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Cloud computing

Numerical integration

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1. Introduction

Purpose of this project is to use computational power of VM (virtual machines) provided by Amazon company to calculate definite integrals. Thanks to AWS, it is possible to check multicore impact on calculations. In this project I will check difference in time between integrating function using my own PC and VM provided by Amazon.

2. Numerical integration

2.1. Calculation of definite integrals

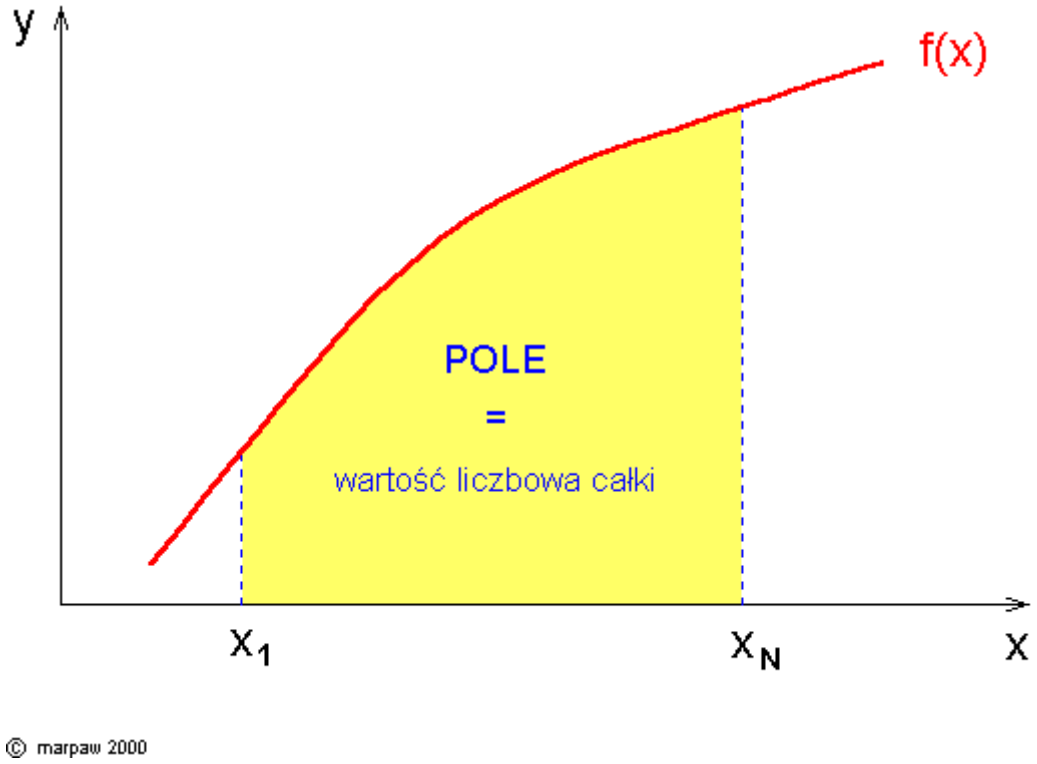


Figure 1 Calculation of definite integral
http://marpaw.elisa.pl/wsti/roznosci/calka/int-pp/int-1.htm

Calculation of definite integral is area below the $f(x)$ function.

2.2. Numerical integration techniques

There are few techniques of calculating integrals, but most common are:

- Midpoint rule
- Trapezoidal rule
- Simpson's rule

In this project, trapezoidal rule will be used. It is more accurate then popular midpoint rule.

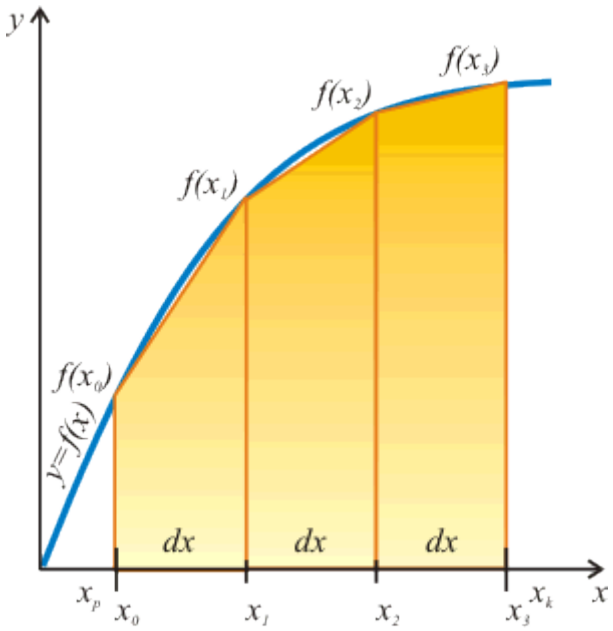


Figure 2 Trapezoidal rule
https://eduinf.waw.pl/inf/alg/004_int/0003.php

2.3. Trapezoidal rule

Integration section

$$[x_p, x_k]$$

Points x_i

Points are designated by dividing section.

$$x_i = x_p + \frac{i}{n}(x_k - x_p)$$

dx – length between two points

$$dx = \frac{x_k - x_p}{n}$$

Value of integral

Value of the integral is are below the function. It is calculated by adding areas of trapezoids.

$$s = P_1 + P_2 + \cdots + P_n$$

$$s = \frac{f_0 + f_1}{2}dx + \frac{f_1 + f_2}{2}dx + \cdots + \frac{f_{n-1} + f_n}{2}dx$$

$$s = \frac{dx}{2}(f_0 + 2f_1 + 2f_2 + \cdots + f_n)$$

$$s = dx\left(f_1 + f_2 + \cdots + \frac{f_0 + f_n}{2}\right)$$

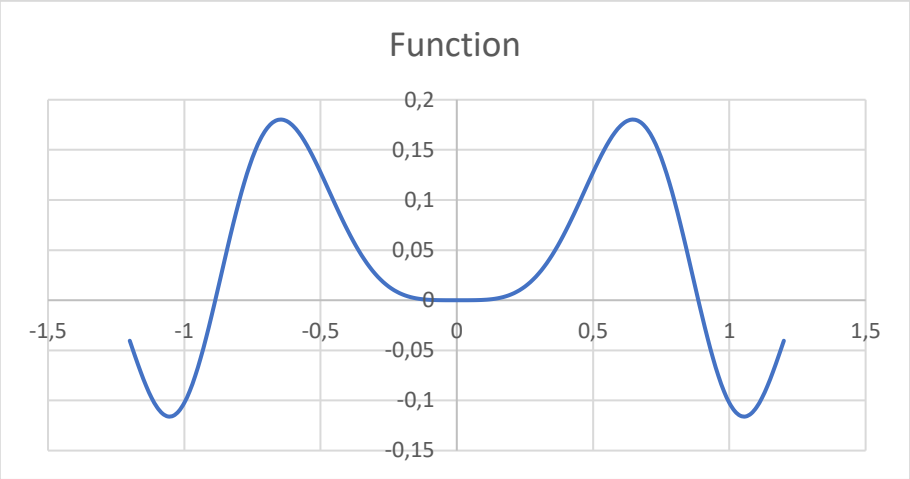
3. Solution

3.1. Function

To calculate the difference between integration on my own PC and using AWS I used following function:

$$f(x) = e^{-2x^2} x^2 sin4x^2 \cos(-4x^2)$$

Function is shown below on graph 1. It is drawn from -1,2 to 1,2.



Graph 1 Function

3.2. Inegration

$$\int_{-1,2}^{1,2} e^{-2x^2} x^2 sin4x^2 \cos(-4x^2)\ dx$$

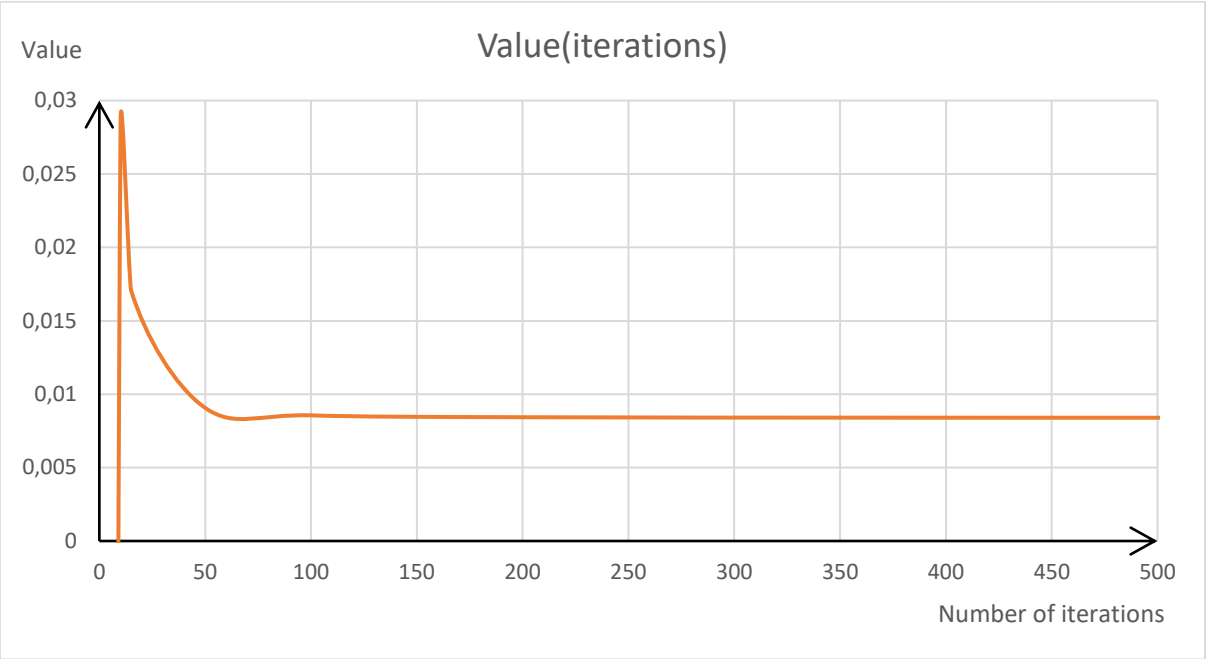
3.3. Solution in Wolfram Alpha

Using Wolfram Alpha I got following solution:

$$\int_{-1,2}^{1,2} e^{-2x^2} x^2 sin4x^2 \cos(-4x^2)\ dx = 0{,}0839019$$

3.4. Solution using my PC

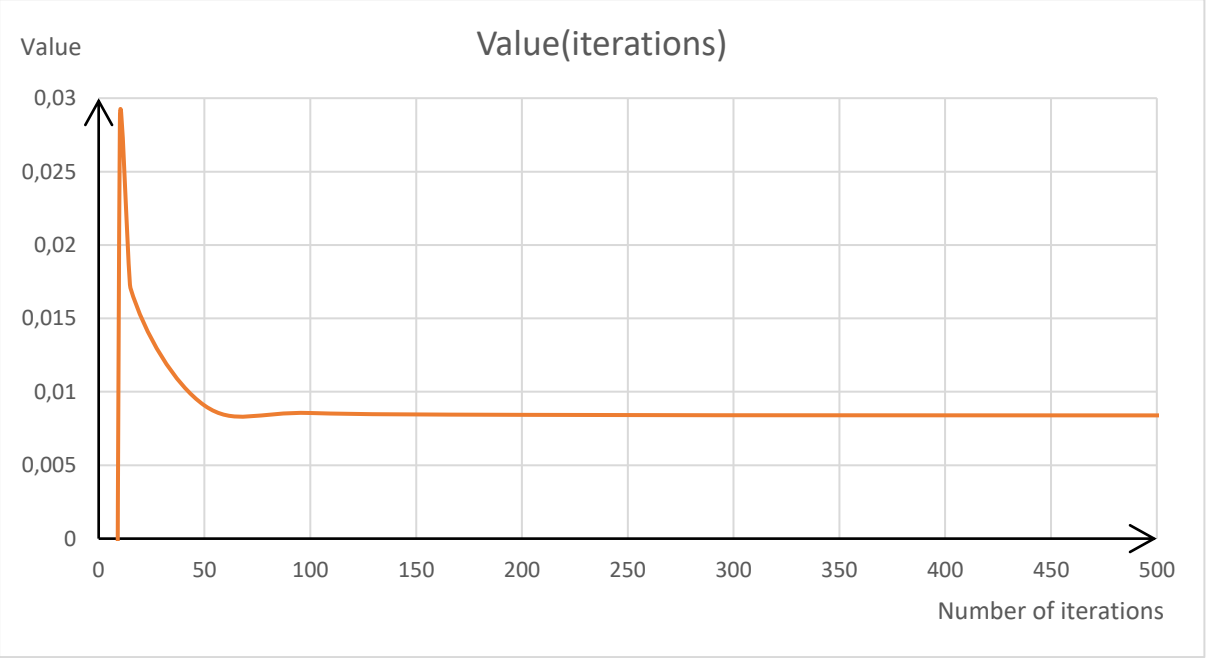
Depending on number of iterations, solution got much more precise. After 100 iterations value didn't change significantly.



Graph 2 Value using my PC

3.5. Solution using AWS

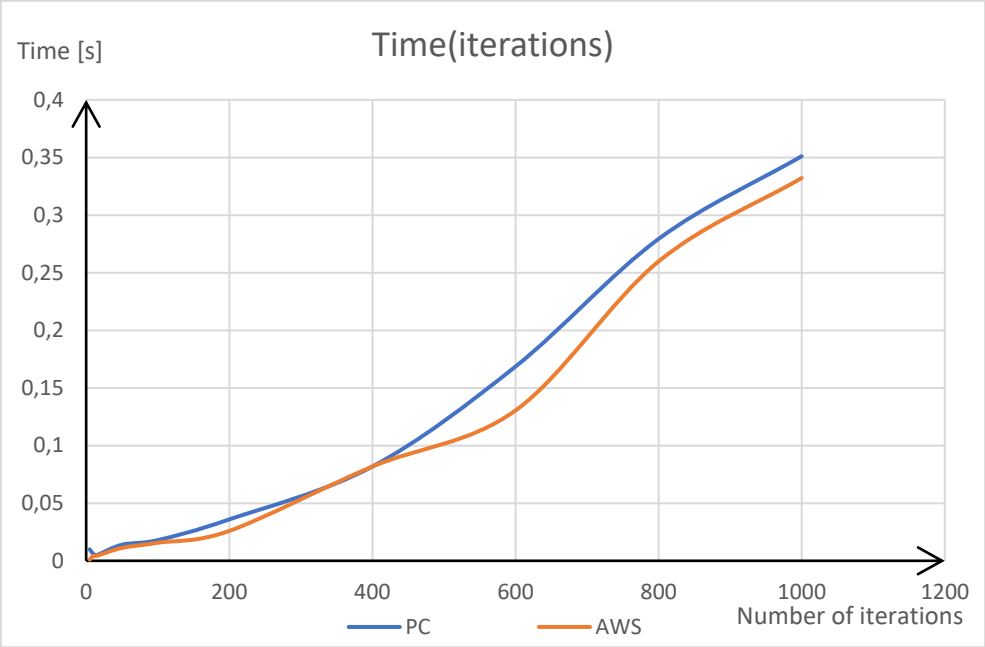
Solution in AWS was the same. No surprise here, as the code was the same. Only difference could be in time to calculate.



Graph 3 Value using AWS

3.6. Comparing 2 methods

The outcome between PC and AWS calculations must be the same, because it is only code related. The reason behind this project was to check, if it is more valuable to use AWS to services to calculate faster. Graph 3 shows the speeds of calculations depending on number of iterations. As you can see there is the difference, but it is very marginal.



Graph 4 Time

3.7. Testing

I've tested even more complex functions to see if is there any significant difference. The more iterations the bigger was the difference, but it was still marginal. Despite time difference, it was pointless to give very big number of iterations (>1000), because the value of iteration didn't change at all.

It is worth mentioning that the time given to calculate one case wasn't always the same.

4. Summary

There was only marginal difference between cases (PC vs AWS solution). There are few reasons behind it:

- too easy task
- cost of communication between CPUs

As the task was too easy, there was no benefit of using more CPUs. Using more and faster CPUs, there were many losses due to communication time. Thus there was only marginal difference.