HYPOTHESES TESTS

Hypotheses Tests for the Mean of the Normal Population

Let $X_1, X_2, ..., X_n$ be a random sample from normal distribution with the mean μ and the variance σ^2. It is shown as $X_1, X_2, ..., X_n \sim N(\mu, \sigma^2)$, then

Two-sided	One(right)-sided	One(left)-sided
$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$
$H_1 : \mu \neq \mu_0$	$H_1 : \mu > \mu_0$	$H_1 : \mu < \mu_0$
	$H_0 : \mu \leq \mu_0$	$H_0 : \mu \geq \mu_0$
	$H_1 : \mu > \mu_0$	$H_1 : \mu < \mu_0$

If population variance σ^2 is <u>known,</u>

Test statistic, $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \geq z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \geq z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \leq -z_\alpha$, H_0 is rejected.

If population variance σ^2 is <u>unknown</u>,

If the sample size n is enough large ($n \geq 30$), <u>(Large sample size)</u>

Test statistic, $z = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$ table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \geq z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \geq z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \leq -z_\alpha$, H_0 is rejected.

NOT: If the population distribution is different from normal distribution, when $n \geq 30$ (Central Limit Theorem) the test statistics given above are used.

If the sample size n is not enough large ($n<30$), <u>(Small sample size)</u>

Test statistic, $t = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$ table value, $t_{\alpha/2, n-1}$, $t_{\alpha, n-1}$ and $-t_{\alpha, n-1}$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|t| \geq t_{\alpha/2, n-1}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $t \geq t_{\alpha, n-1}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $t \leq -t_{\alpha, n-1}$, H_0 is rejected.

Here, S is the standard deviation of the sample.

Hypotheses Tests for the Population Variance σ^2

Let $X_1, X_2, ..., X_n$ be a random sample from normal distribution, shown as $X_1, X_2, ..., X_n \sim N(\mu, \sigma^2)$

Two-sided	One(right)-sided	One(left)-sided
$H_0 : \sigma^2 = \sigma_0^2$	$H_0 : \sigma^2 = \sigma_0^2$	$H_0 : \sigma^2 = \sigma_0^2$
$H_1 : \sigma^2 \neq \sigma_0^2$	$H_1 : \sigma^2 > \sigma_0^2$	$H_1 : \sigma^2 < \sigma_0^2$
	$H_0 : \sigma^2 \leq \sigma_0^2$	$H_0 : \sigma^2 \geq \sigma_0^2$
	$H_1 : \sigma^2 > \sigma_0^2$	$H_1 : \sigma^2 < \sigma_0^2$

Test statistic, $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$.

Table value, $\chi^2_{\alpha/2, n-1}$, $\chi^2_{1-\alpha/2, n-1}$, $\chi^2_{\alpha, n-1}$, $\chi^2_{1-\alpha, n-1}$

Decision: According to alternative hypothesis given above, If alternative hypothesis is two-sided: If $\chi^2 \geq \chi^2_{\alpha/2,n-1}$ or $\chi^2 \leq \chi^2_{1-\alpha/2,n-1}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $\chi^2 \geq \chi^2_{\alpha,n-1}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $\chi^2 \leq \chi^2_{1-\alpha,n-1}$, H_0 is rejected.

The Hypotheses Tests for the Comparison of Two Normal Population Variances

Let $X_{11}, X_{12},..., X_{1n_1}$ and $X_{21}, X_{22},..., X_{2n_2}$ be independent random samples from normal distributions, shown as $X_{11}, X_{12},..., X_{1n_1} \sim N(\mu_1,\sigma_1^2)$ and $X_{21}, X_{22},..., X_{2n_2} \sim N(\mu_2,\sigma_2^2)$.

$H_0 : \sigma_1^2 = \sigma_2^2$
$H_1 : \sigma_1^2 \neq \sigma_2^2$

Test statistic is: if $s_1^2 \geq s_2^2$, then $f = \dfrac{s_1^2}{s_2^2} \geq f_{\alpha/2,\ n_1-1,n_2-1}$

and

if $s_2^2 \geq s_1^2$, then $f = \dfrac{s_2^2}{s_1^2} \geq f_{\alpha/2,\ n_2-1,n_1-1}$, H_0 is rejected.

$H_0 : \sigma_1^2 = \sigma_2^2$ $\quad (H_0 : \sigma_1^2 \leq \sigma_2^2)$
$H_1 : \sigma_1^2 > \sigma_2^2$ $\quad (H_1 : \sigma_1^2 > \sigma_2^2)$

Test statistic is: if $f = \dfrac{s_1^2}{s_2^2} \geq f_{\alpha,\ n_1-1,n_2-1}$, H_0 is rejected.

$H_0 : \sigma_1^2 = \sigma_2^2$ $\quad (H_0 : \sigma_1^2 \geq \sigma_2^2)$
$H_1 : \sigma_1^2 < \sigma_2^2$ $\quad (H_1 : \sigma_1^2 < \sigma_2^2)$

Test statistic is: if $f = \dfrac{s_2^2}{s_1^2} \geq f_{\alpha,\ n_2-1,n_1-1}$, H_0 is rejected.

The Hypotheses Tests to Compare the Means of Two Normal Populations

Let $X_{11}, X_{12},..., X_{1n_1}$ and $X_{21}, X_{22},..., X_{2n_2}$ be independent random samples from normal distribution, shown as $X_{11}, X_{12},..., X_{1n_1} \sim N(\mu_1,\sigma_1^2)$ and $X_{21}, X_{22},..., X_{2n_2} \sim N(\mu_2,\sigma_2^2)$

Two-sided	One(right)-sided	One(left)-sided
$H_0 : \mu_1 - \mu_2 = \delta$	$H_0 : \mu_1 - \mu_2 = \delta$	$H_0 : \mu_1 - \mu_2 = \delta$
$H_1 : \mu_1 - \mu_2 \neq \delta$	$H_1 : \mu_1 - \mu_2 > \delta$	$H_1 : \mu_1 - \mu_2 < \delta$
	$H_0 : \mu_1 - \mu_2 \leq \delta$	$H_0 : \mu_1 - \mu_2 \geq \delta$
	$H_1 : \mu_1 - \mu_2 > \delta$	$H_1 : \mu_1 - \mu_2 < \delta$

If σ_1^2 and σ_2^2 are known,

Test statistic, $z = \dfrac{\bar{x}_1 - \bar{x}_2 - \delta}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \geq z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \geq z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \leq -z_\alpha$, H_0 is rejected.

If σ_1^2 and σ_2^2 are unknown,

If the sample sizes are n_1 and $n_2 \geq 30$, (Large sample sizes)

Test statistics, $z = \dfrac{\bar{x}_1 - \bar{x}_2 - \delta}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \geq z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \geq z_{\alpha}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \leq -z_{\alpha}$, H_0 is rejected.

NOT: If the populations' distributions are different from normal distribution, when n_1 and $n_2 \geq 30$ (Central Limit Theorem) the test statistics given above are used.

If the sample sizes are n_1 and $n_2 < 30$, (Small sample sizes)

Firstly, whether $\sigma_1^2 = \sigma_2^2 = \sigma^2$ or not must be tested.

If $\sigma_1^2 = \sigma_2^2 = \sigma^2$,

Test statistics, $t = \dfrac{\bar{x}_1 - \bar{x}_2 - \delta}{s_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

table value, $t_{\alpha/2, n_1+n_2-2}$, t_{α, n_1+n_2-2} and $-t_{\alpha, n_1+n_2-2}$

Pooled variance $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|t| \geq t_{\alpha/2, n_1+n_2-2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $t \geq t_{\alpha, n_1+n_2-2}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $t \leq -t_{\alpha, n_1+n_2-2}$, H_0 is rejected.

If $\sigma_1^2 \neq \sigma_2^2$,

Test statistic, $t = \dfrac{\bar{x}_1 - \bar{x}_2 - \delta}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ table value, $t_{\alpha/2, \nu}$, $t_{\alpha, \nu}$ and $-t_{\alpha, \nu}$

Degrees of freedom $\nu = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\left(\dfrac{s_1^2}{n_1}\right)^2\left(\dfrac{1}{n_1-1}\right) + \left(\dfrac{s_2^2}{n_2}\right)^2\left(\dfrac{1}{n_2-1}\right)}$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|t| \geq t_{\alpha/2, \nu}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $t \geq t_{\alpha, \nu}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $t \leq -t_{\alpha, \nu}$, H_0 is rejected.

NOT: If the population distributions are different from normal distribution, when n_1 and $n_2 \geq 30$, Central Limit Theorem is used.

The Hypotheses Tests for Paired Samples

$D_i = X_{1i} - X_{2i} \sim N(\mu_1 - \mu_2, \sigma_D^2)$ i=1,2,…,n

If σ_D^2 is known,

Two-sided	One(right)-sided	One(left)-sided
$H_0: \mu_1 - \mu_2 = d_0$	$H_0: \mu_1 - \mu_2 = d_0$	$H_0: \mu_1 - \mu_2 = d_0$
$H_1: \mu_1 - \mu_2 \neq d_0$	$H_1: \mu_1 - \mu_2 > d_0$	$H_1: \mu_1 - \mu_2 < d_0$
	$H_0: \mu_1 - \mu_2 \leq d_0$	$H_0: \mu_1 - \mu_2 \geq d_0$
	$H_1: \mu_1 - \mu_2 > d_0$	$H_1: \mu_1 - \mu_2 < d_0$

Test statistic, $z=\dfrac{\bar{d}-d_0}{\sigma_D/\sqrt{n}}$. Table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z|\ge z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z\ge z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z\le -z_\alpha$, H_0 is rejected.

If σ_D^2 is <u>unknown</u>,

Two-sided	One(right)-sided	One(left)-sided
$H_0:\mu_1-\mu_2=d_0$	$H_0:\mu_1-\mu_2=d_0$	$H_0:\mu_1-\mu_2=d_0$
$H_1:\mu_1-\mu_2\ne d_0$	$H_1:\mu_1-\mu_2>d_0$	$H_1:\mu_1-\mu_2<d_0$
	$H_0:\mu_1-\mu_2\le d_0$	$H_0:\mu_1-\mu_2\ge d_0$
	$H_1:\mu_1-\mu_2>d_0$	$H_1:\mu_1-\mu_2<d_0$

Test statistic, $t=\dfrac{\bar{d}-d_0}{s_D/\sqrt{n}}$. Table value $t_{\alpha/2,n-1}$, $t_{\alpha,n-1}$ and $-t_{\alpha,n-1}$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|t|\ge t_{\alpha/2,n-1}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $t\ge t_{\alpha,n-1}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $t\le -t_{\alpha,n-1}$, H_0 is rejected.

The Hypotheses Tests for the Population Proportion

$X\sim Binom(n,p)$

Two-sided	One(right)-sided	One(left)-sided
$H_0:p=p_0$	$H_0:p=p_0$	$H_0:p=p_0$
$H_1:p\ne p_0$	$H_1:p>p_0$	$H_1:p<p_0$
	$H_0:p\le p_0$	$H_0:p\ge p_0$
	$H_1:p>p_0$	$H_1:p<p_0$

Test statistic, $z=\dfrac{\frac{x}{n}-p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$ Table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z|\ge z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z\ge z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z\le -z_\alpha$, H_0 is rejected.

The Hypotheses Tests to Compare Two Population Proportions

X_1 and X_2 are two random variables from a binomial distribution, shown as $X_1\sim Binom(n_1,p_1)$ and $X_2\sim Binom(n_2,p_2)$.

Two-sided	One(right)-sided	One(left)-sided
$H_0:p_1-p_2=0$	$H_0:p_1-p_2=0$	$H_0:p_1-p_2=0$
$H_1:p_1-p_2\ne 0$	$H_1:p_1-p_2>0$	$H_2:p_1-p_2<0$
	$H_0:p_1-p_2\le 0$	$H_0:p_1-p_2\ge 0$
	$H_1:p_1-p_2>0$	$H_2:p_1-p_2<0$

Test statistic, $z=\dfrac{\frac{x_1}{n_1}-\frac{x_2}{n_2}}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$ Table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z|\ge z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z\ge z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z\le -z_\alpha$, H_0 is rejected.

Here, $\hat{p}=\dfrac{x_1+x_2}{n_1+n_2}$.

CONFIDENCE INTERVALS

The Confidence Interval for the Mean of the Normal Population

Let $X_1, X_2, ..., X_n$ be a random sample from normal distribution with the mean μ and the variance σ^2. It is shown as $X_1, X_2, ..., X_n \sim N(\mu, \sigma^2)$, then

If population variance σ^2 is known, $P\left(\bar{x} - z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}\right) = 1-\alpha$

If population variance σ^2 is unknown and then

when the sample size n is enough large ($n \geq 30$), $P\left(\bar{x} - z_{\alpha/2}\dfrac{s}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\dfrac{s}{\sqrt{n}}\right) = 1-\alpha$

NOT: If the population distribution is different from normal distribution, if $n \geq 30$ (Central Limit Theorem) the interval estimations' formulas given above are used.

If the sample size n is not enough large ($n<30$), $P\left(\bar{x} - t_{\alpha/2,(n-1)}\dfrac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2,(n-1)}\dfrac{s}{\sqrt{n}}\right) = 1-\alpha$

The Confidence Interval for a Population Variance σ^2

Let $X_1, X_2, ..., X_n$ be a random sample from normal distribution, shown as $X_1, X_2, ..., X_n \sim N(\mu, \sigma^2)$

$$P\left(\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},(n-1)}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},(n-1)}}\right) = 1-\alpha$$

The Confidence Interval for the Difference of the Means of Two Normal Populations

Let $X_{11}, X_{12}, ..., X_{1n}$ and $X_{21}, X_{22}, ..., X_{2n_2}$ be independent random samples from normal distribution, shown as $X_{11}, X_{12}, ..., X_{1n_1} \sim N(\mu_1, \sigma_1^2)$ and $X_{21}, X_{22}, ..., X_{2n_2} \sim N(\mu_2, \sigma_2^2)$

If σ_1^2 and σ_2^2 are known,

$$P\left(\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right) = 1-\alpha$$

If σ_1^2 and σ_2^2 are unknown and then

when the sample sizes are enough large (<u>n_1 and $n_2 \geq 30$),</u>

$$P\left(\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right) = 1-\alpha$$

NOT: If the population distribution is different from normal distribution, if n_1 and $n_2 \geq 30$ (Central Limit Theorem) the interval estimations' formulas given above are used.

<u>If the sample sizes are n_1 and $n_2 < 30$,</u>

then when $\sigma_1^2 = \sigma_2^2 = \sigma^2$,

$$P\left(\bar{x}_1 - \bar{x}_2 - t_{\alpha/2,(n_1+n_2-2)} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < \bar{x}_1 - \bar{x}_2 + t_{\alpha/2,(n_1+n_2-2)} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right) = 1-\alpha$$

Pooled Variance: $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$

when $\sigma_1^2 \neq \sigma_2^2$,

$$P\left(\bar{x}_1-\bar{x}_2-t_{\alpha/2,(\nu)}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}<\mu_1-\mu_2<\bar{x}_1-\bar{x}_2+t_{\alpha/2,(\nu)}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\right)=1-\alpha$$

$$\nu=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\left(\frac{s_1^2}{n_1}\right)^2\left(\frac{1}{n_1-1}\right)+\left(\frac{s_2^2}{n_2}\right)^2\left(\frac{1}{n_2-1}\right)}$$

The Confidence Interval for Paired Samples

$D_i = X_{1i} - X_{2i} \sim N(\mu_1-\mu_2, \sigma_D^2)$ i=1,2,…,n,

For observed values $d_i = x_{1i}-x_{2i}$, $i=1,2,...,$ n

If σ_D^2 is <u>known</u>, $P\left(\bar{d}-z_{\alpha/2}\frac{\sigma_D}{\sqrt{n}}<\mu_1-\mu_2<\bar{d}+z_{\alpha/2}\frac{\sigma_D}{\sqrt{n}}\right)=1-\alpha$

If σ_D^2 <u>is unknown</u>, and also <u>n<30</u> $P\left(\bar{d}-t_{\alpha/2,(n-1)}\frac{s_D}{\sqrt{n}}<\mu_1-\mu_2<\bar{d}+t_{\alpha/2,(n-1)}\frac{s_D}{\sqrt{n}}\right)=1-\alpha$

Where, $\bar{d}=\frac{\sum_{i=1}^{n} d_i}{n}$, $s_D^2=\frac{\sum_{i=1}^{n}\left(d_i-\bar{d}\right)^2}{n-1}$

The Confidence Interval for a Proportion

$X \sim Binom(n,p)$

$$P\left(\hat{p}-z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}<p<\hat{p}+z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)=1-\alpha \qquad \hat{p}=\frac{x}{n}$$

The Confidence Interval for the Difference of Two Proportions of Two Binomial Populations

$$P\left(\hat{p}_1-\hat{p}_2-z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}<p_1-p_2 <\hat{p}_1-\hat{p}_2+z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}\right)=1-\alpha$$

$$\hat{p}_1=\frac{x_1}{n_1} \qquad \hat{p}_2=\frac{x_2}{n_2}$$

The Confidence Interval for the Proportions of Population Variances

Let $X_{11}, X_{12},..., X_{1n_1}$ and $X_{21}, X_{22},..., X_{2n_2}$ be independent random samples from normal distributions, shown as $X_{11}, X_{12},..., X_{1n_1} \sim N(\mu_1,\sigma_1^2)$ and $X_{21}, X_{22},..., X_{2n_2} \sim N(\mu_2,\sigma_2^2)$

$$P\left(\frac{1}{f_{\frac{\alpha}{2},(n_1-1),(n_2-1)}}\frac{s_1^2}{s_2^2}<\frac{\sigma_1^2}{\sigma_2^2}<\frac{1}{f_{1-\frac{\alpha}{2},(n_1-1),(n_2-1)}}\frac{s_1^2}{s_2^2}\right)=1-\alpha \qquad f_{1-\frac{\alpha}{2},(n_1-1),(n_2-1)}=\frac{1}{f_{\frac{\alpha}{2},(n_2-1),(n_1-1)}}$$