Computer Engineering, FIZ137 1st Midterm Exam | Inst.: R. Ellialtıoğlu & E. Taşcı | 22 / 11 / 2016

Equations of motion for constant acceleration:

$$v=v_0+at;(x-x_0)=v_0t+\frac{1}{2}at^2;v^2=v_0^2+2a(x-x_0);(x-x_0)=\frac{1}{2}(v_0+v)t;(x-x_0)=vt-\frac{1}{2}at^2$$

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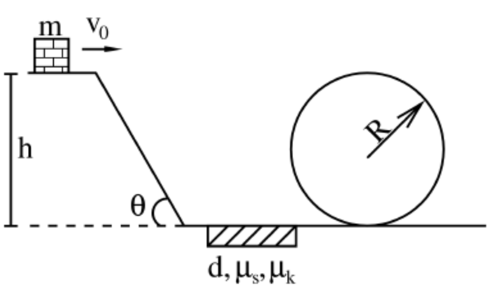
1) Some of the physical laws belonging to alternative universes/computer games are listed below where $\alpha, \beta, \gamma, \delta, \epsilon$ are constants. Analyze each of them and derive the units of the constants in the equations in order to have *in principle* correct equations. (4 points each)
[F: Force; K: Kinetic Energy; U: Potential Energy; ρ: density; μ_k: Kinetic friction coefficient; P: Power]

i. $K=\alpha mv$
ii. $P=\beta\frac{ma}{t^2}$
iii. $F=\gamma mv^2$
iv. $v=\sqrt{v_0^2+\delta\frac{F}{m}}$
v. $\mu_k=\epsilon\frac{F\rho x}{ma^2}$

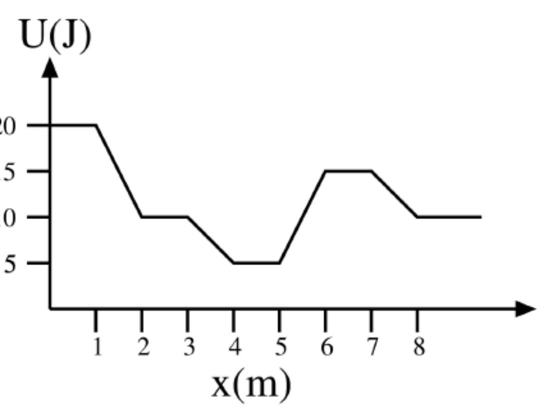
2)



A block of bricks, mass *m* is released from a height of *h* with an initial horizontal speed of v_0. Shortly after it slides down an inclined plane of angle θ and enters a region of *d* length where there is a friction characterized by static and kinetic frictional coefficients of us μ_s and μ_k, respectively, present between the block and the floor. This region is followed by a circular loop of radius *R*. *(Other than the shaded d region, there is no friction present)*

For the particle to be able to complete the loop *just* without falling down, derive an equation between R and other relevant quantities. Indicate the unrelated quantities among the given ones, if there are any. (20 points)

3)



c)

Regarding the given Potential Energy vs. Position graph:

a) Plot the corresponding Force vs. Position graph (7 points)

b) Calculate the work done on the particle by the given interactions to move it from x=0m to x=8m (6 points)

c) For a particle with mechanical energy E_{mec}=13J, released from x=4m and initially moving in the positive x-direction, identify the turning and equilibrium points, if there are any. (7 points)

Automotive Engineering, FIZ137 1st Midterm Exam | Instructor: Emre S. Taşcı | 13 / 11 / 2015
Choose 5 questions among the 7

1) Some of the physical laws belonging to alternative universes/computer games are listed below, among the wrong ones. Analyze each of them and mark the wrong ones, giving your reasons.
[E: Mechanical Energy; K: Kinetic Energy; U: Potential Energy; ρ: density; μ_k: Kinetic friction coefficient; P: Power]
(4 points each)

a) $x = vt^2 + at$

b) $\frac{dE}{dt} = mv\frac{dv}{dt}$

c) $v = \sqrt{\left(\frac{3mat}{\rho V}\right)}$

d) $K = \frac{Pt}{3}\frac{mv^2}{Fx}$

e) $\mu_k = \frac{mv^2 x}{U^2 at^2}$

2) A particle's trajectory is given as: $y(x) = -\frac{1}{2}x^2 + 6x + 5$.

a) If at t=0s, the object is located at (0,5)m and at t=5s located at (10,15); what are the components and magnitude of the average velocity for this interval? (7 Points)

b) If the x-component of its velocity is constant, then derive the y component of the velocity and acceleration with respect to time ($v_y(t) = ?, a_y(t) = ?$) (13 points)

3) A ball is dropped from rest from a height h. Each time it bounces off the ground, it loses half of its mechanical energy.

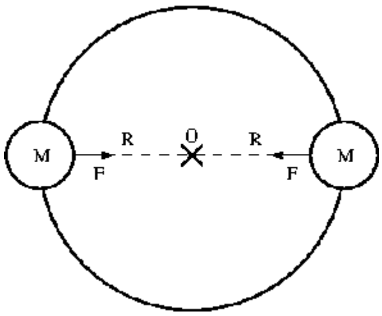
a) After n^{th} bounce, what is the ratio of its speed with respect to the speed it had just before the 1^{st} bounce? (10 Points)

b) Roughly draw the plots of x-t, v-t and a-t. (10 Points)

4) Two identical planets, each having a mass of M, orbit in a circle (radius R) around their mid point O. The force between them is attractive and related to their mass and the distance between them (2R) as :

$$F = \frac{GM^2}{4R^2}$$

where G is a constant.



a) Find the orbital speed of each planet. (6 Points)

b) Find the period of orbit. (4 Points)

c) Find the energy required to separate the two planet to infinity (in terms of G, M and R). (10 Points)

$$\int \frac{1}{x}dx = \ln|x| + C ; \int \frac{1}{x^2}dx = -\frac{1}{x} + C$$

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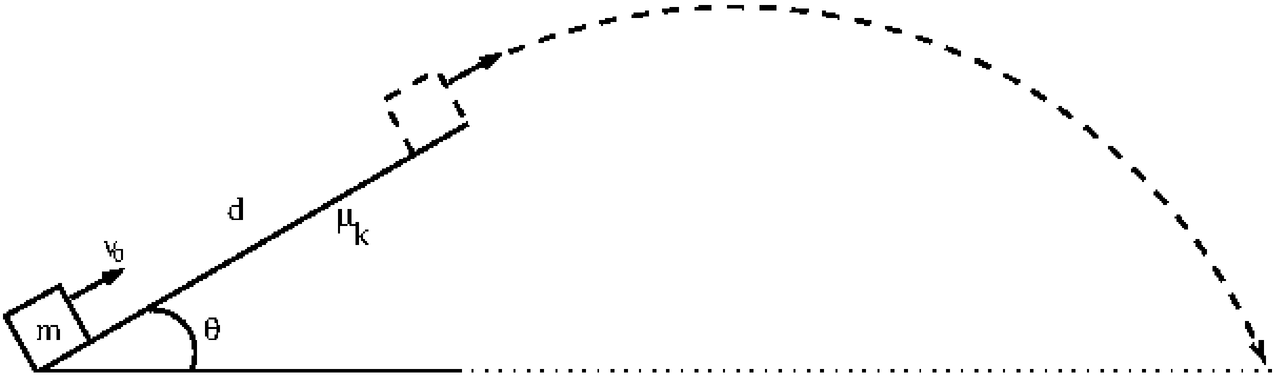
5) Two cars are coming toward each other, each with a velocity of magnitude v as observed from a person standing by the side of the road.

a) Calculate the energy according to the observer that will be released when the two cars crash. (5 Points)

b) Calculate the energy that will be released when the two cars crash, according to one of the drivers. (5 Points)

c) If you have calculated the results of (a) and (b) as same, does this point to the conservation of energy? If you have calculated different results for (a) and (b), then what is the difference between the two parts? If the event is recorded by two cameras, one by the observer standing still at the side of the road, another by a camera attached to one of the cars, which recording will appear as strange and why? (10 Points)

6) A block of mass m=3kg has been sent upwards on an inclined plane making an angle of $\theta=37^o$ by an initial velocity of $v_0=16m/s$. The coefficient of kinetic friction between the block and the plane, μ_k is 0.5 and the total length of the plane's side on which the block moves is d=1m.



a) Show that after being launched from the end of the plane, the object will travel a duration of t in the air before hitting the ground where t satisfies the equation:

$5t^2 - 1.2t - 0.6 = 0$ (having a root at t ≈ 0.5s). (15 Points)

b) How much further along the x-direction will the block have moved at the end of its flight with respect to its starting position? (5 Points)

7) Tarzan, of weight of mg, swings from the vine. The vine's length is r and the lowest point of the vine's arc is h=r/2 from Tarzan's starting point. Until the lowest part is covered (i.e., half distance of the ideal swing) everything is fine but upon passing, the vine starts to weaken and the moment a force of half of Tarzan's weight is applied, it breaks. What is the angle with the vertical when it breaks? (20 Points)

Question		Grade
1	20p	
2	a)15p b)15p	
3	20p	
4	20p	
5	20p	
Total		

FİZ 137 – 25 / 26 PHYSICS I
2^{nd} MIDTERM
19.12.2017 (13:00 – 14:45)

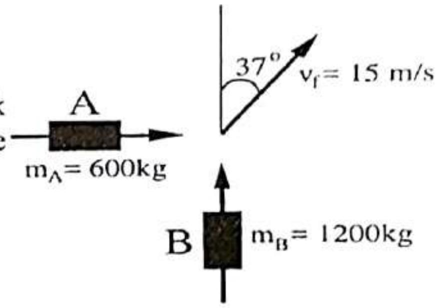
SECTION:

NAME-SURNAME:............................... NUMBER:................. SIGNATURE:...............

1. At t = 0, a motor has an angular velocity 24 rad/s with a constant angular acceleration of 30 rad/s^2. At t = 2 s, its power is shut down and from then on, it rotates through 420 radians as it slows down and eventually stops under a constant angular acceleration (deceleration).

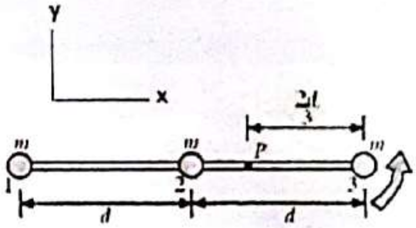
a. Between t = 0 and the time it stops, what is the total angle (in radians) it has turned?
b. At what time does it stop?
c. What is the angular acceleration as it slows down?

2. a. At an intersection, two cars collide with each other and they lock together sliding at 15 m/s at angle of 37° east-due-north as shown in the figure. Calculate the speed of each vehicle before the collision.



2. b. A diver at summer olympics, with his arms straight up and legs straight down has a moment of inertia of 15 kg.m^2 with respect to his rotation axis. When he tucks into a small ball, his moment of inertia decreases to 3.6 kg.m^2. While tucked, he makes two complete revolutions in 1 second. If he hadn't tucked at all, how many revolutions would he have made in 1.5 s from the board to the water?

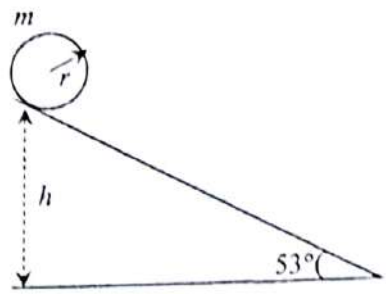
3. A rigid, massless rod has three particles with equal masses attached to it as shown in the figure. The rod is free to rotate in a vertical plane about a frictionless axle perpendicular to the rod through the point P, and is released from rest in the horizontal position at t = 0. Calculate



a. the moment of inertia of the system about the pivot,
b. the net torque (in unit vector notation) acting on the system at t = 0,
c. the angular acceleration of the system at t = 0,
d. the linear acceleration of the particle labeled 3 at t = 0,
e. the angular momentum of the system at t = 7 s.

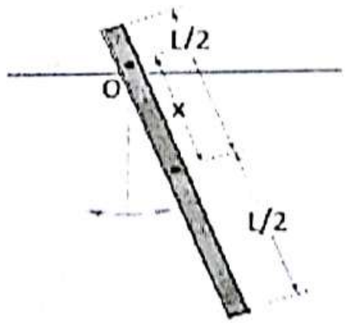
$g = 10$ m/s^2	$\pi = 3$	$I_{com(cylinder)} = (1/2)MR^2$	$I_{com(stick)} = (1/12)ML^2$
$\sin 37° = \cos 53° = 0.6$	$\sin 53° = \cos 37° = 0.8$	$\omega = \omega_0 + \alpha t$	$\vec{\tau}_{net} = I_{com}\vec{\alpha}$
$\alpha_{avg} = (\omega_2 - \omega_1)/(t_2 - t_1)$	$\theta = \theta_0 + \omega_0 t + (1/2)\alpha t^2$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	$L = I\omega$

4. A solid cylinder with mass m = 4 kg and radius r = 0.5 m is released from a frictionless inclined plane at a vertical height h = 4.8 m and started *rolling down without sliding*, as given in the figure.



a. Draw the free body diagram for the cylinder.
b. Write equations of motion for the cylinder.
c. Find the linear acceleration of the center of mass of the cylinder.
d. Find the linear velocity of the center of mass at the bottom of the inclined plane.
e. Find the kinetic energy of the cylinder at the bottom of the inclined plane.

5. A stick with mass M and length L has attached to the wall at point O, that is x apart from its center of mass and establishes a physical pendulum. For small angle approximation ($sin\ \theta \cong \theta$).



a. Write all the forces acting on the system.
b. Write the restoring torque.
c. Find the angular frequency of this physical pendulum.
d. Find the period of this physical pendulum.

GOOD LUCK

Dr. E. TAŞCI (Fiz 137-25) – Dr. Ş. ÇOLAK (Fiz 137-26)

$g = 10\ m/s^2$	$\pi = 3$	$I_{com(cylinder)} = (1/2)MR^2$	$I_{com(stick)} = (1/12)ML^2$
$\sin 37° = \cos 53° = 0.6$	$\sin 53° = \cos 37° = 0.8$	$\omega = \omega_0 + \alpha t$	$\vec{\tau}_{net} = I_{com}\vec{\alpha}$
$\alpha_{avg} = (\omega_2 - \omega_1)/(t_2 - t_1)$	$\theta = \theta_0 + \omega_0 t + (1/2)\alpha t^2$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	$L = I\omega$

1) $\Theta = \Theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\Theta_0 = 0$; $\omega = 24 rad/s$; $\alpha = 30 rad/s^2$

a) $\Theta(t=2s) = 0 + 24 rad/s \cdot 2s + \frac{1}{2} 30 rad/s^2 \cdot (2s)^2$

$= 48 rad + 60 rad = \underline{108 rad}$

→ total angular disp. until it stops:

$\Theta_{TOT} = (108 + 420) rad = 528 rad$

$\boxed{\begin{array}{l} \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \quad (1) \\ \omega = \omega_0 + \alpha t \quad (2) \\ \rightarrow \alpha = \frac{\omega_f - \omega_0}{t} \\ \rightarrow \theta - \theta_0 = \omega_0 t + \frac{1}{2}\left(\frac{\omega - \omega_0}{\cancel{t}}\right) t^{\cancel{2}} \\ \theta - \theta_0 = \frac{2\omega_0 + \omega - \omega_0}{2} t \\ \theta - \theta_0 = \frac{\omega + \omega_0}{2} t \end{array}}$

b) $t = 2s$: $\Theta(t=2s) = 108 rad$

$\omega = \omega_0 + \alpha t = 24 rad/s + 30 rad/s \cdot 2s$

$= 84 rad/s$

$t = t_f$: $\Theta(t_f) = 528 rad$, $\omega(t_f) = 0$

$\Theta(t_f) - \Theta(t=2s) = \frac{\omega(t_f) + \omega(t=2s)}{2}(t_f - 2s)$

$\frac{(528 rad - 108 rad) \cdot 2}{84 rad/s + 0} + 2s = t_f \Rightarrow t_f = 12s$

c) $\alpha = \frac{\omega(t_f) - \omega(t=2)}{t_f - 2s} = \frac{0 - 84 rad/s}{10s} = -8.4 rad/s^2$

2) a) $\vec{P}_i = \vec{P}_f$

$P_{i,x} = P_{f,x}$, $P_{i,y} = P_{f,y}$, $m_f = m_A + m_B$

$v_{f,x} = 15\,m/s \cdot \sin 37° = 15\,m/s \cdot 0{,}6 = 9\,m/s$

$v_{f,y} = 15\,m/s\ \cos 37° = 15\,m/s \cdot 0{,}8 = 12\,m/s$

$P_{f,x} = (m_A + m_B)\,v_{f,x} = \underbrace{m_A v_{A,i}}_{P_{i,x}} \Rightarrow (600\,kg + 1200\,kg) \cdot 9\,m/s = 600\,kg \cdot v_{A,i}$

$\rightarrow v_{A,i} = \dfrac{1800\,kg \cdot 9\,m/s}{600\,kg} = 27\,m/s$

$P_{f,y} = (m_A + m_B)\,v_{f,y} = \underbrace{m_B v_{B,i}}_{P_{i,y}} \Rightarrow 1800\,kg \cdot 12\,m/s = 1200\,kg \cdot v_{B,i}$

$v_{B,i} = \dfrac{1800\,kg \cdot 12\,m/s}{1200\,kg} = 18\,m/s$

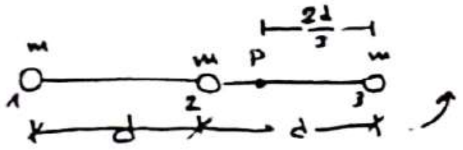
b) $\vec{L}_1 = \vec{L}_2$

$I_1 \omega_1 = I_2 \omega_2$

$(15\,kg\,m^2)\,\omega_1 = (3.6\,kg\,m^2)\left(2\,\frac{rev}{s}\right)$

$\rightarrow \omega_1 = \dfrac{3.6 \times 2}{15}\,\dfrac{rev}{s} \Rightarrow (1.5\,s) \cdot \omega_1 = \dfrac{\cancel{15}^{1} \cdot 3.6 \times 2}{\cancel{15} \times 10} = 0.72\,\underline{rev}$

3)



$I = \Sigma m r_i^2$

a) $I = m\left(d+\frac{d}{3}\right)^2 + m\left(\frac{d}{3}\right)^2 + m\left(\frac{2d}{3}\right)^2 = \frac{16md^2}{9} + \frac{md^2}{9} + \frac{4}{9}md^2 = \frac{21md^2}{9}$

$= \frac{7}{3}md^2$

b) $\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3$

$\tau = mg \cdot \frac{4d}{3} + mg\frac{d}{3} - \frac{2d}{3}mg = mgd$

$\vec{\tau}_{net} = mgd\,\hat{k}$

c) $\alpha = \frac{\tau_{net}}{I} = \frac{mgd}{\frac{7}{3}md^2} = \frac{3}{7}\frac{g}{d}$

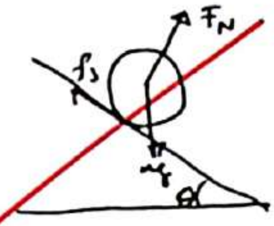
d) $a_3 = \alpha r_3 = \left(\frac{3}{7}\frac{g}{d}\right)\left(\frac{2d}{3}\right) = \frac{2}{7}g$

e) $L = I\omega = \frac{7}{3}md^2\omega$

$\omega = \omega_0 + \alpha t = 0 + \frac{3}{7}g/d \cdot 7 = \frac{3g}{d}$

$L = \left(\frac{7}{3}md^2\right)\left(\frac{3g}{d}\right) = 7mgd \quad (Js)$

1) a)



b) $\Sigma F_y = F_N - mg\cos\theta = 0 \rightarrow F_N = mg\cos\theta$ 1p

$\Sigma F_x = mg\sin\theta - f_s = m a_{com}$ 1p

$\Sigma \tau = -r . f_s = I(-\alpha)$ 1p

$$f_s = \frac{I\alpha}{r} = \frac{[\frac{1}{2}mr^2][\frac{a_{com}}{r}]}{r} = \frac{1}{2} m a_{com}$$

CANCELLED!

Everybody gets points due to a, b, c!

c) $mg\sin\theta - \frac{1}{2} m a_{com} = m a_{com}$

$a_{com}(\frac{3}{2}m) = mg\sin\theta$

$a_{com} = \frac{2}{3} g\sin\theta = \frac{2}{3} g . \sin 53° = \frac{2}{3} 10 m/s^2 \cdot \frac{4}{5} = \frac{16}{3} m/s^2$

d) $mgh = \frac{1}{2} m v_{com}^2 + \frac{1}{2} I\omega^2$

$mgh = \frac{1}{2} m v_c^2 + \frac{1}{2}\left(\frac{1}{2} Mr^2\right)\left(\frac{v_c}{R}\right)^2$

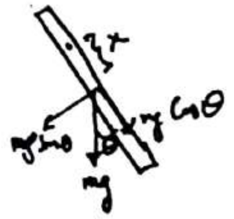
$v_{com} = \sqrt{\frac{4}{3} gh} = 8 m/s$

e) $K_f = U_i = mgh = 4kg . 10 m/s^2 . 4.8m = 192 J$

or

$K = \frac{1}{2} m v_c^2 + \frac{1}{2} I\omega^2 = \frac{3}{4} m v_c^2 = \frac{3}{4} . 4 . 8^2 J = 192 J$

5)

a) 

b) $\tau = x \cdot mg \sin\theta$

c) $I = I_0 + Mx^2$

$= \frac{1}{12} ML^2 + Mx^2$

$\tau = I\alpha$

$x mg \underbrace{\sin\theta}_{\approx \theta} = I \cdot \ddot{\theta}$

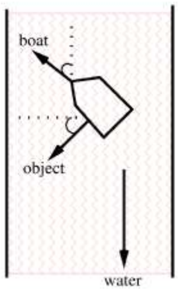
$\ddot{\theta} = \underbrace{\frac{x mg}{I}}_{\omega^2} \theta \rightarrow \omega^2 = \frac{mgx}{\left(\frac{ML^2}{12} + Mx^2\right)}$

d) $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{\frac{ML^2}{12} + Mx^2}{mgx}} = 2\pi \sqrt{\frac{L^2/12 + x^2}{gx}}$

Automotive Engineering, FIZ137 Final Exam | Instructor: Emre S. Taşcı | 5 / 1 / 2016

Choose 5 questions among the 7

1) A boat is moving towards 37° west of north at a speed 30m/s relative to water. The water flows southwards at 5m/s relative to the shore. An object is thrown from the boat towards 37° south of west at 5m/s relative to the boat.



What is the velocity of the object relative to shore?
(Sin 37° = Cos 53° = 0.6 | Cos 37° = Sin 53° = 0.8)

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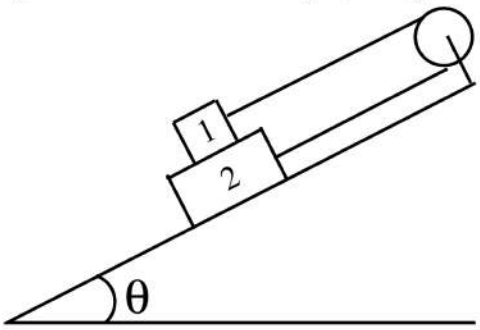
2) Two blocks 1 and 2 ($m_1 << m_2$) are connected by a massless rope wrapped around a massless and frictionless pulley, with block 1 positioned on top of block 2. The plane makes an angle of theta with the horizontal. The coefficient of kinetic friction between the blocks is μ_k and there is no friction between the plane and block 2.



Show that the magnitude of the blocks' accelerations is given by:

$$a = \frac{2\mu_k m_1 g\cos\theta + (m_1 - m_2) g\sin\theta}{m_1 + m_2}$$

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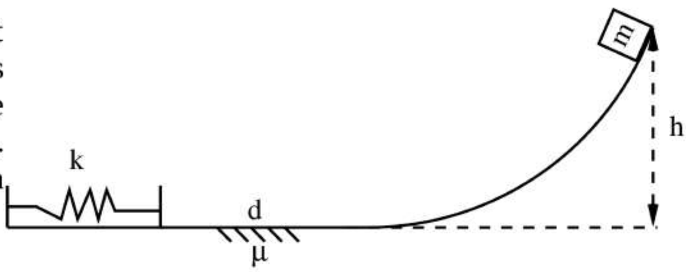
3) An object with mass m is released from rest from a height of h. The path is frictionless except a region of length d as shown in the figure, with a coefficient of kinetic friction μ_k. At the end of the path is a relaxed spring with spring constant k.



Calculate the maximum compression of the spring as the block pushes it upon collision.

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4) A wheel of radius R rotates with constant average acceleration α. At t=0, it starts from rest, and a marked point P on the wheel makes an angle of θ with the horizontal. At a given time $t=t_1$, find:

a) Angular speed of the wheel
b) Tangential speed and total acceleration of point P
c) Angular position of P

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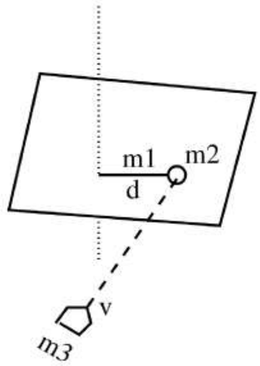
Choose 5 questions among the 7

5) A rod with length d and mass m_1 is fixed to a point such that it can rotate around a vertical line. A particle of mass m_2 is attached to the end of the rod. While this system is stationary, a bullet of mass m_3 hits the particle (sticks to it) with a speed of v.



Calculate:

a) the angular speed of the system just after the collision (in terms of m_1, m_2, m_3, v, d)

b) fractional loss in the kinetic energy

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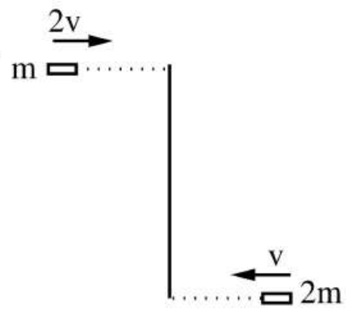
6) A bullet of mass m, with a speed of 2v collides from left with the top of a rod of d length, while another bullet of mass 2m with a speed of v collides from right with the bottom of the rod. The rod is massless.



Just after the collisions, calculate

a) the position of the center of mass of the system

b) the speed of the center of mass of the system

c) the angular speed of the system around its center of massless

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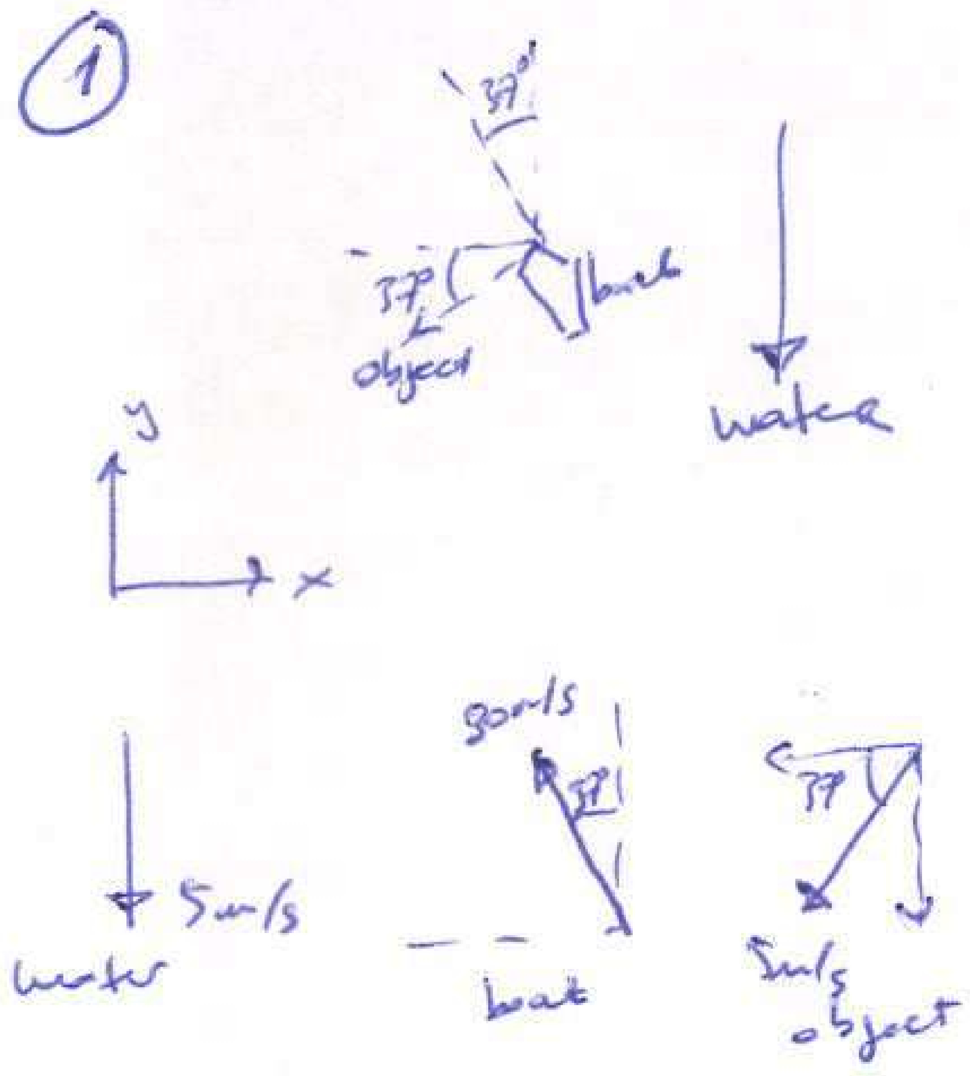
7) A bowling ball of mass M and radius R is thrown with an initial speed v_0 and backspin with speed ω_0 such that $v_0 > R\omega_0$. Calculate the speed v_f of the bowling ball when it starts to roll (without slipping).

FIZ 137 Final Exam (20151)

(1)



$$\vec{v}_{os} = \vec{v}_{ob} + \vec{v}_{bw} + \vec{v}_{ws}$$

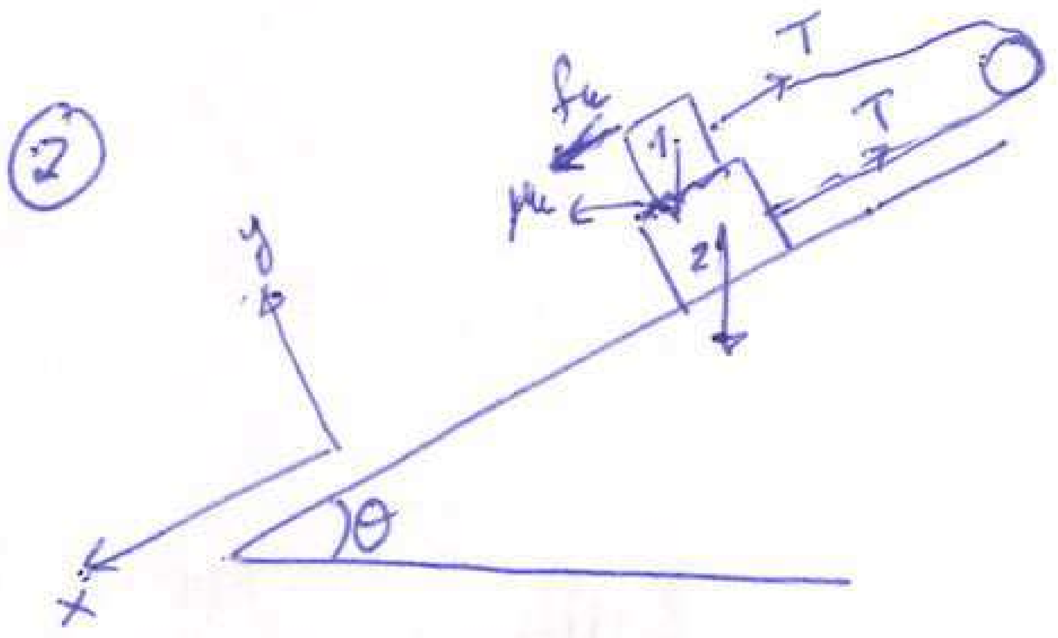
$$\vec{v}_{os} = \left[5(-\cos 37\hat{i} - \sin 37\hat{j}) + 30(-\sin 37\hat{i} + \cos 37\hat{j}) - 5\hat{j} \right] m/s$$

$$= \left[(-5 \cdot 0.8 - 30 \cdot 0.6)\hat{i} + (-5 \cdot 0.6 + 30 \cdot 0.8 - 5)\hat{j} \right] m/s$$

$$\vec{v}_{os} = (-22\hat{i} + 16\hat{j})\, m/s$$

$$|\vec{v}_{os}| = \sqrt{(22)^2 + (16)^2} = 27.2\, m/s \quad , \text{atan}\left(\frac{16}{22}\right) \simeq 37°$$

(2)



$$x_1: \; f_k - T + m_1 g \sin\theta = m_1 a$$

$$y: \; f_k = \mu_k m_1 g \cos\theta$$

$$\rightarrow \mu_k m_1 g \cos\theta - T + m_1 g \sin\theta = m_1 a$$

$$x_2: \; -T - f_k + m_2 g \sin\theta = -m_2 a$$

both blocks have the same $|\vec{a}|$ since the rope connecting them stays at the same length at all times. (but $\vec{a}_1, \vec{a}_2$ have opposite directions)

$\rightarrow$ eliminate T :

$$\mu_k m_1 g \cos\theta + m_1 g \sin\theta - m_1 a$$

$$= \underbrace{-\mu_k m_1 g \cos\theta}_{f_k} + m_2 g \sin\theta + m_2 a$$

$$\rightarrow (m_1 + m_2) a = 2\mu_k m_1 g \cos\theta + m_1 g \sin\theta - m_2 g \sin\theta$$

$$\Rightarrow a = \frac{2\mu_k m_1 g \cos\theta + (m_1 - m_2) g \sin\theta}{(m_1 + m_2)}$$

(3)

$$mgh = \frac{1}{2} m v^2 + \mu_k m g d$$

$$\frac{1}{2} m v^2 = (mgh - \mu_k m g d) = mg(h - \mu_k d)$$

$$\frac{1}{2} k x^2 = \frac{1}{2} m v^2 = mg(h - \mu_k d)$$

$$\rightarrow x^2 = \frac{2mg}{k}(h - \mu_k d) \Rightarrow x = \sqrt{\frac{2mg}{k}} \sqrt{h - \mu_k d}$$

FIZ137 Final Exam (20151)

④ a) $\omega_f = \omega_i + \alpha t = \alpha t_1$

b) $v = R\omega$

$|a_r| = R\omega^2$

$a_t = R\alpha$

$a = \sqrt{a_r^2 + a_t^2} = R\sqrt{\omega^4 + \alpha^2}$

c) $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = \theta_i + \frac{1}{2}\alpha t^2$

⑤ a) $L_i = d\, m_3 v$, $L_f = I\omega = \left[\frac{1}{3}m_1 d^2 + (m_2 + m_3)d^2\right]\omega$

$L_i = L_f \longrightarrow \omega = \dfrac{d m_3 v}{\left(\frac{1}{3}m_1 + m_2 + m_3\right)d^2} = \dfrac{m_3}{\frac{1}{3}m_1 + m_2 + m_3}\,\dfrac{v}{d}$

b) $K_i = \frac{1}{2}m_3 v^2$, $K_f = \frac{1}{2}I\omega^2 = \frac{1}{2}I\,\dfrac{(m_3 v d)^2}{I^2} = \dfrac{1}{2}\,\dfrac{m_3^2 v^2}{\frac{1}{3}m_1 + m_2 + m_3}$

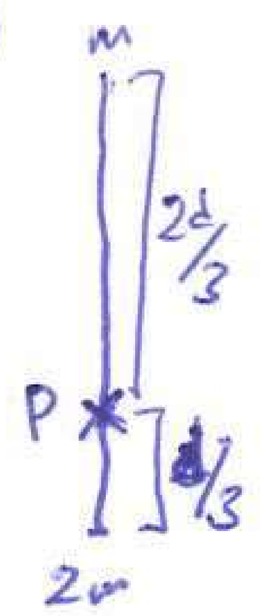
$\Delta K = \frac{1}{2}m_3 v^2\left(\dfrac{m_3}{\frac{1}{3}m_1 + m_2 + m_3} - 1\right) = -K_i \dfrac{1/3\, m_1 + m_2}{\frac{1}{3}m_1 + m_2 + m_3}$

$\hookrightarrow$ fractional Loss

⑥ b) $\vec{P}_{i,tot} = \vec{P}_{f,tot}$

$\vec{L}_{i,tot} = \vec{L}_{f,tot}$

a.)



$P_{tot} = m 2v - 2m.v = 0 = M_{tot}\cdot v_{com} \Rightarrow \underline{\underline{v_{com} = 0}}$

c) $L_{i,com} = m.2v\,\frac{2d}{3} + 2mv\frac{d}{3} = 2mvd$

$L_{f,com} = I_{com}.\omega$; $I_{com} = \dfrac{m4d^2}{9} + 2m\dfrac{d^2}{9} = \frac{6}{9}md^2 = \frac{2}{3}md^2$

$\Rightarrow 2mvd = \frac{2}{3}md^2 . \omega \Rightarrow \underline{\underline{\omega = \dfrac{3v}{d}}}$

⑦ $L_i = m v_0 R - I_{com}\omega_0$

$L_f = m v_f R + I_{com}\omega_f$

rolling without spinning $\rightarrow$ $\omega_f = v_f/R$, $I_{com} = \frac{2}{5}mR^2$

$v_f = (5v_0 - 2\omega_0 R)/7$

Computer Engineering, FIZ137 1st Midterm Exam | Inst.: R. Ellialtıoğlu & E. Taşcı | 22 / 11 / 2016 | 90 minutes

Name:

Student Number:

Section: (RE 25, ET 26)

Equations of motion for constant acceleration:

$v = v_0 + at;\quad (x - x_0) = v_0 t + \frac{1}{2}at^2;\quad v^2 = v_0^2 + 2a(x - x_0);$

$(x - x_0) = \frac{1}{2}(v_0 + v)t;\quad (x - x_0) = vt - \frac{1}{2}at^2$

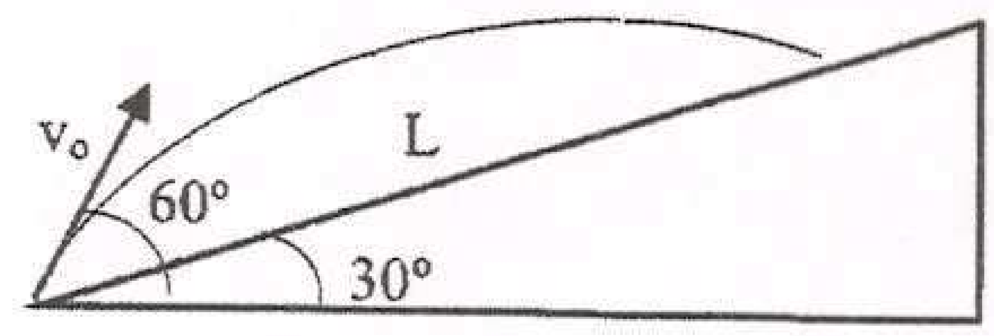
Questions

1. Some of the physical laws belonging to alternative universes/computer games are listed below where $\alpha, \beta, \gamma, \delta, \epsilon$ are constants. Analyze each of them and derive the units of the constants in the equations in order to have in principle correct equations. (4 points each)
[F: Force; K: Kinetic Energy; U: Potential Energy; ρ: density; μ_k: Kinetic friction coefficient; P: Power]

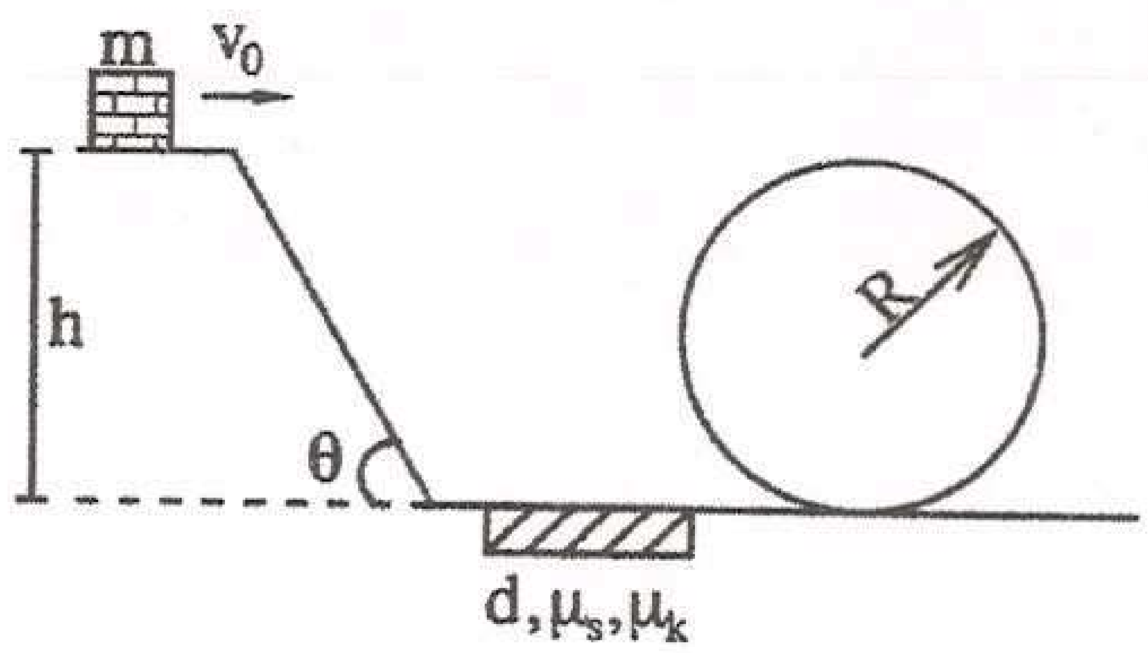
$$K = \alpha m v \qquad P = \beta \frac{ma}{t^2} \qquad F = \gamma m v^2 \qquad v = \sqrt{v_0^2 + \delta \frac{F}{m}} \qquad \mu_k = \epsilon \frac{F\rho x}{ma^2}$$

2. At a certain instant, a 2 kg object is acted on by a force **F** = 4**i** – 2**j** + 5**k** [N] while having a velocity **v** = – 2**i** + 3**k** [m/s]. What is the instantaneous rate at which the force does work on the object? At a later time, the velocity consists of only a y component and the instantaneous power is – 8 W. If the force is unchanged, what is the kinetic energy of the object just then? (20 points)

3. A ball is thrown with an initial velocity of magnitude v_o = 10 m/s, which makes an angle 60° with the horizontal, from the lowest point of a 30° incline plane. Find the distance L travelled by the ball on the incline. (cos30° = sin60° = √3/2; sin30° = cos60° = 1/2; g = 10 m/s²). (20 points)

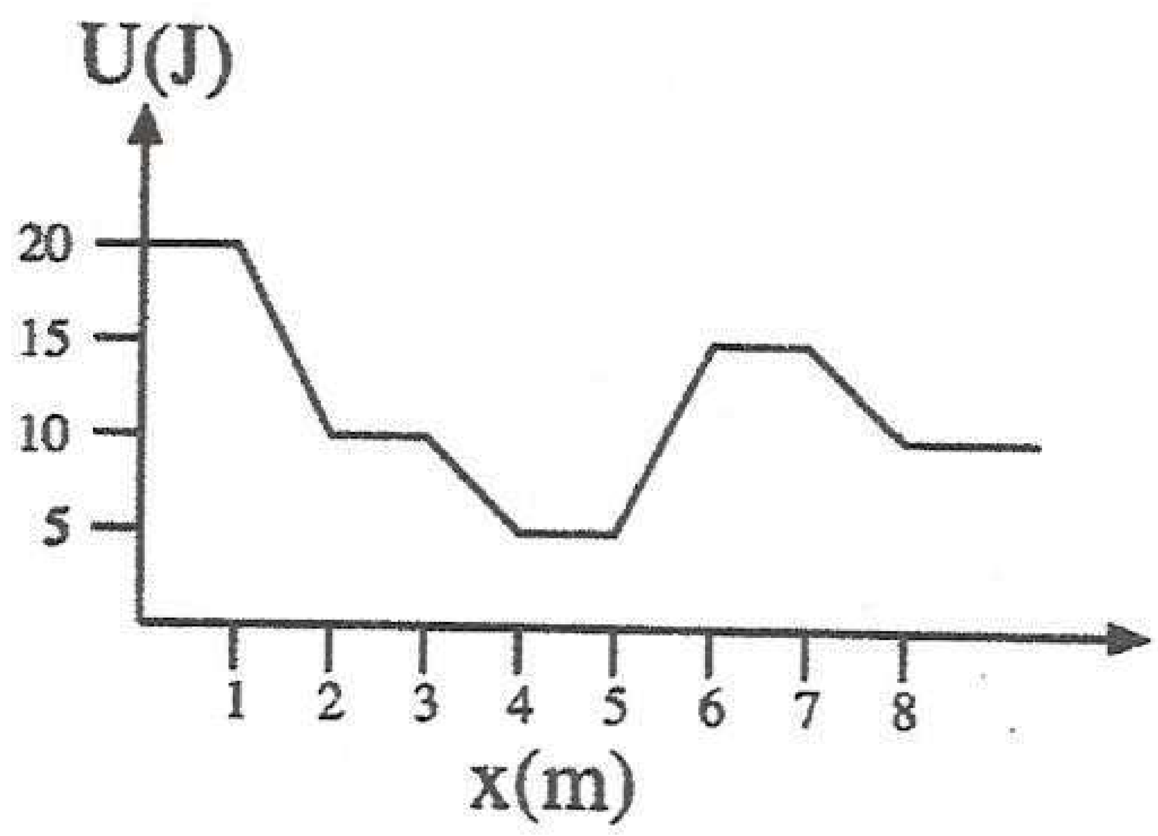


4. A block of bricks, mass m is released from a height of h with an initial horizontal speed of v_o. Shortly after it slides down an inclined plane of angle θ and enters a region of d length where there is a friction characterized by static and kinetic frictional coefficients of us μ_s and μ_k, respectively, present between the block and the floor. This region is followed by a circular loop of radius R. (Other than the shaded d region, there is no friction present)



For the particle to be able to complete the loop just without falling down, derive an equation between R and other relevant quantities. Indicate the unrelated quantities among the given ones, if there are any. (20 points)

5. Regarding the given Potential Energy vs. Position graph:
a) Plot the corresponding Force vs. Position graph (7 points)
b) Calculate the work done on the particle by the given interactions to move it from x = 0 m to x = 8 m (6 points)
c) For a particle with mechanical energy E_{mec} = 13 J, released from x = 4 m and initially moving in the positive x-direction, identify the turning and equilibrium points, if there are any. (7 points)



1) a) $K = \alpha m v$

$$J \equiv kg\frac{m^2}{s^2} = [|\alpha|]\, kg\frac{m}{s}$$

$$[|\alpha|] = \frac{m}{s} \Rightarrow [|\alpha|] = \frac{[|L|]}{[|T|]}$$

b) $P = \beta \frac{ma}{t^2}$

$$W \equiv \frac{J}{s} = \frac{kg\, m^2/s^2}{s} = [|\beta|]\,\frac{kg\, m/s^2}{s^2}$$

$$[|\beta|] = m\cdot s \Rightarrow [|\beta|] = [|L|][|T|]$$

c) $F = \gamma m v^2$

$$N \equiv kg\, m/s^2 = [|\gamma|]\, kg\frac{m^2}{s^2}$$

$$\Rightarrow [|\gamma|] = \frac{1}{[|L|]}$$

d) $[|\delta \frac{F}{m}|] = [|v^2|]$

$$[|\delta|]\frac{kg\, m/s^2}{kg} = \frac{m^2}{s^2} \Rightarrow [|\delta|] = m \Rightarrow [|\delta|] = [|L|]$$

e) $\mu_k = \epsilon \frac{F\rho x}{m a^2}$

$$1 = [|\epsilon|]\frac{kg\, m/s^2\; kg/m^3 \cdot m}{kg\; m^2/s^4} = [|\epsilon|]\frac{kg\, s^2}{m^3}$$

$$\to [|\epsilon|] = \frac{m^3}{kg\, s^2} \Rightarrow [|\epsilon|] = \frac{[|L|]^3}{[|M|][|T|]^2}$$

2) $P = \frac{W}{\Delta t} = \frac{\Delta K}{\Delta t}$

$\Delta K = -\Delta U$

$F = -\frac{\Delta U}{\Delta x} \Rightarrow -\Delta U = F\Delta x$

$\Rightarrow P = \frac{F\Delta x}{\Delta t}$, $\Delta t \to 0$: $P = \vec{F}\frac{d\vec{x}}{dt} = \vec{F}\vec{v}$

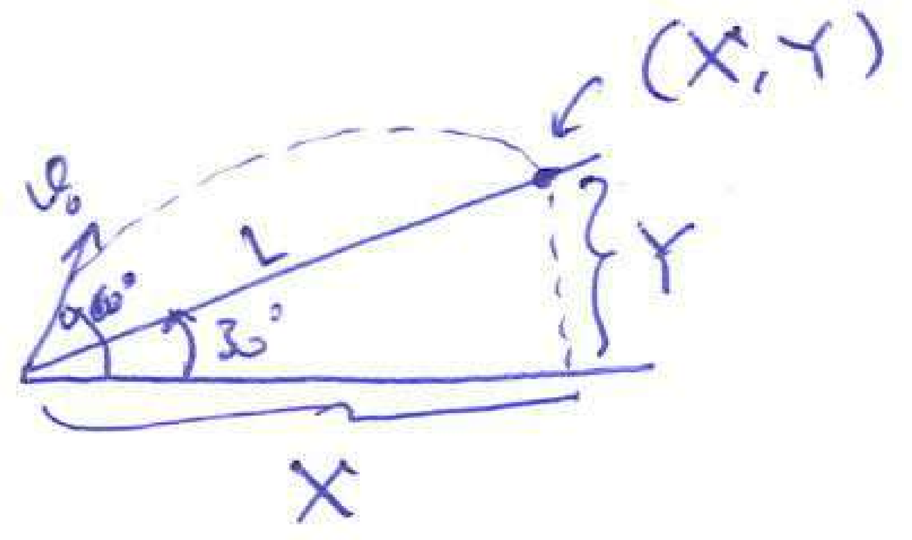
a) $\left.\begin{array}{l}\vec{F} = (4\hat{i} - 2\hat{j} + 5\hat{k})\,N \\ \vec{v} = (-2\hat{i} + 3\hat{k})\,m/s\end{array}\right\} \to P = \vec{F}\cdot\vec{v} = (-8+15)W = \underline{7W}$

b) $\vec{v} = v_y\hat{j} \Rightarrow -8\,W = (4\hat{i} - 2\hat{j} + 5\hat{k})N\,(v_y\hat{j})$

$$-8W = -2v_y N \Rightarrow v_y = \frac{-8W}{-2N} = 4\,m/s$$

$$\Rightarrow K = \frac{1}{2}mv^2 = \frac{1}{2}(2kg)(4\,m/s)^2 = \underline{\underline{16J}}$$

3)



$X = L\cos 30°$

$Y = L\sin 30°$

$v_x = v_0 \cos 60°$

$v_y = v_0 \sin 60°$

$X = v_x t = v_0\cos 60°\, t \to L\cos 30° = v_0 \cos 60°\, t$

$\Rightarrow t = \frac{L\cos 30°}{v_0\cos 60°}$ (1)

$Y = v_y t - \frac{1}{2}g t^2 = L\sin 30°$

(1) $\Rightarrow \cancel{v_0}\sin 60°\left(\frac{\cancel{L}\cos 30°}{\cancel{v_0}\cos 60°}\right) - \frac{1}{2}g\left(\frac{L^{\cancel{2}}\cos^2 30°}{v_0^2\cos^2 60°}\right) = \cancel{L}\sin 30°$

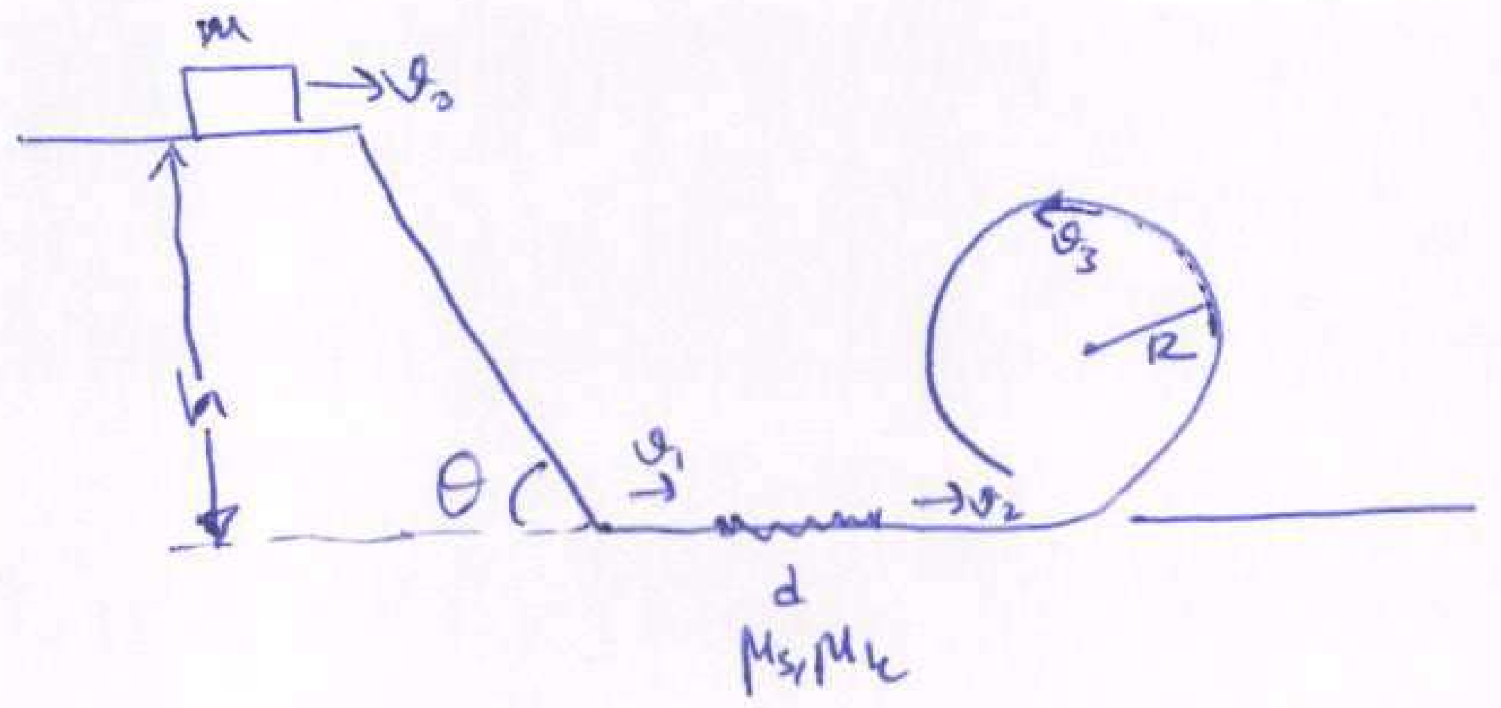
$$\frac{\sin 60°\cos 30°}{\cos 60°} - \frac{1}{2}g\frac{L\cos^2 30}{v_0^2\cos^2 60°} = \sin 30°$$

$$\frac{\sqrt{3}/2\;\sqrt{3}/2}{1/2} - \frac{1}{2}(10\,m/s^2)\frac{L\,(\sqrt{3}/2)^2}{(10\,m/s)^2\,(1/2)^2} = \frac{1}{2} \to \frac{3}{2} - \frac{L\;3/4}{(10m)\;1/2} = \frac{1}{2}$$

$$\frac{3}{20m}L = \frac{3}{2} - \frac{1}{2} = 1$$

$$\Rightarrow L = \frac{20m}{3} = 6.\bar{6}\,m$$

4)



$$\cancel{m}gh + \frac{1}{2}\cancel{m}v_0^2 = \frac{1}{2}\cancel{m}v_1^2 \quad (v_1 > v_0)$$

$$2gh + v_0^2 = v_1^2 \quad (1)$$

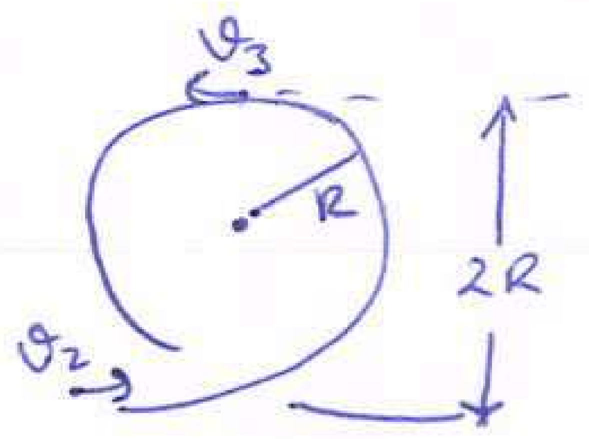
$\Delta K = W_f$, $W_f < 0$ (Energy is transferred from the block by the friction Force)

($v_2 < v_1$)

$$\frac{1}{2}\cancel{m}v_2^2 - \frac{1}{2}\cancel{m}v_1^2 = -f_k d$$
$$= -\cancel{m}g\mu_k d$$

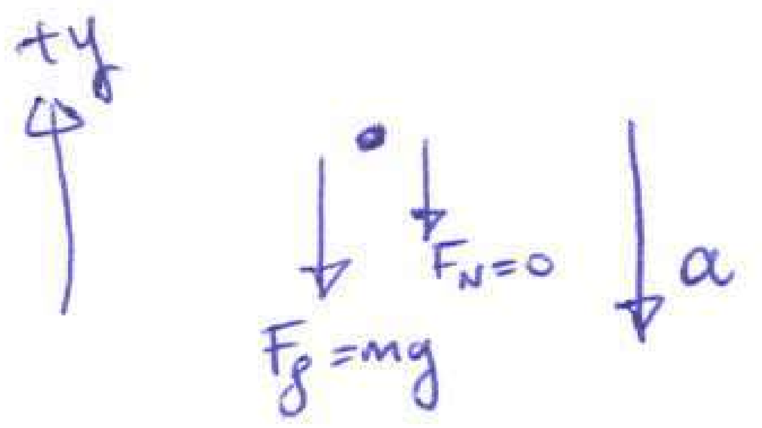
$$v_2^2 = -2g\mu_k d + v_1^2$$

(1) $\Rightarrow$ $$= -2g\mu_k d + 2gh + v_0^2 \quad (2)$$



$$\frac{1}{2}mv_2^2 = mg(2R) + \frac{1}{2}mv_3^2$$

$$v_3^2 = v_2^2 - 4gR \quad (3)$$



$$0 - \cancel{m}g = \cancel{m}(-a) = \cancel{m}\left(-\frac{v_3^2}{R}\right)$$

$$g = \frac{v_3^2}{R} \Rightarrow v_3^2 = gR$$

(3) $\Rightarrow$ $$v_2^2 - 4gR = gR$$

$$v_2^2 = 5gR \quad (4)$$

(2) + (4) : $$5gR = v_0^2 + 2gh - 2g\mu_k d$$

$$\boxed{R = \frac{v_0^2}{5g} + \frac{2}{5}(h - \mu_k d)}$$

↳ R is independent of m, θ and μ_s.

5) a) $F = -\frac{dU}{dx}$, U linear $\Rightarrow$ $F = -\frac{\Delta U}{\Delta x}$

i) x: 0 to 1m : $U = \text{const.} \rightarrow F_{01} = 0$

ii) x: 1m to 2m : $F_{12} = -\frac{(20-10)\,J}{(1-2)\,m} = 10N$

iii) x: 2m to 3m : $U = \text{const} \longrightarrow F_{23} = 0$

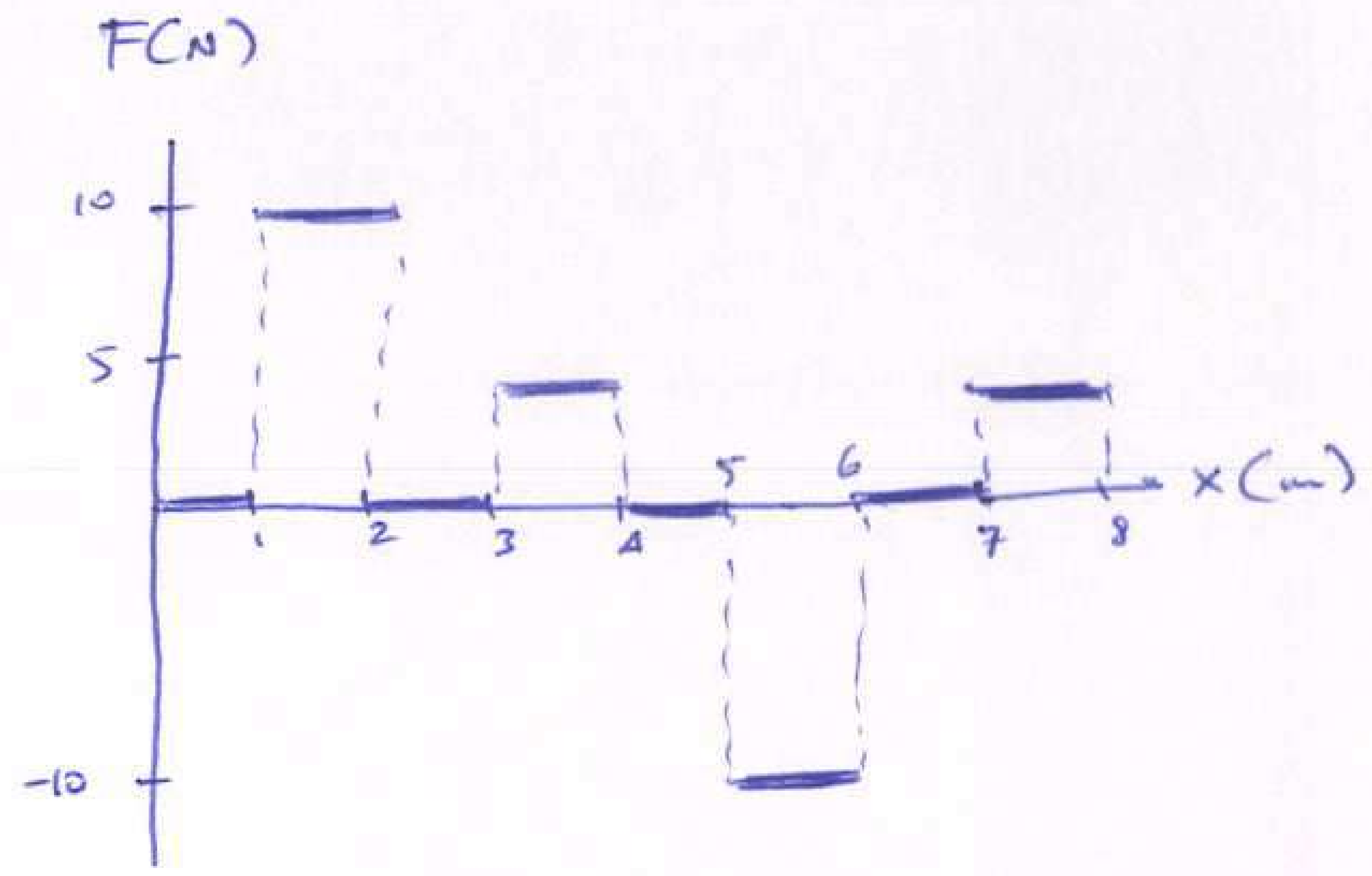
iv) x: 3m to 4m : $F_{34} = -\frac{(10-5)\,J}{(3-4)\,m} = 5N$

v) x: 4m to 5m : $U = \text{const} \longrightarrow F_{45} = 0$

vi) x: 5m to 6m : $F_{56} = -\frac{(5-15)\,J}{(5-6)\,m} = -10N$

vii) x: 6m to 7m : $U = \text{const} \longrightarrow F_{67} = 0$

viii) x = 7m to 8m : $F_{78} = -\frac{(15-10)\,J}{(7-8)\,J} = 5N$

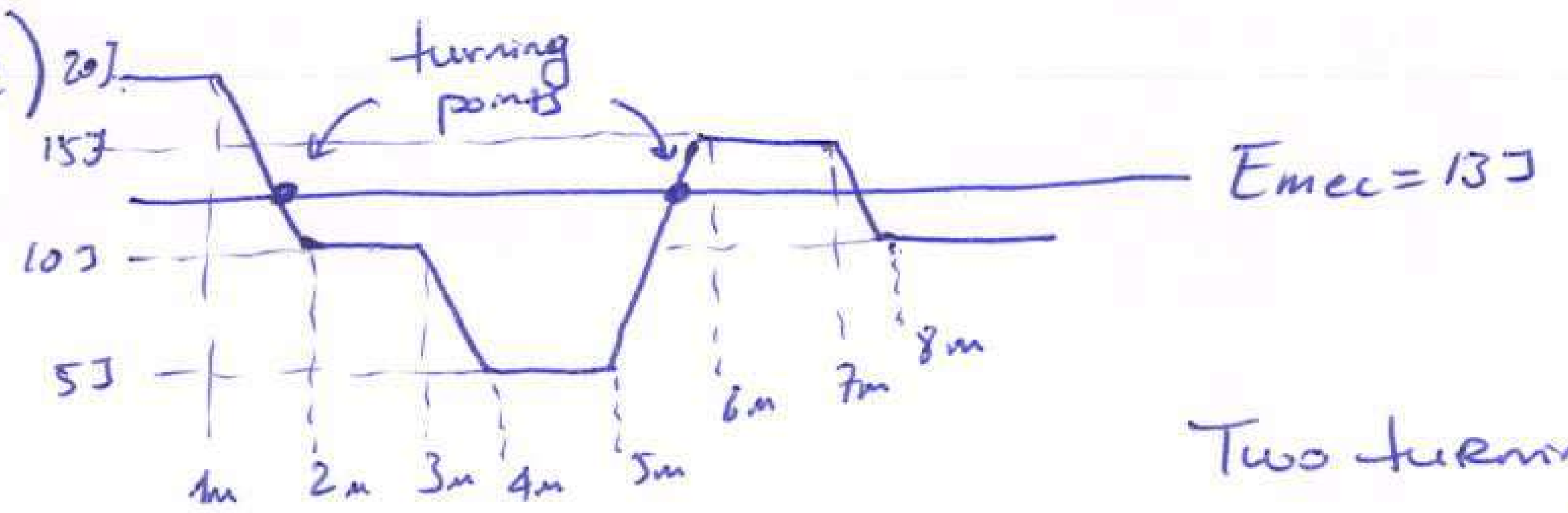


b) $W = \sum F(x)\,\Delta x = (10.1 + 5.1 - 10.1 + 5.1)\,J = 10\,J$

– or –

$W = -\Delta U = -\big(U(x=8m) - U(x=0m)\big) = -(10J - 20J) = 10J$

c)



Two turning points:

i) x_{t1-2} : between x = 1m and x = 2m

ii) x_{t5-6} : between x = 5 and x = 6m

No equilibrium point in between

$K(x) + U(x) = E_{mec}$

at turning point, $K = 0$ $(v=0)$, $F \neq 0$

$$x_{t1-2} : \frac{\overset{13J}{y_{t1-2}} - 20J}{x_{t1-2} - 1m} = \frac{(20-10)\,J}{(1-2)\,m} \rightarrow \frac{-7J}{x_{t1-2} - 1m} = \frac{10J}{-1m}$$

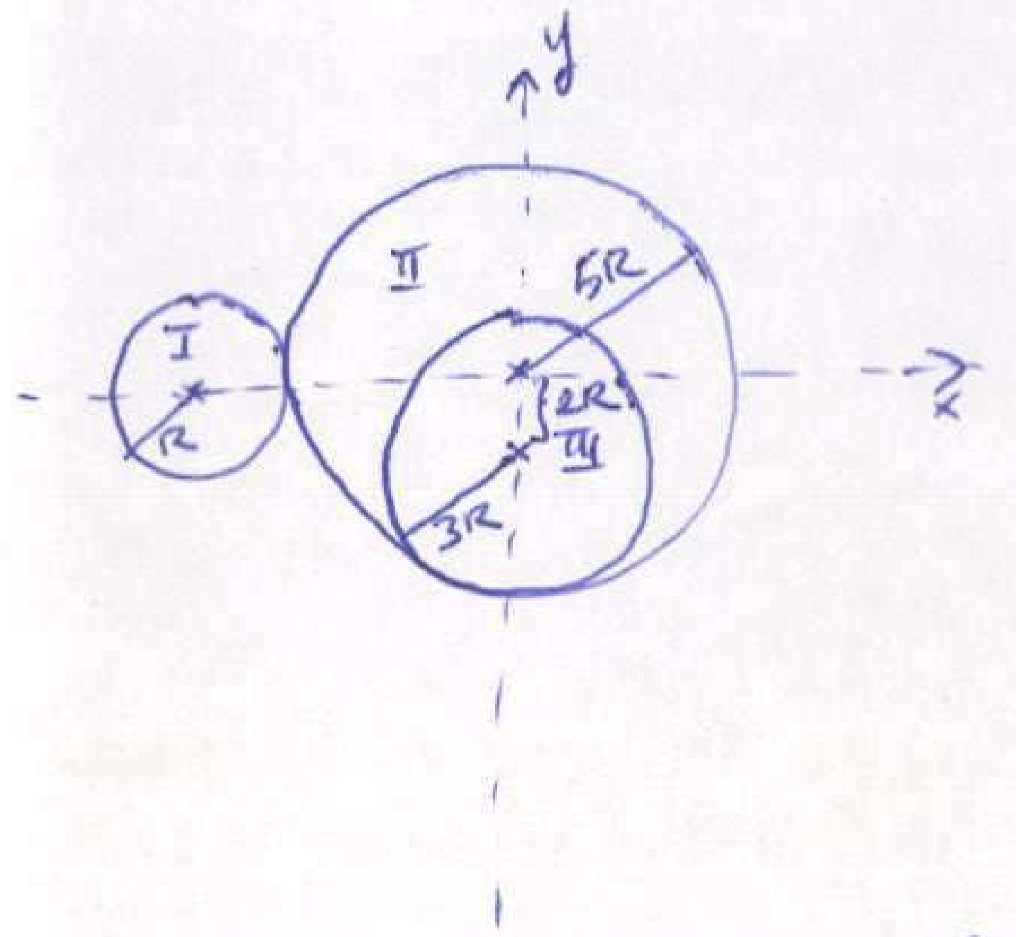
$$\Rightarrow x_{t1-2} - 1m = \frac{7m}{10} \rightarrow x_{t1-2} = \frac{17m}{10} = \underline{1.7m}$$

$$x_{t5-6} : \frac{\overset{13J}{y_{t5-6}} - 15J}{x_{t5-6} - 6m} = \frac{(5-15)\,J}{(5-6)\,m} \longrightarrow \frac{-2J}{x_{t5-6} - 6m} = \frac{-10J}{-1m}$$

$$\Rightarrow x_{t5-6} - 6m = -\frac{2m}{10}$$

$$x_{t5-6} = \frac{58m}{10} = \underline{\underline{5.8m}}$$

1)



a.) $m_I = m$

$$\frac{m_I}{m_{II}} = \frac{A_I}{A_{II}} \Rightarrow \frac{m}{m_{II}} = \frac{\pi R^2}{25\pi R^2 - 9\pi R^2} \Rightarrow m_{II} = 16m$$

$$\frac{m_I}{m_{III}} = \frac{\pi R^2}{9\pi R^2} \Rightarrow m_{III} = 9m$$

$$X_{com} = \frac{(-6R)m + \overset{\to 0}{x_{II}}\,16m}{17m} = -\frac{6}{17}R$$

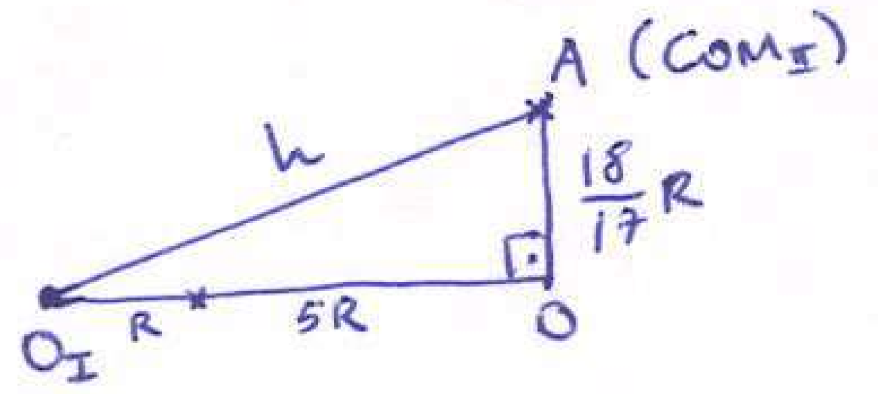
$$Y_{com} = \frac{y_I \cdot m + y_{II} \cdot 16m}{17m} \quad , \; y_{II} = ?$$

$$y_{II} + y_{III} = 0 = \frac{y_{II}\,16m + (-2R) \cdot 9m}{25m} \Rightarrow y_{II} = \frac{18}{16}R = \frac{9}{8}R$$

$$\Rightarrow Y_{com} = \frac{\frac{9}{8}R \cdot 16m}{17m} = \frac{18}{17}R$$

$$(X_{com}, Y_{com}) = \left(-\frac{6}{17}R, \frac{18}{17}R\right)$$

b.)

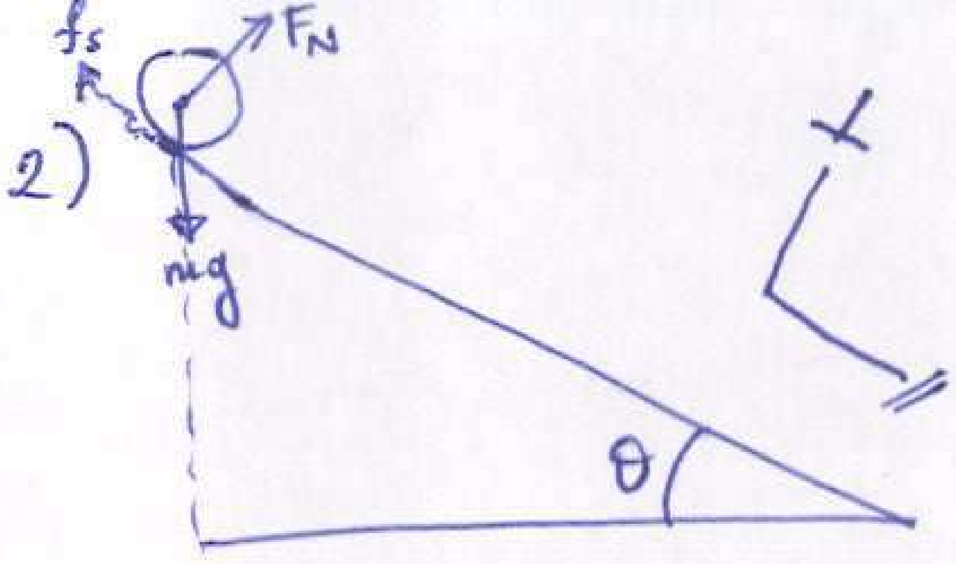
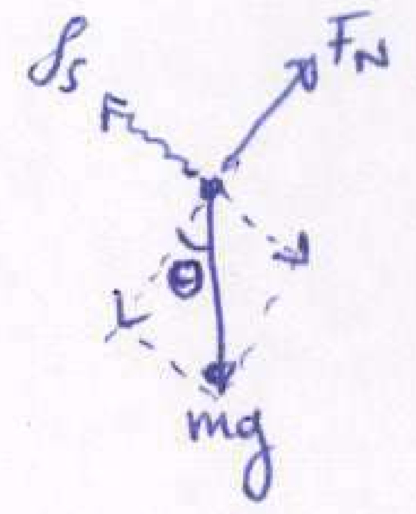


$$I_{tot,O_I} = I_I + I_{II,O_I} = \frac{1}{2}MR^2 + I_{II,A} + M_{II}h^2 \quad , \; I_{II,A} = ?$$

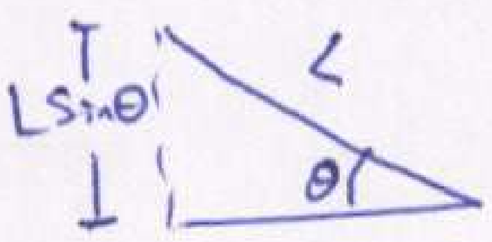
$$I_{II,A} : I \quad \rightarrow \quad I + I_{\times A} = I_{\times A}$$

$$\rightarrow \frac{1}{2}\,9m.9R^2 + 9m\left(3 + \frac{18}{17}\right)^2 R^2 \qquad \rightarrow \frac{1}{2}\,25m \cdot 25R^2 + 25m\left(\frac{18}{17}R\right)^2$$

$$\Rightarrow I_{II,A} = I = I_{\times A} - I_{\times A}$$

2)  

a.) $U_i = K_f$



$mgL\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\omega = \frac{v}{R}$, $I = \frac{1}{2}mR^2$

$\rightarrow mgL\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\cdot\frac{v^2}{R^2}$

$gL\sin\theta = \frac{3}{4}v^2 \Rightarrow v = \sqrt{\frac{4}{3}gL\sin\theta}$

$v = \sqrt{\frac{4}{3}\cdot 10.10.\frac{1}{2}}\ m/s = \sqrt{\frac{400}{6}}\ m/s = \underline{\underline{8.16\ m/s}}$

b) $K_{final} = U_{initial}$

$= mgL\sin\theta$

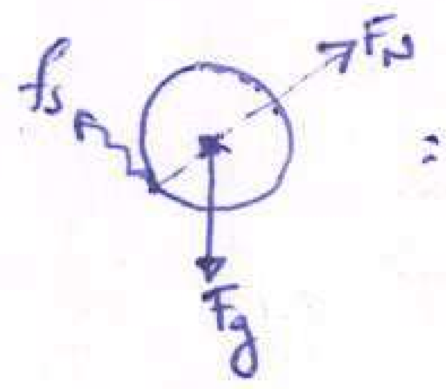
$= 0.5kg \cdot 10 m/s^2 \cdot 10m \cdot \frac{1}{2}$

$= \underline{\underline{25}}\ J$

c) $a_\perp = 0$ (no movement along the $\perp$ direction)

$\rightarrow \sum F_\perp = F_N - mg\cos\theta = 0 \Rightarrow F_N = mg\cos\theta$ (1)

$\sum F_{//} = mg\sin\theta - f_s = ma$ (2)

 : Rotation is only due to the Friction

$\longrightarrow \tau = f_s . R = I\alpha \rightarrow f_s = \frac{I\alpha}{R} = \frac{mR^2}{2}\cdot\frac{1}{R}\cdot\frac{a}{R} = \frac{ma}{2}$ (3)

(3) $\rightarrow$ (2) : $mg\sin\theta - f_s = 2f_s \Rightarrow f_s = \frac{mg\sin\theta}{3}$

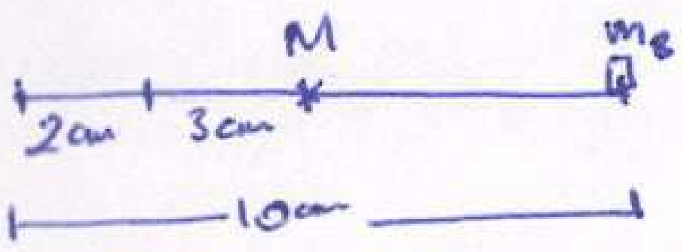
(1) $\rightarrow f_s = \mu_s F_N = \frac{mg\sin\theta}{3}$

$\mu_s . mg\cos\theta = \frac{1}{3}mg\sin\theta \rightarrow \mu_s = \frac{1}{3}\tan\theta = \frac{1}{3}\ 0.58 = \underline{\underline{0.19}}$

d) Otherwise, it would imply that there is slip involved.

3) a.) Since there is no external torque, L is conserved.

b)



$M = 100\times10^{-3}\,kg = 10^{-1}\,kg$

$m = 1g = 10^{-3}\,kg$

$\omega_A = 0.5\,rad/s$

$L = 10\times10^{-2}\,m = 10^{-1}\,m$

$L_i = L_f$

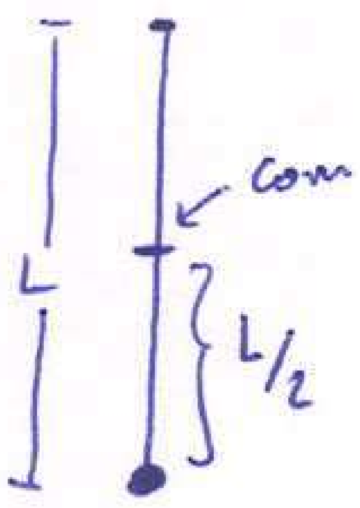
$I_i \cdot \omega_A = I_f \cdot \omega_B$

$$I_i = \overbrace{\frac{1}{12}ML^2 + M(3\times10^{-2}m)^2}^{I_{rod}} + \overbrace{m_B(8\times10^{-2}m)^2}^{I_{bug,A}} = \underbrace{\frac{1}{12}10^{-1}kg\,(10^{-1}m)^2 + 10^{-1}kg\cdot9\times10^{-4}m^2}_{\downarrow} + 10^{-3}kg\cdot64\times10^{-4}m^2$$

$$I_f = \underbrace{\frac{1}{12}ML^2 + M(3\times10^{-2}m)^2}_{I_{rod}} + \underbrace{m_B(3\times10^{-2}m)^2}_{I_{bug,B}} = \ldots + 10^{-3}kg\cdot9\times10^{-4}m^2$$

$$\omega_B = \frac{I_i\cdot\omega_A}{I_f} = \frac{\left(\frac{10}{12}+\frac{9}{10}+\frac{64}{10^3}\right)\times10^{-4}\,kg\,m^2\cdot0.5\,rad/s}{\left(\frac{10}{12}+\frac{9}{10}+\frac{9}{10^3}\right)\times10^{-4}\,kg\,m^2}$$

4)



$U_i = K_f$

$mg\frac{L}{2} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2\right)\cdot\left(\frac{v}{L}\right)^2$

$\cancel{m}g\frac{L}{\cancel{2}} = \frac{1}{\cancel{2}}\cdot\frac{\cancel{m}\cancel{L^2}}{3}\cdot\frac{v^2}{\cancel{L^2}} \rightarrow v^2 = 3gL$

$v = \sqrt{3gL} = \sqrt{3\cdot10\,m/s^2\cdot30\times10^{-2}\,m} = 3\,m/s$

5) $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$t=0:\ \left.\begin{matrix}\theta_0 = 0\\ \omega_0 = 0\end{matrix}\right\} \rightarrow \theta(t) = \frac{1}{2}\alpha t^2$

$\theta(t_0+4s) - \theta(t_0) = 120\,rad$

$\frac{1}{2}\alpha(t_0+4s)^2 - \frac{1}{2}\alpha(t_0)^2 = 120\,rad$

$\frac{1}{\cancel{2}}\cdot3\,\cancel{rad}/s^2\left[\cancel{t_0^2} + 8s\cdot t_0 + 16s^2 - \cancel{t_0^2}\right] = \cancel{120}^{\cancel{40}\ 80}\,\cancel{rad}$

$(8s)\cdot t_0 + 16s^2 = 80s^2 \rightarrow (8s)t_0 = 64s^2$

$\underline{t_0 = 8s}$