

Digital Cash

Cryptography – fall semester 2024,2025











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Sajjad Ranjbar Yazdi

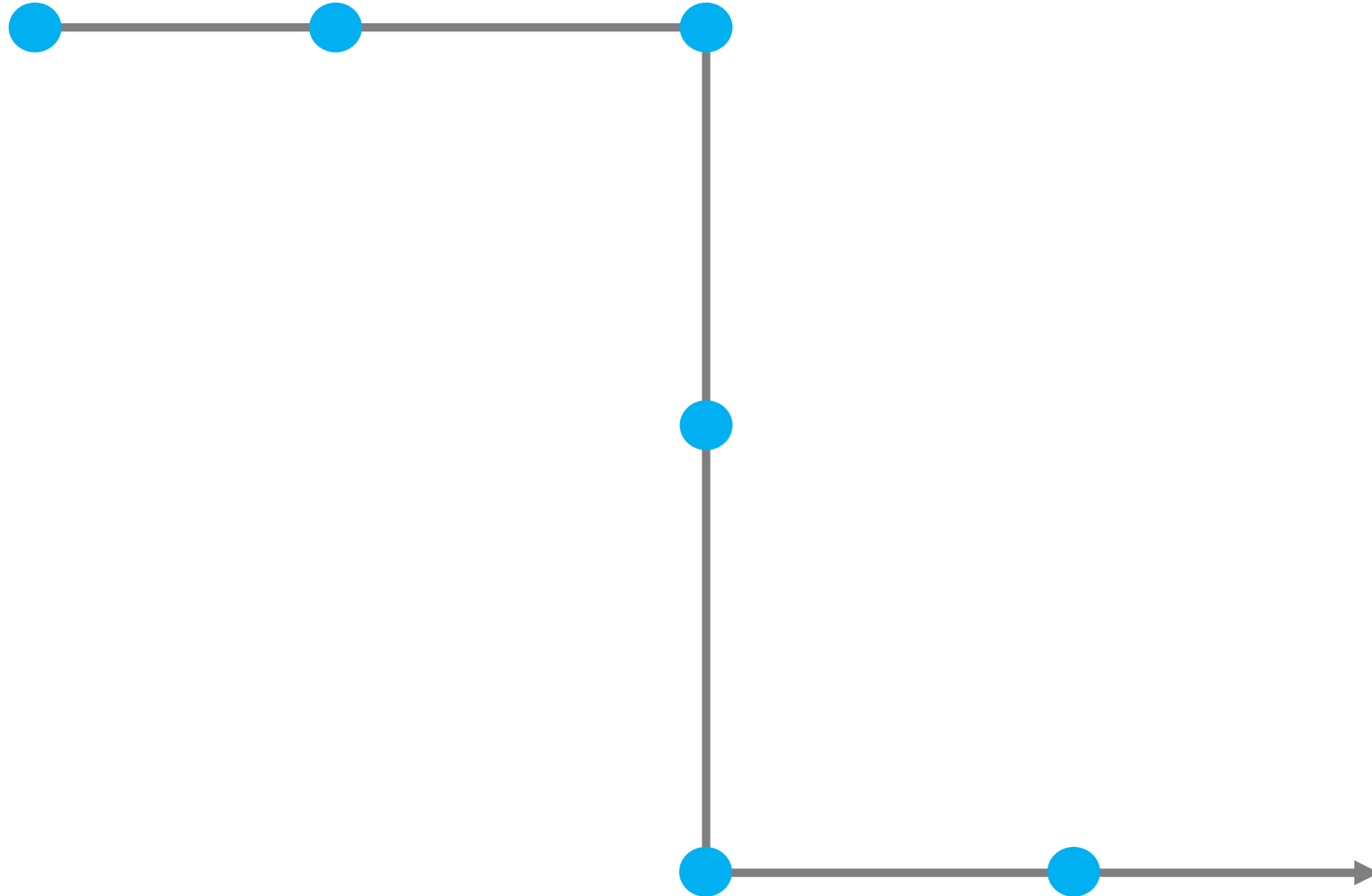


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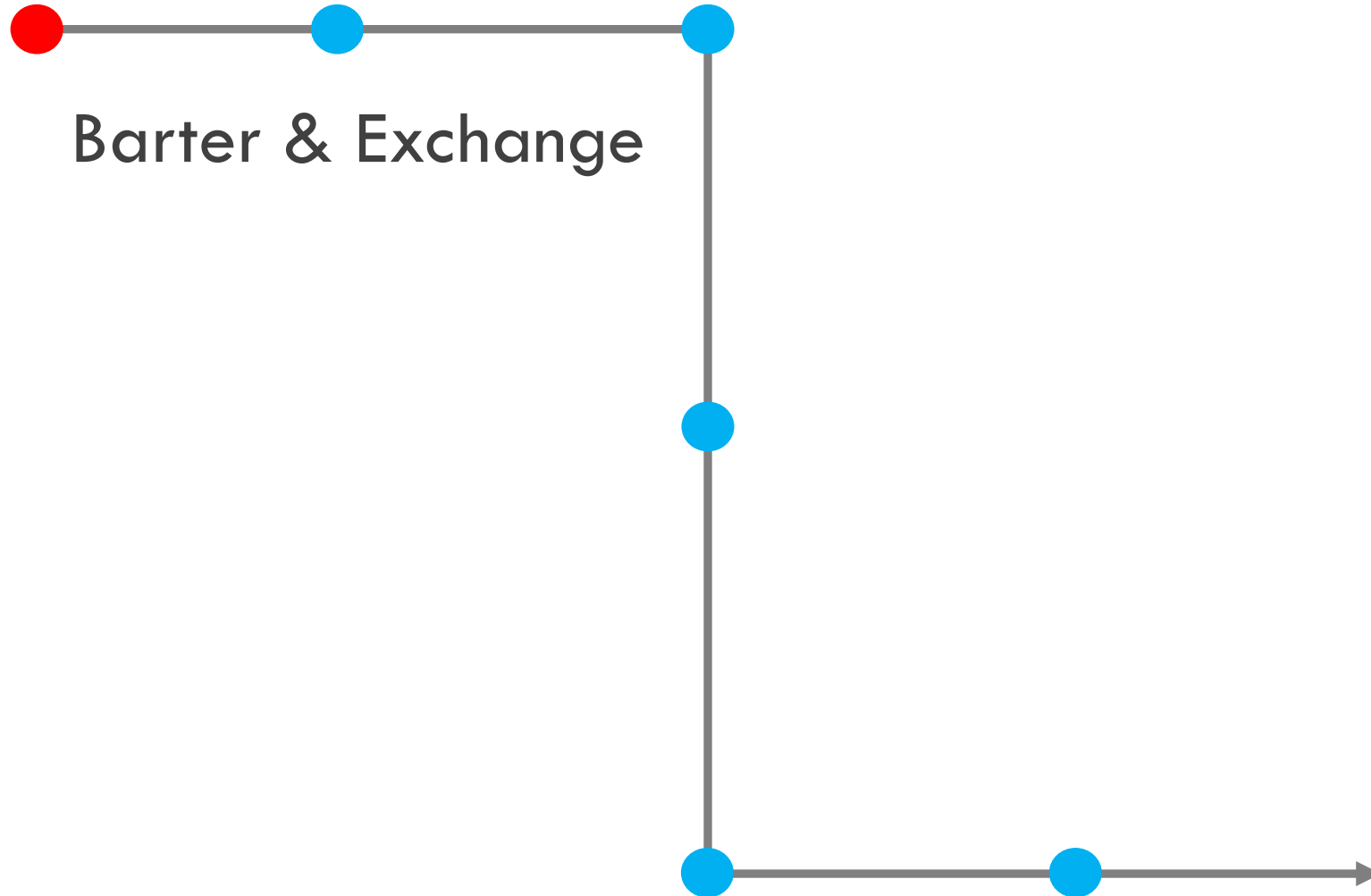
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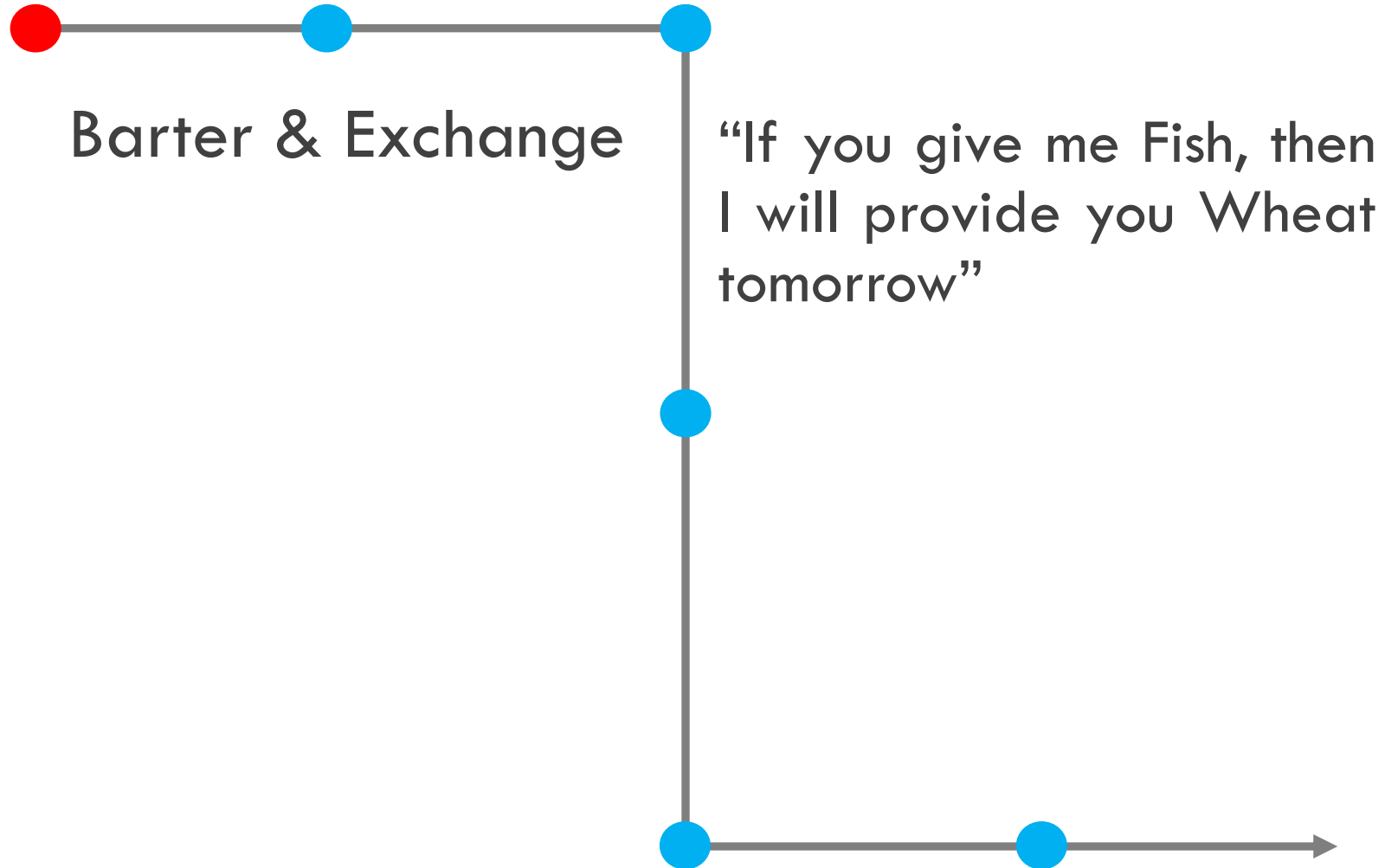
Setting the stage for **Digital Economies**



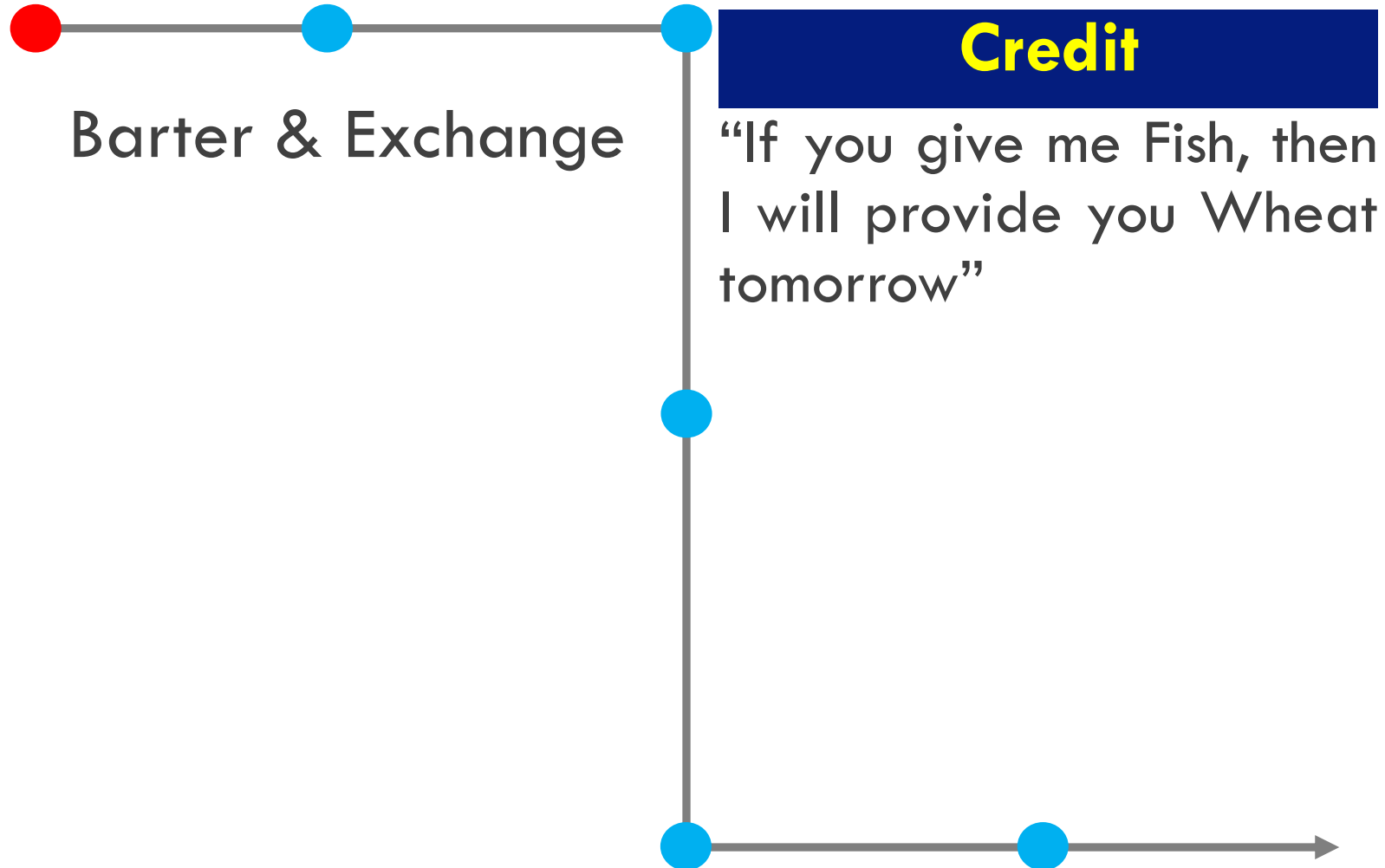
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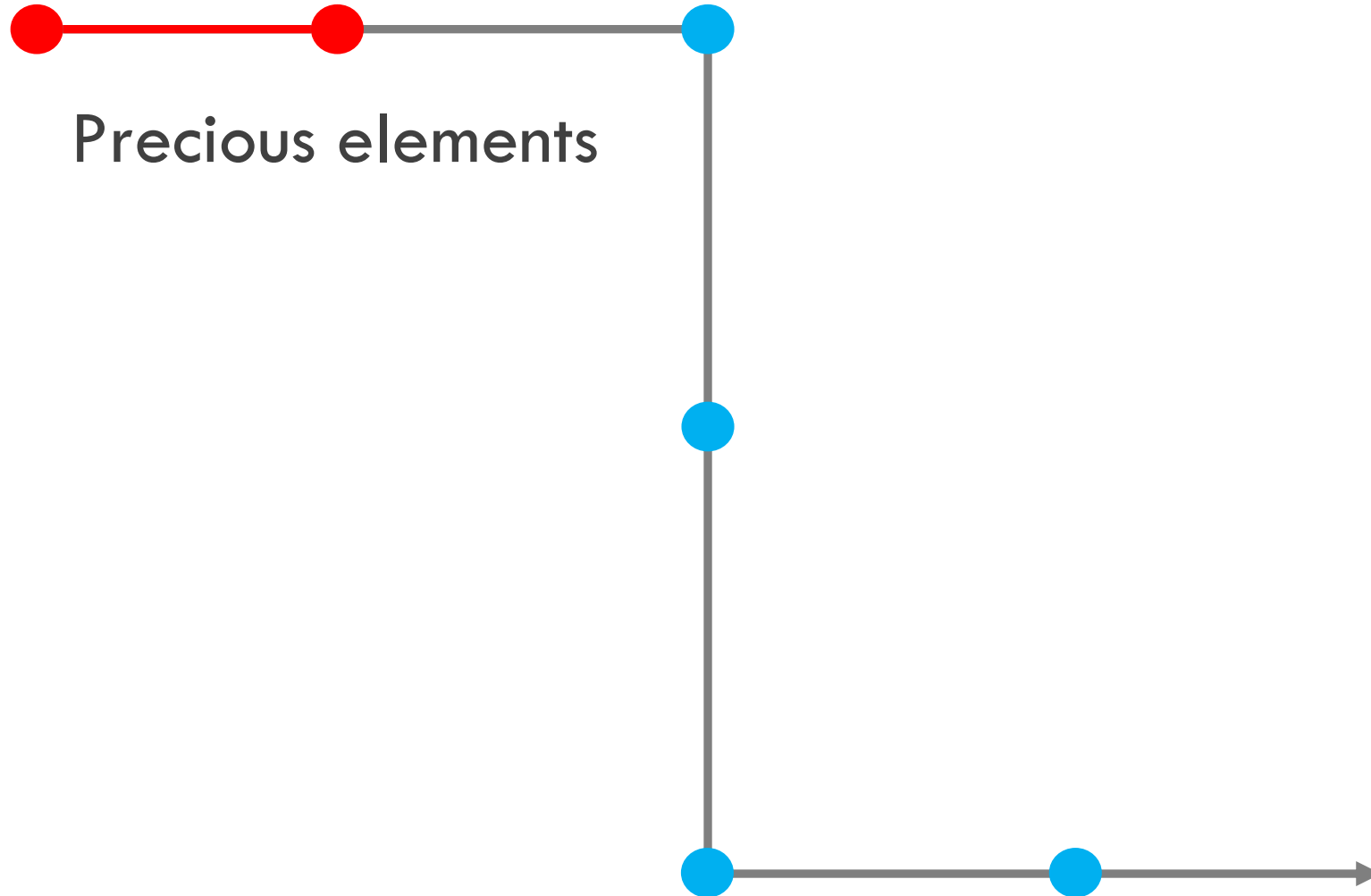
Setting the stage for **Digital Economies**



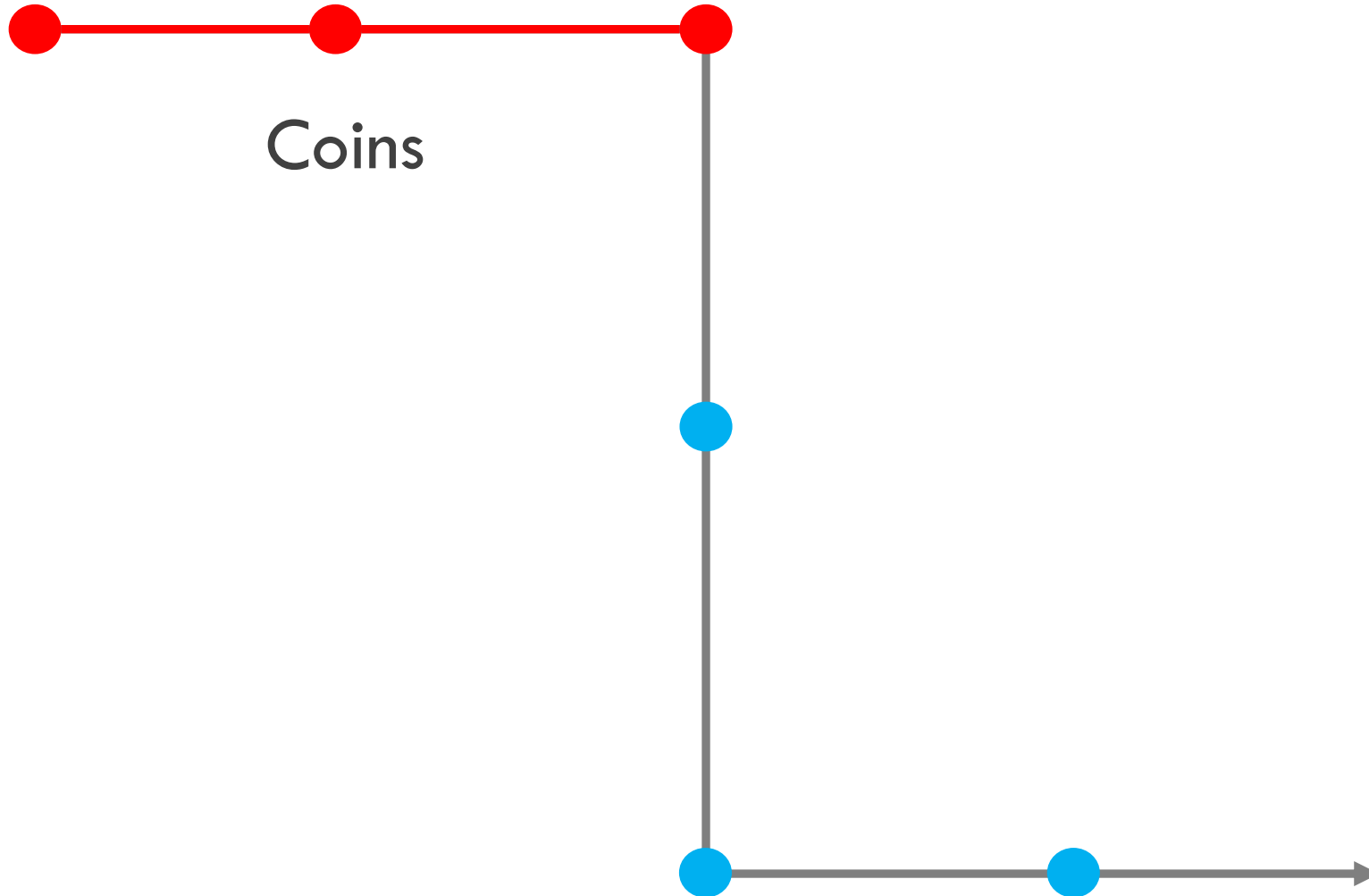
Setting the stage for **Digital Economies**



Setting the stage for **Digital Economies**



Setting the stage for **Digital Economies**



Setting the stage for **Digital Economies**

Coins



Lydian Lion

First



Setting the stage for **Digital Economies**

Coins

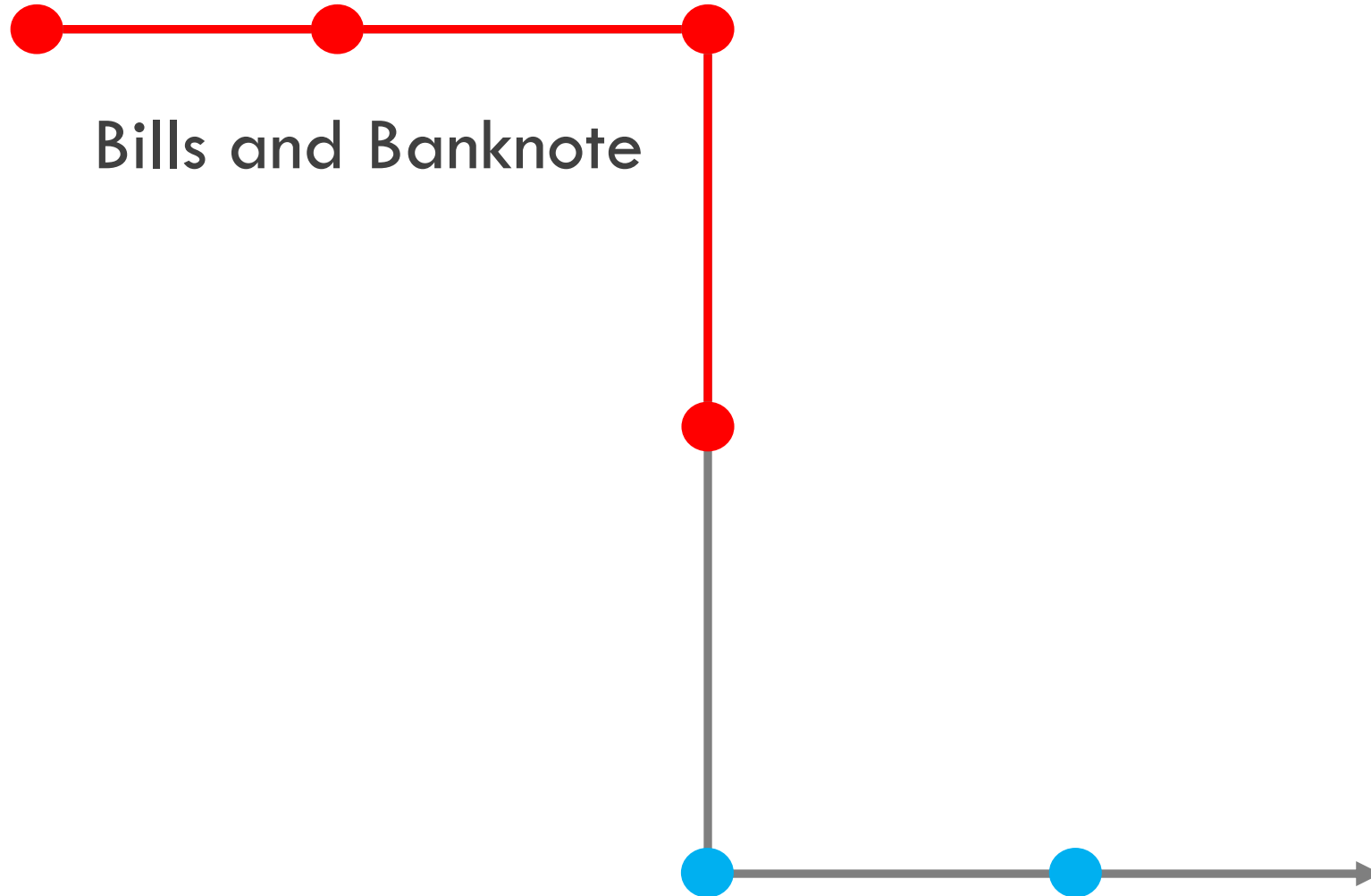


First

Lydian Lion



Setting the stage for **Digital Economies**



Setting the stage for **Digital Economies**

Bills and Banknote

Jiaozi

First



Setting the stage for Digital Economies

Bills and Banknote

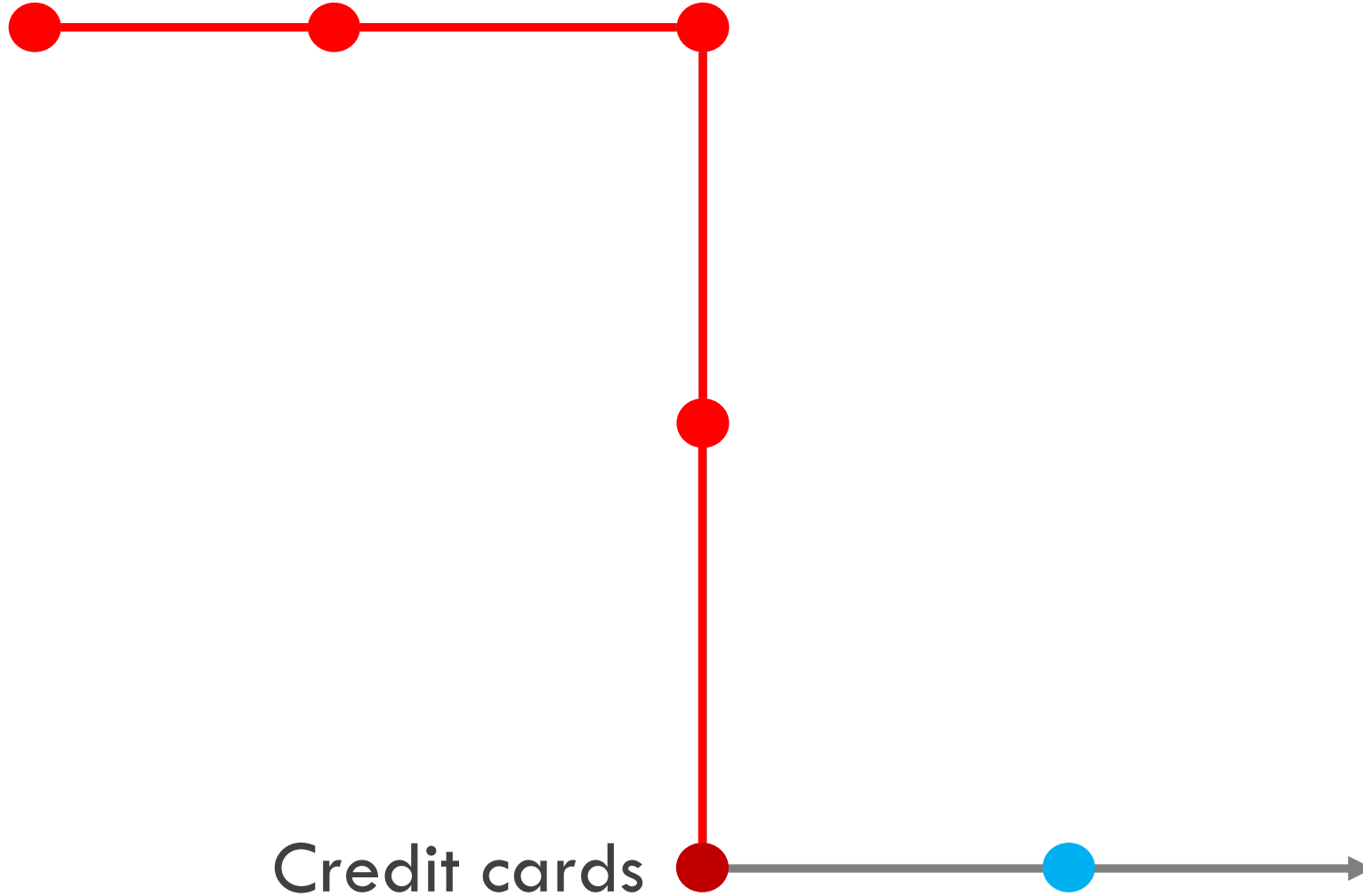


Jiaozi

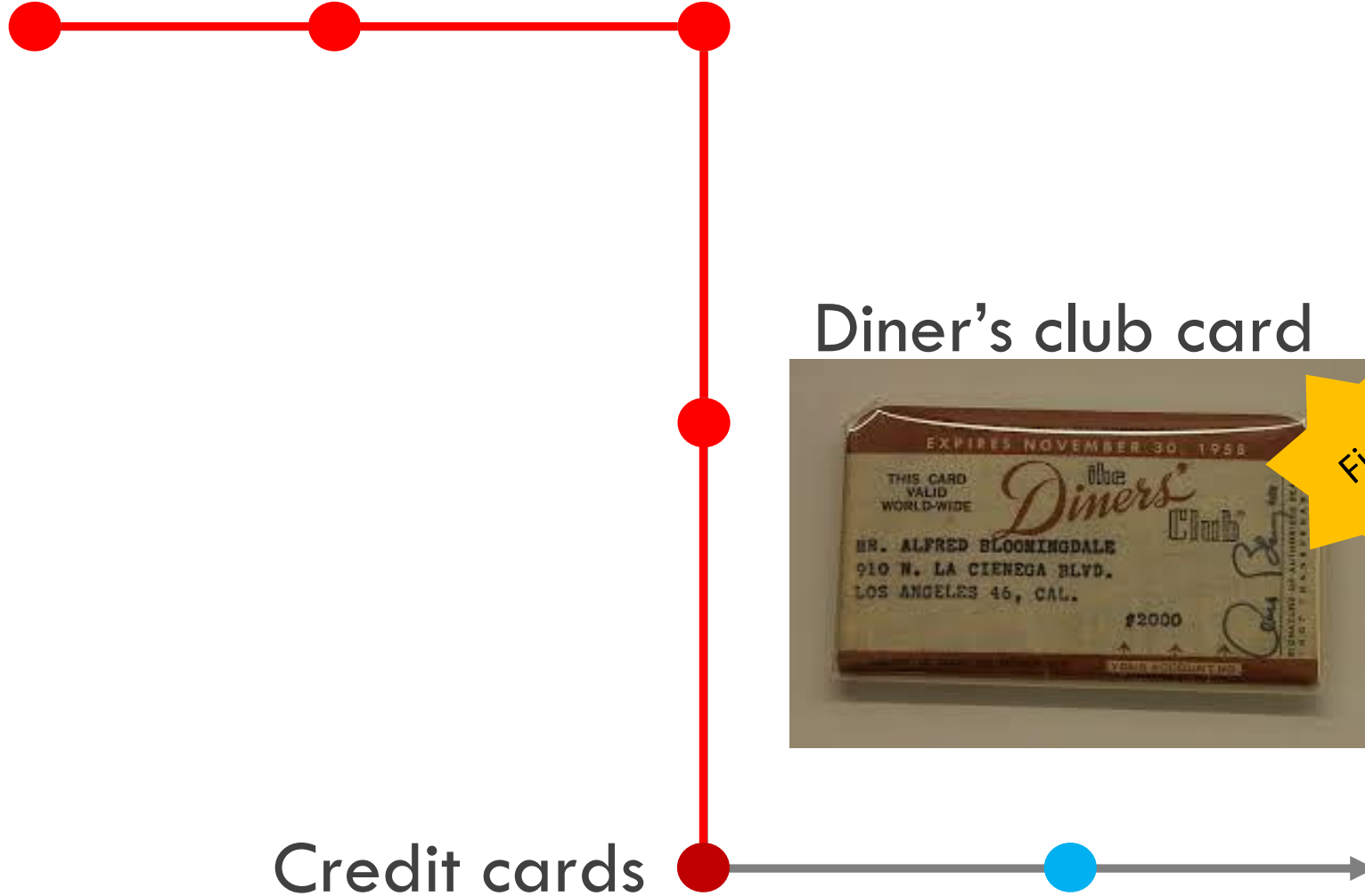
First



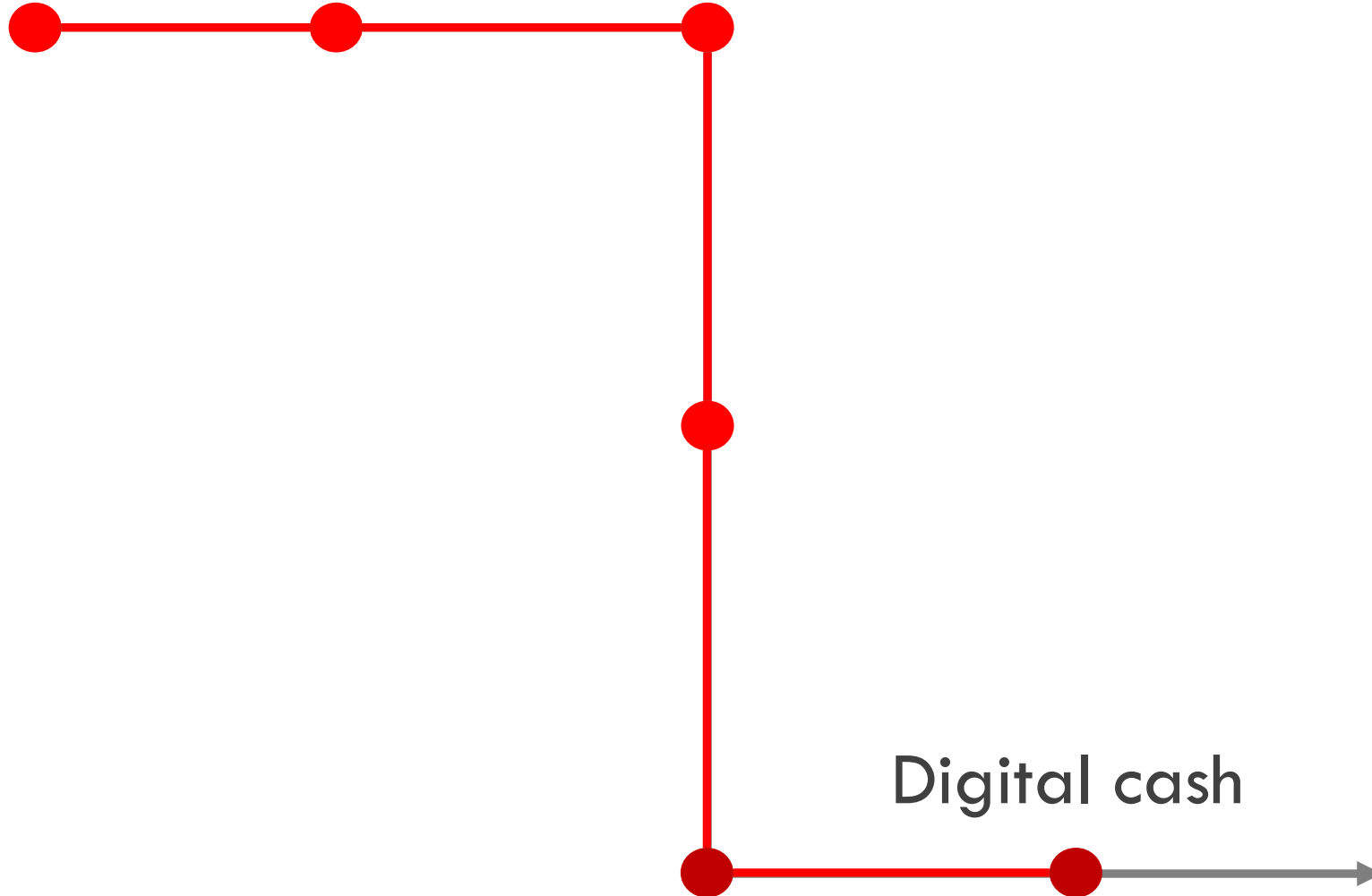
Setting the stage for **Digital Economies**



Setting the stage for **Digital Economies**



Setting the stage for **Digital Economies**



Setting the stage for **Digital Economies**

It took several **millennia** from the first coin to the first banknote



Setting the stage for **Digital Economies**

It took several **millennia** from the first coin to the first banknote

It took several **centuries** from the first banknote to the first credit card



Setting the stage for **Digital Economies**

It took several **millennia** from the first coin to the first banknote

It took several **centuries** from the first banknote to the first credit card

It took only a few **decades** from the first credit card to digital cash



Setting the stage for **Digital Economies**

Science is advancing at a very fast pace. With this outlook, perhaps in a few years, everything related to our business will change. Therefore, it is incumbent upon us to move to the edge of science.



Digital cash system



Participants



Participants



Bank



Participants



Spender



Bank



Digital cash system

Participants



Merchant



Spender



Bank



Bank

Issuance: The bank issues digital cash in Central Bank Digital Currencies (CBDCs) or as private digital money like stablecoins.

Verification: It verifies the authenticity of digital transactions to prevent fraud and counterfeiting.

Security: Banks use cryptographic measures to secure transactions and prevent unauthorized access or double-spending issues.

Regulation: Acts as a regulator, ensuring compliance with anti-money laundering (AML) and know-your-customer (KYC) norms.





Spender

User of Digital Cash: The spender uses digital cash to pay for goods or services.

Authentication: They must authenticate themselves, often through digital wallets or other secure methods. Transaction

Initiation: Initiates transactions by transferring digital cash to merchants, ensuring they have the required balance.

Privacy: In certain systems, spenders can remain anonymous, depending on the cryptographic and policy frameworks in use.



Merchant

Acceptance of Payments: Merchants receive payments in digital cash for goods or services provided.

Integration with Payment Systems: They must integrate with digital payment infrastructure, like wallets and point-of-sale (POS) systems.

Settlement Requests: Merchants request settlement from the bank or payment intermediary, converting digital cash into their desired form (e.g., fiat currency).

Compliance: Ensure compliance with taxation and regulatory frameworks for digital transactions.



Initialization a **Digital cash system**



Initialization a **Digital cash system**

Choose large prime number p , such that $q=(p-1)/2$ is also prime.



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Choose large prime number p , such that $q=(p-1)/2$ is also prime.
Let g be the square of a primitive root mod p .



Initialization a Digital cash system

Choose large prime number p , such that $q=(p-1)/2$ is also prime.
Let g be the square of a primitive root mod p .

```
def find_square_of_primitive_root(p):  
    g0 = primitive_root(p) # imported from sympy  
    # Calculate the square of the primitive root modulo p  
    g = pow(g0, 2, p)  
    return g
```



Initialization a **Digital cash system**

Choose large prime number p , such that $q=(p-1)/2$ is also prime.
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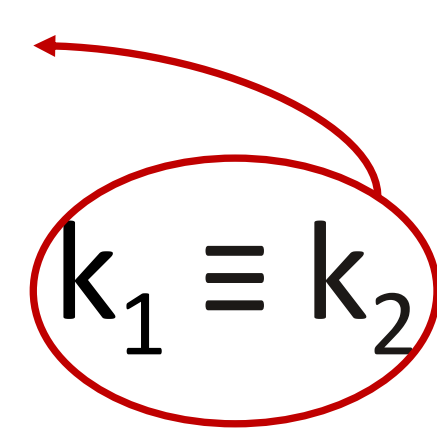
$$g^{k_1} \equiv g^{k_2} \pmod{p} \iff k_1 \equiv k_2 \pmod{q}$$



Initialization a **Digital cash system**

Choose large prime number p , such that $q=(p-1)/2$ is also prime.
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Two secret random exponent!

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Initialization a **Digital cash system**

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g_1, g_2



Initialization a **Digital cash system**

We need two public hash functions.



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H: takes a 5-tuple integers and outputs an integer mod q .



Initialization a **Digital cash system**

We need two public hash functions.

H: takes a 5-tuple integers and outputs an integer mod q .

```
def hash_H(input_tuple):  
    if len(input_tuple) != 5:  
        raise ValueError("Input must be a 5-tuple of integers.")  
    input_bytes = ','.join(map(str, input_tuple)).encode('utf-8')  
    digest = Hash(SHA256(), backend=default_backend())  
    digest.update(input_bytes)  
    hash_digest = digest.finalize()  
    return int.from_bytes(hash_digest, 'big') % q
```



Initialization a **Digital cash system**

We need two public hash functions.

H : takes a 5-tuple integers and outputs an integer mod q .

H_0 : takes a 4-tuple integers and outputs an integer mod q .



Initialization a **Digital cash system**

We need two public hash functions.

H: takes a 5-tuple integers and outputs an integer mod q .

H_0 : takes a 4-tuple integers and outputs an integer mod q .

```
def hash_H0(input_tuple):  
    if len(input_tuple) != 4:  
        raise ValueError("Input must be a 4-tuple of integers.")  
    input_bytes = ','.join(map(str, input_tuple)).encode('utf-8')  
    digest = Hash(SHA256(), backend=default_backend()) digest.update(input_bytes)  
    hash_digest = digest.finalize()  
    return int.from_bytes(hash_digest, 'big') % q
```



Initialization a **Digital cash system**

So after initialization steps we have:



Initialization a **Digital cash system**

So after initialization steps we have:

$$p, q, g, g_1, g_2, H, H_0$$


The Bank



The Bank



The bank choose its secret identity number “ x ”



The Bank



The bank choose its secret identity number “x”

$$h \equiv g^x \pmod{p}$$



The Bank



The bank choose its secret identity number “**x**”

$$h \equiv g^x \pmod{p}$$

The number “**h**” is made public and identifies the bank



The Spender



The Spender



The spender choose its secret identity number “ u ”



The Spender



The spender choose its secret identity number “u”

$$I \equiv g_1^u \pmod{p}$$



The Spender



The spender choose its secret identity number “u”

$$\mathbf{I} \equiv g_1^u \pmod{p}$$

Account number of the spender



The Spender



The spender choose its secret identity number “u”

$$\mathbf{I} \equiv g_1^u \pmod{p}$$

Account number of the spender



The spender sends “I” to the bank and the bank stores it with other information like name, address, etc.



The Merchant



The Merchant



The Merchant choose an identification number “M”



The Merchant

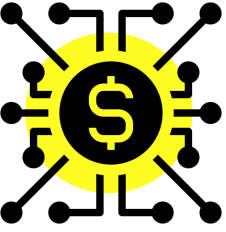


The Merchant choose an identification number “M”

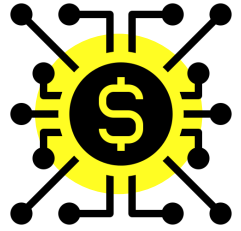
The Merchant registers “M” with the bank



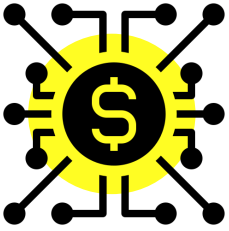
Creating a Coin



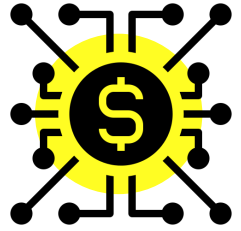
Creating a Coin



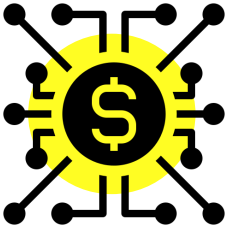
The Spender contacts the bank, asking for a coin



Creating a Coin



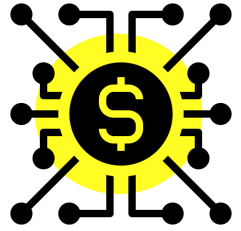
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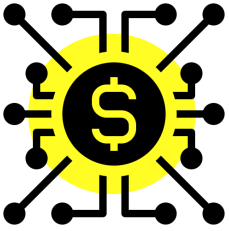
The bank requires proof of identity, just as when someone is withdrawing classical cash from an account!



Creating a Coin



The Spender contacts the bank, asking for a coin



The bank requires proof of identity, just as when someone is withdrawing classical cash from an account!

All coins in the present scheme have the same value!

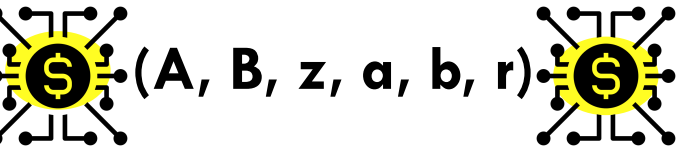


We can present a coin with 6-tuple of numbers

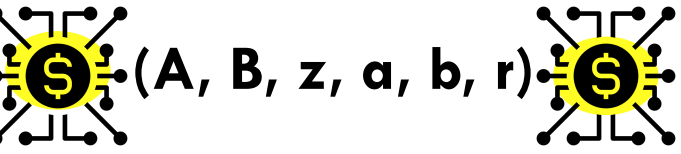
 (A, B, z, a, b, r) 



Creating a Coin



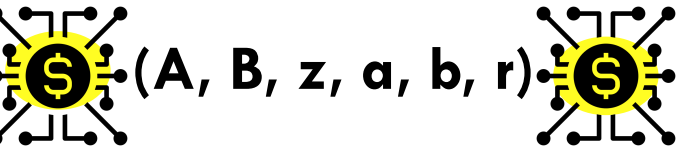
Creating a Coin



The Bank chooses a random number “ ω ”



Creating a Coin



The Bank chooses a random number “ ω ”
(ω is a different number for each coin)



Creating a Coin

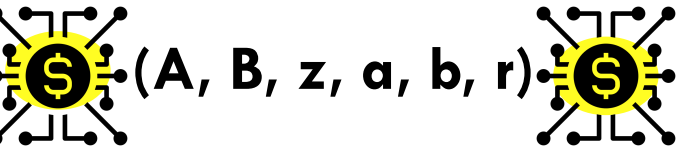


The Bank chooses a random number “ ω ”
(ω is a different number for each coin)

$$g_{\omega} \equiv g^{\omega} \pmod{p}$$



Creating a Coin



The Bank chooses a random number “ ω ”
(ω is a different number for each coin)

$$g_{\omega} \equiv g^{\omega} \pmod{p}$$
$$\beta \equiv (I g_2)^{\omega} \pmod{p}$$



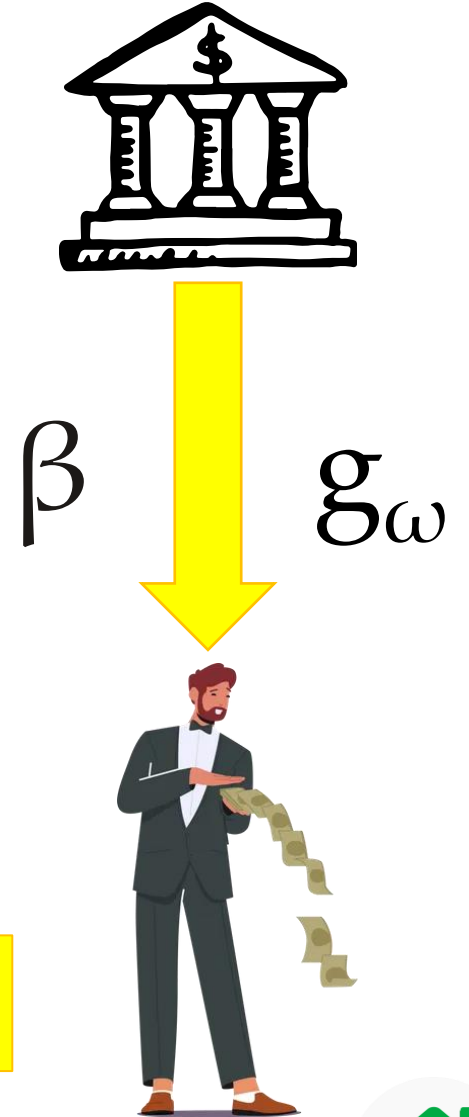
Creating a Coin



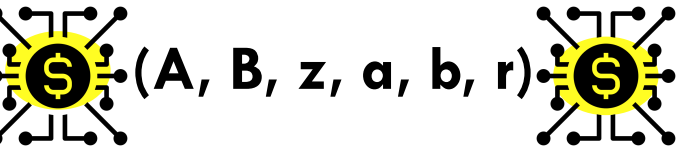
The Bank chooses a random number “ ω ”
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$$g_{\omega} \equiv g^{\omega} \pmod{p}$$
$$\beta \equiv (I g_2)^{\omega} \pmod{p}$$

The bank sends β and g_{ω} to the spender



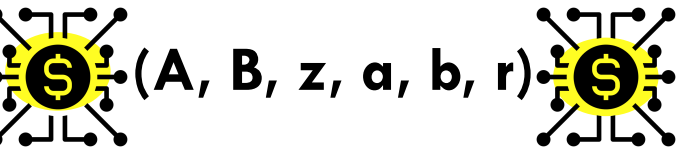
Creating a Coin



The Spender chooses 5 random integers.



Creating a Coin



The Spender chooses 5 random integers.

(s, x_1, x_2, a_1, a_2)



Creating a Coin



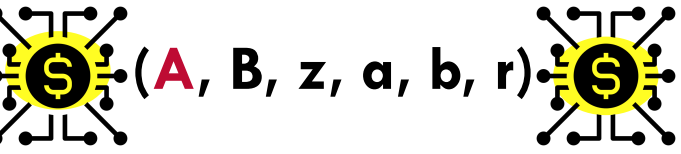
The Spender chooses 5 random integers.

(s, x_1, x_2, a_1, a_2)

$$A \equiv (\lg 2)^s \pmod{p}$$



Creating a Coin



The Spender chooses 5 random integers.

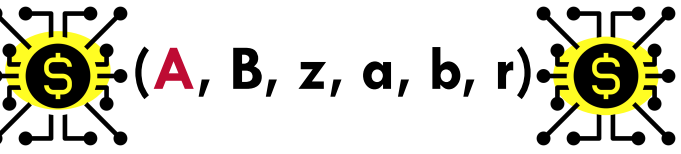
$(s, x_1, x_2, \alpha_1, \alpha_2)$

Coins with $A = 1$ are not allowed

$$A \equiv (\lg 2)^s \pmod{p}$$



Creating a Coin



The Spender chooses 5 random integers.

$$(s, x_1, x_2, \alpha_1, \alpha_2)$$

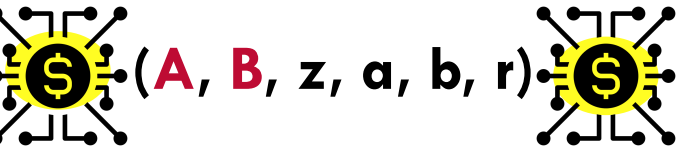
This can happen in only two ways. One is when $s \equiv 0 \pmod{q}$, so we require $s \not\equiv 0$

Coins with $A = 1$ are not allowed

$$A \not\equiv (\lg 2)^s \pmod{p}$$



Creating a Coin



The Spender chooses 5 random integers.

(s, x_1, x_2, a_1, a_2)

$$A \equiv (\lg 2)^s \pmod{p}$$

$$B \equiv g_1^{x_1} g_2^{x_2} \pmod{p}$$



Creating a Coin



The Spender chooses 5 random integers.

$(s, x_1, x_2, \alpha_1, \alpha_2)$

$$A \equiv (\lg 2)^s \pmod{p}$$

$$a \equiv g_{\omega}^{\alpha_1} g^{\alpha_2} \pmod{p}$$

$$B \equiv g_1^{x_1} g_2^{x_2} \pmod{p}$$



Creating a Coin



The Spender chooses 5 random integers.

$(s, x_1, x_2, \alpha_1, \alpha_2)$

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$$B \equiv g_1^{x_1} g_2^{x_2} \pmod{p}$$

$$b \equiv \beta^{s\alpha_1} A^{\alpha_2} \pmod{p}$$



Creating a Coin



The Spender chooses 5 random integers.

$(s, x_1, x_2, \alpha_1, \alpha_2)$

$$A \equiv (\lg 2)^s \pmod{p}$$

$$a \equiv g_{\omega}^{\alpha_1} g^{\alpha_2} \pmod{p}$$

$$z \equiv z'^s \pmod{p}$$

$$B \equiv g_1^{x_1} g_2^{x_2} \pmod{p}$$

$$b \equiv \beta^{s\alpha_1} A^{\alpha_2} \pmod{p}$$



Creating a Coin

 (A, B, z, a, b, r) 



Creating a Coin

 (A, B, z, a, b, ) 



Creating a Coin



Creating a Coin



$$c \equiv \alpha_1^{-1} H(A, B, z, a, b) \pmod{p}$$



Creating a Coin



$$c \equiv \alpha_1^{-1} H(A, B, z, a, b) \pmod{p}$$



$$c_1 \equiv cx + \omega \pmod{p}$$



Creating a Coin



$$c \equiv \alpha_1^{-1} H(A, B, z, a, b) \pmod{p}$$



$$c_1 \equiv cx + \omega \pmod{p}$$



$$r \equiv \alpha_1 c_1 + \alpha_2 \pmod{p}$$

$$r \equiv \alpha_1 c_1 + \alpha_2 \pmod{p}$$



Creating a Coin

 (A, B, z, a, b, r) 



Spending a Coin



Merchant



Spender



Spending a Coin



Merchant



Goods or services

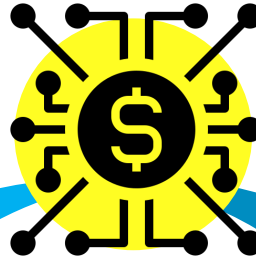


Spender

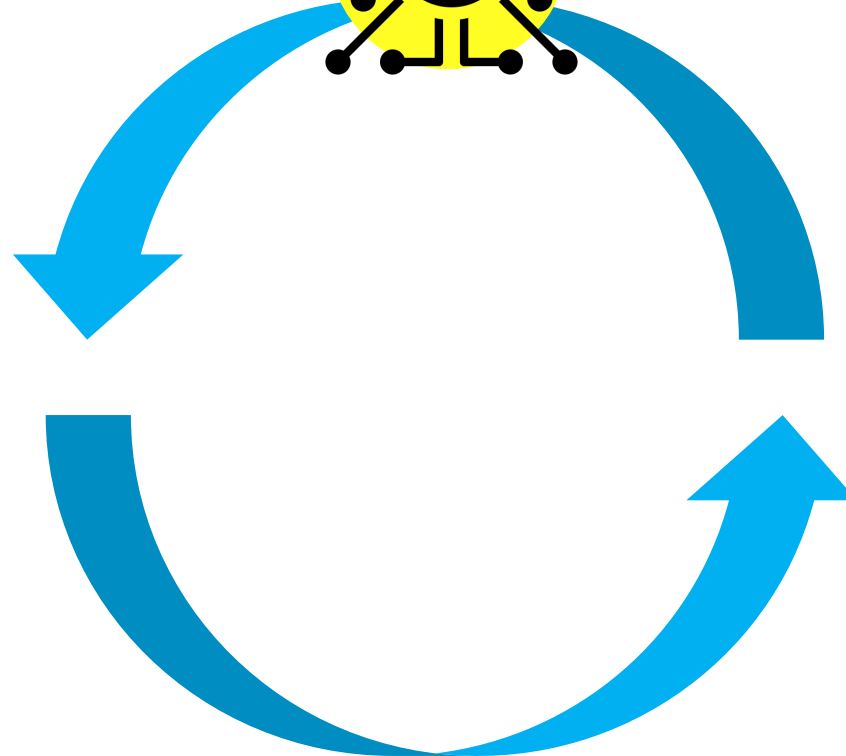


Spending a Coin

(A, B, z, a, b, r)



Merchant



Goods or services

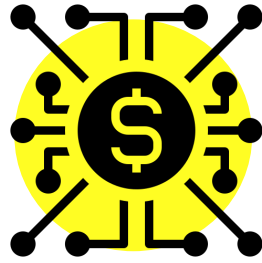


Spender



Spending a Coin

(A, B, z, a, b, r)



Merchant

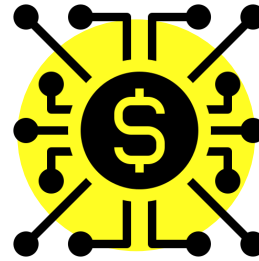
$$g^r \equiv a h^{H(A, B, z, a, b)} \pmod{p}$$

$$A^r \equiv b z^{H(A, B, z, a, b)} \pmod{p}$$



Spending a Coin

(A, B, z, a, b, r)



Merchant

$$g^r \equiv a h^{H(A, B, z, a, b)} \pmod{p} \quad \checkmark$$

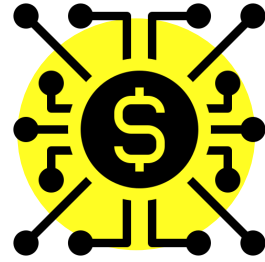
$$A^r \equiv b z^{H(A, B, z, a, b)} \pmod{p} \quad \checkmark$$

The merchant makes sure of the authenticity of the coin



Spending a Coin

(A, B, z, a, b, r)



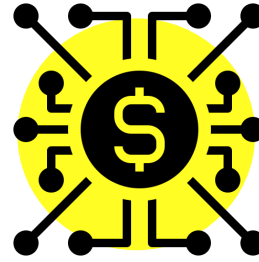
Merchant

$d \equiv H_0(A, B, M, t)$



Spending a Coin

(A, B, z, a, b, r)



Merchant

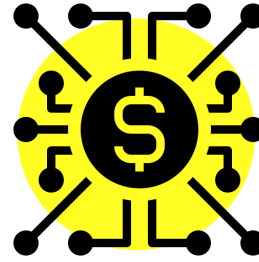
$$d \equiv H_0(A, B, M, \textcircled{t})$$

t is a number representing date and time of transaction.



Spending a Coin

(A, B, z, a, b, r)



Merchant

d

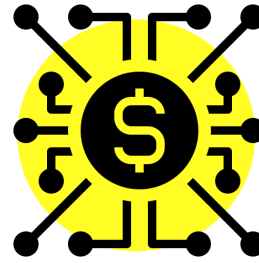


Spender



Spending a Coin

(A, B, z, a, b, r)



$$r_1 \equiv ds + x_1 \pmod{q}$$

$$r_2 \equiv ds + x_2 \pmod{q}$$

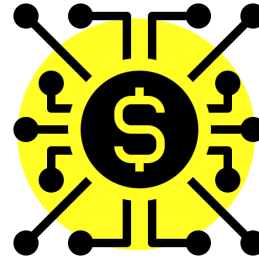


Spender



Spending a Coin

(A, B, z, a, b, r)



Merchant

$r_1 r_2$

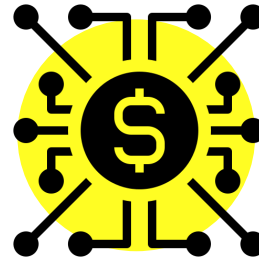


Spender



Spending a Coin

(A, B, z, a, b, r)



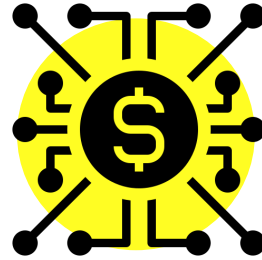
Merchant

$\left\{ \begin{array}{l} \text{if } g_1^{r_1} g_2^{r_2} \equiv A^d B \pmod{p} \\ \text{ACCEPT} \\ \text{else} \end{array} \right.$

REJECT



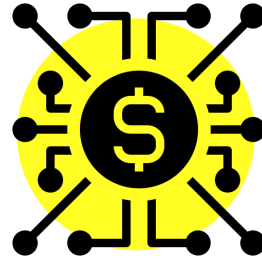
Deposit a **Coin** in the bank



A few days after receiving the coin, the Merchant wants to deposit it in the Bank



Deposit a **Coin** in the bank

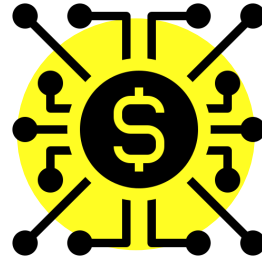


A few days after receiving the coin, the Merchant wants to deposit it in the Bank

The Merchant submits the coin plus the triple of transaction



Deposit a **Coin** in the bank



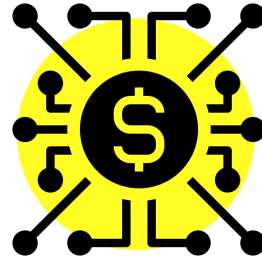
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(A, B, z, a, b, r)



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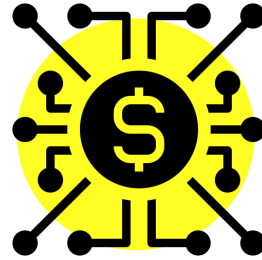
The Merchant submits the coin plus the triple of transaction

(A, B, z, a, b, r)

(r_1, r_2, d)



Deposit a **Coin** in the bank



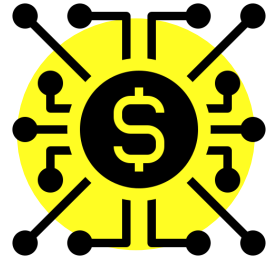
Merchant



Bank



Deposit a **Coin** in the bank



Merchant

(A, B, z, a, b, r)

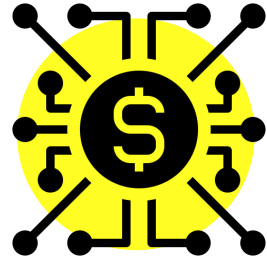
(r_1, r_2, d)



Bank



Deposit a **Coin** in the bank



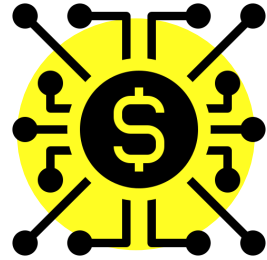
The checks that the coin (A, B, z, a, b, r) has not been previously deposited.



Bank



Deposit a **Coin** in the bank



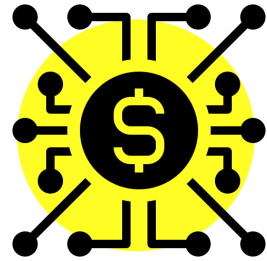
Is coin valid?



Bank



Deposit a **Coin** in the bank



Is coin valid?

$$g^r \equiv a h^{H(A, B, z, a, b)} \pmod{p}$$

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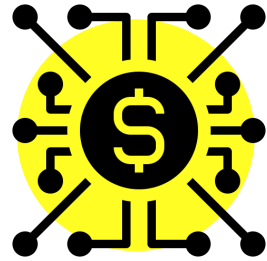
$$g_1^{r_1} g_2^{r_2} \equiv A^d B \pmod{p}$$



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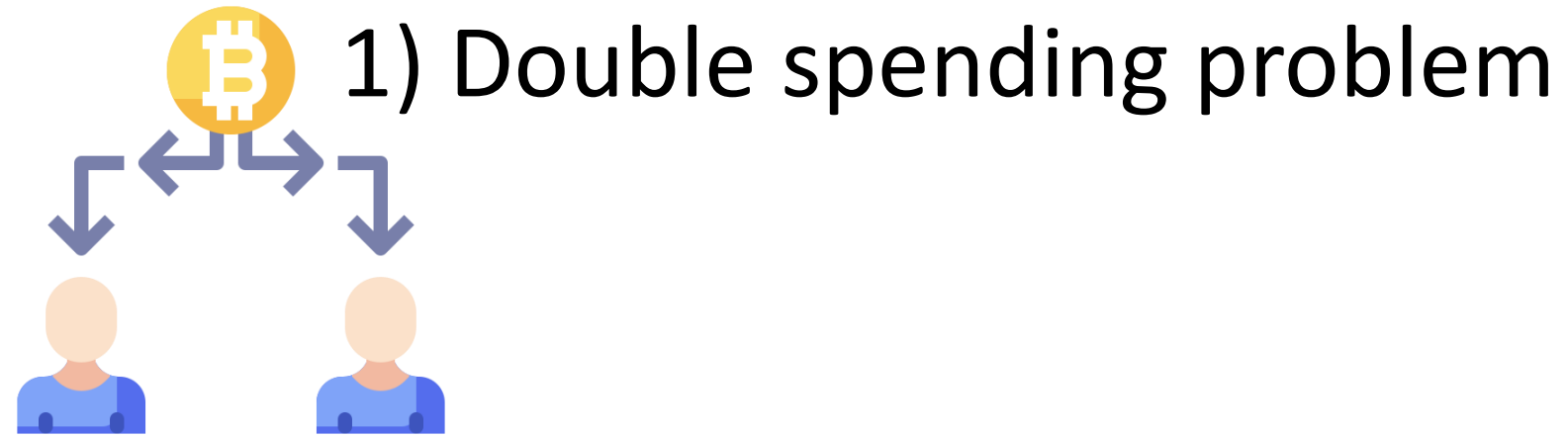
coin is valid and the Merchant's account is credited.



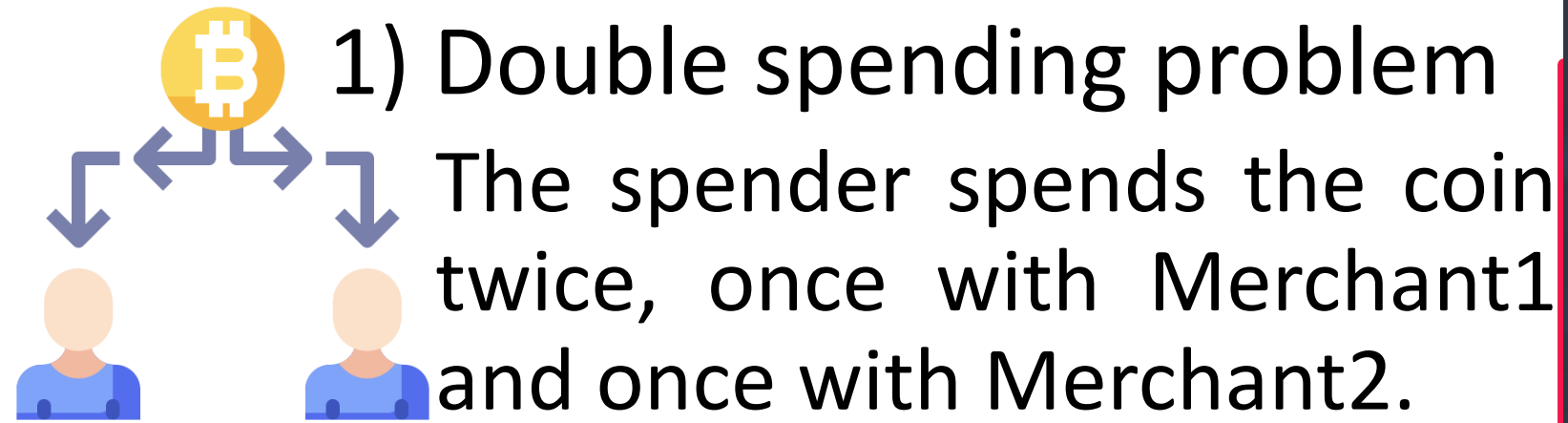
Fraud control



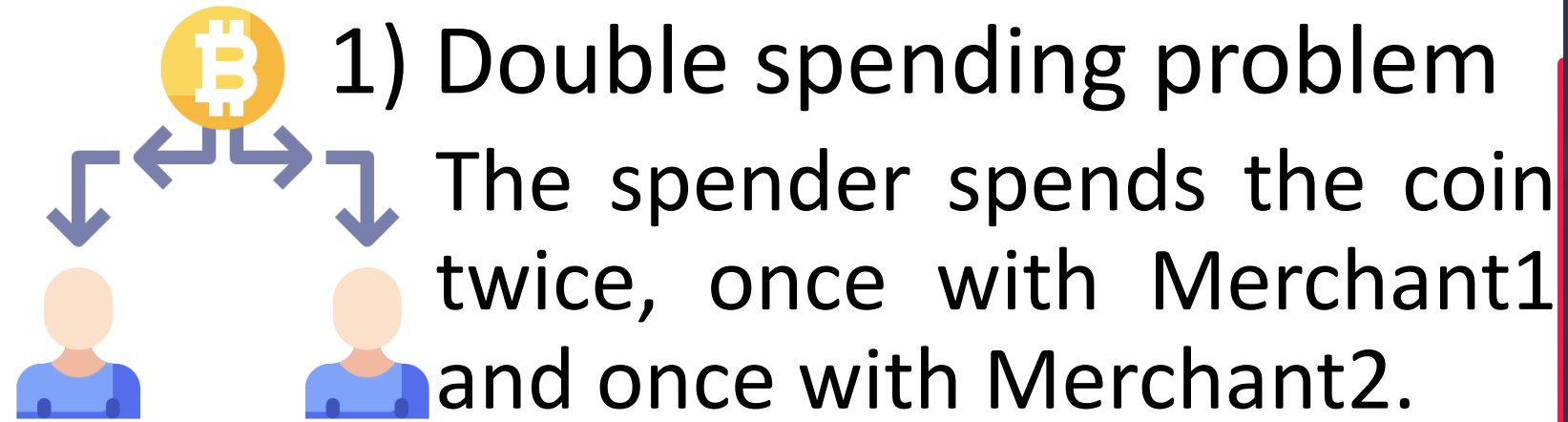
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Fraud control



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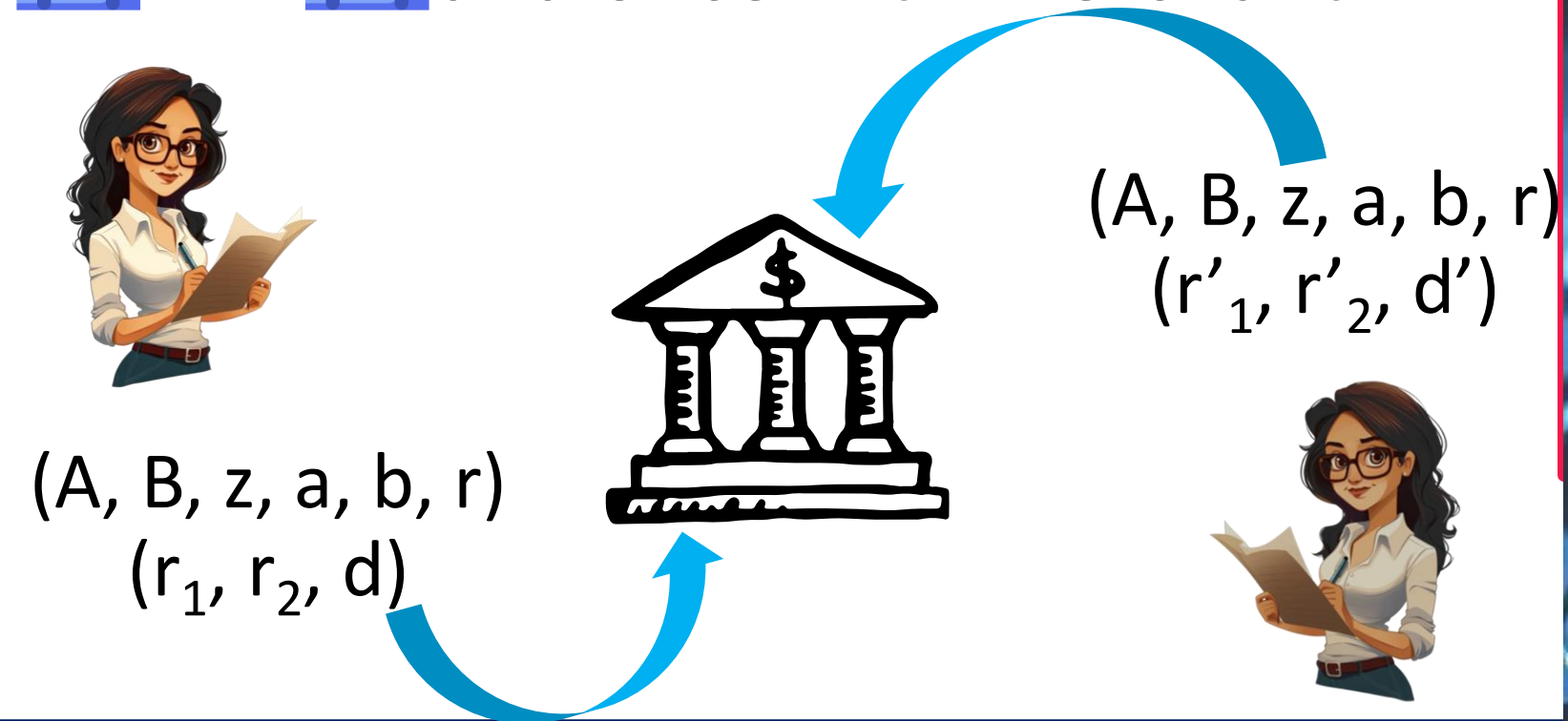
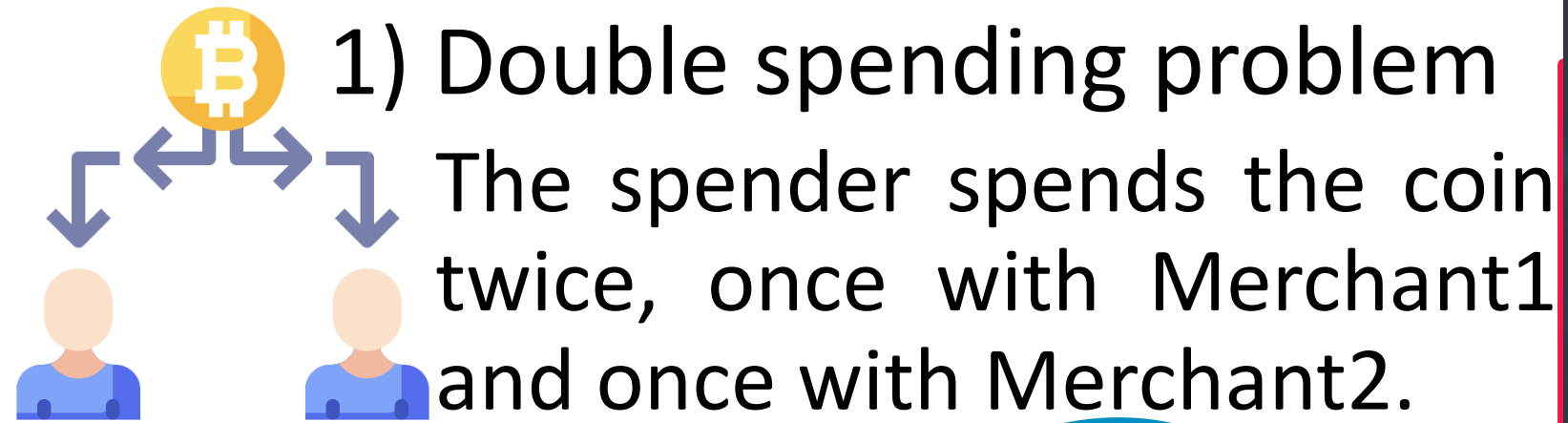
(A, B, z, a, b, r)
 (r'_1, r'_2, d')



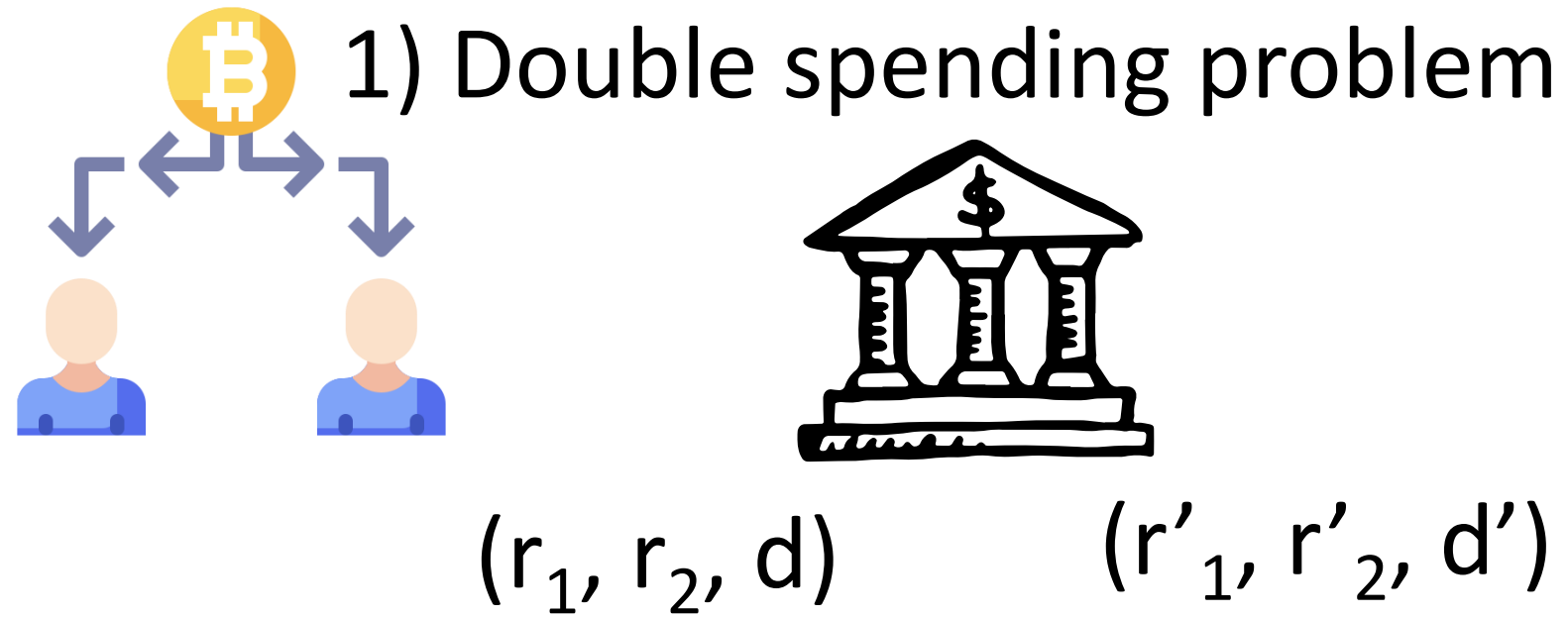
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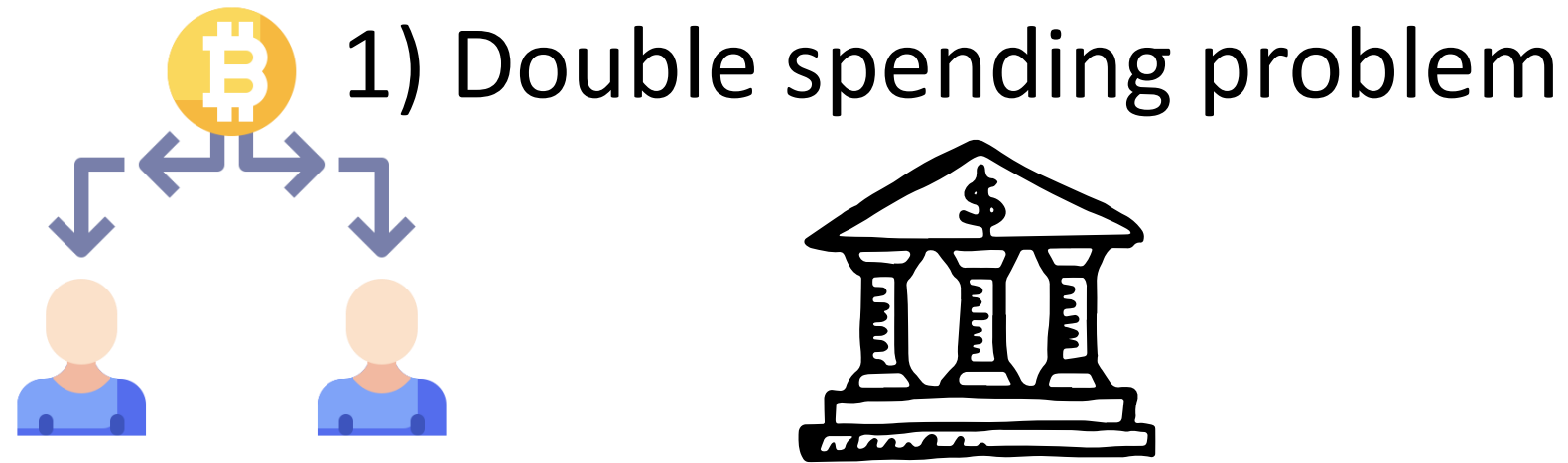
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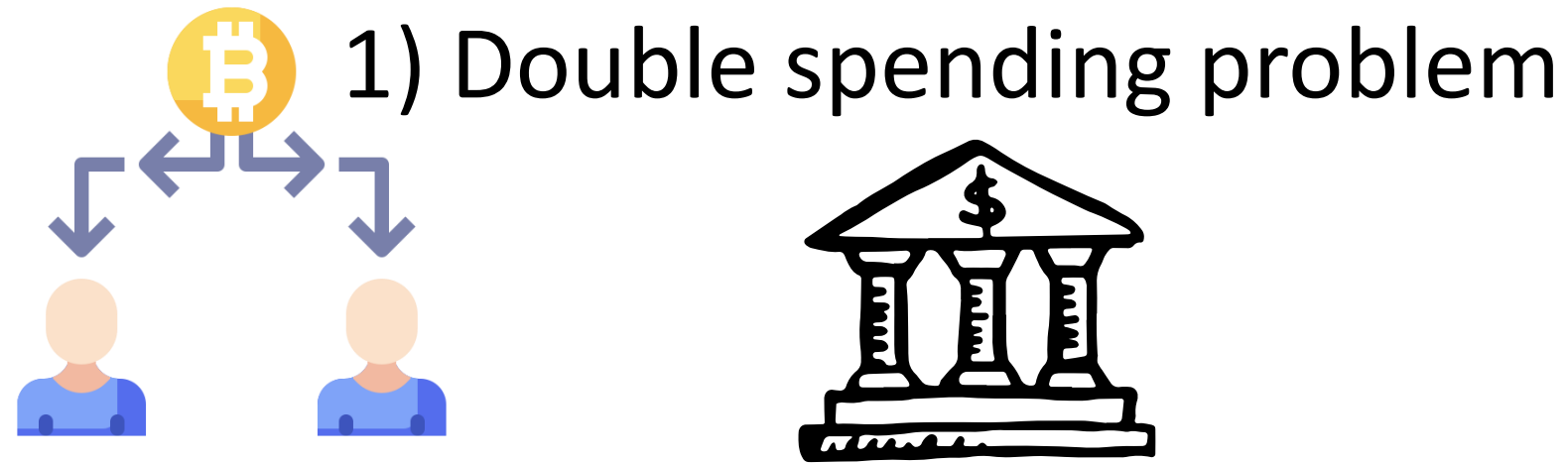
$$(r_1, r_2, d) \quad (r'_1, r'_2, d')$$

$$r_1 - r'_1 \equiv us(d-d') \pmod{p}$$

$$r_2 - r'_2 \equiv s(d-d') \pmod{p}$$



Fraud control



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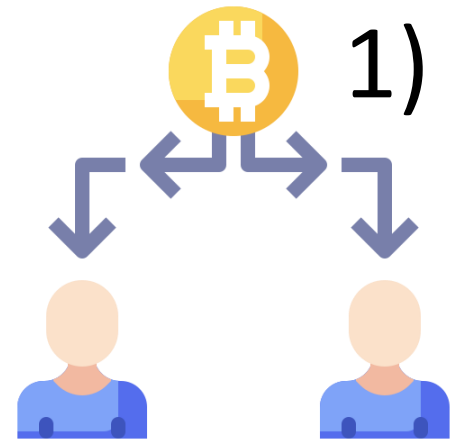
$$r_2 - r'_2 \equiv s(d-d') \pmod{p}$$

$$u \equiv (r_1 - r'_1)(r_2 - r'_2)^{-1} \pmod{q}$$



Fraud control

1) Double spending problem

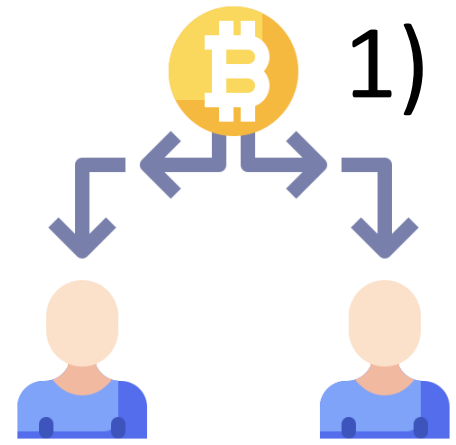


$$\mathbf{I} \equiv g^u_1(\text{mod } p)$$



Fraud control

1) Double spending problem



The spender who did this will be found

$$\mathbf{I} \equiv g^u_1(\text{mod } p)$$



Fraud control

2) The Merchant tries to submitting the coin twice.



Fraud control

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(r_1, r_2, d) (r'_1, r'_2, d')



Fraud control

2) The Merchant tries to submitting the coin twice.

$$(r_1, r_2, d) \quad (r'_1, r'_2, d')$$

This is essentially **impossible** for the Merchant to do since it is complicated for the Merchant to produce numbers such that:

$$g_1^{r'_1} g_2^{r'_2} \equiv A^{d'} B \pmod{p}$$



Fraud control

3) Someone tries to make an unauthorized coin



Fraud control

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This requires finding numbers such that:
 $g^r \equiv a \cdot h^{H(A, B, z, a, b)}$ and $A^r \equiv z \cdot H(A, B, z, a, b)$



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discrete logarithm problem



Fraud control

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However, since the Spender has kept u secret, the person in the bank cannot produce a suitable r_1

Of course, this would be possible if $s = 0$ were allowed!



Flash back



$$(s, x_1, x_2, \alpha_1, \alpha_2)$$

This can happen in only two ways.
One is when $s \equiv 0 \pmod{q}$, so we
require $s \not\equiv 0$

$$A \equiv (\lg 2)^s \pmod{p}$$

Coins with $A = 1$ are not allowed



Fraud control

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This is one reason $A = 1$ is not allowed!!!



Fraud control

5) Someone steals the coin from the Spender and tries to spend it



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The first verification equation is still satisfied.



Fraud control

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The first verification equation is still satisfied.

but the thief does not know u and therefore will not be able to produce r_1, r_2 such that $g_1^{r_1} g_2^{r_2} \equiv A^{d'} B$



Anonymity

Anonymity in digital cash refers to the degree to which a user's identity and transactions are concealed when they interact with a digital currency system. Traditional cash transactions are inherently anonymous; they don't link to a user's identity unless explicitly tracked. In the digital realm, achieving a similar level of privacy is complex due to the technological infrastructure and regulatory frameworks involved!



Anonymity

Sender Anonymity: The sender's identity is hidden from the recipient or any third party.



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Sender Anonymity: The sender's identity is hidden from the recipient or any third party.

Receiver Anonymity: The recipient's identity is concealed from the sender or others.

Transaction Anonymity: The details of the transaction, including the parties involved and the amount, are obfuscated.



Technologies Enabling Anonymity



Technologies Enabling Anonymity

1. Zero-Knowledge Proofs (ZKPs)
2. Ring Signatures
3. Mixing Services
4. Blind Signatures



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Bitcoin



Bitcoin



Bitcoin is presented as a decentralized digital currency introduced by **Satoshi Nakamoto** in 2008 through a white paper titled *"Bitcoin: A Peer-to-Peer Electronic Cash System."*



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Unlike traditional currencies, Bitcoin does not rely on central authorities (like banks) for transactions or issuance.

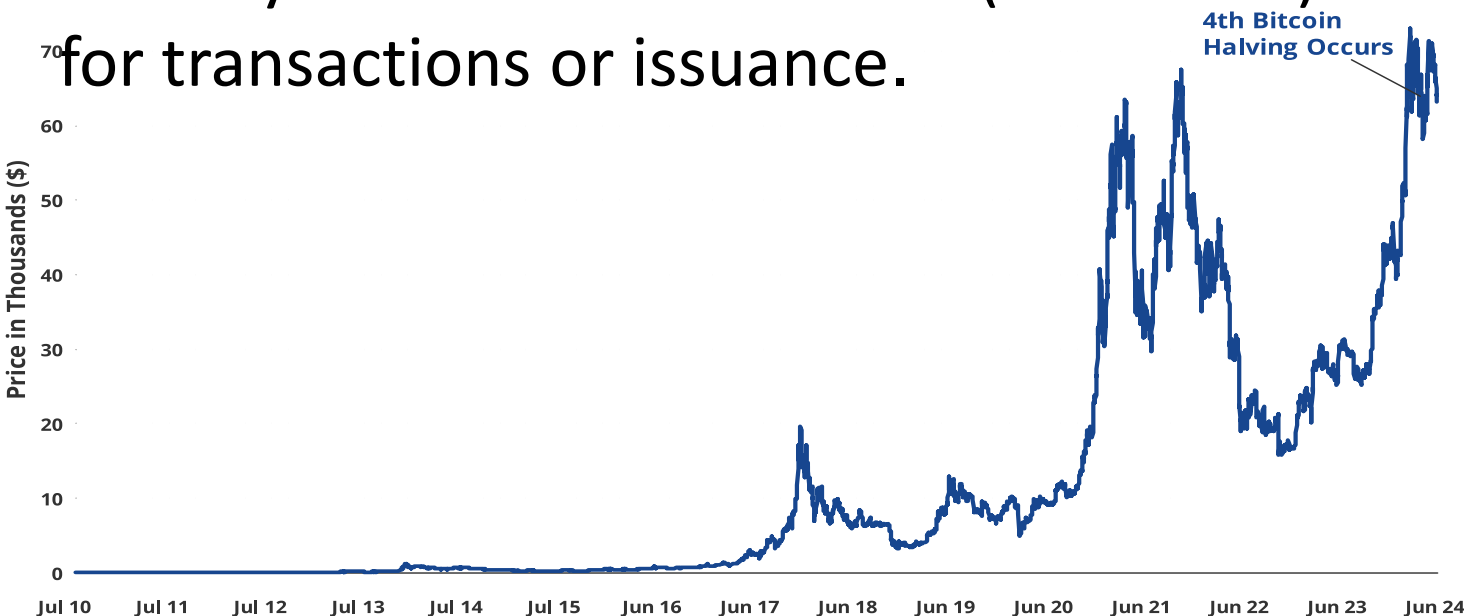


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Key Cryptographic Principles in Bitcoin



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- **Public-Key Cryptography** cilbup esu snoticasnarT : gningis dna notiacfitinedi eruces rof syek etavirp dna



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- **Hash Functions**: Cryptographic hash functions (like SHA-256) ensure the integrity of data in transactions and are essential to the mining process



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gningis dna notiacfitinedi eruces rof syek etavirp dna
- **Hash Functions**: Cryptographic hash functions (like SHA-256) ensure the integrity of data in transactions and are essential to the mining process
- **Digital Signatures** :
fo ytictinehtua eht yfirev ot desU :
a fo renwo lutfhgir eht ylno taht gnirusne ,snoticasnart
ti dneps nac niocitB



Bitcoin

Blockchain



Bitcoin

Blockchain



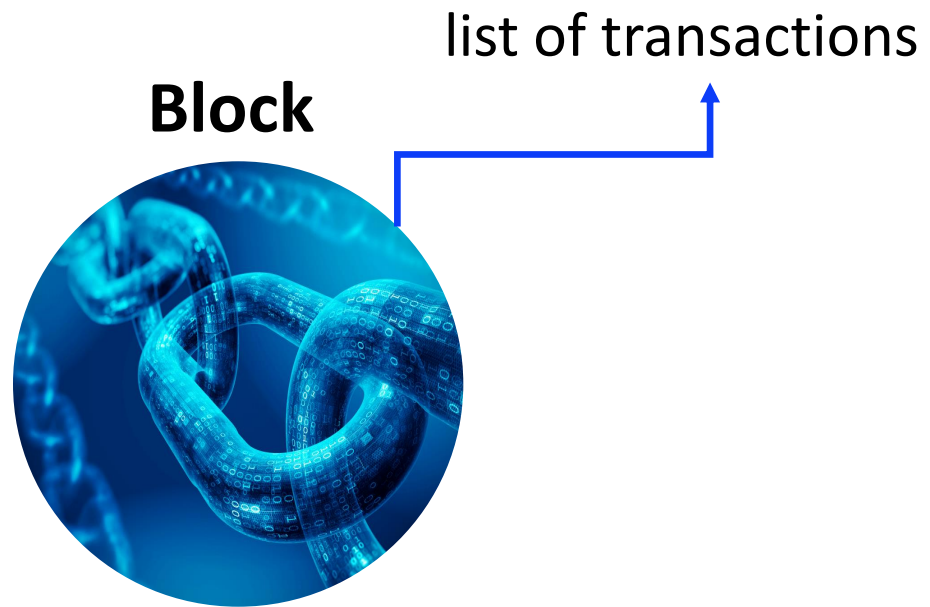
The blockchain is the backbone of Bitcoin and other cryptocurrencies. It is a distributed ledger that records all transactions in a transparent, tamper-proof, and decentralized manner. Transactions are grouped into blocks, which are linked sequentially in a chain using cryptographic hashes



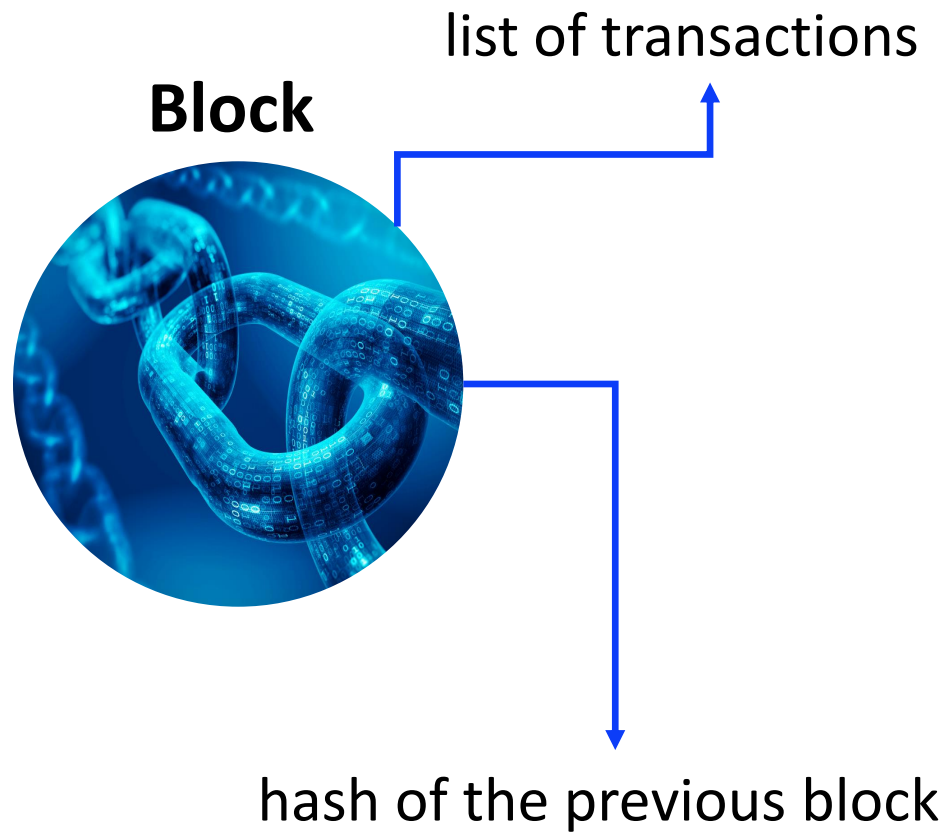
Block



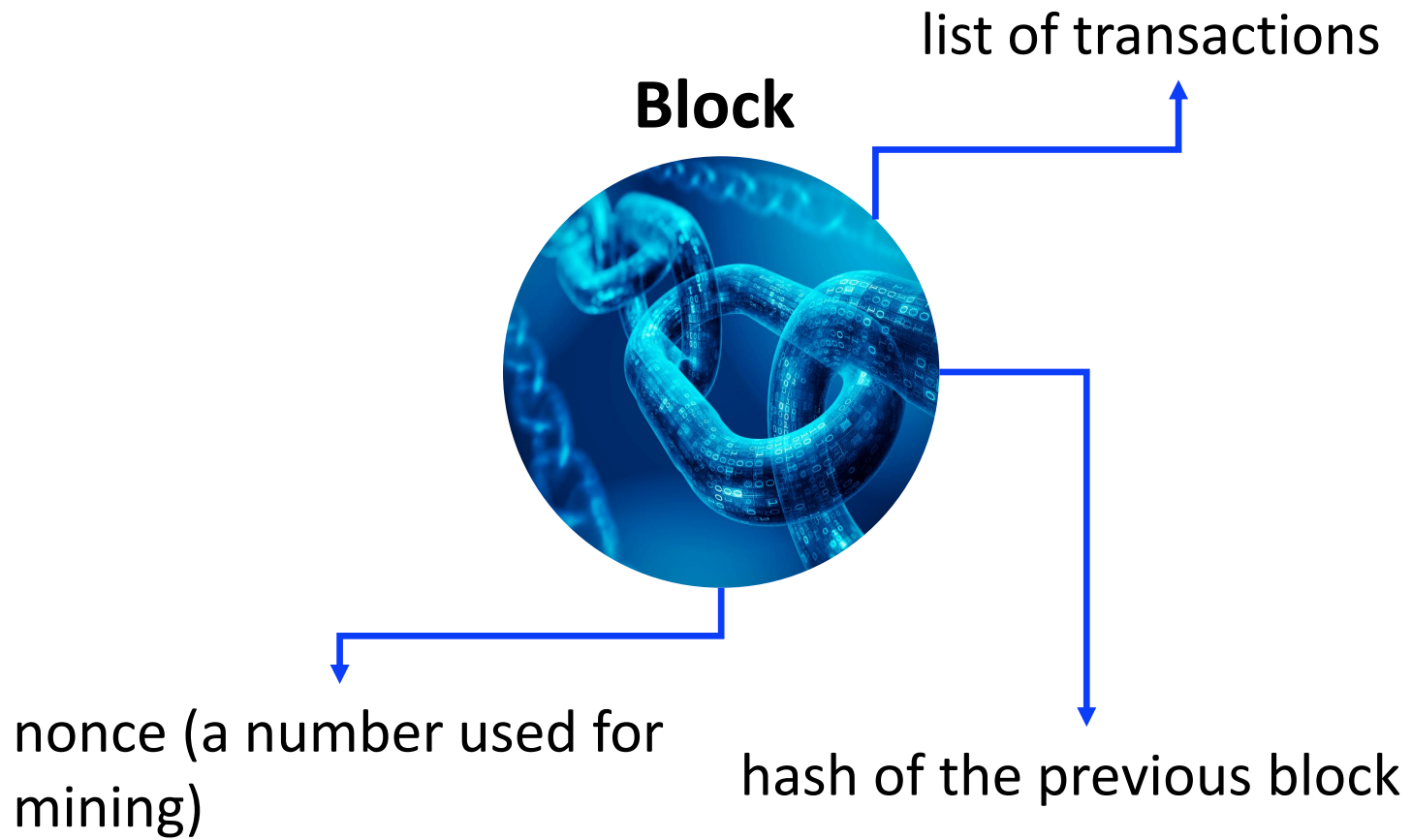
Bitcoin



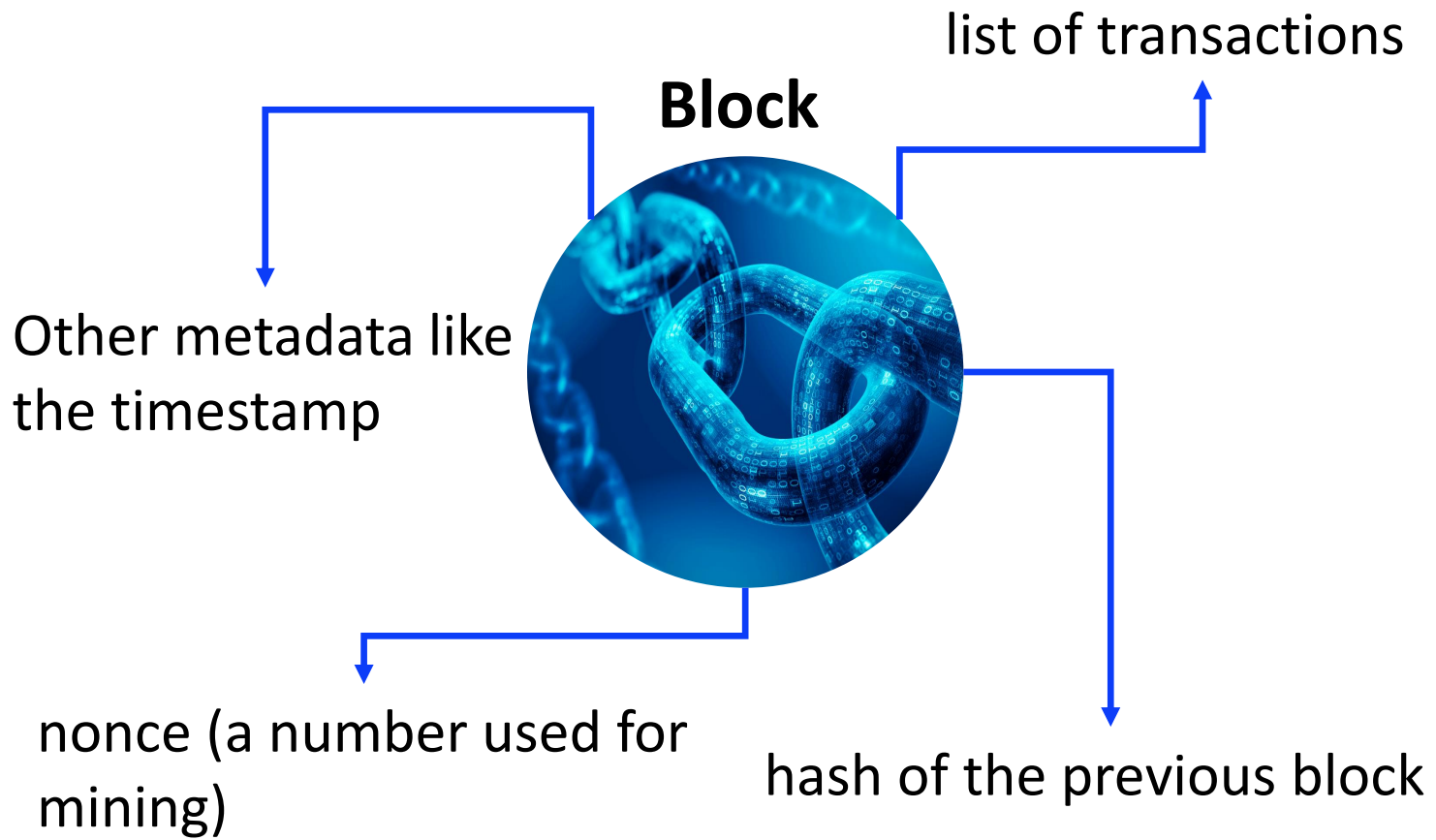
Bitcoin



Bitcoin



Bitcoin



Bitcoin

how does the blockchain work when Bob wants to send Bitcoin to Alice?



Bitcoin

Bob create transaction



Bitcoin

Bob create transaction

Bob wants to send 2 BTC to Alice.



Bob create transaction

Bob wants to send 2 BTC to Alice.

1. **Sender Address:** Bob's Bitcoin address (derived from her public key).
2. **Recipient Address:** Alice's Bitcoin address.
3. **Amount:** 2 BTC.
4. **Digital Signature:** Bob uses his private key to sign the transaction, ensuring it's authentic and can't be altered.



Bitcoin

Broadcasting the transaction



Bitcoin

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The transaction is broadcast to the Bitcoin network
(a peer-to-peer network of nodes)



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- **Signature Validity:** Using Bob's public key, nodes verify that Bob signs the transaction.
- **Sufficient Balance:** Nodes confirm Bob has at least 2 BTC to spend.



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If valid, the transaction is added to the mempool (a pool of pending transactions)



Bitcoin

Mining (Adding to the Blockchain)



Bitcoin

Mining (Adding to the Blockchain)

Miners group pending transactions into a new block. They compete to solve a Proof-of-Work (PoW) puzzle by finding a special number (the nonce) such that:

$$\text{Hash}(\text{Block Data} + \text{Nonce}) < \text{Target Value}$$



Mining (Adding to the Blockchain)

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$$\text{Hash}(\text{Block Data} + \text{Nonce}) < \text{Target Value}$$

Once a miner finds a solution:

1. The new block (containing Bob's transaction) is broadcast to the network
2. Other nodes verify the solution and the validity of the block



Block added to Blockchain



If the block is valid, it is added to the blockchain. The blockchain now has a record of Bob sending 2 BTC to Alice



Bitcoin

Alice receives bitcoin



Bitcoin

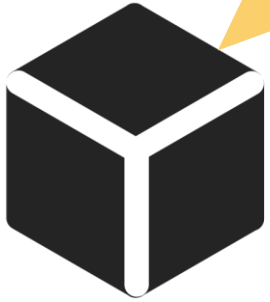
Alice receives bitcoin

Alice's Bitcoin wallet checks the blockchain and sees a transaction crediting him with 2 BTC. The transaction is considered confirmed after multiple blocks are added after the block containing this transaction (this ensures security against potential forks)



Bitcoin

Block 1001
Transactions: [Charlie → Dana: 1 BTC]
Hash: H1001



Bitcoin

Block 1001

Transactions: [Charlie → Dana: 1 BTC]

Hash: H1001



Block 1002

Transactions: [Bob → Alice : 2 BTC]

The hash of Previous Block: H1001

Nonce: 982371

Hash: H1002



Bitcoin

Advantages and challenges



Bitcoin

Advantages and challenges

- Transparency
- security
- the ability to operate without intermediaries

- Scalability
- energy of mining
- regulatory concerns





Edge of science





Divisible E-Cash for Billing in Private Ad Retargeting



Kevin Liao
Henry Corrigan-Gibbs
Dan Boneh

2024

 [Link](#)

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Divisible E-Cash for Billing in Private Ad Retargeting

It examines a solution to solve the privacy problem in Ad Retargeting. The main goal of this research is to design a system that allows anonymous payment and maintains user privacy in the digital advertising process.



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Specifically, the authors propose a system for divisible electronic cash (Divisible E-Cash) that allows users to make smaller transactions without revealing personal information or previous transactions. The system aims to achieve a balance between economic efficiency and privacy.



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Zero-knowledge proofs



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Divisible E-Cash for Billing in Private Ad Retargeting

Zero-knowledge proofs

This system allows users to withdraw a large amount of currency and anonymously divide it into smaller portions.



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Zero-knowledge proofs

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Anonymization protocol

Compatibility with advertising infrastructure



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The system was evaluated based on simulation and real-world experiments, and the results showed that:



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Divisible E-Cash for Billing in Private Ad Retargeting

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- The system latency is only about 63 milliseconds, which is quite acceptable for integration into online advertising.



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- The system performs better than previous methods in terms of anonymization and money segmentation.
- The scalability and security of the system are guaranteed, and it can be used on a large scale for advertising networks.



HybCBDC: A Design for Central Bank Digital Currency(CBDC) Systems Enabling Digital Cash



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The main purpose of CBDC is to support retail and wholesale.



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Transparency



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The main purpose of CBDC is to support retail and wholesale.

Transparency

Transparency in this system conflicts with these rules.

Anti-money laundering(AML)

Countering the finance of terrorism(CFT)



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HybCBDC can solve this problem!



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Layer privacy Model:



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Transactions { Low-risk
High-risk



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Role of intermediaries: central banks don't store sensitive data and some regulated intermediaries check transactions.



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Limits:

1. HybCBDC increases Complexity and operational costs.
2. It doesn't support offline payments.
3. In implementing protocols, it is assumed that the attacker does not have access to transaction information (such as IP, etc.), while in the real world, this assumption is not true.





Thank You

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