Hadamard Submatrix

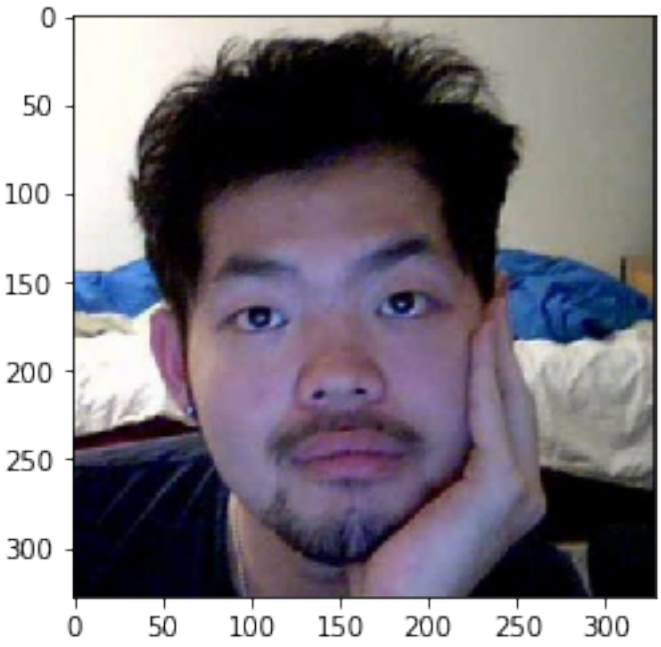
February 20, 2018

What happens when you start with a matrix M, and you define M'_{ij} to be the Hadamard product (sum of the element products) of M by a matrix which is mostly 0 except for a 3x3 submatrix of all () centered around (i,j)? (this is simultaneously done for all i, j. Yikes!) Test your hypothesis by doing it to an image file of your face.

In [1]:
```
from PIL import Image
import math
import numpy as np
from matplotlib.pyplot import imshow
%matplotlib inline
```

In [2]:
```
#original = Image.open('../image_tilt/aj_face.jpeg')
original = Image.open('yanxzhang.jpeg')
image_tensor = np.asarray(original)
imshow(image_tensor)
```

Out[2]:
```
<matplotlib.image.AxesImage at 0x7f864024f898>
```



```
In [3]: type(image_tensor), image_tensor.shape, image_tensor.dtype

Out[3]: (numpy.ndarray, (328, 328, 3), dtype('uint8'))

In [4]: w = image_tensor.shape[0]
        h = image_tensor.shape[1]
        D = image_tensor.shape[2] # D is capital to indicate it as a constant. we won't be mod
```

Create a matrix to contain the transformed result that is the same shape as the original.

For each pixel in the image, apply the transformation individually on each rgb layer. Note that values are capped at 255.

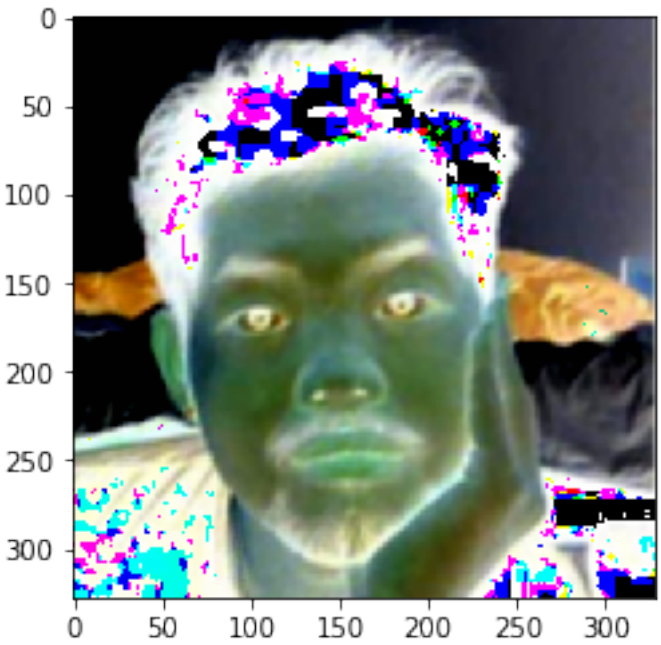
```
In [5]: def apply_transformation(size, factor):
            """
            Applies a transformation to each pixel by
            size: size of the matrix, positive odd integer. if even, will effectively round up
            factor: a ratio to multiply by
            """
            margin = int(size/2)
            transformed = np.zeros(image_tensor.shape)
            transformed.shape, transformed.dtype
            for x in range(w):
                for y in range(h):
                    # define the region to sample from
                    x_min = x-margin if x-margin >= 0 else 0
                    x_max = x+margin if x+margin < w else w-1
                    y_min = y-margin if y-margin >= 0 else 0
                    y_max = y+margin if y+margin < h else h-1
                    # apply the transformation per rgb layer
                    layers = []
                    for z in range(D):
                        value = np.rint(np.sum(image_tensor[np.ix_([x_min,x_max],[y_min,y_max]
                        # cap the value at 255
                        if value > 255:
                            value = 255
                        layers.append(value)
                    transformed[x][y] = np.asarray(layers)
            return transformed

In [6]: # def save_image(image, size, factor):
        #     "Saves an image with the filename transform_sizexsize_1-factor"
        #     filename = f'transform_{size}x{size}_1-{1/factor}.jpg'
        #     savable = Image.fromarray(image, 'RGB')
        #     image.save(savable)
```

Try the transformation with different parameters, matrix size and multiply factor.

```
In [7]: size = 3
        factor = 1/3
```

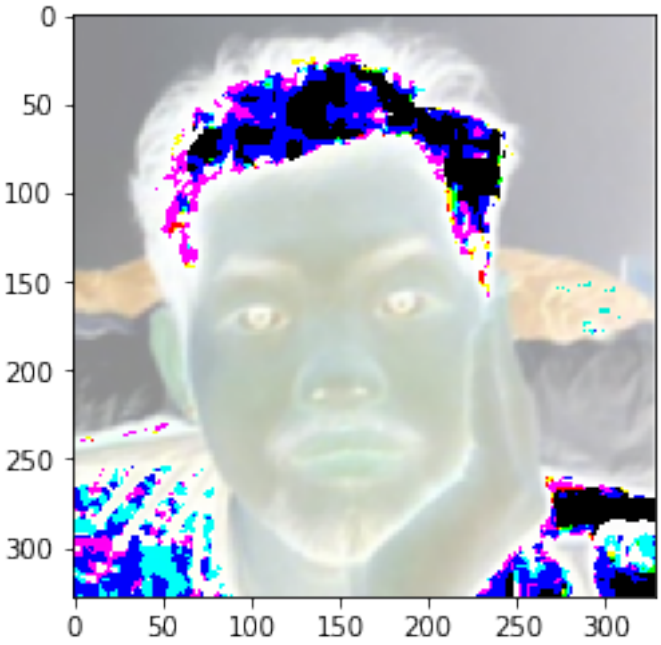
```
one_third = apply_transformation(size,factor)
imshow(one_third)
#save_image(one_third)
```

Out[7]: <matplotlib.image.AxesImage at 0x7f86401834a8>



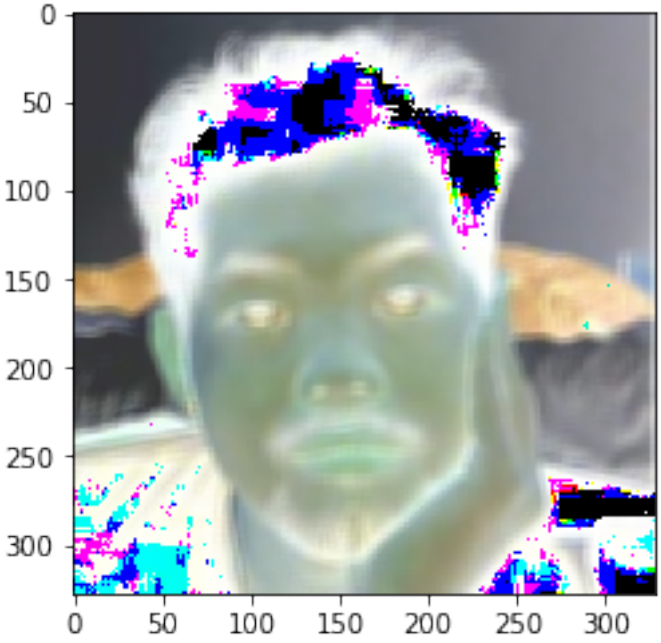
In [9]:
```
size = 3
factor = 1/9
threebythree_ninth = apply_transformation(size, factor)
imshow(threebythree_ninth)
#save_image(threebythree_ninth)
```

Out[9]: <matplotlib.image.AxesImage at 0x7f864007aa20>



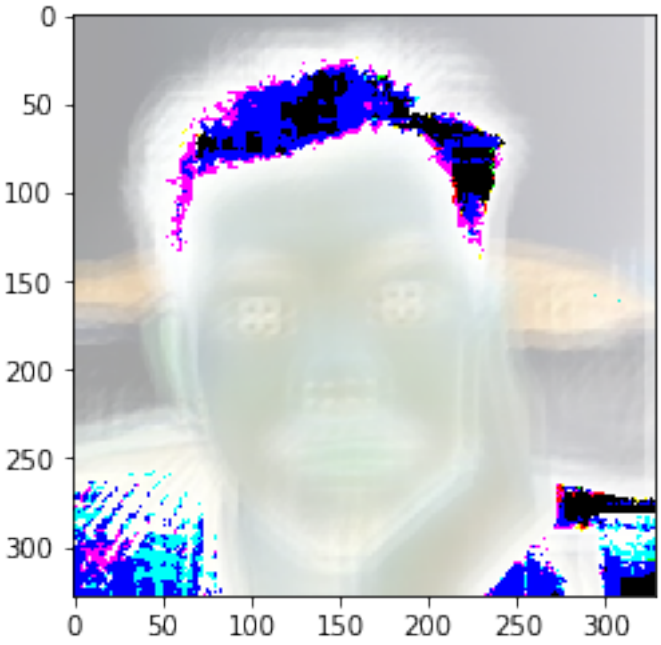
In [10]:
```
size = 5
factor = 1/5
five = apply_transformation(size, factor)
imshow(five)
#save_image(five, size, factor)
```

Out[10]: <matplotlib.image.AxesImage at 0x7f8640059e48>



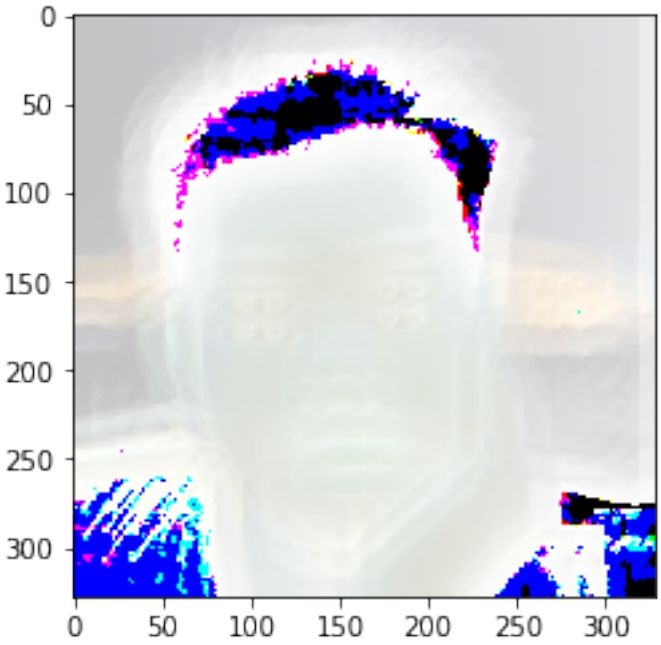
In [11]:
```
size = 11
factor = 1/11
eleven = apply_transformation(size, factor)
imshow(eleven)
```

Out[11]: <matplotlib.image.AxesImage at 0x7f862ea992e8>



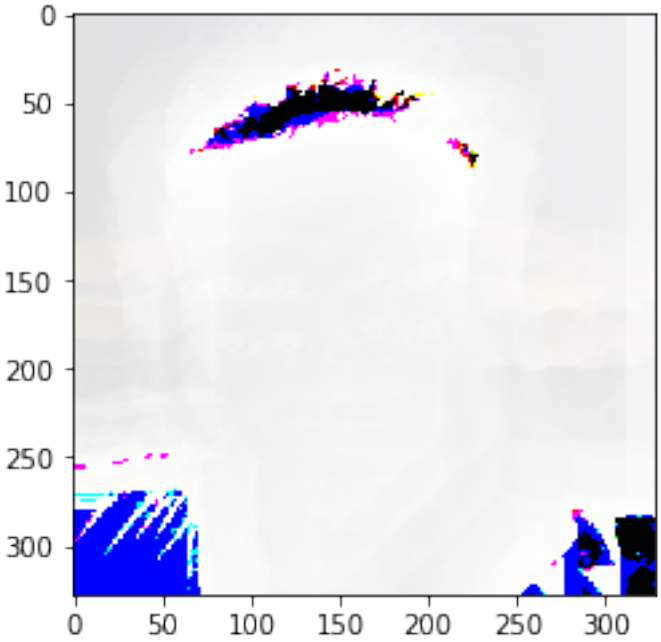
In [12]:
```
size = 17
factor = 1/17
seventeen = apply_transformation(size, factor)
imshow(seventeen)
#save_image(seventeen, size, factor)
```

Out[12]: <matplotlib.image.AxesImage at 0x7f864016d390>



In [13]:
```
size = 33
factor = 1/33
imshow(apply_transformation(size, factor))
```

Out[13]: <matplotlib.image.AxesImage at 0x7f862ea68c88>



In [14]:
```
size = 65
factor = 1/65
imshow(apply_transformation(size, factor))
```

Out[14]:
```
<matplotlib.image.AxesImage at 0x7f862e9d3080>
```