A Parametric Study of Opinion Progression in a Divided Society

Farshad Salimi Naneh Karan and Subhadeep Chakraborty

Department of Mechanical, Aerospace, and Biomedical Engineering,
University of Tennessee, Knoxville, Tennessee

Abstract. In this paper, a probabilistic finite state automaton framework is used to model the temporal evolution of opinions of individuals in an ideologically divided society in the presence of social interactions and influencers. In such a society, even quantifiable and verifiable facts are not unqualified absolute but are only viewed through the prism of the individual's biases which are almost always strongly aligned with one of the few prominent actors' viewpoint. The gradual progression of divisiveness and clustering of opinion or formation of consensus in a scale free network is studied within the framework of bounded-confidence interaction between nodes. Monte Carlo simulations were conducted to study the effect of different model parameters, such as the initial distribution of opinion, confidence bound, etc. in the behavior of the society. We have shown that in absence of influencers, government policies are the important factors in the final distribution of the society unless a specific group has higher number of members initially. Also, even very small groups of influencers proved to be highly effective in changing the dynamics.

Keywords: Decision making, Opinion dynamics, Influence

1 Introduction

The field of social psychology has a rich history of studies in social influence and group behavior. Such work generally investigates a number of disparate motives and contextual factors to explain individual-level and group behavior, traditionally in small-scale laboratory settings, but increasingly motivated by data mined from online resources, such as social media. In fact, social media, with its always-on persistence and open infrastructural base has quickly developed into a platform for news-storytelling, collaborative filtering and curating of news [1]. Twitter has enabled non-elites to emerge as gatekeepers of information within networked, crowdsourced environments, but at the same time, has provided a forum for conflict, confrontation and polarization. This has never been so relevant as now and particularly in the political scene in the US in the pre-and post-time period of the 2016 presidential election.

While the extremity of the current political rhetoric may feel unprecedented, confrontation has always focused on the singular goal of winning over the other side. As an entire population become consumed by this mindset, we reenact partisan patterns of conflict that may comfort our fears, promote strong clustering between like-minded people, but undermine cooperation across society and the chance of ever achieving unity and consensus.

Studies suggest that the operation of homophily - the tendency to follow like-minded individuals and to shun those with opposing opinions -is strongly prevalent in social media applications [2]. Furthermore, shared geo-locality and communal bonds are strengthened via Twitter posts, permitting forms of "peripheral awareness and ambient community" [3]. To model these findings, several non-linear interaction models among individuals, have been studied which illustrate polarized decision, the self-organization of behavioral conventions, and the transition from individual to mass behavior. In one such study by Shutters [4], the cultural polarization phenomenon has been studied on different network structures in the presence of extremists in the framework of bounded confidence. In this study, the change of opinion after each interaction is guaranteed for non-extremists, rendering the imitation and simulation of human decision making rather unrealistic.

This paper is an attempt to construct a parametric study that can address at least a portion of these ideas within a mathematically tractable agent-based modeling (ABM) framework. The nodes in an extended Barabási-Albert (BA-extended) network are modeled as rational actors, their choices and logical mechanism are related to the historically dominant expected utility family of theories made popular by von Neumann and Morgenstern [5]. The heart of the theory, sometimes called the rational expectations principle [6], proposes that each alternative course of action or choice should be evaluated by weighting its global expected satisfaction-dissatisfaction with the probabilities that the component consequences will occur and be experienced. The Probabilistic Finite State Automata (PFSA) based discrete choice model, proposed and studied in [7] has been modified with one key difference.

The scenario under discussion and this key difference are explained in the next sections. First, the PFSA framework and its assumptions are briefly explained. Next, we discuss how the Bounded Confidence interaction model is applied in the simualtions. Then the results of simulations for different scenarios are presented and discussed. In the end, all findings of this paper are summarized and concluded.

2 Simulation Setup

2.1 Probabilistic Finite state Automata

In this paper, a Probabilistic Finite State Automata (PFSA) framework is used to represent the decision making routine. In this framework, it is assumed that every agent has the same *finite set of discrete choices (or states)* at each time instant. Also, we assume that agents subscribe to the normative perspective of the group they belong to; social norms of groups can differ from each other, but inside a group the same social norm is shared with everybody. Moreover, it is assumed that, even when presented with the exact same choices with the

States	Description	Events	Description
I	State of being undecided/neutral	g	A popular act by the government
R	State of supporting the Revolution	$\tilde{g}$	An unpopular government act
G	State of supporting the Government	ε	An internal decision
A	State of political advantage	s	Success of the revolution
D	State of political disadvantage	f	Failure of the revolution

Table 1: List of PFSA States and Events

exact same pay-offs, different individuals, and possibly even the same individual, may probabilistically make alternate decisions. Furthermore, it is assumed that there are two types of events, *external/global* and *internal*; when a global event happens all the population is affected, however, an internal event is similar to an individual's personal choice. A complete description of the mentioned assumptions can be found in [7].

The assumption of *normative perspective* allows rational behavior, to be encoded as a PFSA. In our simplified depiction of the situation, each individual faces the *internal* decision of supporting the existing government, supporting the rebelling group, or remaining in a state of indecision. Additionally, the individual can reach a state of political advantage, or disadvantage, but the uncontrollable transition to these two states can only occur through an *external* event, namely, the success or failure of the revolution. The five PFSA states and events are described in table 1.

Assumption. Different normative perspective for different groups
In this study, it is assumed that the society is divided in three groups: the Independent group, the group with preexisting bias towards Revolution (R-Leaning), and the group biased towards the government (G-Leaning). The way each group perceives the actions of the government is different. Members of the Independent group perceive the popular actions as good and the unpopular actions as bad. However, members of the R-Leaning group perceive all actions of the government as bad. On the other hand, members of the G-Leaning group perceive all actions of the government as good. This assumption results in having three different normative perspectives in the population, Fig. 1.

Figure 1a gives a schematic of the assumed normative perspective encoded as a PFSA for the Independent group . It may be noticed that transitions such as $g : G \rightarrow R$ or $g : I \rightarrow R$ are unauthorized, since it is assumed that a favorable act by the government should not make anyone decide to join the opposing group. Also, the same event can cause alternate transitions from the same state; the actual transition will depend probabilistically on the measure of attractiveness of the possible target states. In Fig. 1b, all actions of the government are clubbed together and recognized as good for the G-Leaning group. The elimination of some transitions between states in Figs. 1b and c is rooted in the biased views that the G-Leaning and R-Leaning groups have. For example, the only way the transition $G \rightarrow R$ can be authorized is when an unpopular action by the government, $\tilde{g}$, happens. Since unpopular actions are perceived as good by the G-Leaning group, $G \rightarrow R$ can never happen for this group and must be eliminated.

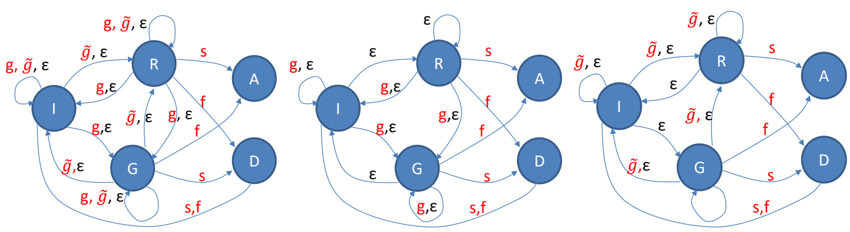
In the PFSA framework, the probability of transitioning to a different state is dependent on the reachability of that state from the current state, the current event (external or internal), and also the relative degree of attractiveness of the target state. The state attractiveness measure is calculated using the concept of positive real measure attributed to a string of events [8]. It depends on the reward from each state (χ), the state transition matrix (Π), and the distribution of states ($\bar{v}_i$). A real measure ν_θ^i for state i is defined as

$$\nu_\theta^i = \sum_{\tau=0}^{\infty} \theta \left(1-\theta\right)^\tau \bar{v}^i \Pi^\tau \bar{\chi} \tag{1}$$

where $\theta \in (0, 1]$ is a user-specified parameter. Mathematical structure of the mentioned parameters are available in [7].

It should be noted at this point that the premise for these assumptions are the authors' hypotheses and conjectures based on observations from the political sphere, but these are as of now unsubstantiated by studies or data analytics. The basic observation is that in response to the SAME political event two groups of people respond in diametrically opposite manner (strongly in favor, or strongly against) - each group with the same passion and conviction that they are correct. Thus, although within their own logical construct, the two groups are not dissimilar, but the perturbation that drives the two groups manifest itself in two completely different ways - to the G-leaning group each action by the government seems perfect, while to the R-leaning group, the same actions are detestable.

The other interesting dynamic in these composite groups is the possibility of gradual drift of opinion due to interaction and inter and intra group communications. It seems that the KH bounded confidence model of interactions is appropriate since this model predicates that two nodes only communicate if they are not too dissimilar. This is described next.



(a) Independent (b) G-Leaning (c) R-Leaning

Fig. 1: *Normative perspective of different groups*

Number of vertices	100
Number of initial disconnected nodes	3
Number of added/rewired edges at a time	2
Probability to add new lines	0.3333
Probability to rewire edges	0.33335

Table 2: List of parameters used for BA scale-free network

2.2 Bounded Confidence Dynamic

In the Krause-Hegselmann (KH) bounded confidence model, an agent is chosen at random; then, the agent interacts with its compatible neighbors. Compatibility between two nodes is determined by the distance between the current opinions held by the two nodes. The procedure is repeated by selecting another agent randomly and so on. The type of final configuration reached by the system depends on the value of the confidence bound d. In this paper, it is assumed that the interaction is entirely through the characteristic function χ of the states. The update rule for the reward vector of agent i, due to interactions with its neighbors is as follows:

$$\bar{\chi}^i_{t+1} = \bar{\chi}^i_t + \mu.(\bar{\chi}^i_{t_{neighbors}} - \bar{\chi}^i_t) \tag{2}$$

where $\bar{\chi}^i_{t_{neighbors}}$ is the *mean reward vector* of the first-order neighbors in the network of agent i at time step t. The averaging process is used to combat the the minor fluctuations in the local reward vector [9]. Here μ (or the convergence parameter) is the weight which determines how much an agent is influenced by the other one.

Since many networks in the real-world are conjectured to be scale-free, including the World Wide Web, biological networks, and social networks, in this study, a BA extended model network created by the Pajek software program is used [10]. Table 2 presents the parameters of the network. In addition, one of the experiments is repeated on a complete graph topology for comparing the results related to different networks.

Influence Model: The influencers are treated as indistinguishable except for the fact that they never update or change their $\bar{\chi}$ values; moreover, they do not make decisions, and stay in the same state of mind during the entire simulation. Also, it is typical that the influencers are serving a certain agenda, in this case, trying to mobilize forces to join the Revolution. But, this influence is exerted very passively, by advertising a higher value for $\chi(R)$ and lower value for all other states.

$$\chi^I(q_j) = \begin{cases} \chi_m(q_j) - \Delta & \text{if } j = 1, 3, 4, 5, \\ \chi_m(q_j) + \Delta & \text{if } j = 2. \end{cases} \tag{3}$$

in which $\chi^I(q_j)$ represents the reward associated with state q_j for influence nodes, and $\bar{\chi}_m$ is an estimate of the reward values expressed by the whole society on an average. Δ is a parameter adjusting the strength of influencers (control input).

Simulation Process: A population of 100 people are divided in three groups with specific ratios (G_i, R_i, I_i where $G_i + R_i + I_i = 1$). Each group is initialized and given the respective normative perspective. All agents are assigned a random number drawn from a uniform distribution $U(0, 1)$, representing the time remaining before that person makes a decision. This imposes an ordering on the list of people in the network. As soon as someone makes a decision, the time to his next decision, drawn from $U(0, 1)$, is assigned and the list is updated. Additionally, external events g and $\tilde{g}$ are also associated with a random time drawn from $U(a, b)$.

At the time epoch t_k, when it is the i^{th} person's turn to make a decision, he updates his personal estimate of the reward vector according to the update equation (Eqn. 2). He then calculates the degree of attractiveness of the states based on the normalized measure, using Eqn. 1. The only difference in the case of an external event such as $g, \tilde{g}, s$ or f is that everyone simultaneously updates their states rather than asynchronously, as in the case of internal events. Each simulation is run 50 times and the average of all the runs is analyzed.

3 Simulation Scenarios and Results

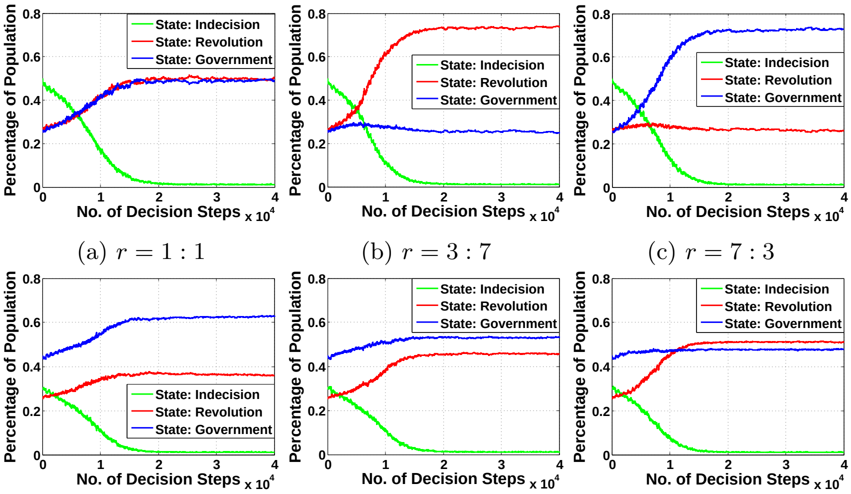
3.1 Effect of Global Events

This experiment studies the effect of the ratio of good to bad external events, or equivalently $r = \frac{P(g)}{P(\tilde{g})}$, on the opinion dynamic of the population. First, we consider equal initial distributions for G-Leaning and R-Leaning groups ($G_i = R_i = 0.25$). The first row of Fig. 2 provides the results of this type of initial distribution. In all three cases, as soon as events happen, members of the Independent group change their opinions.

In Fig. 2a, the same probability of good and bad external events causes the Independent group to equally divide between the G-Leaning and R-Leaning groups. However, in Fig. 2b, because of the higher probability of bad events, people from the Independent group lean towards the Revolution group. The exact same reasoning can be used for Fig. 2c where good actions by the government outnumber the bad ones causing people from the Independent group to support the Government. It can be concluded that in the absence of influencers, for equal initial distribution, r is the determining factor of the state distribution of the population at equilibrium.

In a second set of experiments, we consider unequal initial distributions for G-Leaning and R-Leaning groups ($G_i = 0.45, R_i = 0.25$). Similar to the previous experiment, when equal number of good and bad events happen, the Independent group leans towards either groups somewhat equally, Fig. 2d.

In Fig. 2e, when bad events slightly outnumber the good events, the Revolution state starts to increase but it is not the dominant opinion of the population as it was in Fig. 2b. The reason is that a high percentage of the population



(a) $r = 1 : 1$ (b) $r = 3 : 7$ (c) $r = 7 : 3$

(d) $r = 1 : 1$ (e) $r = 4 : 6$ (f) $r = 3 : 7$

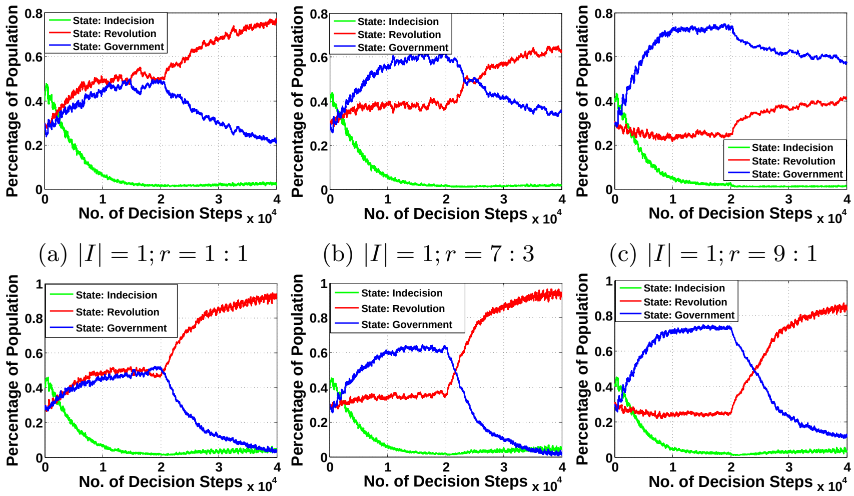
Fig. 2: *Effect of external events on population opinion without influence group*

is initially in the G-Leaning group. As a result, Independent members have a higher chance of interacting with G-Leaning members in their network, and consequently, changing their opinions to G. Nonetheless, when bad policies outnumber the good policies significantly, the Revolution state rises to the top and becomes the dominant opinion of the population, Fig. 2f. The slight increase in the G state in the beginning of the simulation is because of this phenomenon. In conclusion, in absence of influencers, for unequal initial distribution both initial distribution and r are important in the steady state opinion distribution of the population.

3.2 Effect of Influencers

This experiment investigates the effects of presence of Influencers and their quantity on the steady state behavior of the population. In order to specifically observe the effect of Influencers, they are activated at decision step 10000, and they are biased towards the Revolution state. The influencers randomly choose people to form a links with, where the probability of forming a link is 0.25. Figure 3 presents the results of this experiment for $G_i = R_i = 0.3$. In the first experiment, only one Influencer is available in the society, Figs. 3a, b, c.

When $r = 1 : 1$, as discussed earlier, Independent members start joining the R or G state equally, Fig. 3a. However, as soon as the Influencer is activated, the percentage of people in the R state starts rising drastically because of the interactions which happen among the population. The interesting point is that there is a slight increase in the state of Indecision too. The reason can be found in the normative perspective of the G-Leaning group, Fig. 1b. As a result of



(a) $|I| = 1; r = 1:1$ (b) $|I| = 1; r = 7:3$ (c) $|I| = 1; r = 9:1$

(d) $|I| = 5; r = 1:1$ (e) $|I| = 5; r = 7:3$ (f) $|I| = 5; r = 9:1$

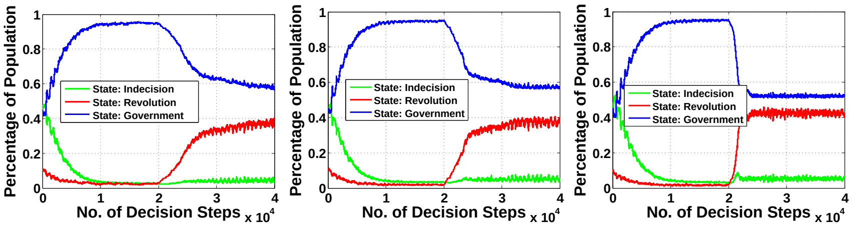
Fig. 3: *Effect of external events on population opinion in presence of Influencers*

continuous interactions with influencers, even a G-Leaning member might change his opinion to R, and the only path he can change his opinion from the G state is through state I. This causes a slight increase in the number of agents in the state of Indecision.

As the number of popular acts by the government increases, in the first phase of the simulation, the Independent members join the G state making it the dominant opinion of the whole population, Fig. 3b. However, the presence of the influencer affects peoples opinions through interactions causing the R state to be the dominant opinion of the society although there are more popular acts by the government. In this scenario, higher probability of the good actions just makes the transition to the R state slower. The same reasoning is applicable for Fig. 3c, with the only exception that presence of one influencer is not able to overcome the effect of significantly higher probability of the popular acts, to destabilize the society.

In another set of experiments, higher number of influencers were added to the population, Figs. 3d, e, f. It is observed that with more influencers present, the transition is faster, and almost all the populations joins the R state. Also, higher number of popular act by the government cannot prevent the destabilization of the society. This raises the question that "Can high number of Influencers guide the population towards destabilization under any condition?"

To answer this question, we consider an extreme case where the initial distribution is highly in favor of the G-Leaning group ($G_i = 0.45, R_i = 0.1$), and the probability of popular actions by the government is significantly higher. Figure 4 represents the results of simulation for such a case with varying number of influencers. As number of Influencers increases, the percentage of population



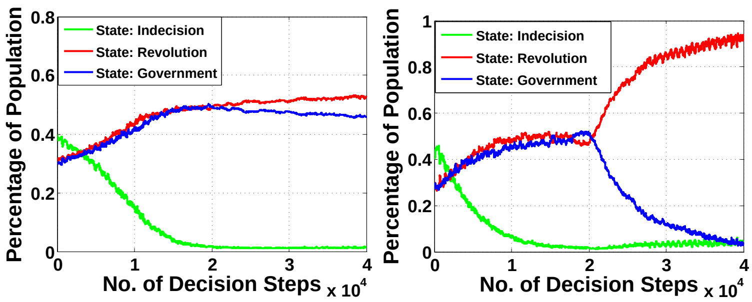
(a) $|I| = 5$ (b) $|I| = 10$ (c) $|I| = 50$

Fig. 4: *Effect of number of influencers on a G-dominant society with* $r = 9 : 1$

in the R state increases. Moreover, this transition is faster. However, the Influencers are not able to dominate the majority opinion. Both the dominant initial G-Leaning distribution and the high number of popular acts by the government cause this behavior. So, a large group of influencers is not a guarantee for guiding a population towards a predetermined state under all conditions.

3.3 Effect of Distance Parameter (d)

Influencers deliberately advertise biased reward values in an attempt to pull the population slowly towards the state of their choice (R, in this study). Nonetheless, they would be successful in doing so if they are reachable for agents in the society. The parameter which controls reachability is the distance parameter d. In this section, we study the effect of the distance parameter on the steady state behavior of the population. Figure 5 presents the behavior of a society with two values of d. In Fig. 5a, because of the low distance parameter, most of the agents are not able to interact with Influencers. As a result the change in the population behavior is not significant. However, in Fig. 5b, the distance parameter is higher, more agents are affected by the Influencers as a result of interacting with them, and the change in the behavior of the society is significant.



(a) $d = 0.5$ (b) $d = 1$

Fig. 5: *Effect of distance parameter,* $|I| = 5$, $r = 1 : 1$

4 Conclusion and Future Work

This paper studies the temporal evolution of opinions of individuals in an ideologically divided society by incorporation a PFSA framework along with the KH bounded confidence model. Three ideological groups called Independent, G-Leaning and R-Leaning, form a population in which people are connected. Indistinguishable influencers are also present in some experiments. There are two motives for individuals to change their decisions: popular/unpopular acts by the government, and interactions between people.

Results show that, in absence of influencers, ratio is the determining parameter in the equilibrium state of the population unless one of the groups includes a significantly higher number of members. The results also reveal that although very small in number, influencers are capable of creating drastic changes in the opinion dynamic of the society. Higher number of influencers result in faster transitions and attracting more people to the target state. It is shown that in the presence of a major group, there are situations where influencers, although very high in number, are not able to push the population's opinion towards the opinion of the minor group. Finally, it is presented that with a low distance parameter, the societies behavior is not greatly affected by the influencers.

References

1. E. Schonfeld, "Costolo: Twitter now has 190 million users tweeting 65 million times a day," *Techcrunch. June*, vol. 8, 2010.
2. L. A. Adamic and N. Glance, "The political blogosphere and the 2004 US election: divided they blog," in *Proceedings of the 3rd international workshop on Link discovery.* ACM, 2005, pp. 36–43.
3. I. Erickson, "Geography and community: New forms of interaction among people and places," *American Behavioral Scientist*, vol. 53, no. 8, pp. 1194–1207, 2010.
4. S. T. Shutters, "Cultural polarization and the role of extremist agents: A simple simulation model," in *International Conference on Social Computing, Behavioral-Cultural Modeling, and Prediction.* Springer, 2013, pp. 93–101.
5. J. v. Neumann, O. Morgenstern *et al.*, "Theory of games and economic behavior," 1944.
6. M. Fishbein and I. Ajzen, "Belief, attitude, intention, and behavior: An introduction to theory and research," 1977.
7. S. Chakraborty and M. M. Mench, "Socio-cultural evolution of opinion dynamics in networked societies," in *International Conference on Social Computing, Behavioral-Cultural Modeling, and Prediction.* Springer, 2012, pp. 78–86.
8. A. Ray*, "Signed real measure of regular languages for discrete event supervisory control," *International Journal of Control*, vol. 78, no. 12, pp. 949–967, 2005.
9. S. Boyd, A. Ghosh, B. Prabhakar, and D. Shah, "Randomized gossip algorithms," *IEEE/ACM Transactions on Networking (TON)*, vol. 14, no. SI, pp. 2508–2530, 2006.
10. V. Batagelj and A. Mrvar, "Pajek: Program for analysis and visualization of large networks," *Timeshift-The World in Twenty-Five Years: Ars Electronica*, pp. 242–251, 2004.