Image Tilt Notebook

February 12, 2018

1 0: Task and Tools

The goal is to code the rotation of an image by 30 degrees using a rotation matrix.

We'll use *PIL* for importing images, *math* for sin/cos, *numpy* for linear algebra, and *imshow* to display our images in the notebook.

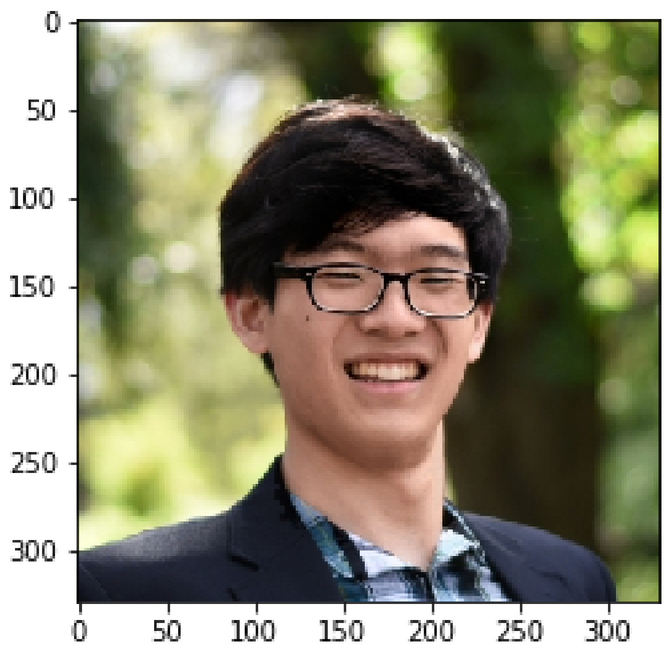
```
In [1]: from PIL import Image
        import math
        import numpy as np
        from matplotlib.pyplot import imshow
```

2 1: Transforming our image (rgb tensor) to cartesian coordinates mapped to rgb values.

First we load in our image:

```
In [2]: %matplotlib inline
        original = Image.open('aj_face.jpeg')
        image_tensor = np.asarray(original)
        imshow(image_tensor)

Out[2]: <matplotlib.image.AxesImage at 0x7f48141159e8>
```



As we can see below, the image loads as a numpy ndarray with 329 rows, 330 columns, and 3 depths. That's 329 in *width*, 330 pixels in *height*, and 3 rgb values in *Depth*. At this point, the coordinate information for each rgb is stored implicitly. We want to extract the coordinates explicitly so we can perform a rotation upon each.

```
In [3]: type(image_tensor), image_tensor.shape, image_tensor.dtype

Out[3]: (numpy.ndarray, (329, 330, 3), dtype('uint8'))
```

To do this, I'm going to flatten our numpy array, so that it's a long list of aggregated rgb values. That way I can attach coordinates to each later.

```
In [4]: w = image_tensor.shape[0]
        h = image_tensor.shape[1]
        D = image_tensor.shape[2] # D is capital to indicate it as a constant. we won't be mod

        flattened_pixels = image_tensor.reshape((w * h, D)) # directly from np.array(im.getdat
        flattened_pixels, flattened_pixels.shape, flattened_pixels.dtype

Out[4]: (array([[ 87,  90,  59],
                [ 93,  98,  66],
                [100, 105,  73],
                ...,
                [146, 165,  83],
                [144, 166,  83],
                [144, 166,  84]], dtype=uint8), (108570, 3), dtype('uint8'))
```

To attach the coordinates correctly, let's make sure we understand how the reshaping works. From the original tensor of rgb values, we'll select the first 5 rgb groups along the y axis at the first x. By comparing to the flattened output above, we know the flattening goes by columns.

```
In [5]: image_tensor[0,:5,:]
```

```
Out[5]: array([[ 87,  90,  59],
               [ 93,  98,  66],
               [100, 105,  73],
               [115, 122,  91],
               [131, 140, 109]], dtype=uint8)
```

Now that we know the order in which the pixels flattened, we can appropriately map each pixel's coordinate as a vector to its rgb value. We also translate our coordinates so that the origin is at the center of the image.

```
In [6]: # create a bunch of x,y vectors
        dx = int(w/2)
        dy = int(h/2)
        coordinate_generator = (np.array([x - dx, y - dy]) for x in range(w) for y in range(h)
        coordinates_and_rgb = list(zip(coordinate_generator, flattened_pixels)) # take out the
        print('Transformed output, formatted (coordinate, rgb):')
        coordinates_and_rgb[:5]
```

```
Transformed output, formatted (coordinate, rgb):
```

```
Out[6]: [(array([-164, -165]), array([87, 90, 59], dtype=uint8)),
         (array([-164, -164]), array([93, 98, 66], dtype=uint8)),
         (array([-164, -163]), array([100, 105,  73], dtype=uint8)),
         (array([-164, -162]), array([115, 122,  91], dtype=uint8)),
         (array([-164, -161]), array([131, 140, 109], dtype=uint8))]
```

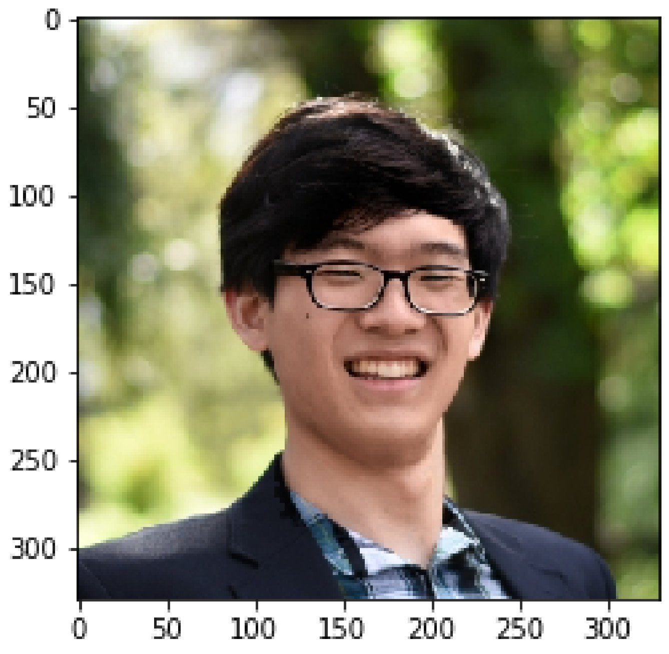
Before going further, I'll write a function to transform our mapped data structure back to the original, so we know it works as intended.

```
In [7]: def points_to_tensor(points, shape):
            tensor = np.zeros(shape, np.uint8)
            dx = shape[0] / 2
            dy = shape[1] / 2
            for point in points:
                (x, y), rgb = point # might be something wonky with row/col
                # add back half the width and height to center the image
                x, y = int(x + dx), int(y + dy)
                tensor[x][y] = rgb
                #print(f'Row: {row}. Col: {col}')
            return tensor
```

```
In [8]: should_go_back = points_to_tensor(coordinates_and_rgb, image_tensor.shape)
        print(f'Original shape: {image_tensor.shape}. Return shape: {should_go_back.shape}')
        print(f'Original first 5:\n{image_tensor[0,:5,:]} \nReturn first 5:\n{should_go_back[0
        imshow(Image.fromarray(should_go_back, 'RGB'))
```

```
Original shape: (329, 330, 3). Return shape: (329, 330, 3)
Original first 5:
[[ 87  90  59]
 [ 93  98  66]
 [100 105  73]
 [115 122  91]
 [131 140 109]]
Return first 5:
[[ 87  90  59]
 [ 93  98  66]
 [100 105  73]
 [115 122  91]
 [131 140 109]]
```

```
Out[8]: <matplotlib.image.AxesImage at 0x7f48056fdbe0>
```



Great! We get back the original. Now we can start rotating points.

3 2: Applying the rotation

First we need to define a function to get a rotation matrix for a specified angle.

```
In [9]: def make_rotation_matrix(degree):
            " Return a rotation matrix that rotates by the given angle in degrees "
            theta = math.radians(degree)
            return np.array([
                [math.cos(theta), -math.sin(theta)],
                [math.sin(theta), math.cos(theta)]
            ])
```

Let's test this by creating a rotation matrix for 30 degrees counterclockwise.

```
In [10]: # make sure rotation 30 degrees is accurate
         rotate_30_test = make_rotation_matrix(30)
         rotate_30_test # we won't use this variable again

Out[10]: array([[ 0.8660254, -0.5      ],
                [ 0.5      ,  0.8660254]])
```

Now we can apply the matrix transformation to each of the coordinates.

```
In [11]: def rotate_func(degrees):
             """
             Returns a rotation function that can be used with map.
             """
             rotation_matrix = make_rotation_matrix(degrees)
             def rotate(point):
                 "Applies the rotation transformation"
                 location, rgb = point
                 new_location = rotation_matrix.dot(location)
                 return (new_location, rgb)
             return rotate


         rotated_points = np.array(list(map(rotate_func(30), coordinates_and_rgb)))
         rotated_points[:5]

Out[11]: array([[array([ -59.52816622, -224.89419162]),
                 array([87, 90, 59], dtype=uint8)],
                [array([ -60.02816622, -224.02816622]),
                 array([93, 98, 66], dtype=uint8)],
                [array([ -60.52816622, -223.16214082]),
                 array([100, 105,  73], dtype=uint8)],
                [array([ -61.02816622, -222.29611541]),
                 array([115, 122,  91], dtype=uint8)],
                [array([ -61.52816622, -221.43009001]),
                 array([131, 140, 109], dtype=uint8)]], dtype=object)
```

Get the min and max x and y values, so we know how big to make the tensor that will hold our rotated points.

```
In [12]: min_x = min(rotated_points, key=lambda a: a[0][0]) [0][0] # coordinate, x
         max_x = max(rotated_points, key=lambda a: a[0][0]) [0][0] # coordinate, x
         min_y = min(rotated_points, key=lambda a: a[0][1]) [0][1] # coordinate, y
         max_y = max(rotated_points, key=lambda a: a[0][1]) [0][1] # coordinate, y

         rot_width = math.ceil(max_x - min_x)
         rot_height = math.ceil(max_y - min_y)
         print(f'Width: {rot_width}. Height: {rot_height}')
```

```
Width: 449. Height: 449
```

Our image dimensions grew. That's to be expected, as rotating moves the corners farther up and out.

Iterate through our rotated points, and use their location to place them at the appropriate index in the target tensor.

```
In [13]: tilted_image = points_to_tensor(rotated_points, (rot_width+1, rot_height, D)) # for s
         tilted_image[:2]
```

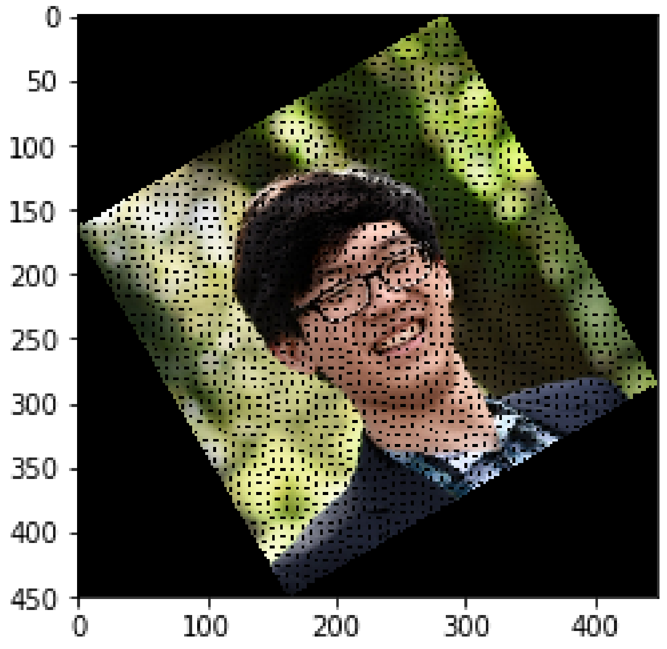
```
Out[13]: array([[[0, 0, 0],
                 [0, 0, 0],
                 [0, 0, 0],
                 ...,
                 [0, 0, 0],
                 [0, 0, 0],
                 [0, 0, 0]],

                [[0, 0, 0],
                 [0, 0, 0],
                 [0, 0, 0],
                 ...,
                 [0, 0, 0],
                 [0, 0, 0],
                 [0, 0, 0]]], dtype=uint8)
```

Make an image from the tensor and see what we get!

```
In [14]: img = Image.fromarray(tilted_image, 'RGB')
         img.save('tilted.png')
         imshow(img)
```

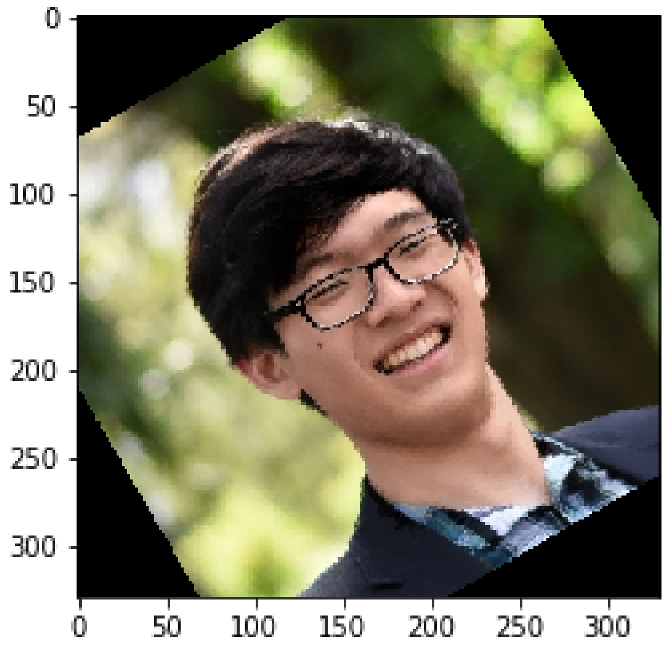
```
Out[14]: <matplotlib.image.AxesImage at 0x7f48056578d0>
```



We'll perform a true rotation using a built in function to check.

```
In [16]: true_rotate = original.rotate(30)
         imshow(true_rotate)

Out[16]: <matplotlib.image.AxesImage at 0x7f4805396710>
```



Hurray!