Hadamard Submatrix

February 24, 2018

1 Problem

What happens when you start with a matrix M, and you define M'_{ij} to be the Hadamard product (sum of the element products) of M by a matrix which is mostly 0 except for a 3x3 submatrix of all (1/9) centered around (i,j)? (this is simultaneously done for all i, j. Yikes!) Test your hypothesis by doing it to an image file of your face.

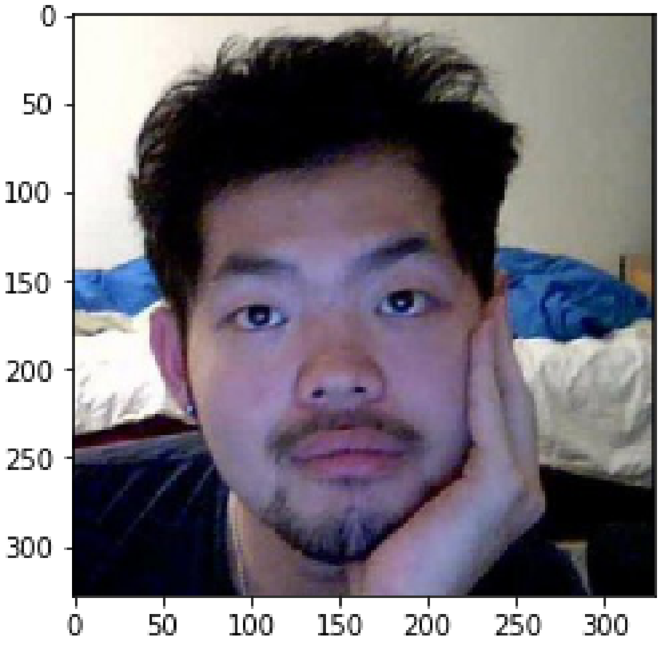
2 Setup

Import libraries and load the image.

```
In [1]: from PIL import Image
        import math
        import numpy as np
        from matplotlib.pyplot import imshow
        %matplotlib inline
```

```
In [2]: #original = Image.open('../image_tilt/aj_face.jpeg')
        original = Image.open('yanxzhang.jpeg')
        image_tensor = np.asarray(original)
        # print(image_tensor, image_tensor.dtype)
        imshow(image_tensor)
```

```
Out[2]: <matplotlib.image.AxesImage at 0x7f17040188d0>
```



We have a 328x328 image with 3 color channels, dtype is unsigned 8 bit integer.

```
In [3]: type(image_tensor), image_tensor.shape, image_tensor.dtype

Out[3]: (numpy.ndarray, (328, 328, 3), dtype('uint8'))

In [4]: w = image_tensor.shape[0]
        h = image_tensor.shape[1]
        D = image_tensor.shape[2]

In [5]: # twobytwo = image_tensor[0:2,0:2,0] * 1/4
        # print(twobytwo)
        # np.sum(twobytwo)
```

3 Create a function to apply the transformation

For each pixel in the image, apply the transformation individually on each rgb layer. Note that values are capped at 255.

```
In [6]: def apply_transformation(size):
            """
            Applies a transformation to each pixel by averaging from pixels in its surrounding
            size: size of the matrix, positive odd integer. if even, will effectively round up
            """
            margin = int(size/2)
            transformed = np.zeros(image_tensor.shape, dtype=np.uint8) # setting dtype=np.uint
```

```
    for x in range(w):
        for y in range(h):
            # define the region to sample from
            x_min = x-margin if x-margin >= 0 else 0
            x_max = x+margin+1 if x+margin+1 < w else w
            y_min = y-margin if y-margin >= 0 else 0
            y_max = y+margin+1 if y+margin+1 < h else h
            factor = 1 / ((x_max - x_min) * (y_max-y_min)) # set the factor to properl
            # apply the transformation per rgb layer
            layers = []
            for z in range(D):
                submatrix = np.sum(image_tensor[x_min:x_max, y_min:y_max, z] * factor)
#                 print(submatrix) DEBUG
                # cap the value at 255
                submatrix = np.clip(submatrix, 0, 255)
                layers.append(submatrix)
            transformed[x_min:x_max, y_min:y_max] = np.asarray(layers)
    return transformed
```

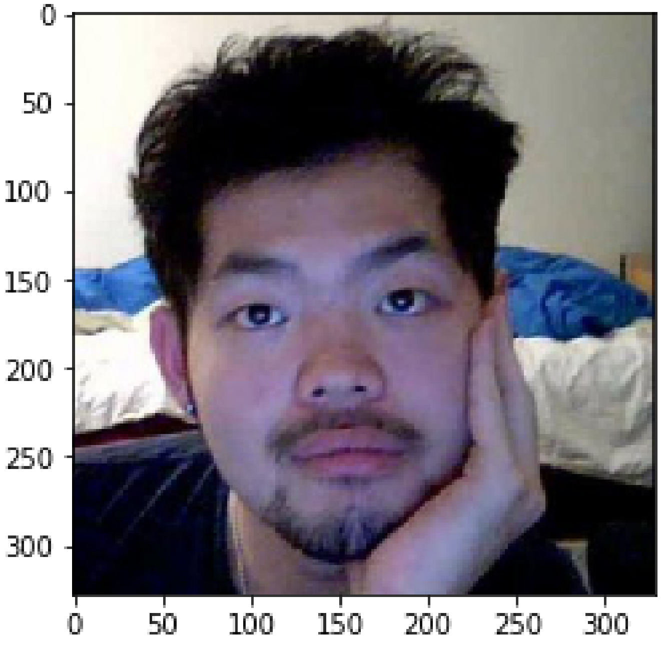
```
In [7]: # def save_image(image, size, fa3ctor):
        #     "Saves an image with the filename transform_sizexsize_1-factor"
        #     filename = f'transform_{size}x{size}_1-{1/factor}.jpg'
        #     savable = Image.fromarray(image, 'RGB')
        #     image.save(savable)
```

4 Try the transformation with different parameters.

A transformation with submatrix size 1 should return back the same image.

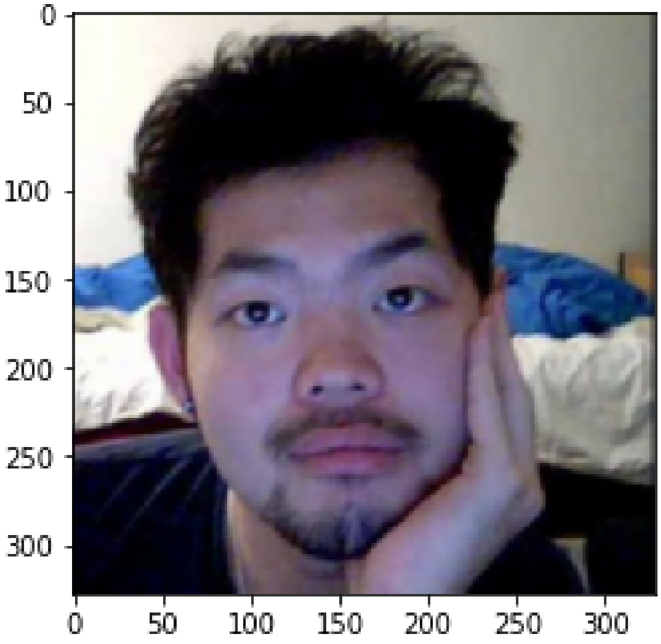
```
In [8]: one = apply_transformation(1)
        imshow(one)
        #save_image(one_third)
```

```
Out[8]: <matplotlib.image.AxesImage at 0x7f16f6d50198>
```



Nice. How about for size 3?

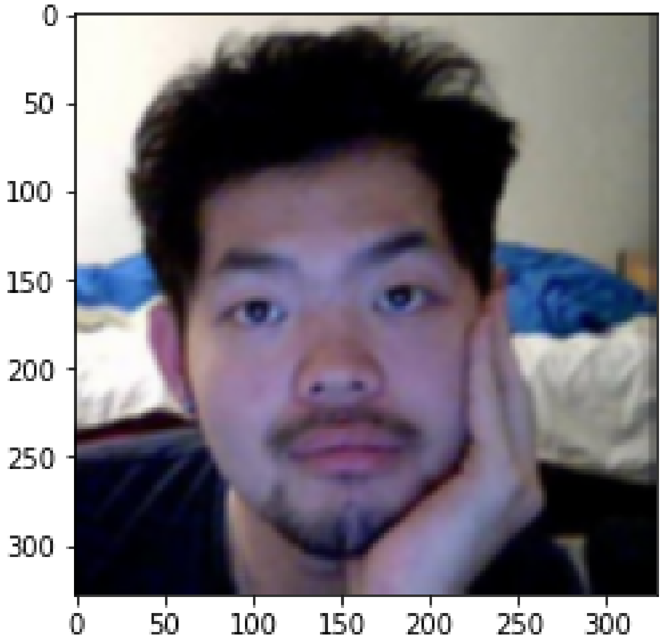
```
In [9]: three = apply_transformation(3)
        imshow(three)

Out[9]: <matplotlib.image.AxesImage at 0x7f16f6d33c18>
```



Hm... not much difference. What if we go bigger?

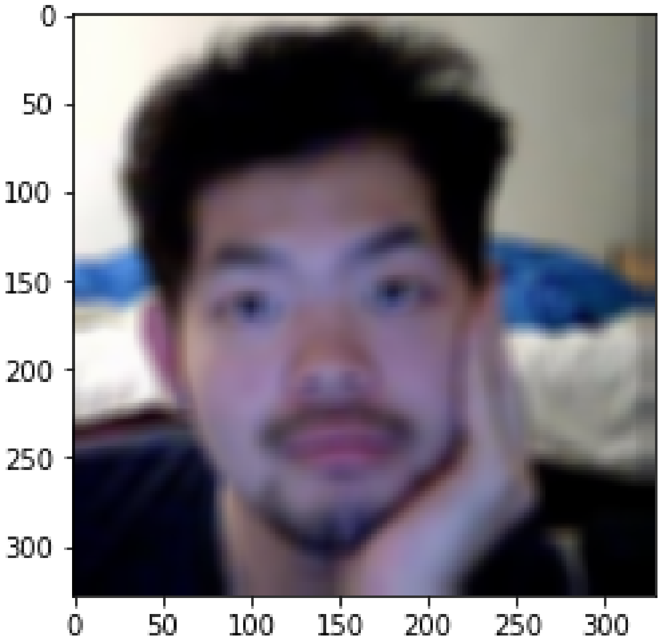
```
In [10]: five = apply_transformation(5)
         imshow(five)
```

```
Out[10]: <matplotlib.image.AxesImage at 0x7f16f6c99080>
```



```
In [11]: eleven = apply_transformation(11)
         imshow(eleven)
```

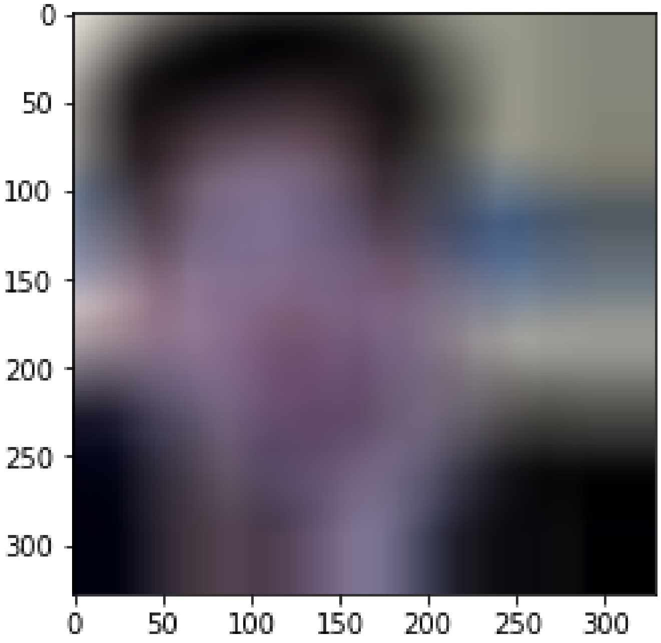
```
Out[11]: <matplotlib.image.AxesImage at 0x7f16f6c7b400>
```



We're starting to get some significant results!

```
In [12]: imshow(apply_transformation(65))
```

```
Out[12]: <matplotlib.image.AxesImage at 0x7f16f6bd7860>
```



5 Conclusion: The transformation blurs the image. Cool!