Hadamard Submatrix

February 24, 2018

0.1 Problem

What happens when you start with a matrix M, and you define M'_{ij} to be the Hadamard product (sum of the element products) of M by a matrix which is mostly 0 except for a 3x3 submatrix of all (1/9) centered around (i,j)? (this is simultaneously done for all i, j. Yikes!) Test your hypothesis by doing it to an image file of your face.

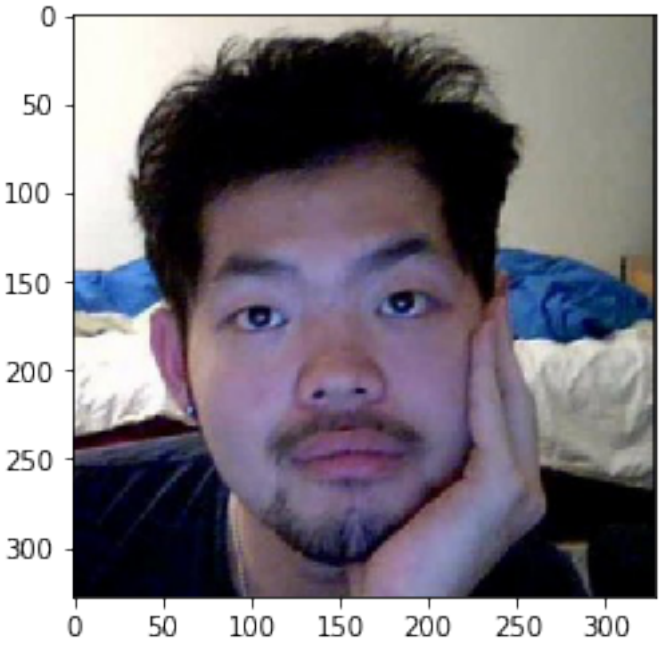
0.1.1 Setup

Import libraries and load the image.

```
In [1]: from PIL import Image
        import math
        import numpy as np
        from matplotlib.pyplot import imshow
        %matplotlib inline
```

```
In [2]: #original = Image.open('../image_tilt/aj_face.jpeg')
        original = Image.open('yanxzhang.jpeg')
        image_tensor = np.asarray(original)
        # print(image_tensor, image_tensor.dtype)
        imshow(image_tensor)
```

```
Out[2]: <matplotlib.image.AxesImage at 0x7f17040188d0>
```



We have a 328x328 image with 3 color channels, dtype is unsigned 8 bit integer.

```
In [3]: type(image_tensor), image_tensor.shape, image_tensor.dtype

Out[3]: (numpy.ndarray, (328, 328, 3), dtype('uint8'))

In [4]: w = image_tensor.shape[0]
        h = image_tensor.shape[1]
        D = image_tensor.shape[2]

In [5]: # twobytwo = image_tensor[0:2,0:2,0] * 1/4
        # print(twobytwo)
        # np.sum(twobytwo)
```

0.1.2 Create a function to apply the transformation

For each pixel in the image, apply the transformation individually on each rgb layer. Note that values are capped at 255.

```
In [6]: def apply_transformation(size):
            """
            Applies a transformation to each pixel by averaging from pixels in its surrounding
            size: size of the matrix, positive odd integer. if even, will effectively round up
            """
            margin = int(size/2)
            transformed = np.zeros(image_tensor.shape, dtype=np.uint8) # setting dtype=np.uint
            for x in range(w):
```

```
            for y in range(h):
                # define the region to sample from
                x_min = x-margin if x-margin >= 0 else 0
                x_max = x+margin+1 if x+margin+1 < w else w
                y_min = y-margin if y-margin >= 0 else 0
                y_max = y+margin+1 if y+margin+1 < h else h
                factor = 1 / ((x_max - x_min) * (y_max-y_min)) # set the factor to properl
                # apply the transformation per rgb layer
                layers = []
                for z in range(D):
                    submatrix = np.sum(image_tensor[x_min:x_max, y_min:y_max, z] * factor)
#                     print(submatrix) DEBUG
                    # cap the value at 255
                    submatrix = np.clip(submatrix, 0, 255)
                    layers.append(submatrix)
                transformed[x_min:x_max, y_min:y_max] = np.asarray(layers)
        return transformed
```

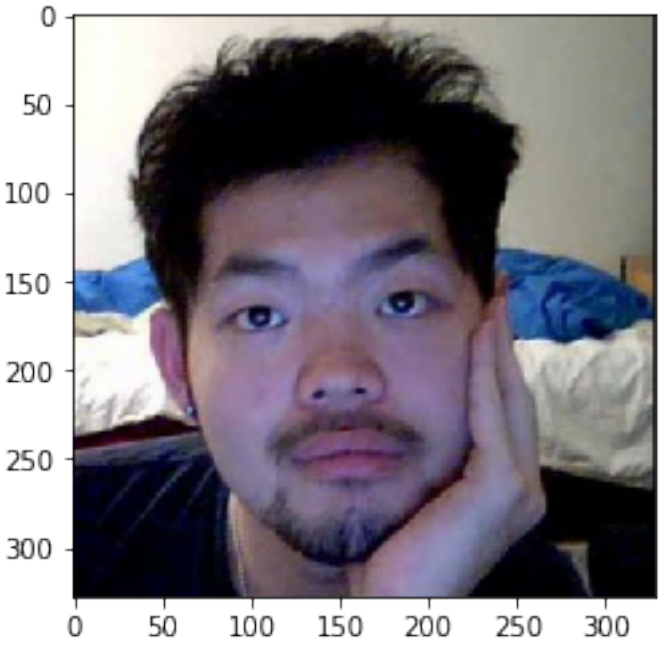
```
In [7]: # def save_image(image, size, fa3ctor):
        #     "Saves an image with the filename transform_sizexsize_1-factor"
        #     filename = f'transform_{size}x{size}_1-{1/factor}.jpg'
        #     savable = Image.fromarray(image, 'RGB')
        #     image.save(savable)
```

0.1.3 Try the transformation with different parameters.

A transformation with submatrix size 1 should return back the same image.

```
In [8]: one = apply_transformation(1)
        imshow(one)
        #save_image(one_third)
```

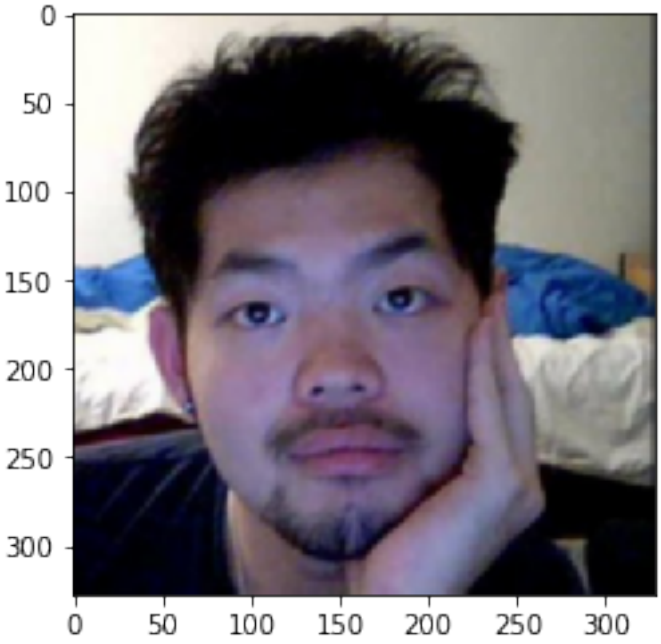
```
Out[8]: <matplotlib.image.AxesImage at 0x7f16f6d50198>
```



Nice. How about for size 3?

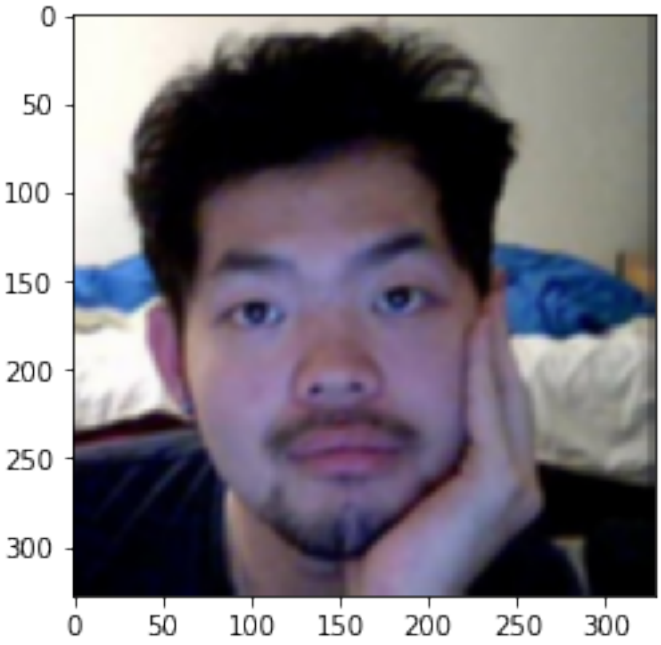
```
In [9]: three = apply_transformation(3)
        imshow(three)
```

```
Out[9]: <matplotlib.image.AxesImage at 0x7f16f6d33c18>
```

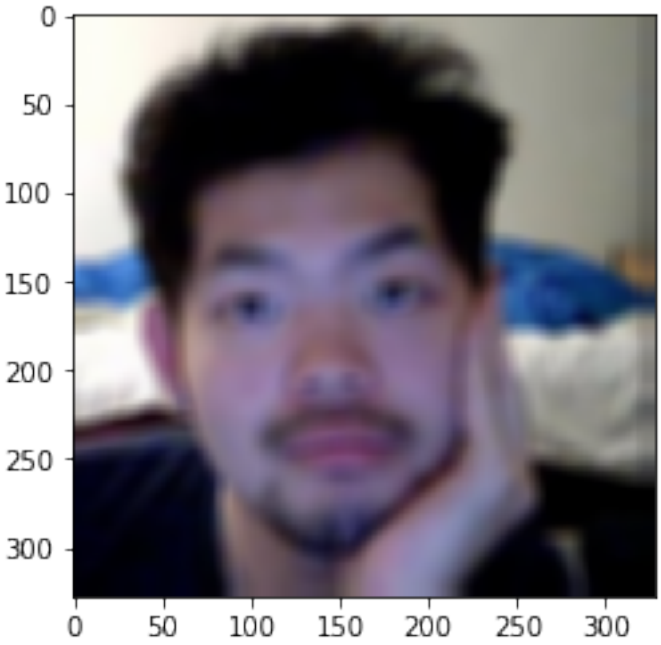


Hm... not much difference. What if we go bigger?

```
In [10]: five = apply_transformation(5)
         imshow(five)

Out[10]: <matplotlib.image.AxesImage at 0x7f16f6c99080>
```



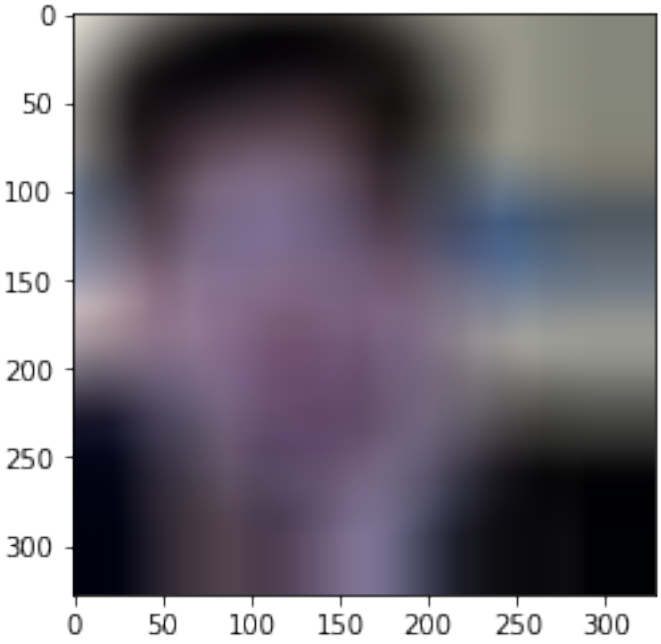

```
In [11]: eleven = apply_transformation(11)
         imshow(eleven)

Out[11]: <matplotlib.image.AxesImage at 0x7f16f6c7b400>
```



We're starting to get some significant results!

```
In [12]: imshow(apply_transformation(65))
```

```
Out[12]: <matplotlib.image.AxesImage at 0x7f16f6bd7860>
```



Conclusion: The transformation blurs the image. Win!