Graph Coloring

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Given, a planar graph $G = (V, E)$ and a set of four colors, $C = \{1, 2, 3, 4\}$
Variables:

$$x_{ic} = \begin{cases} 1, & \text{if node } i \text{ is colored with color } c \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$z_c = \begin{cases} 1, & \text{if } c \text{ is assigned to at least one node} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Constraints:

1. One color for each node.

$$\sum_{j \in C} x_{ij} = 1 \qquad \forall i \in V \tag{3}$$

2. The set of neighbor nodes is denoted by $N(i)$ for node i. Nodes $j \in N(i)$ cannot have the same color as i for all node i.

$$\sum_{k \in N(i)} x_{kc} \leq |N(i)|(1 - x_{ic}) \qquad \forall c \in C \tag{4}$$

3. A color is assigned if at least one node uses that color.

$$z_c \geq x_{ic} \qquad \forall c \in C \; \forall i \in V \tag{5}$$

4. Binary constraints

$$x_{ic} \in \{0, 1\} \tag{6}$$

$$z_c \in \{0, 1\} \tag{7}$$

Objective:

$$\min_{c \in C} z_c \tag{8}$$