datasheet [17]. In the further description and derivations, the coefficients and all the technical considerations are based on this datasheet.

4.1.2.1 Valve Conductivity Function $k_v(OD)$

Instead of k_{v100}, more generally $k_v(OD)$ can be used which is a function of the opening degree, where $OD \in [0, 1]$. In case of user-operated valves, k_v does not remain constant, it ranges over a compact set of values as the opening degree varies. [12].

All valves in the system share the same characteristics, therefore the following characteristics of k_v are valid for all of them.

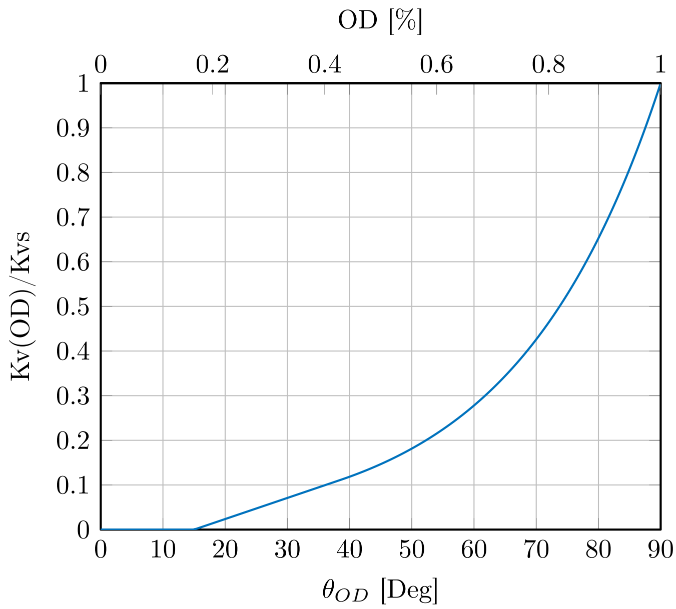


Figure 4.2. Valve characteristics - Valve conductivity in the function of OD

According to [18], the following definition can be written up generally for the conductivity function, $k_v(OD)$:

$$k_v(OD) = \begin{cases} k_{v100}\, \frac{\theta_{OD}}{\theta_{max}}\, n_{gl}\, e^{(1-n_{gl})}, & \text{if } \frac{\theta_{OD}}{\theta_{max}} \leq \frac{1}{n_{gl}}; \\ k_{v100}\, e^{(n_{gl}\,(\frac{\theta_{OD}}{\theta_{max}}-1))}, & \text{if } \frac{\theta_{OD}}{\theta_{max}} \geq \frac{1}{n_{gl}} \end{cases} \tag{4.25}$$

Where

θ_{OD}	is the opening degree	[°]
θ_{max}	is the upper interval of the opening degree where the conductivity does not change $\in [90°, 100°]$	[°]
n_{gl}	is the valve characteristic curve factor.	[·]

A new parameter, θ_{max}, is introduced which describes the maximum angle where the actuator closes the valve. The same can be stated for a minimum angle. The valve is closed when the position of the actuator $\in [0°, 15°]$. As a consequence, there is an offset in the curve as it is shown in *Figure 4.2*. Introducing the following angle:

$$\gamma = \frac{\theta_{OD} - \theta_{off}}{\theta_{max} - \theta_{off}} \tag{4.26}$$

Where

θ_{off}	is the minimum angle where the valve opens.	[°]

In case of the water distribution system *Equation: (4.25)* modifies to:

$$k_v(OD) = \begin{cases} k_{v100}\, \gamma\, n_{gl}\, e^{(1-n_{gl})}, & \text{if } \gamma \leq \frac{1}{n_{gl}}; \\ k_{v100}\, e^{(n_{gl}\,\gamma)}, & \text{if } \gamma \geq \frac{1}{n_{gl}} \end{cases} \tag{4.27}$$

As it is shown, the conductivity function of the valve consists of two types of functions:

$$k_v(OD) = \begin{cases} k_v(\theta_{OD}) \sim linear(), & \text{if } \gamma \leq \frac{1}{n_{gl}}; \\ k_v(\theta_{OD}) \sim exponential(), & \text{if } \gamma \geq \frac{1}{n_{gl}} \end{cases} \tag{4.28}$$

Since exponential functions never cross the zero point, it is reasonable to use linear characteristics in the lower range. The transition from linear to exponential has to be continuously differentiable and predetermined by n_{gl} [12, 18]

4.1.2.2 Complete valve model

Using *Equation: (4.24)* with the conductivity function $k_v(OD)$ and expressing Δp yields:

$$\Delta p = \frac{1}{k_v(OD)^2}|q|q \tag{4.29}$$

Describing it in a compact form for the k^{th} valve in the network yields:

$$\Delta p_k = \mu_k(q_k, k_v(OD)) \tag{4.30}$$

4.1.3 Pump Model

In order to move water from the reservoirs to the costumers, pumping is required. To guarantee that the water reaches every end-user with the appropriate pressure, different pumps can be used in the water distribution system.

Centrifugal pumps are ideal for this purpose. A model describing the pressure drop is derived which is presented in detail in [19]. The pressure provided by the pump is given by:

The list bellow should be with units.

$$\Delta p = -a_{h2}{q_i}^2 + a_{h1}\omega_r q_i + a_{h0}{\omega_r}^2 \tag{4.31}$$

Where

Δp	is the head produced by the pump
q_i	is the volume flow through the impeller
ω_r	is the impeller speed

We need the model of the power consumption of the pump to minimize the cost of power consumed - Tom

4.1.4 Water Tower

Water towers are used to maintain the correct pressure level in the system, ensure reliability and to improve the optimality of the water supply. The WT plays a determinative role in the flow control, therefore its dynamic model must be derived.

Similarly to the modelling of the other components, the relation between the two dual variables, pressure difference and flow are derived. The structure of the WT is illustrated in *Figure 4.3*.

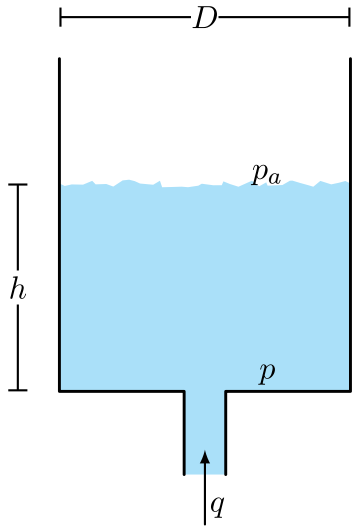


Figure 4.3. Sketch of the open water tower

In *Figure 4.3*, p_a represents the pressure on the surface of the water, therefore it is the atmospheric pressure at all time. p equals to the pressure value on the bottom of the tank. The rate of change of the fluid volume in the water tower is proportional to the volumetric flow at which water enters or leaves the tank.

$$q = \frac{dV_t}{dt} = A\frac{dh}{dt} \tag{4.32}$$

Where

h	is the height of the fluid in the WT	$[\mathrm{m}]$
V_t	is the volume of the WT	$[\mathrm{m}^3]$
A	is the cross section of the WT which is assumed to be constant	$[\mathrm{m}^2]$
q	for $y \in [0, h]$	$\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$
	is the volumetric flow	

The force on the bottom of the WT is due to the weight of water. According to Newton's second law:

$$F = mg = \rho g V_t \tag{4.33}$$

Where

ρ	is the density of water.	$\left[\frac{\text{kg}}{\text{m}^3}\right]$

Equation: (4.33) can be rewritten in terms of pressure such as:

$$\frac{F}{A} = \rho g h = p - p_a = \Delta p \tag{4.34}$$

The total pressure on the bottom of the WT is a result of the pressure difference due to the fluid (p) and the atmospheric pressure (p_a). However, the model is derived in such a way that the atmospheric pressure is set to zero. Therefore, if the water is assumed to be incompressible(density does not change with pressure), *Equation: (4.32)* can be written as:

$$q = \frac{dV}{dt} = \frac{A}{\rho g}\frac{d}{dt}\Delta p = C_H \Delta \dot{p} \tag{4.35}$$

Where

C_H	is the hydraulic capacitance.	$\left[\frac{\text{m}^3}{\text{N/m}^2}\right]$

This equation shows proportionality between pressure and the volume of water, which is exactly the defining characteristic of a fluid capacitor. When the fluid capacitance is large, corresponding to a tank with a large area, a large increase in volume is accompanied by a small increase in pressure.

An analogy can be made between an electronic circuit and the hydraulic system, where the WT acts as a capacitor. Deriving the relationship between the voltage and the charge of the capacitor:

$$I = C\frac{dU}{dt} \tag{4.36}$$

Where

U	is the voltage	[V]
C	is the capacitance	[F]

In the *Equation: (4.35)* the volume flow rate (q) is equivalent to the current (I) in a circuit and the constant term $\left(\frac{A}{\rho g}\right)$ is equivalent to the capacitance of a capacitor (C). The voltage drop is analogous to the pressure drop in the water system.

4.2 Component Model

Using the final expression of each component, a complete system model can be obtained. This model includes pipe, valve, pump components and the WT.

The model of the water tower is described by a first order differential equation, consisting of the first time derivative of the pressure drop. The final expression is shown in Section *4.1.4: Water Tower*:

$$\Delta \dot{p}_{WT;k} = \frac{1}{C_{H;k}} q_k \tag{4.37}$$

The complete model consists of the pipe model, *Equation: (4.30)* the valve model, *Equation: (4.31)* the pump model and *Equation: (4.37)* the WT model. For the pressure drop of the k^{th} component the following expression can be written:

$$\Delta p_k = \underbrace{\lambda_k(q_k) + \zeta_k + J_k \dot{q}_k}_{\text{Pipe}} + \underbrace{\mu_k(q_k, k_v)}_{\text{Valve}} - \underbrace{\alpha_k(u)}_{\text{Pump}} + \underbrace{\Delta p_{WT;k}}_{\text{Water tank}} \tag{4.38}$$

The complete component model, *Equation: (4.38)* is used to represent the pressure contribution of each component. In order to describe each part of the system by *Equation: (4.38)* the parameters and functions that correspond to the specific components are chosen. The remaining are set to zero if they do not match the model of the specific part of the network. The relation seen in *Table: 4.1* shows the parametrization of the system.

Component	J_k	λ_k	μ_k	α_k	ζ_k	$\Delta p_{WT;k}$
kth Pipe	J_k	λ_k	0	0	ζ_k	0
kth Valve	0	0	μ_k	0	0	0
kth Pump	0	0	0	α_k	0	0
kth Water tower	0	0	0	0	0	$\Delta p_{WT;k}$

Table 4.1. Complete model parametrization.

4.2.1 Unit Transformation

During the derivation of the dynamic model, the unit of the physical variables are considered as pascals and seconds. However, it will be concluded in a later chapter that the flow is significantly small compared to the pressure if the SI-units are kept. Therefore a unit conversion is carried out from Pascal[Pa] to bars and from seconds[s] to hours[h]. Another reason which makes this conversion reasonable is that the conductivity function,k_v100 in Section *4.1.2.1: Valve Conductivity Function $k_v(OD)$*, relies on the condition that the pressure drop is one bar. The time scaling is due to conventions. Among the research community in hydraulics, a convenient way to handle the volumetric flow in $[m^3/h]$ instead of$[m^3/s]$.
The detailed derivation of the unit conversion can be found in *Appendix: A*. The result is stated here:

$$\frac{L\rho}{A \cdot 10^5} \frac{d}{dt} \frac{q}{3600} = \Delta \frac{p}{10^5} - \left(\frac{8fL}{\pi^2 g D^5 \cdot 10^5} + k_f \frac{8}{\pi^2 g D^4 \cdot 10^5}\right) \rho g \frac{|q|}{3600} \frac{q}{3600} - \frac{\Delta z \rho g}{10^5} \tag{4.39}$$

4.3 Network Model

After deriving the dynamics of all components in the network, the complete system can be drawn. In *Appendix: C.2*, the topology of the test system is described in more detail. In the following discussion, all statements and notations are based on the figure in the appendix.
The way of modelling a hydraulic system is in some way analogous to an electric circuit. Most of the various hydraulic components can be represented as electronic equivalents and vice versa, however there are some differences too. It should be emphasized that there are not any analogous phenomenon as magnetic flux in hydraulic networks.
In the block diagram of the system, nodes are introduced which represent different potential points in the system. This is equivalent to hydraulic pressures. Nodes represent points in the system where pressure might have different values. These points represent interconnection between hydraulic components and take into account the fact that each

individual component in the system has an effect on the pressure drop on their two corresponding endpoints. Therefore nodes are placed at all places where the modelling requires it.
In the network, volume flow rate is equivalent to current and the quantity of water has similar representation as charge in an electric circuit. Again, it should be noted that e.g. the water quantity cannot be affected by magnetic fields, therefore the word: similar.
Although nodes can be placed across all the endpoints of each component, some simplifications are introduced. There are two different types of simplification in the network. For a better transparency, these parts of the system are shown in *Figure 4.3* and *Figure 4.3*:

wrong reference, two figures are similar, correction needed!

Figure 4.4. Simplifications: In the left when $\omega = 0$ and in the right one when $\omega \neq 0$

In *Figure 4.3* three components are shown between n_3 and n_{18}. The latter is the node where the WT connects to the system. In this particular case the pump is turned off, however contributes to the pressure drop due to its resistance. The same can be said for the valves, except that they are fully open at all time but they modify the flow. Extra nodes are not introduced between the valves and the pump, instead the series connection is seen as one component. This can be modelled by lumping the resistance of the valve, *Equation: (4.29)*, into the model of the pump, *Equation: (4.31)*, when the rotational speed is zero. Thus the following model yields for the case when $\omega = 0$:

$$\Delta p = \left(\frac{2}{k_{v100}^2} - a_{h2}\right)|q|q \tag{4.40}$$

And the following when $\omega \neq 0$

$$\Delta p = \left(\frac{2}{k_{v100}^2} - a_{h2}\right)|q|q + a_{h1}\omega_r q + a_{h0}{\omega_r}^2 \tag{4.41}$$

The case when $\omega = 0$ applies to components between (n_4-n_8), (n_3-n_{18}) and to (n_5-n_{13}). The case when $\omega \neq 0$ applies to the two main pumps, between (n_1-n_2) and (n_1-n_7). All these subsystems are modelled as described above.
Since the components influence the pressure between the endpoints, as it was mentioned before, they can be represented as electric components. Valves are considered as nonlinear resistors, since the pressure is the quadratic function of the flow and it has a resistance depending on the OD. The model of the pipes is equivalent to a series connection of a linear inductor, a non-linear resistor and a voltage source which stands for the elevation, gravity term. The pumps provide pressure and therefore flow to the system. They can be seen as voltage generators. The WT is a simple capacitor, as it was described previously. The equivalence between the hydraulic and electric system is summarized in *Table: 4.2*.

Hydraulic system	Electrical system
Valve	Nonlinear resistor
Pipe	Linear inductor with a nonlinear drift term
WT	Capacitor
Pressure	Voltage
Flow	Current
Pumps	Voltage source

Table 4.2. Equivalence of electrical and hydraulic network.

4.4 Graph Representation

A graph is a formal mathematical way for representing a network which can be applicable, among others, in engineering or scientific research, mechanical systems, electrical circuits or hydraulic networks [20].

In order to make the modelling of the water distribution network, Graph Theory is used. Each terminal of the network is associated with a node, and the components of the system correspond to edges [21].

4.4.1 Incidence Matrix

The incidence matrix, H, of a graph with n nodes and e edges is defined by $H = [a_{ij}]$. Where the number of rows and columns are defined by the amount of nodes and edges respectively. Additionally, the particular node and edge is denoted with the indices i and j, respectively.

In case of hydraulic networks, edges have a direction which results in a directed incidence matrix:

$$a_{ij} = \begin{cases} 1 & \text{if the } j^{th} \text{ edge is incident out of the } i^{th} \text{ node} \\ -1 & \text{if the } j^{th} \text{ edge is incident into the } i^{th} \text{ node} \\ 0 & \text{otherwise} \end{cases} \tag{4.42}$$

In *Appendix: C.5* the corresponding incidence matrix of the system is shown.

4.4.2 Cycle Matrix

A spanning tree, $T \in \mathcal{G}$ is a subgraph which contains all nodes of $\mathcal{G}$ but has no cycles [22]. In order to obtain the spanning tree it is necessary to remove an edge from each cycle of the graph. The removed edges are called chords. To obtain the number of chords, l, the following applies:

$$l = e - n + 1 \tag{4.43}$$

Adding any additional chord to T, creates one cycle which is called fundamental cycle. A graph is conformed by as many fundamental cycles as number of chords it has [22]. The set of fundamental cycles correspond to the fundamental cycle matrix B, such as the number of rows and columns are defined by the amount of chords and edges, respectively.

The cycle matrix of a directed graph can be expressed with $B = [b_{ij}]$ where i and j denote the specific chords and edges respectively.

$$b_{ij} = \begin{cases} 1 & \text{if the edges } j^{th} \text{ is in the cycle } i^{th} \text{ and the directions match} \\ -1 & \text{if the edges } j^{th} \text{ is in the cycle } i^{th} \text{ and the directions are opposite} \\ 0 & \text{otherwise} \end{cases} \tag{4.44}$$

In *Appendix: C.6* the corresponding incidence matrix of the system is shown.

4.4.3 Kirchhoff's Law

The directed graph of a hydraulic network assigns two variables to each edges: the pressure, $\Delta p_k(t)$, and the flow, $q_k(t)$. These two variables are vectors containing the individual flows through the edges and the pressure across them:

$$\Delta p(t) = \begin{bmatrix} \Delta p_c \\ \Delta p_f \\ \vdots \\ \Delta p_e \end{bmatrix} \text{ and } \quad q(t) = \begin{bmatrix} q_c \\ q_f \\ \vdots \\ q_e \end{bmatrix} \tag{4.45}$$

In order to derive a model of the hydraulic network a set of independent flow variables has to be identified [23]. These flow variables have the characteristic that their values can be set independently from other flows in the network, and coincide with the flows through the chords. Therefore, it has been decided to choose the numbering of the columns of the H and B matrix, such as:

$$H = [H_c \quad H_f] \text{ and } \quad B = [B_c \quad B_f] = [I \quad B_f] \tag{4.46}$$

Where

H_c	and B_c	are the matrices corresponding to the chords
H_f	and B_f	are the matrices corresponding to the spanning tree

Since the edges variables are elements interconnected to form a network, they must obey the law of conservation of mass and pressure [22].

Kirchhoff´s Current Law (KCL) states that the net sum of all the flows leaving and entering the node is zero. Formulating this statement in matrix form:

$$Hq(t) = 0 \tag{4.47}$$

Furthermore, regarding Kirchhoff´s Voltage Law (KVL) it is stated that at any time the net sum of the pressure around a cycle is zero. In terms of matrix form:

$$B\Delta p(t) = 0 \tag{4.48}$$

Where the fundamental loops have a reference direction given by the direction of the chords.

4.5 Model for the Parameter Estimation

Once the corresponding incidence and cycle matrix have been identified and the analogy between hydraulic and electrical circuits has been made, it is possible to derive a model for the hydraulic network.

In the parameter estimation the WT will be discarded from the hydraulic system, in order to facilitate the process of estimation. Consequently, the WT will be isolated from the other components of the system.

In Section *C.5: Incidence Matrix* the form of the incidence matrix has been shown. It is worth mentioning that the last column of the H matrix agrees with the WT edge. Hence, the H matrix can be written as

$$H = [H_1 \quad H_0] \tag{4.49}$$

Where

$H_1 \epsilon \mathbb{R}^{n \times e-1}$	is the H matrix without the edge corresponding to the WT
$H_0 \epsilon \mathbb{R}^{n \times 1}$	is the H matrix with the column corresponding to the WT

In the same way, the fundamental cycle matrix, B, has been structured such as the last column agrees with the WT edge.

$$B = [B_1 \quad B_0] \tag{4.50}$$

Where

$B_1 \epsilon \mathbb{R}^{l \times e-1}$	is the B matrix without the edge corresponding to the WT
$B_0 \epsilon \mathbb{R}^{l \times 1}$	is the B matrix with the column corresponding to the WT

As mentioned above, q is a vector containing all the individual flows, which is re-structured as following

$$q = \begin{bmatrix} q_1 \\ q_0 \end{bmatrix} \tag{4.51}$$

Where

$q_1 \epsilon \mathbb{R}^{e-1 \times 1}$	is the flow through all edges expect for WT
$q_0 \epsilon \mathbb{R}$	is the flow through the WT

The vector containing the pressures at the nodes is also re-structured as

$$p = \begin{bmatrix} p_1 \\ p_0 \end{bmatrix} \tag{4.52}$$

Where

$p_1 \epsilon \mathbb{R}^{n-1 \times 1}$	is the pressure at all the nodes expect for WT
$p_0 \epsilon \mathbb{R}$	is the pressure in the WT

In *Equation: (4.47)* KCL law is applied for a connected directed graph, where it is stated that the sum of all the flows entering into a node must be equal the the sum of all the nodes out of the node.

By choosing an independent set of flows, corresponding to the chords of a spanning tree, the flow through every edge of the hydraulic system can be expressed in terms of the flow through the chords, z [22]. Thus, reducing the number of unknowns in the system. These elements of z are called the free flows of the system and are independent variables [21].

$$q_1 = B_1^T z \tag{4.53}$$

In order to extract the component model into a more generalized form, it is rewritten as a function of flow, q_1, angular velocity, ω, and conductivity factor, k_v as follows:

$$\tilde{f}_i(q_1, \omega, k_v) = \lambda_i(q_1) + \zeta_i + \nu_i(q_1, k_v) - \alpha_i(\omega) \tag{4.54}$$

Where

$\tilde{f}_i = -C_{pi} q_i |q_i|$ if i = 2,3,4,5,6,7,10,11,12,14,17,18,19,21,23

$\tilde{f}_i = -C_{vi} q_i |q_i|$ if i = 13,15,20,22

$\tilde{f}_i = \left(\frac{2}{k_{v100}^2} - a_{h2i}\right)|q_i|q_i + a_{h1i}\omega_i q_i + a_{h0i}{\omega_i}^2$ if i = 1,8,9,16

Fixe structuring error

The following hydraulic network model shows an overall model along with the above considerations. Now recall that the inertia matrix, J, was defined in Section *4.1.1.3: Complete Pipe Model*.

$$\Delta p_1 = J\dot{q}_1 + f(q_1, w, k_v) \tag{4.55}$$

In *Equation: (4.55)* the hydraulic network model is described in terms of the flow through all the nodes. In order to reduce the order of the model and hence, the amount of unknowns *Equation: (4.53)* is applied.

$$\Delta p_1 = J{B_1}^T\dot{z} + f(z, w, k_v) \tag{4.56}$$

Making use of the identity shown in *Equation: (4.48)*, the following is obtained

$$0 = B_1\Delta p_1 = B_1[J{B_1}^T\dot{z} + f(z, w, k_v)] \tag{4.57}$$

Isolating the inertia matrix to the left side

$$-B_1 J{B_1}^T\dot{z} = B_1 f(z, w, k_v) \tag{4.58}$$

It is desired to know the value of the flow through the chords, hence the above equation is solved for $\dot{z}$. Nevertheless, in order to invert $(B_1 J{B_1}^T)$ it has to be nonsingular i.e. invertible.

Setting $\mathcal{J} = B_1 J{B_1}^T$, for $\mathcal{J}$ to be positive-definite it has to be a square matrix and its determinant has to be nonzero. Observe that $\mathcal{J}$ is

$$\mathcal{J} = \begin{pmatrix} I & B_f \end{pmatrix} \begin{pmatrix} J_c & 0 \\ 0 & J_f \end{pmatrix} \begin{pmatrix} I \\ {B_f}^T \end{pmatrix} = J_c + B_f J_f {B_f}^T \tag{4.59}$$

Where

$J_c \in \mathbb{R}^{l\times l}$ is the inertia in the chords components

$J_f \in \mathbb{R}^{f\times f}$ is the inertia in the component of the spanning tree

J_c is the diagonal inertia matrix containing the chords elements, since all the components corresponding to a chord in $\mathcal{G}$ are pipes, all the diagonal term are positive. Thus, $J_c > 0$.

Nevertheless, if there would be at least a chord corresponding to a non-pipe element, *Equation: (4.59)* will continue being positive-definite as long as there is a possibility to create a spanning tree containing all chord as pipes elements from $\mathcal{G}$ [23].

For the remaining term $B_f J_f {B_f}^T$, J_f is non-negative matrix as all its elements are zero or pipe´s inertia. Multiplying $B_f J_f {B_f}^T$ by a non-zero vector column $\mathbf{x}$ and its transpose $\mathbf{x^T}$

$$\mathbf{x^T} B_f J_f {B_f}^T \mathbf{x} \tag{4.60}$$

Creating a new variable $y = {B_f}^T\mathbf{x}$ and applying the positive semi-definite matrix definition [24]

$$y^T J_f y \geqslant 0 \tag{4.61}$$

Thus, *Equation: (4.59)* is positive definite a sufficient condition for $\mathcal{J}$ being invertible.

Therefore, the system model can be written as follows

$$\dot{z} = -(B_1 J {B_1}^T)^{-1} B_1 f(z, w, k_v) \tag{4.62}$$

4.5.1 Model Relations

As a consequence of the system model established above, a new set of relations for the parameter estimation can be obtained. Starting from the component complete model

$$\Delta p_1 = J\dot{q}_1 + f(q_1, w, k_v) \tag{4.63}$$

Recall that previously, in *Equation: (4.53)*, a new state, z, has been defined for the independent flows in the graph

$$\Delta p_1 = J{B_1}^T \dot{z} + f(z, w, k_v) \tag{4.64}$$

The flow rate through the chords is found in *Equation: (4.62)*, thus the expression for Δp_1 can be rewritten as

$$\Delta p_1 = J{B_1}^T[-(B_1 J {B_1}^T)^{-1} B_1 f(z, w, k_v)] + f(z, w, k_v) \tag{4.65}$$

Rewritten it in a shorter form

$$\Delta p_1 = (-J{B_1}^T(B_1 J {B_1}^T)^{-1} B_1 + \mathcal{I}) f(z, w, k_v) \tag{4.66}$$

4.5.2 The Estimation Method

Applying KVL to the hydraulic model, the following expression is obtained. Moreover, the parameter estimation will be carried out for steady-state situation where the inertia, J, will not act. Thus, the term containing the diagonal matrix J can be disregarded from the equation.

$$0 = B_1 \Delta p_1 = B_1[J{B_1}^T \dot{z} + f(z, w, k_v)] = B_1(f(z, w, k_v)) \tag{4.67}$$

The pressure difference in the pumps have been specified as inputs, by reason of those pressures can be obtained from the setup available so they are considered as known parameters. Thereby, in *Equation: (4.67)* the term can be split up as following:

$$0 = f(z, k_v) + f(z, w) \tag{4.68}$$

The term $f(z, w)$ is set as input, it is renamed as U and it represents the pressure difference in the 4 pumps acting on the system. Therefore, *Equation: (4.68)* is considered as the input equation where the flow through the chords and the friction parameter of the pipes are unknown.

An output equation is defined, which represents the pressure difference known from the system setup. In this way, the output equation can be compared to the data measured in the setup and proceed to estimate the unknown parameters.

From the system setup 8 different relative pressures can be measured, following *Figure C.2* notation the sensors are placed in: n_2 n_4 n_5 n_7 n_{10} n_{11} n_{15} n_{16}.

In order to compare the measurements from the system setup and the data obtained from the simulation in Matlab, a reference point has to be set in the simulation to calculate the desired data.

The atmospheric is set as reference point and the pressures obtained from the simulation are dependant on it. The relationship between pressures, where DpCXX describes the pressure difference for the XX component, can be defined as:

Node 2

$$DpC2 = y_1 \tag{4.69}$$

Node 7

$$DpC16 = y_2 \tag{4.70}$$

Node 4

$$DpC18 + DpC19 + DpC23 + DpC24 = y_3 \tag{4.71}$$

Node 5

$$DpC25 + DpC26 + DpC30 + DpC31 = y_4 \tag{4.72}$$

Node 10

$$DpC24 = y_5 \tag{4.73}$$

Node 11

$$DpC20 + DpC21 = y_6 \tag{4.74}$$

Node 15

$$DpC31 = y_7 \tag{4.75}$$

Node 16

$$DpC28 + DpC27 = y_8 \tag{4.76}$$

With the 8 equations depicted above the output vector y is obtained.

Part II

Control Design

Controller 5

In this chapter the design of the controller is explained. Furthermore the optimization controller is designed and this is implemented in simulink.

5.1 Control Problem

The water distribution system explained in Section *2.1: System Overview* need to be controlled according the Section *3: Requirements and Constraints*. The requirements can be summarized as:

- Pressure at CP, $0.08 < p_{cp} < 0.14$ [bar]
- Minimum water exchange , $\bar{q}_{wt} > 0.06$ $\left[\frac{m^3}{day}\right]$
- Minimizing the total cost of running the system

The pressure at a given CP can be controlled by the water level in the WT which is controlled by the rotational speed of the pumps. To fulfill the requirement of a minimum pressure at a given CP the water level in the WT have to be within a operations area. A controller have to be develop that takes the pressure actuators into account.

The flow through the WT is controlled by pressure in the outer ring. Whenever the pressure is higher in the outer ring than in the WT, water is being pump into the WT. Furthermore the average flow rate, $\bar{q}_{wt}$, through the WT must meet the minimum requirement to insure the water quality. This can be seen as a constrain of the operation area of the system.

At the same time the total cost of running the system should be minimized. Therefor a cost function is needed. This cost function purpose is to find the optimal control signal which minimize the cost of running the pumps. Thereby spending the least money on running the total system.

Considering both the cost function and the constrain, this leads to a description of the systems operate area, $C_T(\Delta p_i, q_i)$ wherein the system must operate. By considering the total cost of running, C_t this can be seen as a minimization problem:

$$\min_{u,\,q} C_T(\Delta p_i, q_i) = \min_{u,\,q} \sum_{i=1}^{N} C_T(\Delta p_i, q_i) \tag{5.1}$$

s.t

$$\begin{aligned} 0.08 < p_{cp} &< 0.14 \\ \bar{q}_{wt} &> 0.06 \end{aligned}$$

Where

	$C_T(\Delta p_i, q_i)$	is the power consumption of the i^{th} pump,	[W]
	p_{cp}	is the pressure at a given CP,	[Bar]
and	$\bar{q}_{wt}$	is the average flow rate through the WT.	$\left[\frac{\mathrm{m}^3}{\mathrm{day}}\right]$

5.2 Pressure Control

5.3 Minimization Problem

In this section the cost function for the minimization problem and the constraints is explained. Furthermore the optimization controller is designed.

6 Implementation of controller

This chapter will explain how the controller designed in Chapter *5: Controller* is implemented in MATLAB simulink.

Part III

Conclusion and verification

7 Accepttest

8 Discussion

9 Conclusion

Part IV

Appendices

A Unit Conversion

Due to the large difference between the SI-units of flow, [m^3/s], and pressure, [Pa], a conversion from seconds to hours and pascal to bar is made.

The final pipe model from *Equation: (4.21)*, is shown below.

$$\begin{aligned}\frac{L\rho}{A}\frac{dq}{dt} &= \Delta p - \frac{8fL}{\pi^2 gD^5}\rho g|q|q - k_f\frac{8}{\pi^2 gD^4}\rho g|q|q - \Delta z\rho g \\ &= \Delta p - (\frac{8fL}{\pi^2 gD^5} + k_f\frac{8}{\pi^2 gD^4})\rho g|q|q - \Delta z\rho g\end{aligned} \tag{A.1}$$

1[bar] = 10^5[Pa]. Therefore we can rewrite *Equation: (A.1)* to:

$$\begin{aligned}\frac{L\rho}{A\cdot 10^5}\frac{dq}{dt} &= \Delta\frac{p}{10^5} - (\frac{8fL}{\pi^2 gD^5\cdot 10^5} + k_f\frac{8}{\pi^2 gD^4\cdot 10^5})\rho g|q|q - \frac{\Delta z\rho g}{10^5} \\ \frac{L\rho}{A\cdot 10^5}\frac{dq}{dt} &= \Delta p_{bar} - (\frac{8fL}{\pi^2 gD^5\cdot 10^5} + k_f\frac{8}{\pi^2 gD^4\cdot 10^5})\rho g|q|q - \frac{\Delta z\rho g}{10^5}\end{aligned} \tag{A.2}$$

The conversion from $[\frac{m^3}{s}]$ to $[\frac{m^3}{h}]$ is $\frac{m^3}{s}3600 = \frac{m^3}{s}$. *Equation: (A.1)* can be written as:

$$\frac{L\rho}{A\cdot 10^5}\frac{d}{dt}\frac{q}{3600} = \Delta\frac{p}{10^5} - (\frac{8fL}{\pi^2 gD^5\cdot 10^5} + k_f\frac{8}{\pi^2 gD^4\cdot 10^5})\rho g\frac{|q|}{3600}\frac{q}{3600} - \frac{\Delta z\rho g}{10^5} \tag{A.3}$$

Assumption List B

Number	Assumptions	Section reference
1	The fluid in the network is water.	Section *4.1.1: Pipe Model*
2	All pipes in the system are filled up fully with water at all time.	Section *4.1.1: Pipe Model*
3	The pipes have a cylindrical structure and the cross section, $A(x)$, is constant for every $x \in [0, L]$.	Section *4.1.1: Pipe Model*
4	The flow of water is uniformly distributed along the cross sectional area of the pipe and the flow is turbulent.	Section *4.1.1: Pipe Model*
5	Δz, the change in elevation only occurs in pipes.	Section *4.1.2: Valve Model*
6	The pumps in the network are centrifugal pumps.	Section *4.31: Pump Model*
7	The storage of the WT has a constant diameter. In other words, the walls of the WT are vertical.	Section *4.1.4: Water Tower*
8	Valves in the water distribution system are modelled according to the assumption that the length,L, is zero.	Section *4.1.2: Valve Model*

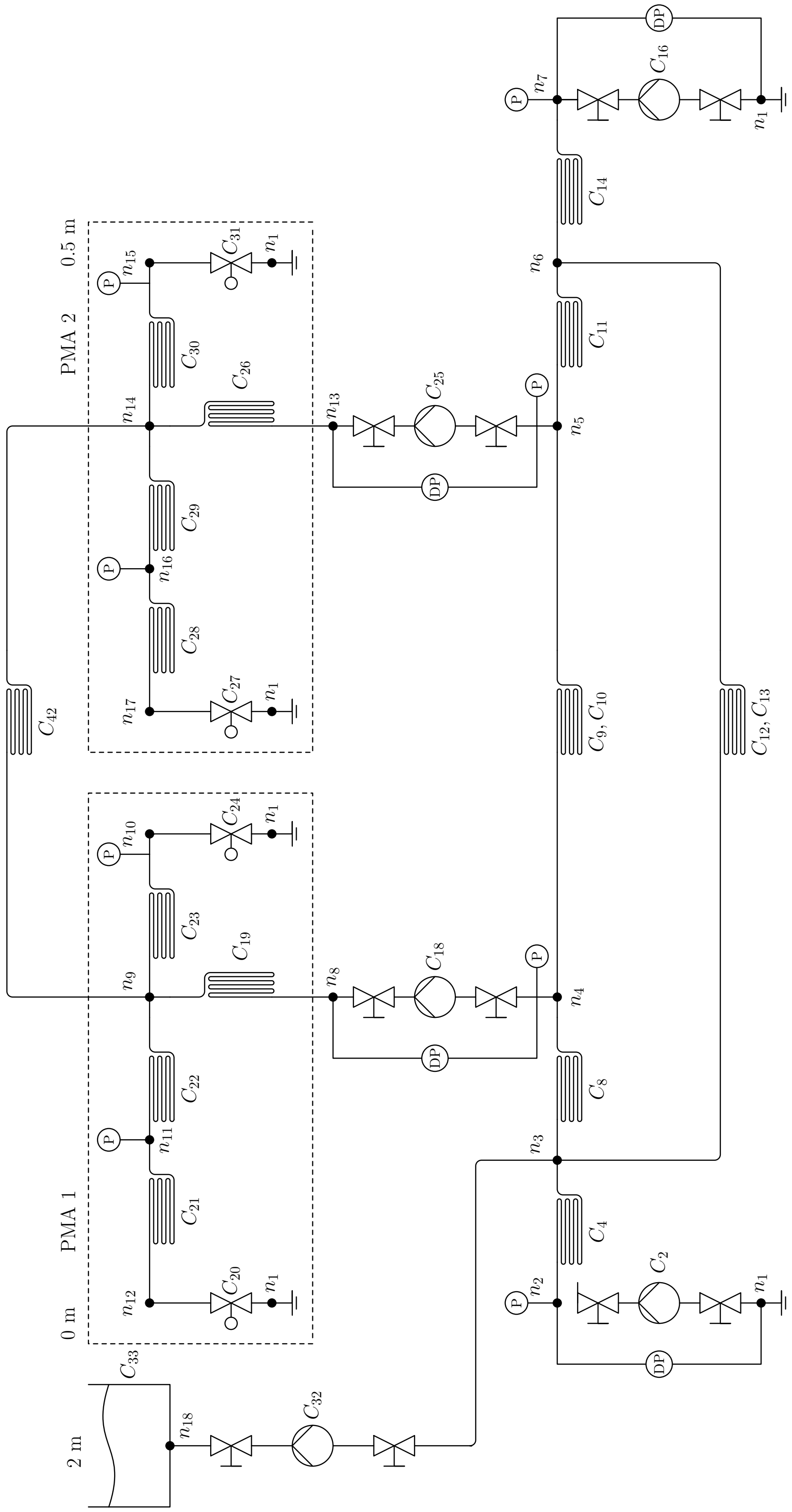
Table B.1. List of assumptions

System Description C

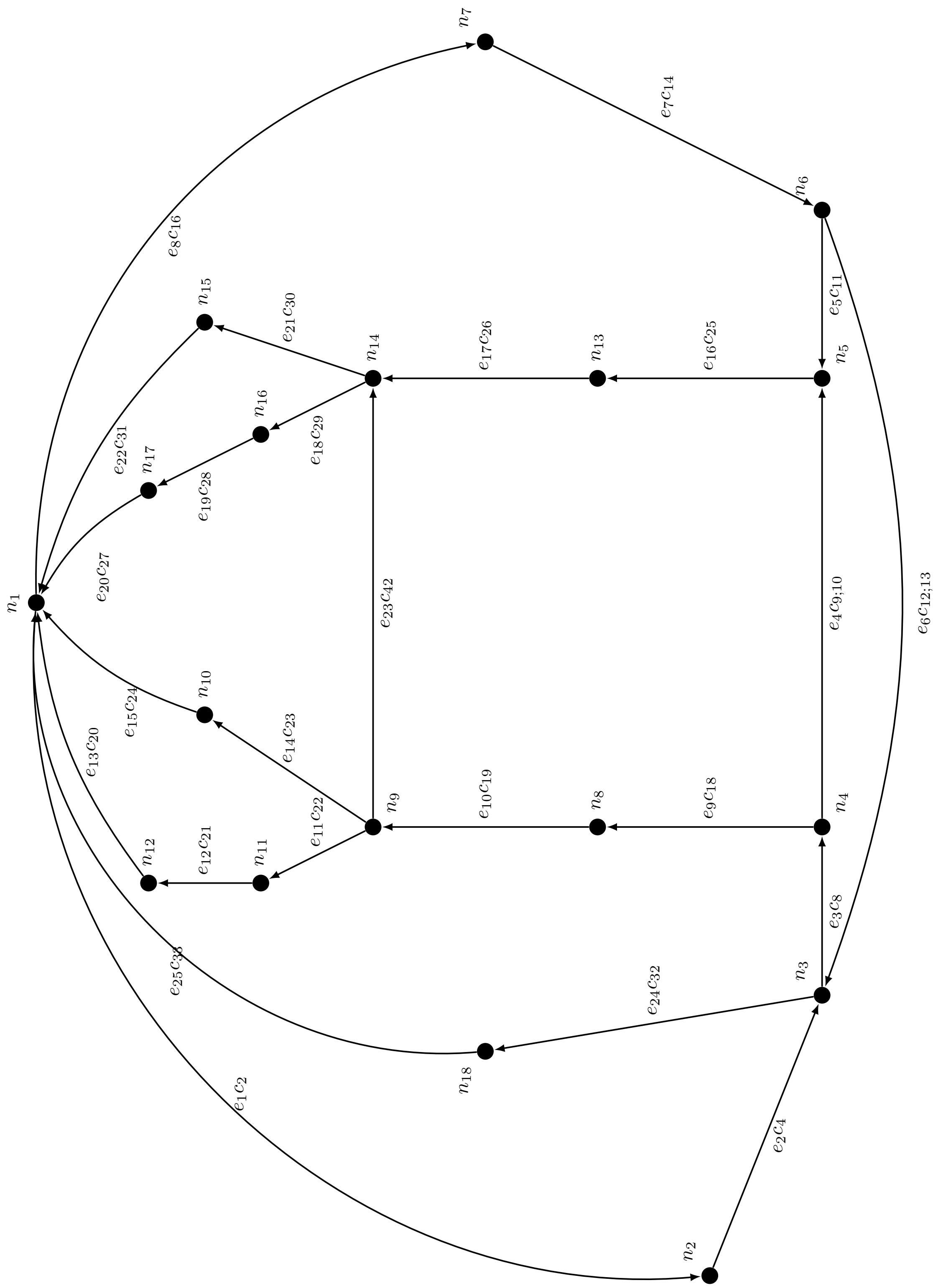
C.1 Components of the System

Here all necessary data(from the datasheets) and notations should be listed about the components (pipes, pumps, valves.. etc)

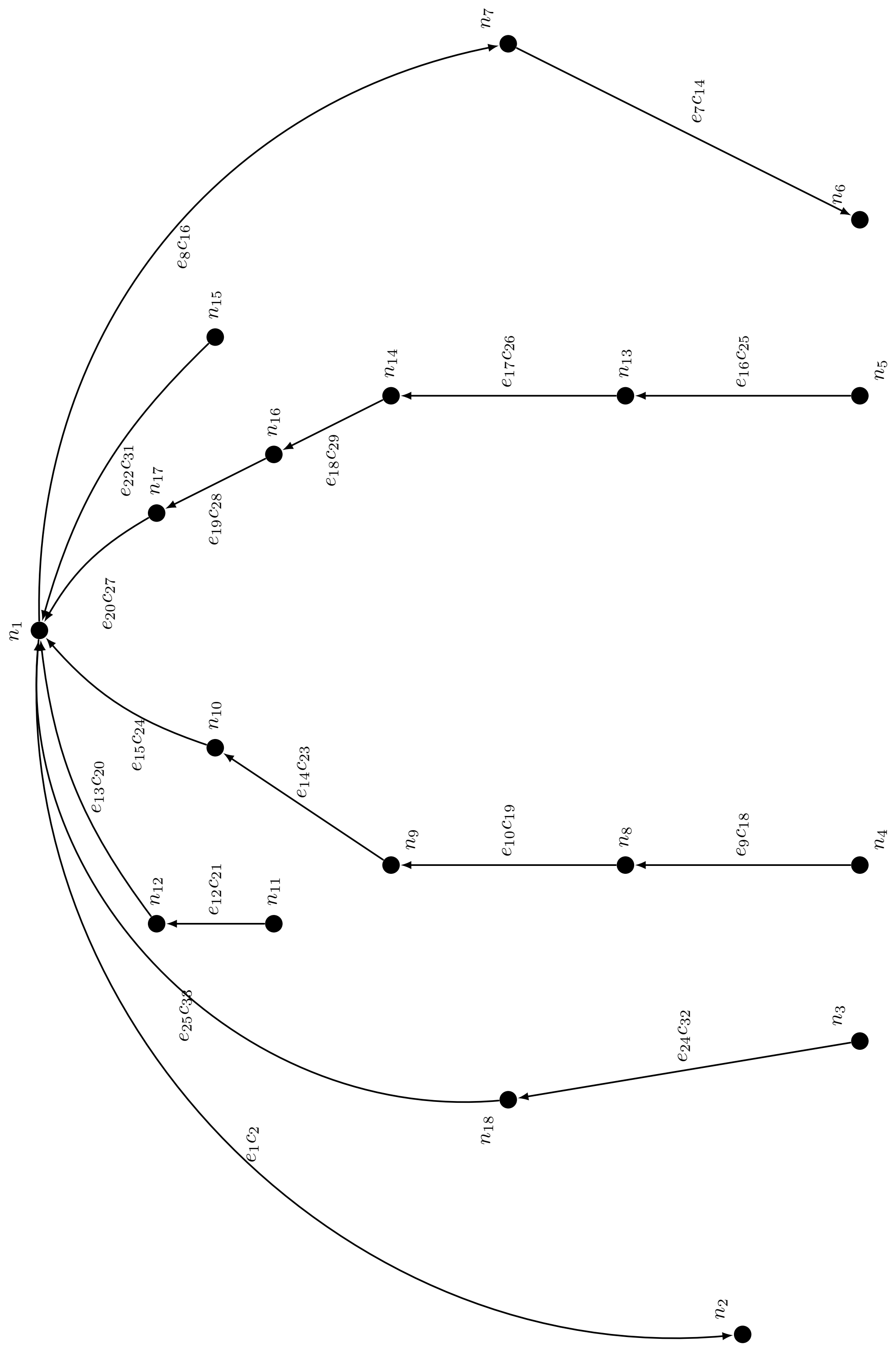
C.2 System Topology



C.3 System Graph



C.4 Spanning Tree



C.5 Incidence Matrix

C.6 Cycle Matrix

$$H = \left[\begin{array}{c|ccccccccccccccccccccccccc}
18x24 & e2 & e3 & e4 & e5 & e6 & e11 & e21 & e23 & e1 & e7 & e8 & e9 & e10 & e12 & e13 & e14 & e15 & e16 & e17 & e18 & e19 & e20 & e22 & e24 & e25 \\
n1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 \\
n2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n3 & -1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
n4 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n5 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n6 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n9 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n11 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
n13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
n14 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
n15 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
n16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
n17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
n18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{array}\right] \tag{C.1}$$

$$B = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0
\end{bmatrix} \tag{C.2}$$

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Rettelser

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