Name-Last name :
Student No :
Section :

FİZ 138 PHYSICS II
MIDTERM EXAM I
March 28, 2017
13:00 – 14:30 (90 min)

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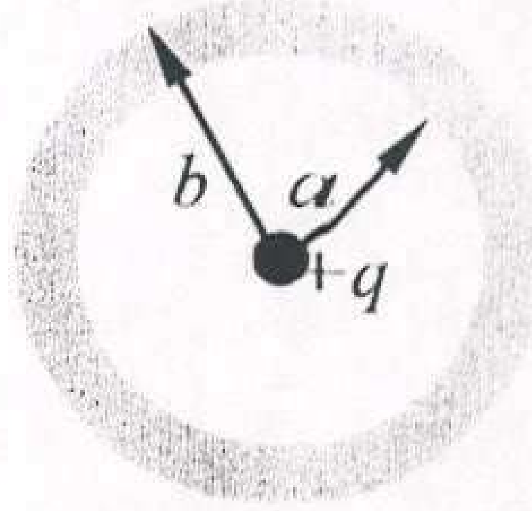
Questions

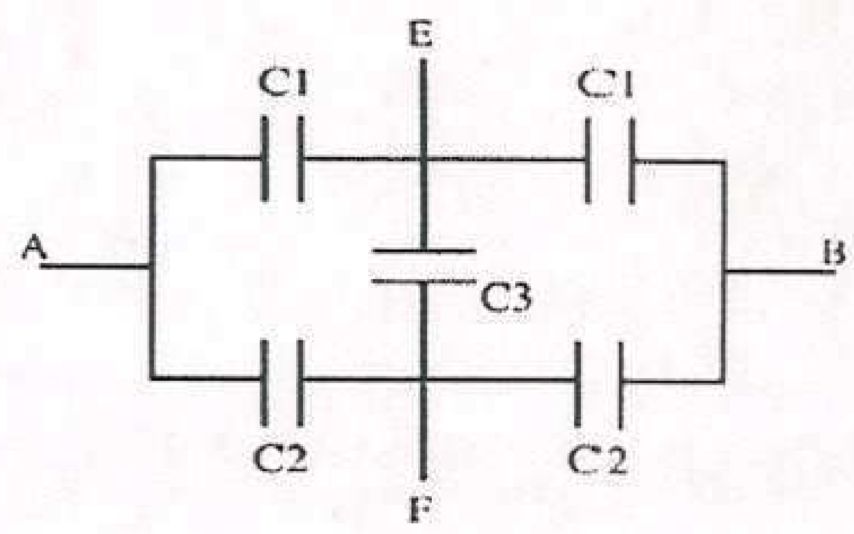
1. Figure shows a nonconducting spherical shell, of inner radius a and outer radius b, has a positive volume charge density $\rho = A/r$ (within its thickness), where A is a constant and r is the distance from the center of the shell. In addition, a positive charge q is located at the center. What value should A have if the electric field in the shell ($a \le r \le b$) is to be uniform?



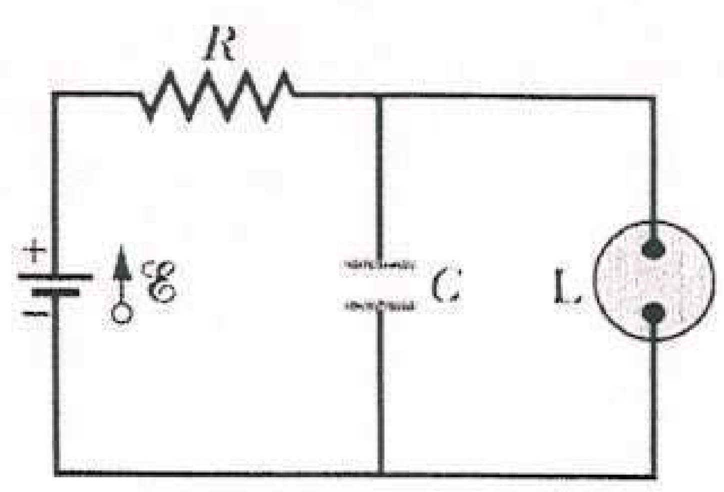
2. In a region where the electric potential is given by $V(x,y,z) = 2x^2 + yz$, find the electric field at the point with coordinates $x = 2$, $y = 1$, and $z = 2$. Everything is in SI units.

3. A spherical capacitor has radii a and b. What is the radius r for which the energy stored within, i.e., the spherical shell from radius a to radius r, is one third (1/3) of the total energy stored in the capacitor?

4. Calculate the equivalent capacitance (in terms of C_1, C_2 and C_3)
   a) C_{AB} between the points A and B,
   b) C_{EF} between the points E and F.



5. The fluorescent lamp L only functions when the potential difference across it reaches V_L— below that value, no current passes through it; then the capacitor discharges completely through the lamp and the lamp flashes briefly.
   a) With C, $\mathcal{E}$ (ideal *emf* device) and V_L given, calculate the necessary R in order to achieve n flashes per second from the lamp.
   b) If the circuit is turned on at $t = 0$, plot the voltage across the lamp's terminals with respect to time.



Charge on a charging capacitor: $q(t) = \mathcal{E}C[1-\exp(-t/RC)]$
Charge on a discharging capacitor: $q(t) = q_o \exp(-t/RC)$

1) $\epsilon_0 E(a) 4\pi a^2 = q$ ← central charge (1)

$$\epsilon_0 E(r) 4\pi r^2 = q + \int_a^r dr' 4\pi r'^2 \frac{A}{r'} = q + 4\pi A \frac{1}{2}(r^2 - a^2) \quad (2)$$

← charge in the shell (a to r)

E : uniform $\longrightarrow E(a) = E(r) = E$

$$E = \underbrace{\frac{1}{4\pi\epsilon_0}\frac{q}{a^2}}_{1} = \underbrace{\frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\left[q + 2\pi A(r^2 - a^2)\right]}_{2}$$

$$\rightarrow \frac{qr^2}{a^2} = q + 2\pi A(r^2 - a^2)$$

$$q\left(\frac{r^2}{a^2} - 1\right) = 2\pi A(r^2 - a^2)$$

$$q\frac{\cancel{(r^2-a^2)}}{a^2}\frac{1}{2\pi\cancel{(r^2-a^2)}} = A \Rightarrow \underline{A = \frac{q}{2\pi a^2}}$$

2) $E_i = -\frac{dV}{dx_i} \rightarrow$

$$E_x = -4x, \quad E_y = -z, \quad E_z = -y$$

$$\vec{E}(x,y,z) = -4x\hat{i} - z\hat{j} - y\hat{k}$$

$$\underline{\vec{E}(2,1,2) = (-8\hat{i} - 2\hat{j} - \hat{k})\,N/C}$$

3) **1st Way**

$u = \frac{1}{2}\epsilon_0 E^2$: Energy Density

$$E(r) = \frac{Q}{4\pi\epsilon_0 r^2} \rightarrow u = \frac{1}{2}\epsilon_0 \frac{Q^2}{16\pi^2\epsilon_0^2 r^4}$$

$$dU = u \cdot dV = \frac{1}{2}\frac{Q^2}{16\pi^2\epsilon_0 r^4} 4\pi r^2 dr$$

$$dU = \frac{Q^2}{8\pi\epsilon_0 r^2} dr$$

$$\frac{1}{3} = \frac{\int_a^r \frac{Q^2}{8\pi\epsilon_0 r'^2} dr'}{\int_a^b \frac{Q^2}{8\pi\epsilon_0 r^2} dr} = \frac{-\left[\frac{1}{r} - \frac{1}{a}\right]}{-\left[\frac{1}{b} - \frac{1}{a}\right]} = \frac{a-r}{\cancel{a}r} \cdot \frac{\cancel{a}b}{a-b}$$

2nd Way

$$U = \frac{1}{2}QV$$

$$E(r) 4\pi r^2 \epsilon_0 = Q \rightarrow E(r) = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$V(r) = -\int_a^r E(r') dr' = -\frac{Q}{4\pi\epsilon_0}\int_a^r \frac{dr'}{r'^2}$$

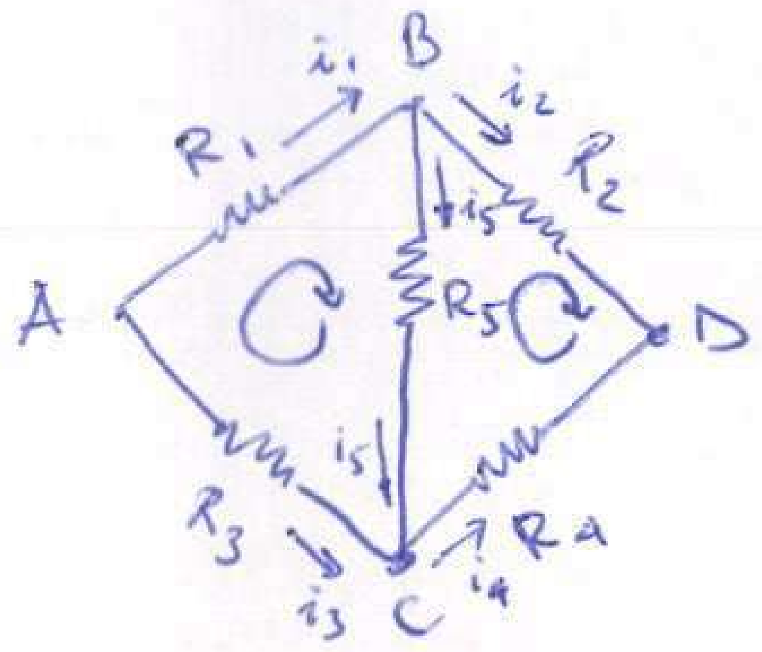
$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{a}\right] = \frac{Q}{4\pi\epsilon_0}\frac{(a-r)}{ar}$$

$$\frac{1}{3} = \frac{\frac{1}{2}QV(r)}{\frac{1}{2}QV(b)} = \frac{V(r)}{V(b)} = \frac{(a-r)}{ar} \cdot \frac{ab}{(a-b)}$$

$$\frac{1}{3} = \frac{(a-r)b}{(a-b)r} \longrightarrow \begin{aligned} 3ab - 3br &= ar - br \\ 3ab &= r(a - b + 3b) \end{aligned}$$

$$\boxed{r = \frac{3ab}{a+2b}}$$

Wheatstone Bridge Revisited



5 unknowns $= i_1, i_2, i_3, i_4, i_5$

$$B: \quad i_1 - i_2 - i_5 = 0$$

$$C: \quad i_3 + i_5 - i_4 = 0$$

$$ABC: \quad -i_1 R_1 - i_5 R_5 + i_3 R_3 = 0$$

$$BDC: \quad -i_2 R_2 + i_4 R_4 + i_5 R_5 = 0$$

4 equations (ABCD is just ABC + BDC, so nothing new)

5 unknowns − 4 equations = Not sufficient

Let's assume $i_5 = 0$. Then: $B: \; i_1 - i_2 = 0 \rightarrow i_1 = i_2 \equiv i_A$

$C: \; i_3 - i_4 = 0 \rightarrow i_3 = i_4 \equiv i_B$

$\rightarrow$ $ABC: \; -i_A R_1 + i_B R_3 = 0 \rightarrow i_A R_1 = i_B R_3$

$BDC: \; -i_A R_2 + i_B R_4 = 0 \rightarrow i_A R_2 = i_B R_4$ $\Big\} \rightarrow \dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$

$$\Rightarrow \quad \boxed{i_5 = 0 \longleftrightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}}$$

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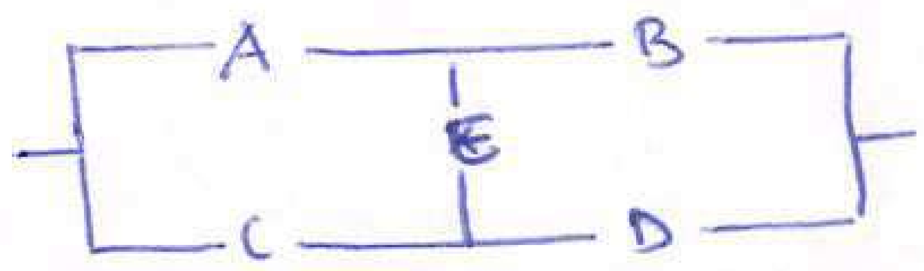
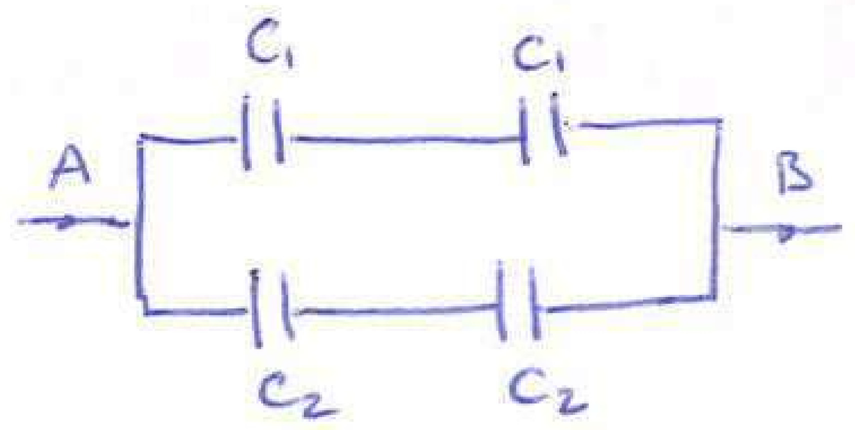
4) a) AB (Wheatstone Bridge)



$$\frac{X_A}{X_B} = \frac{X_C}{X_D} \Rightarrow i_E = 0$$

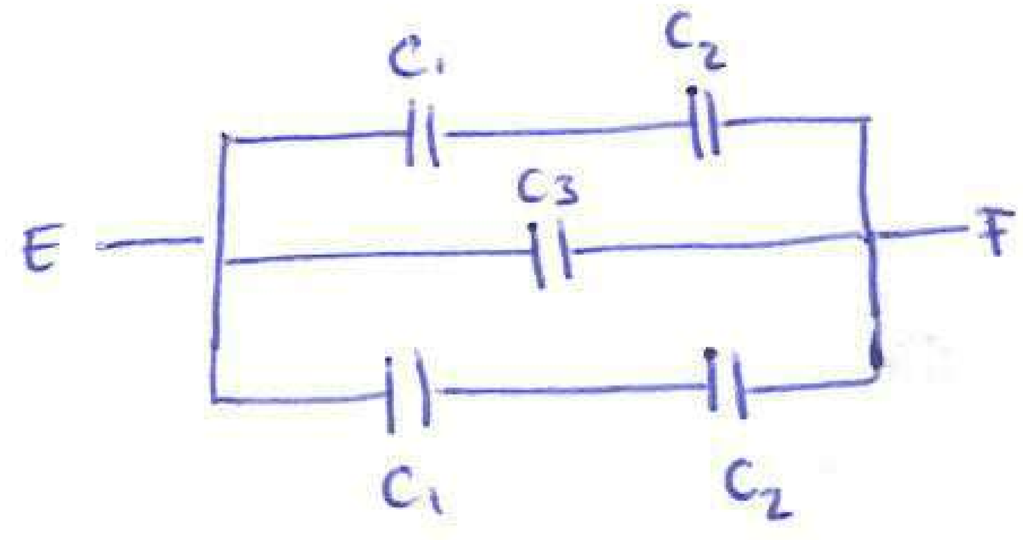
$\rightarrow$ 

$$\frac{C_1}{C_1} = \frac{C_2}{C_2} \rightarrow i_3 = 0$$

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_1}\right)^{-1} + \left(\frac{1}{C_2} + \frac{1}{C_2}\right)^{-1}$$

$$= \frac{C_1}{2} + \frac{C_2}{2} = \underline{\frac{C_1 + C_2}{2}}$$

b) EF



$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} + C_3 + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

$$= 2\frac{C_1 C_2}{C_1 + C_2} + C_3$$

$$= \frac{2C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2}$$

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5) a) $V = \dfrac{Q}{C} \rightarrow V(t) = \dfrac{\mathcal{E} \cancel{C} [1 - \exp(-t/RC)]}{\cancel{C}}$

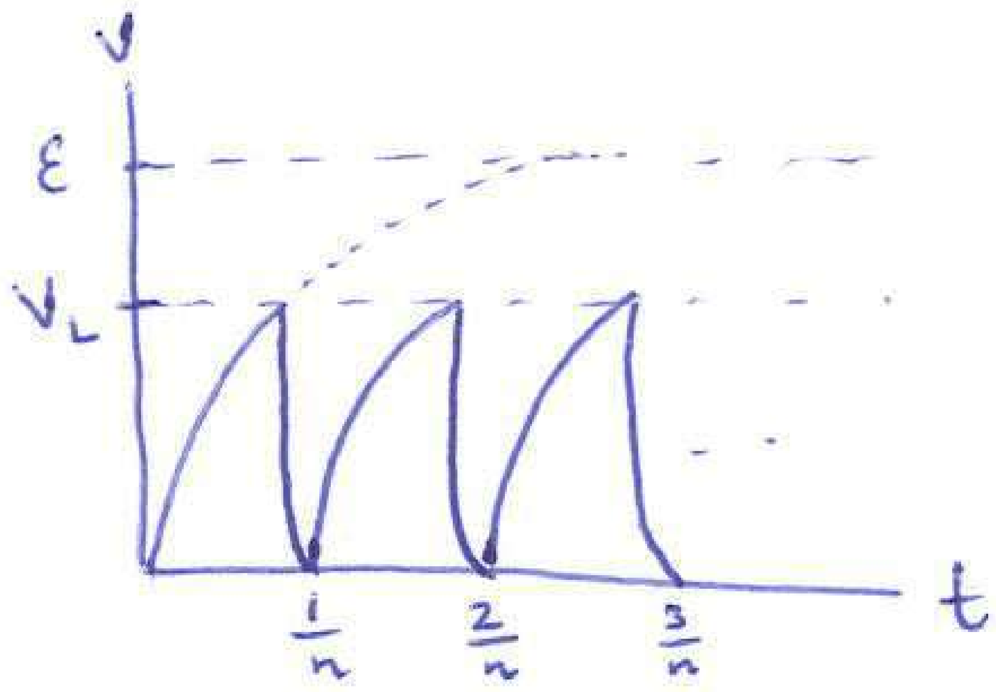
$V = V_L \leftrightarrow t = \frac{1}{n} \Rightarrow V_L = \mathcal{E}\left[1 - \exp\left(-\frac{1}{nRC}\right)\right]$

$$1 - \frac{V_L}{\mathcal{E}} = \exp\left(-\frac{1}{nRC}\right)$$

$$\ln\left(1 - \frac{V_L}{\mathcal{E}}\right) = -\frac{1}{nRC}$$

$$R = -\frac{1}{nC}\frac{1}{\ln\left(1 - \frac{V_L}{\mathcal{E}}\right)} = -\frac{1}{nC}\frac{1}{\ln\left(\frac{\mathcal{E} - V_L}{\mathcal{E}}\right)} = \frac{1}{nC\ln\left(\frac{\mathcal{E}}{\mathcal{E} - V_L}\right)}$$

b)



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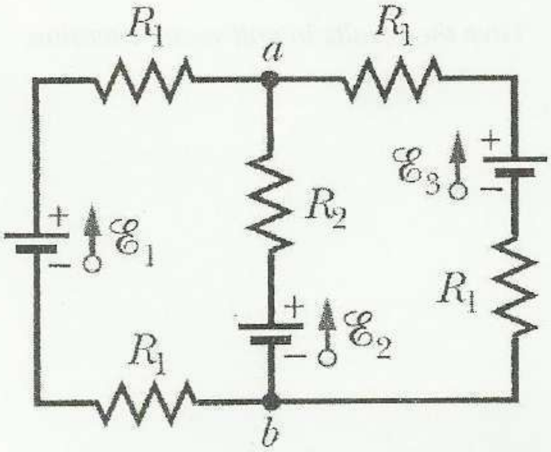
FİZ 138 - 25, 26 PHYSICS I
2nd MIDTERM
08 MAY 2018 / 13:00 – 14:50

NAME-SURNAME:..............................NUMBER:................. SIGNATURE:...............

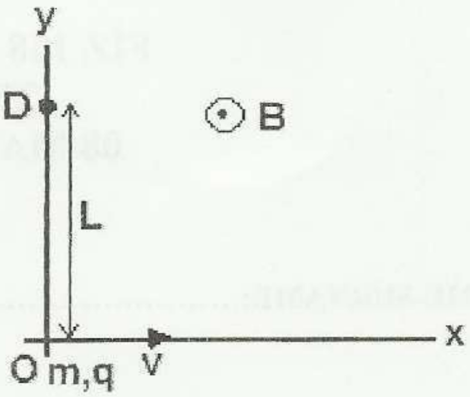
GOOD LUCK

1. In the given figure, the resistances are $R_1 = 2\ \Omega$ and $R_2 = 4\ \Omega$ and the ideal batteries have emfs $\mathcal{E}_1 = 4$ V and $\mathcal{E}_2 = \mathcal{E}_3 = 8$ V.



a) What is the current in battery 1
b) What is the current in battery 2
c) What is the currnet in battery 3
d) What is the potential difference $Va - Vb$?

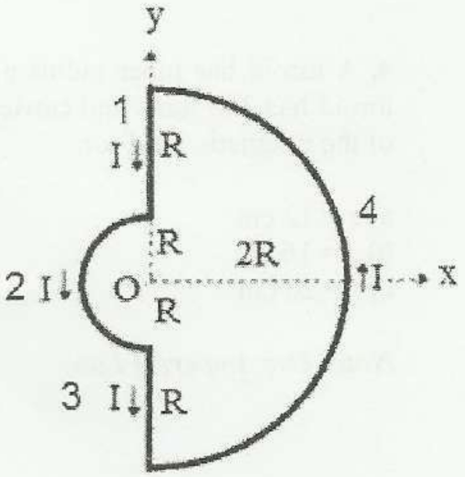
2. A particle of mass $m = 4\ g$ and charge $q = 0.5\ C$ is projected into a uniform magnetic field of $\vec{B} = 4.5\ \hat{k}$ (T). It passes the origin O at t = 0 s with a velocity of $\vec{v} = 9.0\ \hat{\imath}$ (m/s) as shown in the figure. After a time t, the particle passes the point D, on the y axis, at a distance L from the origin.



a) Find the sign of the charge of the particle with a brief explanation.
b) Find the distance L.
c) How long does it take for the particle to reach the point D?
d) What is the velocity of the particle at the point D?

Give the results in unit vector notation.

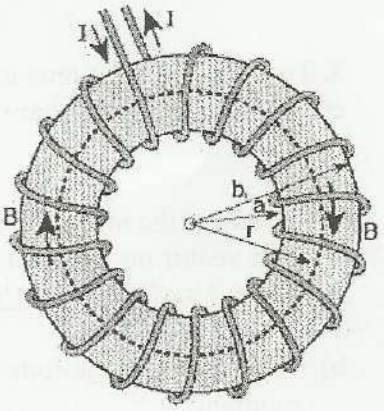
3. Two semicircular arcs have radii R and 2R where R = 4 cm, carry current i = 0.8 A and share the same center curvature as shown in the figure.



a) What are the magnitude and direction of the net magnetic field (in unit vector notation) at the center point O?
Note: <u>*Use Biot Savart's Law*</u>

b) What is the magnitude and direction of the total magnetic dipole moment?

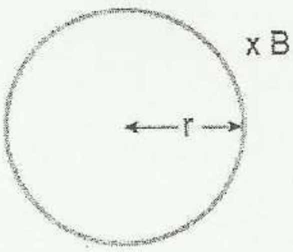
4. A toroid has inner radius $a = 15$ cm and outer radius $b = 18$ cm. The toroid has 250 turns and carries a current of 8 A. Calculate the magnitude of the magnetic field for:



a) $r = 12$ cm
b) $r = 16$ cm
c) $r = 20$ cm

***Note:** Use Ampere's Law.*

5. A circular loop of flexible iron has an initial radius of 160 cm but its radius is decreasing at a constant rate of 10 cm/s due to a tangential pull on the wire. The loop is in a constant magnetic field Orient ed perpendicular (inward) to the plane of the loop with a magnitude of 0.5 T.



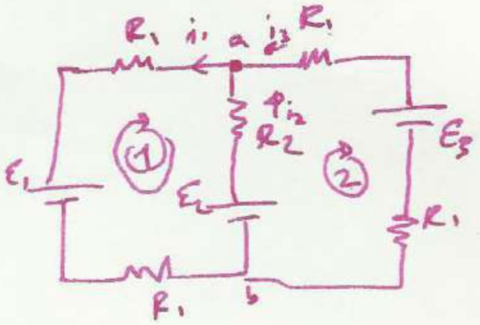
a) Find the emf induced in the loop at the instant when 6 seconds have passed.
b) Find the direction of the induced current in the loop as viewed looking along the direction of the magnetic field (i.e., the magnetic field is going away from you, into the loop)

$\lvert e^- \rvert = 1.6 \times 10^{-19}$ C	$g = 10$ m/s^2	$k = 9\times10^9$ Nm^2/C^2	$\varepsilon_o = 9.0\times10^{-12}$ C^2/N.m^2
$\sin 90° = \cos 0° = 1$ $\cos 90° = \sin 0° = 0$	$\pi = 3$	$\mu_0 = 4\pi\times10^{-7}$ N/A	$m_e = 9\times10^{-31}$ kg
$\vec{\mu} = NiA\hat{n}$	$\varepsilon = -\dfrac{d\Phi_B}{dt}$	$\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B}$	$\varepsilon_L = -L\dfrac{di}{dt}$
$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$	$d\mathbf{F}_B = i\,d\mathbf{L}\times\mathbf{B}$	$U_B = -\boldsymbol{\mu}\cdot\mathbf{B}$	$d\mathbf{B} = \dfrac{\mu_0 i}{4\pi}\dfrac{d\mathbf{s}\times\mathbf{r}}{r^3}$
$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 i$	$\oint \mathbf{E}\cdot d\mathbf{s} = -\dfrac{d\Phi_B}{dt}$	$L = \dfrac{N\Phi_B}{i}$	$\Phi_B = \int \mathbf{B}\cdot d\mathbf{A}$

1)

$R_1 = 2\Omega \quad \mathcal{E}_1 = 4V$

$R_2 = 4\Omega \quad \mathcal{E}_2 = \mathcal{E}_3 = 8V$



(1) $\mathcal{E}_1 + i_1R_1 + i_2R_2 - \mathcal{E}_2 + i_1R_1 = 0$

$4 + 2i_1 + 4i_2 - 8 + 2i_1 = 0$

$4(i_1 + i_2) = 4A$

$\boxed{i_1 + i_2 = 1A}$ (1)

(2) $i_3R_1 - \mathcal{E}_3 + i_3R_1 + \mathcal{E}_2 - i_2R_2 = 0$

$2i_3 - \cancel{8} + 2i_3 + \cancel{8} - 4i_2 = 0$

$4(i_3 - i_2) = 0$

$\boxed{i_3 = i_2}$ (2)

$\mathcal{E}_1 + i_1R_1 + i_3R_1 - \mathcal{E}_3 + i_3R_1$

$+ i_1R_1 = 0$

$4 + 2i_1 + 2i_3 - 8 + 2i_3 + 2i_1 = 0$

$i_1 + i_3 = 1A$

a: $\boxed{i_1 = i_2 + i_3}$ (3)

$\rightarrow i_1 = 2i_2$

$\Rightarrow$ (1): $3i_2 = 1A \Rightarrow i_2 = \frac{1}{3}A$

$i_1 = \frac{2}{3}A$

$i_3 = i_2 = \frac{1}{3}A$

d) $V_a + i_2R_2 - \mathcal{E}_2 = V_b$

$V_a - V_b = -\left(\frac{1}{3}A\right)\cdot(4\Omega) + 8V$

$= \left(8 - \frac{4}{3}\right)V = \frac{20}{3}V$

2)

$m = 4g$, $|q| = 0.5C$, $B = 4.5\hat{k}$ T

$t = 0$ $\vec{v} = 9\hat{i}$ m/s

a.) $\hat{J} = \hat{F} = \hat{v} \times \hat{B} \cdot \mathrm{sgn}(q)$

$= \hat{i} \times \hat{k} \cdot \mathrm{sgn}(q)$

$= -\hat{j} \cdot \mathrm{sgn}(q) \Rightarrow \mathrm{sgn}(q) = -1 \rightarrow q = -0.5C$

b) $L = 2r$

$F = ma \Rightarrow qvB \sin 90 = \frac{mv^2}{r}$

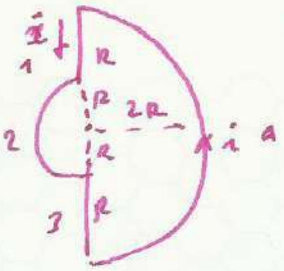
$r = \frac{mv}{qB} = \frac{(4 \times 10^{-3} kg)(9 m/s)}{(0.5C)(4.5T)} = 16 \times 10^{-3}$ m

c) $\cancel{2}\pi r = vt \Rightarrow t = \frac{\pi r}{v} = \frac{\cancel{3} \cdot 16 \times 10^{-3} m}{\cancel{9}_{3} m/s} = \frac{16}{3} \times 10^{-3}$ s

d) Speed is constant (Magnetic field can only act on the direction!)

$\Rightarrow \vec{v}_D = -9\hat{i}$ m/s

3)



$R = 4\,cm$

$i = 0.8\,A$

a.) ① & ③ no contribution to magnetic field. $(d\vec{s} \parallel \vec{r})$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \vec{r}}{r^3}$$

② $d\vec{s} \perp \vec{r}$ $\hat{k}$ ⊙

③ $\hat{k}$ ⊙

$ds = R\,d\theta$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,.\,\hat{k}}{R^2} \int_0^{\pi} R\,d\theta$$

$$= \frac{\mu_0 i}{4R} \hat{k}$$

② : $R = R$, ④ $R = 2R$

$$\vec{B} = \frac{\mu_0 i}{4} \hat{k} \left(\frac{1}{R} + \frac{1}{2R} \right) = \frac{3}{8} \frac{\mu_0 i}{R} \hat{k}$$

$$= \frac{3}{8} \frac{(4\pi \times 10^{-7}\,Tm/A)\,.\,0.8\,A}{4 \times 10^{-2}\,m} \hat{k}$$

$$= 9 \times 10^{-6}\, \hat{k}\ T$$

b) $\mu = N i A = (0.8A) . \left(\frac{\pi}{2} (R^2 + 4R^2) \right)$

$$= \frac{4}{5}A\, \frac{\pi}{2} . 5 . (4 \times 10^{-2}\,m)^2$$

$$= 32 . \pi . \times 10^{-4}\, A\,m^2 \ , \quad ⊙\ \hat{k}$$

4) $a = 15$ $\quad N = 250$ / $i = 8\,A$
$b = 18$

a.) $r = 0.12\,m$ $\quad i_{enc} = 0 \Rightarrow B = 0$

b) $r = 0.16\,m$ $\quad i_{enc} = 250 \cdot 8\,A$

$$B = \frac{\mu_0 N I}{2\pi r} = \frac{\mu_0\, 250 \cdot 8\,A}{2\pi\,(0.16\,m)} = 2.5\times10^{-3}\,T$$

c) $r = 0.2\,m$: $\quad I_{enc} = N\,(I + (-I)) = 0 \Rightarrow B = 0$

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5)

a) $\Phi_B = BA = B \cdot \pi r^2$

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = B\cdot\pi\frac{d}{dt}(r^2) = B\cdot\pi\cdot(2r)\cdot\frac{dr}{dt}$$

$t = 6\,s$: $\quad r = 1.6\,m - (6s)(0.1\,m/s) = 1\,m$

$$\rightarrow |\mathcal{E}| = (0.5\,T)(2\pi)(1\,m)\cdot(0.1\,m/s) = 0.3\,V$$

b) 

A↓ Lenz' law says induced emf will support the magnetic field, hence will produce X direction.

$\Rightarrow \mathcal{E}_{ind.}$

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2 (20p)	
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5 (20p)	
6 (5p)	

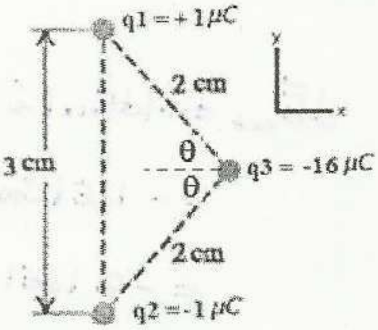
FİZ 138 – 25, 26 PHYSICS II
1ST MIDTERM
27 MARCH 2018 / 13:00 – 14:50

SECTION (25 / 26):

NAME - SURNAME: **NUMBER:** **SIGNATURE:**

GOOD LUCK

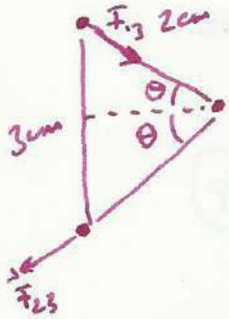
1. Three charges are at the corners of an isosceles triangle, as shown in the figure. The $q_1 = 1\mu C$ and $q_2 = -1\mu C$ charges form a dipole.



a. Find the force (magnitude and direction) of $q_3 = -16\,\mu C$ charge exerts on the dipole.

b. For an axis perpendicular to the line connecting the $\pm 1\mu C$ charges at the midpoint of this line (along the z-direction), calculate the torque (magnitude and direction) exerted on the dipole by the $q_3 = -16\,\mu C$ charge.

a)



$\vec{F}_{net} = \vec{F}_{13} + \vec{F}_{23}$

$= F_{13}\cdot\cos\theta\,\hat{i} - F_{13}\sin\theta\,\hat{j} - F_{23}\cos\theta\,\hat{i} - F_{23}\sin\theta\,\hat{j}$ (5p)

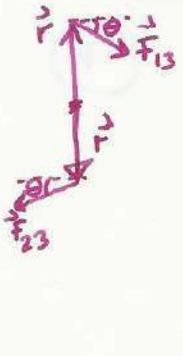
$F_{13} = F_{23} = \dfrac{k|q_1||q_3|}{r^2} = \dfrac{9\times 10^9 \,Nm^2/C^2\,(1\times 10^{-6}C)(16\times 10^{-6}C)}{(2\times 10^{-2}m)^2} = 360N$ (4p)

$\rightarrow \vec{F}_{net} = 360N.\cancel{\cos\theta\,\hat{i}} - 360N\sin\theta\,\hat{j} - 360\,\cancel{N.\cos\theta\,\hat{i}} - 360N\sin\theta\,\hat{j}$

$= -720N.\sin\theta\,\hat{j}$

$\sin\theta = \dfrac{1.5cm}{2\,cm} = \dfrac{3}{4} \Rightarrow = -720N.\dfrac{3}{4}\,\hat{j} = -540\hat{j}\,N$ (4p)

b) $\vec{\tau} = \vec{r}\times\vec{F}$



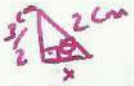
$\tau = 2\left(\dfrac{3}{2}\times 10^{-2}m\right).\,360N.\cos\theta$

$= 3\times \cancel{360}^{90}\times\dfrac{\sqrt{7}}{\cancel{4}}\times 10^{-2}\,Nm$ (7p)

$= 2.7\times\sqrt{7}\,Nm$

$(= 7.14\,Nm)$ ⊗ , $-\hat{k}$

$\cos\theta =$



$\sin\theta = \dfrac{3}{4}$

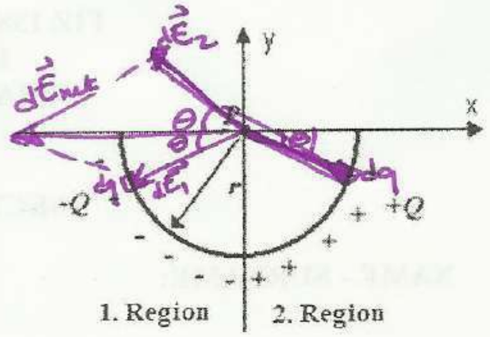
$\sin^2\theta + \cos^2\theta = 1$

$\dfrac{9}{16} + \cos^2\theta = \dfrac{16}{16}$

$\cos^2\theta = \dfrac{7}{16}$

$\cos\theta = \dfrac{\sqrt{7}}{4}$

2. In figure, a thin glass rod is bent into semicircle of radius $r = 2\ cm$. A charge $-Q = 5$ pC is uniformly distributed along the lower left half (1. region) and $+Q = 5$ pC is uniformly distributed along the lower right half (2. Region).



a. Find the magnitude and direction (in unit vector notation) of the electric field **E** at the point P, center of the semicircle.

b. If $q = +3\ pC$ charge is placed at the point P, calculate the electric force acting on this charge.

$$|d\vec{E}| = |d\vec{E}_1| = |d\vec{E}_2| = k\frac{dq}{r^2} \quad \begin{cases} |dE_{1x}| = |dE_{2x}| = |dE| . \cos\theta \\ |dE_{1y}| = |dE_{2y}| = |dE| . \sin\theta \end{cases}$$

$$d\vec{E}_{net} = -|dE_{1x}|\hat{i} - |dE_{1y}|\hat{j} - |dE_{2x}|\hat{i} + |dE_{2y}|\hat{j}$$

$$= -|dE|\cos\theta\,\hat{i} - \cancel{|dE|\sin\theta\,\hat{j}} - |dE|\cos\theta\,\hat{i} + \cancel{|dE|\sin\theta\,\hat{j}}$$

$$= -2\,|dE|\cos\theta\,\hat{i} \longrightarrow d\vec{E}_{net} = -2\left(\frac{k\,dq}{r^2}\right)\cos\theta\,\hat{i}$$

(2p)

$$dq = \lambda\,dl, \quad dl = r\,d\theta \rightarrow dq = \lambda r\,d\theta$$

$$\lambda = \frac{Q}{\left(\frac{2\pi r}{4}\right)} = \frac{2Q}{\pi r} \rightarrow dq = \frac{2Q}{\pi r} r\,d\theta = \frac{2Q}{\pi} d\theta$$

$$d\vec{E}_{net} = -\frac{2k}{r^2}\underbrace{\left(\frac{2Q}{\pi}d\theta\right)}_{dq}\cos\theta\,\hat{i} = -\frac{4kQ}{\pi r^2}\cos\theta\,d\theta\,\hat{i}$$ (5p)

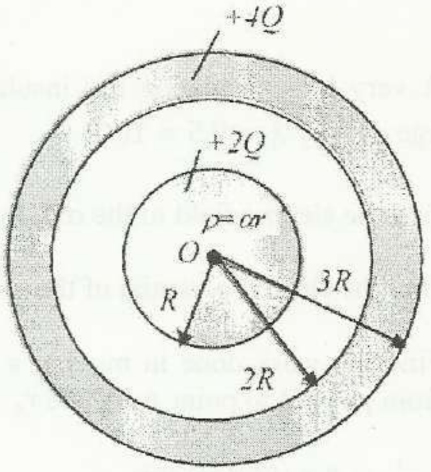
$$\Rightarrow \vec{E}_{net} = -\frac{4kQ}{\pi r^2}\hat{i}\int_0^{\pi/2}\cos\theta\,d\theta = -\frac{4kQ}{\pi r^2}\hat{i}\left(\sin\theta\Big|_0^{\pi/2}\right) = -\frac{4kQ}{\pi r^2}\hat{i}$$ (6p)

$$\vec{E}_{net} = -\frac{\overset{3}{\cancel{4}}(\cancel{9}\times10^{9}\,Nm^2/C^2)(5\times10^{\overset{-3}{\cancel{-12}}}\,C)}{(\cancel{3})(\cancel{2}\times10^{-2}\,m)^2} = -15\times10\,\hat{i}\ N/C = \underline{-150\,\hat{i}\ N/C}$$ (2p)

b) $$\vec{F} = q\vec{E}_{net} = (3\times10^{-12}C)(-150\,\hat{i}\,N/C) = \underline{-450\times10^{-12}\,\hat{i}\ N}$$

(5p)

3. A solid, insulating sphere of radius R has a non-uniform charge density of $\rho = \alpha r$ where α is a positive constant and r is the radial distance from origin and a total charge of $+2Q$. Concentric with this sphere is a charged conducting shell sphere with $+4Q$ total charge has inner and outer radii $2R$ and $3R$, respectively as shown in the figure.



a) Express the charge of the inner sphere in terms of α and R.

b) Find the magnitude of the electric fields defined below in terms of k, Q, r and R.

$E\ (r < R)$,
$E\ (R < r < 2R)$,
$E\ (2R < r < 3R)$,
$E\ (r > 3R)$

c) What are the charges of inner and outer surfaces on conducting shell?

a) $2Q = \int \rho\, dV = \int_0^R (\alpha r)(4\pi r^2 dr) = 4\pi\alpha \int_0^R r^3 dr = 4\pi\alpha \frac{R^4}{4} = \pi\alpha R^4$ (5p)

$\left(2Q = \pi\alpha R^4 \leftrightarrow \alpha = \frac{2Q}{\pi R^4}\right)$

b) * $r<R$: $Q_{enc} = \int_0^r (\alpha r')(4\pi r'^2 dr') = \int_0^r \frac{2Q}{\pi R^4} 4\pi r'^3 dr' = \frac{8Q}{R^4}\frac{r^4}{4} = \frac{2Qr^4}{R^4}$

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E.4\pi \cancel{r^2} = \frac{2Qr^{\cancel{4}\,2}}{\epsilon_0 R^4} \Rightarrow E = \frac{Q}{2\pi\epsilon_0}\frac{r^2}{R^4} \quad \left(= \frac{\alpha r^2}{4\epsilon_0}\right)$ (7p)

* $R<r<2R$: $Q_{enc} = 2Q$

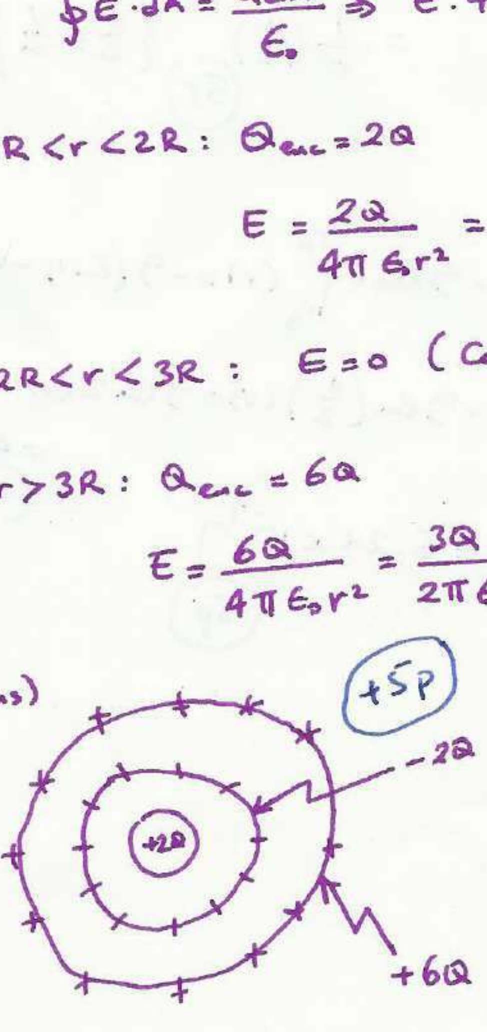
$E = \frac{2Q}{4\pi\epsilon_0 r^2} = \frac{Q}{2\pi\epsilon_0 r^2}$ (3p)

* $2R<r<3R$: $E = 0$ (Conductor) (2p)

* $r>3R$: $Q_{enc} = 6Q$

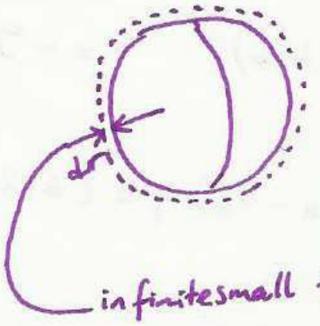
$E = \frac{6Q}{4\pi\epsilon_0 r^2} = \frac{3Q}{2\pi\epsilon_0 r^2}$ (3p)

(Bonus) (+5p)
c)



SIDE INFORMATION

Surface area of a spherical shell : $4\pi r^2$



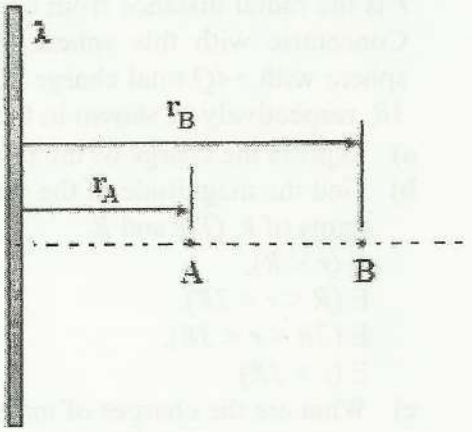
infinitesmall thickness dr

$\Rightarrow$ Volume of the infinitesmall thick shell: $dV = 4\pi r^2 dr$

$\Rightarrow V = \int_0^R 4\pi r^2 dr = 4\pi \frac{R^3}{3}$

(A solid sphere is made of cocentric shells)

4. A very long insulating rod insulating. carries a constant linear charge density $\lambda = 0.5 \times 10^{-9}\,\frac{C}{m}$.



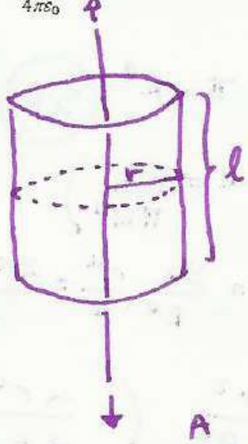
a. Find the electric field of the rod, by using Gauss' Law.

b. Find the electric potential of the rod at point A and at point B.

c) Find the work done in moving a point charge $q = 1 \times 10^{-9}\,C$ from point B to point A, where $r_A = 60\,cm$ and $r_b = 120\,cm$.

$(k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\,N.m^2/C^2)$

a)



$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$, $\begin{cases} \vec{E} \parallel d\vec{A} \text{ on the side} \\ \vec{E} \perp d\vec{A} \text{ on the top \& bottom faces} \end{cases}$

$E\cdot(2\pi r \cancel{\ell}) = \frac{\cancel{\ell}\lambda}{\epsilon_0} \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$

$= \frac{\cancel{2}(9\times10^9)(\frac{1}{\cancel{2}}\times10^{-9})}{r}\left(\frac{N}{C}\right)$

$= \frac{9}{r}\left(\frac{N}{C}\right) \quad \left\{\frac{N}{C} \equiv \frac{V}{m}\right\}$ (5p)

b) $V_A - V_B = -\int_B^A \vec{E}\cdot d\vec{\ell} = -\int_B^A E\,d\ell \underbrace{\cos 180^\circ}_{-1} = \int_B^A E\,d\ell$

$\vec{E} \rightarrow$

$A \xleftarrow{d\vec{\ell}} B$

$= -\int_{r_B}^{r_A} E\,dr = -\int_{r_B}^{r_A} \frac{9}{r}\,dr\,(V) = -9\ln r\Big|_{r_B}^{r_A}(V) = -9(\ln r_A - \ln r_B)\,(V)$

$d\ell = -dr$

$(d\ell\uparrow \Leftrightarrow dr\downarrow)$

$= -9\ln\left(\frac{r_A}{r_B}\right)(V) = -9\ln\left(\frac{1}{2}\right)(V) = 9\ln 2\,(V) = 9\times 0.69\,V$

$= 6.21\,V$ (10p)

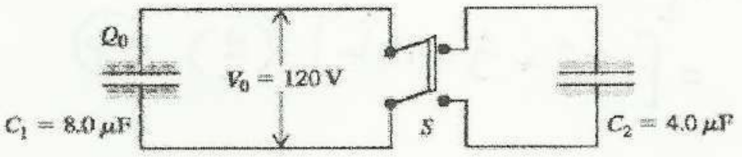
c) $W_{BA} = (V_A - V_B)\,q = (6.21\,V)(1\times10^{-9}\,C) = 6.21\times10^{-9}\,J$ (5p)

5. A capacitor C_1 of capacitance $8\ \mu F$ has been charged by connecting it to a source of potential difference $V_0 = 120\ V$ while the switch S is open.

a. What is the charge Q_0 on C_1 as the switch is kept open?
b. What is the energy stored in C_1 as the switch is kept open?

The capacitor C_2 of capacitance $4\ \mu F$ is initially uncharged. After switch S is closed:

c. What is the equivalent capacitance of the circuit?
d. What is the potential difference across each capacitor?
e. What is the charge on each capacitor?
f. What is the final total energy of the system?



a) $C_1 = \frac{Q_0}{V_0} \rightarrow Q_0 = C_1 V_0 = (8\times10^{-6}F)(120V) = 960\mu C = 9.6\times10^{-4}C$ (3p)

b) $U = \frac{1}{2}C_1V_0^2 = \frac{1}{2}(8\times10^{-6}F)(120V)^2 = 57600\times10^{-6}J = 0.0576J$ (4p)

or: $U = \frac{1}{2}Q_0V_0 = \frac{1}{2}(9.6\times10^{-4}C)(120V) = 0.0576J$

c) $C_{eq} = C_1 + C_2$ (parallel) $\rightarrow C_{eq} = (8+4)\mu F = 12\mu F$ (1p)

d) $V = \frac{Q_0}{C_{eq}} = \frac{960\mu C}{12\mu F} = 80V$ (3p) (Since the capacitors are connected in parallel, the potential difference across each capacitor is same)

e) $Q_1 = C_1V = 8\mu F \cdot 80V = 640\mu C = 6.4\times10^{-4}C$ (5p) ($Q_1+Q_2 = 960\mu C = Q_0$ ✓)
$Q_2 = C_2V = 4\mu F \cdot 80V = 320\mu C = 3.2\times10^{-4}C$

f) $U_{TOT} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1+C_2)V^2 = \frac{1}{2}(12\mu F)(80V)^2 = 38400\times10^{-6}J$
$= 0.0384J$

or: $U_{TOT} = \frac{1}{2}Q_1V + \frac{1}{2}Q_2V = \frac{1}{2}(Q_1+Q_2)V = \frac{1}{2}Q_0V = \frac{1}{2}(9.6\times10^{-4}C)(80V)$ (4p)
$= 0.0384J$

6. The potential in a region is given by $V(x, y, z) = x^2 - 4y + 3yz^2$ (V). Find the direction of electric field in unit vector notation at the point P(1, 2, 1).

$$\vec{E} = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

$$\frac{\partial V}{\partial x} = 2x \; ; \; \frac{\partial V}{\partial y} = -4 + 3z^2 \; ; \; \frac{\partial V}{\partial z} = 6yz$$

$$\rightarrow \vec{E}(x,y,z) = \left[-(2x)\hat{i} - (-4+3z^2)\hat{j} - (6yz)\hat{k}\right] \left(\frac{N}{C}\right)$$

$$\vec{E}(1,2,1) = \left[-2\hat{i} + (4-3)\hat{j} - 6.2\hat{k}\right] \left(\frac{N}{C}\right)$$

$$= \left[-2\hat{i} + \hat{j} - 12\hat{k}\right] \left(\frac{N}{C}\right) \quad 5p$$

$F = k\frac{q_1 q_2}{r^2}$	$E = \frac{F}{q}$	$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$	$\vec{E}_s = -\frac{\partial V}{\partial s}\hat{s}$
$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$	$U_e = k\frac{q_1 q_2}{r}$	$W = q\Delta V$	Micro - μ (10^{-6})
$\oint \vec{E} . d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$	$U_E = \frac{1}{2}CV^2$	$C = \frac{Q}{V}$	Pico – p (10^{-12})
$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \; N.m^2/C^2$	$\varepsilon_o = 9 \times 10^{-12} \; C^2/N.m^2$	$\pi = 3$	$\lvert e^- \rvert = 1.6 \times 10^{-19}\; C$
$\ln 2 = 0.69$	$\sqrt{6} \cong 2.5$	$\vec{\tau} = \vec{r} \times \vec{F}$	$g = 10 \; m/s^2$

FIZ138 MT1 Grading Notes
(27.3.2018) Emre S. Taşcı

Lack of units in the results : −1p (each)

1) a) Calculating the net force on the q_3 instead of dipole : −3p

b) $\vec{p} \times \vec{E} = \vec{\tau}$ . . 2p ($\vec{E}$ is not uniform !)

2) * Treating Q_1 & Q_2 charge distribution as single particles and calculating via $\frac{kQ}{r^2}$,  : 3p (2p if the direction is false)

* Calculation of λ : +1p

* Treating Qs without dq but still correctly including angle dependencies : −7p

$* \int_{\pi}^{3\pi/2} -\cos\theta \, d\theta$ . . −1p | $\int_0^{\pi/2} .. ds$ ... −5p | ... $\oint \vec{E} \, d\vec{A} = \frac{q}{\epsilon_0}$: 3p

b) $F = qE \cdots \rightarrow$ 5p

$F = k\frac{Q_1 Q_3}{r^2} \longrightarrow$ 2p

3) a) $dQ = \rho 4\pi r^2 dr \cdots\rightarrow 2Q = \alpha 4\pi R^3$: 2p

c) "No electrical field inside a conductor." : 2p

4) a) without derivation, $E = \frac{\lambda}{2\pi\epsilon_0 r}$: 1p

b) $V = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda dr}{r}$ ... : 2p

c) $W = q\Delta V \cdots \rightarrow$ 3p

5) "in Series" ... → c, d, e → 0p, f: $U = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 \rightarrow$ 3p

d) $V_1 = V_2$ (≃60V) 1p

6) * $E_x = 2x$
$E_y = -4 + 3z^2$, i.e. "−" omitted : −1p ; $E_y = 4$ (only) : −1p
$E_z = 6yz$

* Omitted $\hat{\imath}, \hat{\jmath}, \hat{k}, \vec{E}$ treated as scalar, e.g. $E = -2 + 1 - 12 = -13$ N/C
−4p

Automotive Engineering, FIZ138 1st Midterm Exam | Instructor: Emre S. Taşcı | 1 / 4 / 2016

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{SOLUTIONS}

Mark the 5 questions you want to be evaluated from (each question is worth 20 points):

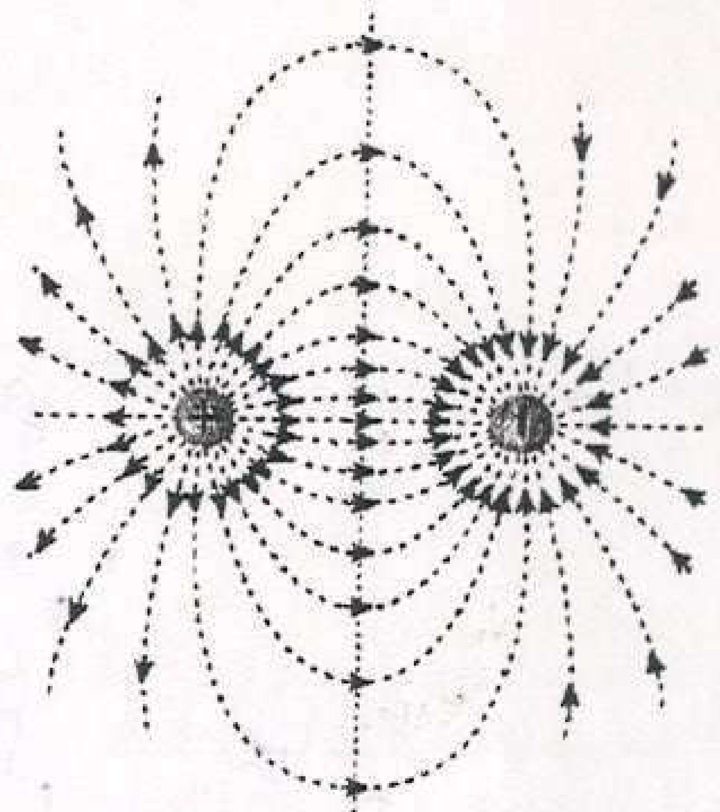
Mark:	Q1	Q2	Q3	Q4	Q5	Q6	Q7
Grade:							

Q1) a) **New Coulomb's Law:** Suppose that we invent a new unit of charge, "newC" which is defined as $1\,newC = \frac{1}{\sqrt{4\pi\epsilon_0}} C$ in order to get rid of the Coulomb's constant k. Discuss the problem with this approach.

b) Show that $R \times C$ multiplication has a unit of seconds.

c) An electric dipole is enclosed in a cubic box of side length a. If the electric flux for this cubic box is Φ_0, what will it be for another cubic box with side length $2a$? (The boxes' centers coincide)

d) An electric dipole is centered at the origin (with the charges placed at $x = \pm d/2$. What is the ratio of the magnitudes of electric field at a distance x and $6x$ from the origin of the dipole where $x >> d$?



a.) If we are to take only the value of ϵ_0 (i.e. without the units), then the result would be in units of $\frac{C^2}{m^2}$ which is not equal to Newton, hence we would have found something other than force, and still would need a constant $k' = 1 \frac{Nm^2}{C^2}$ to fix it.

If, on the other hand we had included the unit of ϵ_0 in our "new C", then we would be using something other than charge, which would be incorrect.

In summary: with such an approach, the units do not match any longer on the two sides of the equation!

b) $[V]$, $V = -\int E\,dx$, $[I]$

$$\Omega = \frac{V}{A} = \left(\frac{N}{C} m\right)\left(\frac{s}{C}\right) = \frac{Nms}{C^2} = \frac{(kgm/s^2)ms}{C^2} = \frac{kgm^2}{C^2 s} \longrightarrow \Omega \times F = \frac{\cancel{kg}\cancel{m^2}}{\cancel{C^2}\cancel{s}} \times \frac{\cancel{C^2}s^{\cancel{2}}}{\cancel{kg}\cancel{m^2}} = \underline{s}$$

$\frac{1}{A}$

$[\epsilon_0]$, $[A]$, $[d]$

$$F = \frac{C^2}{Nm^2} \frac{m^2}{m} = \frac{C^2}{kgm/s^2\, m^2\, m} = \frac{C^2 s^2}{kgm^2}$$

b.) Alternative: $\Omega . F = \frac{V}{A} \cdot \frac{C}{V} = \frac{C}{A} = \frac{C}{C/t} = \underline{s}$

c) Since the net charge inside is equal to 0:

$$\Phi_0(a) = \Phi_0(2a) = 0$$

d.) $\vec{E}_{dipole} = \frac{1}{2\pi\epsilon_0} \frac{p}{x^3}$ $(x >> d)$

$$\rightarrow \frac{E(a)}{E(6a)} = \frac{1/a^3}{1/(6a)^3} = 6^3 = \underline{216}$$

Automotive Engineering, FIZ138 1st Midterm Exam | Instructor: Emre S. Taşcı | 1 / 4 / 2016

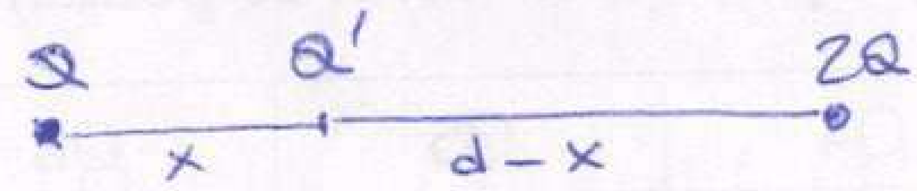
Q2) Two charges Q and 2Q are separated by a distance of *d*.

a) Find the equilibrium point for a third charge of -Q placed between the positive charges.

b) Find the equilibrium point for a third charge of -3Q placed between the positive charges.

c) Analytically (mathematically) show that it is not possible to find an equilibrium point for a third charge lying outside the line passing through the two charges.

a, b)



$$\vec{F}_{Q'} = \vec{F}_{QQ'} + \vec{F}_{2QQ'} = 0 \rightarrow \vec{E}_{Q'} = 0 \leftarrow \text{independent of } Q'$$

$$\Rightarrow (a) \equiv (b)$$

$$k\frac{Q}{x^2} = k\frac{(2Q)}{(d-x)^2}$$

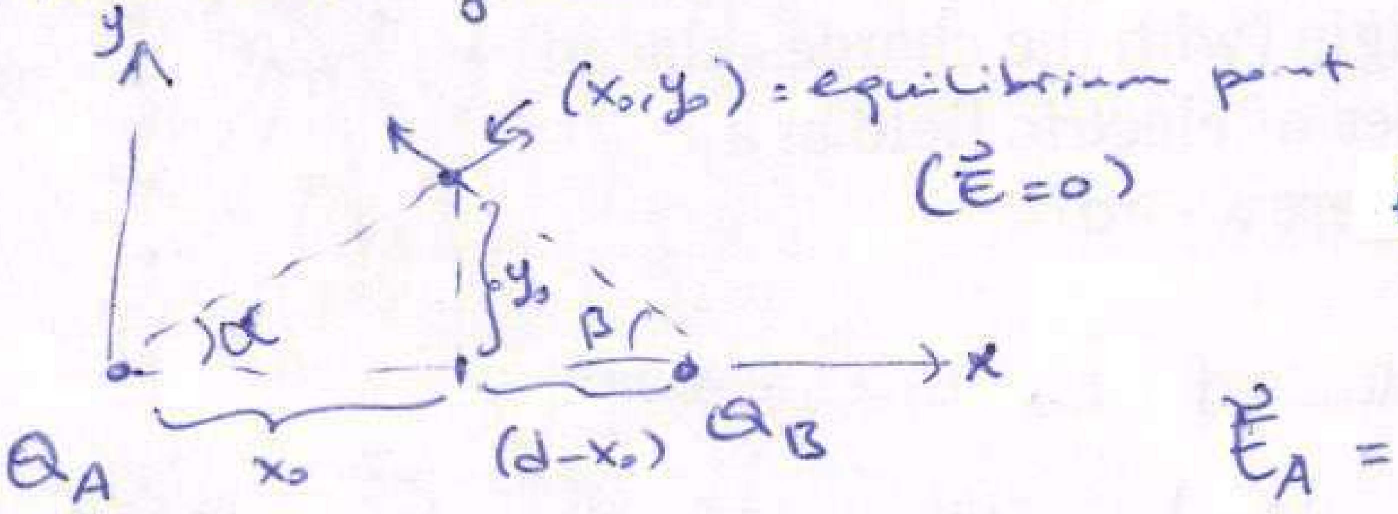
$$(d-x)^2 = 2x^2$$

$$d^2 - 2dx + x^2 = 2x^2 \rightarrow x^2 + 2dx - d^2 = 0$$

$$x = \frac{-2d \pm \sqrt{4d^2+4d^2}}{2} = -d \pm d\sqrt{2} = -d(1 \mp \sqrt{2})$$

$$d(\sqrt{2}-1)$$

c) In the most general case:



$\vec{E}$ components

$$\vec{E}_A = \frac{kQ_A}{(x_0^2+y_0^2)^{3/2}}(x_0\hat{i} + y_0\hat{j})$$

$$|\vec{E}_A| = \frac{kQ_A}{x_0^2+y_0^2}$$

$$\cos\alpha = \frac{x_0}{(x_0^2+y_0^2)^{1/2}}$$

$$\sin\alpha = \frac{y_0}{(x_0^2+y_0^2)^{1/2}}$$

$$\vec{E}_B = \frac{kQ_B}{[(d-x_0)^2+y_0^2]^{3/2}}(-(d-x_0)\hat{i} + y_0\hat{j})$$

$$|\vec{E}_B| = \frac{kQ_B}{(d-x_0)^2+y_0^2}$$

$$\cos\beta = \frac{(d-x_0)}{[(d-x_0)^2+y_0^2]^{1/2}}$$

$$\sin\beta = \frac{y_0}{[(d-x_0)^2+y_0^2]^{1/2}}$$

At the equilibrium point components of $\vec{E}$ must sum to 0:

x-components:

$$\frac{A}{(x_0^2+y_0^2)^{3/2}}x_0 = \frac{B}{[(d-x_0)^2+y_0^2]^{3/2}}(d-x_0) \rightarrow \frac{A}{B} = \left[\frac{x_0^2+y_0^2}{(d-x_0)^2+y_0^2}\right]^{3/2}\frac{(d-x_0)}{x_0} \quad (1)$$

x_0 can't be zero!

y-components:

$$\frac{A}{(x_0^2+y_0^2)^{3/2}}y_0 = -\frac{B}{[(d-x_0)^2+y_0^2]^{3/2}}y_0 \rightarrow \frac{A}{B} = -\left[\frac{x_0^2+y_0^2}{(d-x_0)^2+y_0^2}\right]^{3/2} \quad (2)$$

Substitute (2) in (1):

$$\frac{A}{B} = \left(-\frac{A}{B}\right)\frac{d-x_0}{x_0} \rightarrow x_0 - d = x_0$$

$$\Rightarrow d = 0\ ???$$

$\rightarrow$ There is a consistent solution only when y is taken to be zero, i.e.:

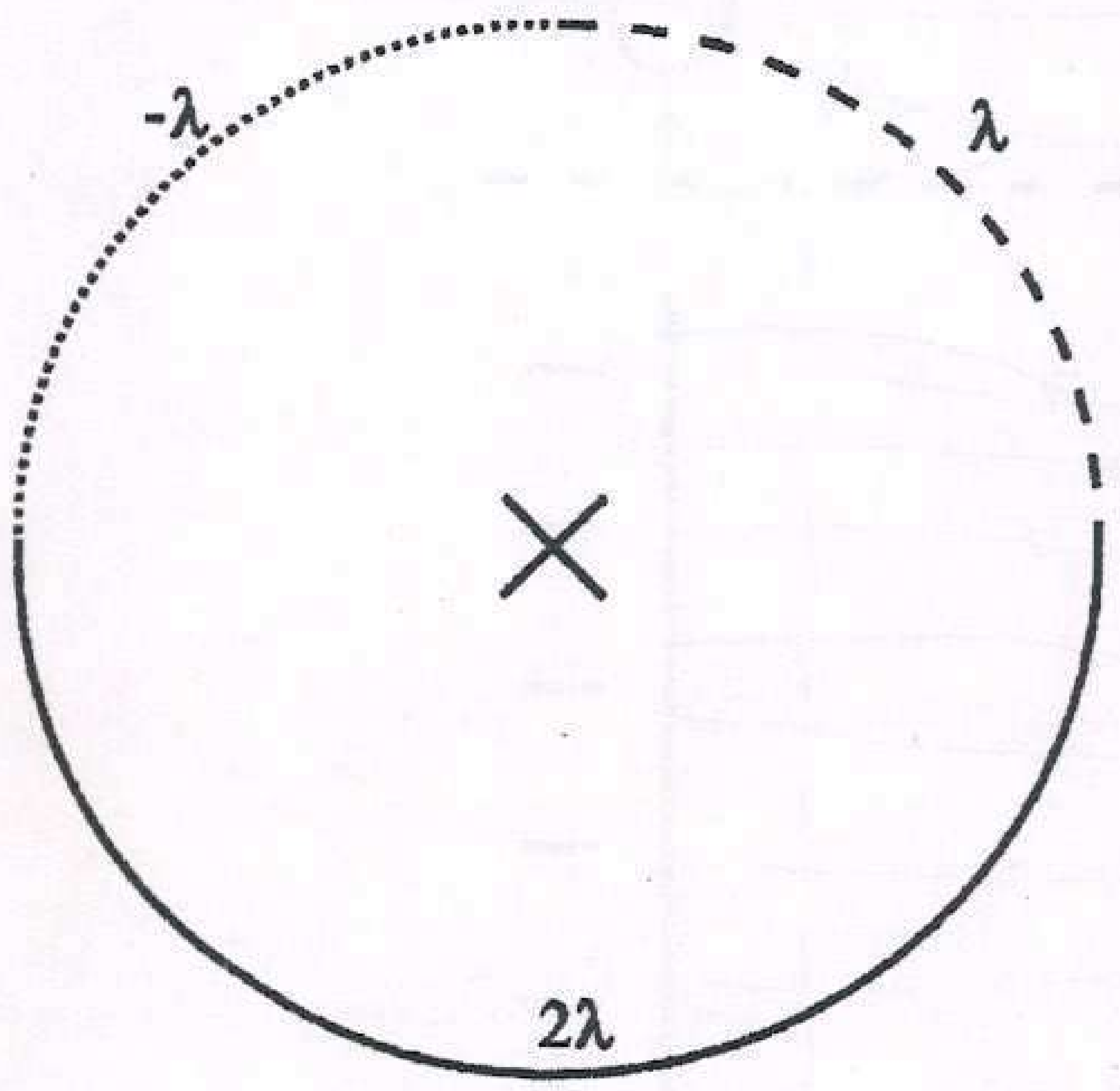
$$y=0 \Rightarrow (1): \quad \frac{A}{B} = \left[\frac{x_0^2}{(d-x_0)^2}\right]^{3/2}\frac{d-x_0}{x_0} = \frac{x_0^2}{(d-x_0)^2} \checkmark$$

$$\left((2): \frac{A}{(\ )^{3/2}}\cdot 0 = -\frac{B}{(\ldots)^{3/2}}\cdot 0 \Rightarrow 0=0 \checkmark\right)$$

Automotive Engineering, FIZ138 1st Midterm Exam | Instructor: Emre S. Taşcı | 1 / 4 / 2016

Name: Number: Signature:

Q3) 3 wires with charge densities λ $(\theta=0\ldots\frac{\pi}{2})$, $-\lambda$ $(\theta=\frac{\pi}{2}\ldots\pi)$ and 2λ $(\theta=\pi\ldots2\pi)$ are arranged into a loop of radius R as shown in the figure. Calculate the electric field at the center in vector form.



A General Case:

$$dq = \ell\, dx = \ell \cdot R\, d\theta$$

$$|d\vec{E}_0| = k\frac{dq}{R^2} = k\,\ell\,\frac{R\,d\theta}{R^2} = \frac{k\ell}{R}\,d\theta$$

$$d\vec{E}_0 = \frac{k\ell}{R}\,d\theta\left[-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right]$$

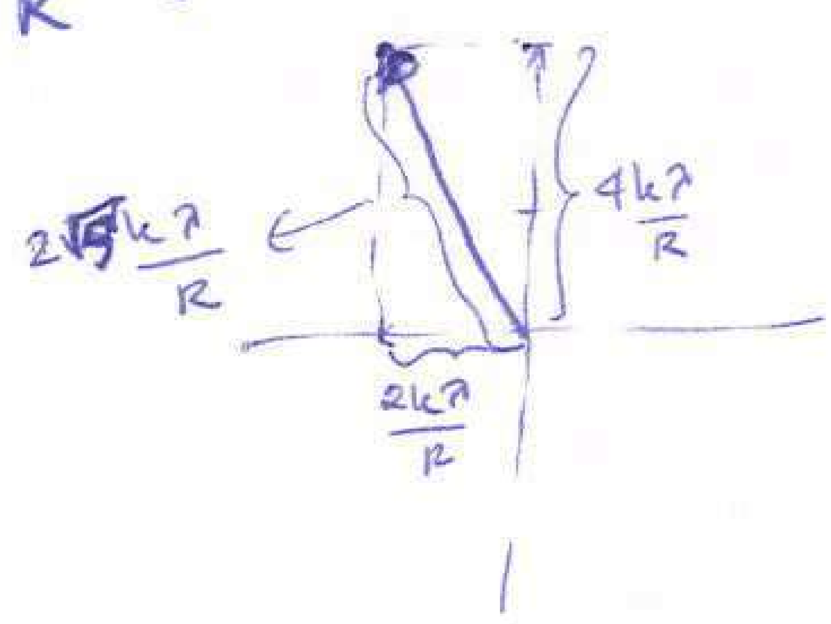
$$\rightarrow \vec{E}_0 = -\frac{k\ell}{R}\int_{\theta_1}^{\theta_2}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})\,d\theta = -\frac{k\ell}{R}\left[\begin{matrix}(\sin\theta_2 - \sin\theta_1)\,\hat{i}\\ -(\cos\theta_2 - \cos\theta_1)\,\hat{j}\end{matrix}\right]$$

θ_1	θ_2	ℓ	$-\frac{k\ell}{R}\left[(\sin\theta_2-\sin\theta_1)\hat{i} - (\cos\theta_2-\cos\theta_1)\hat{j}\right]$
0	$\pi/2$	λ	$-\frac{k\lambda}{R}\left[(1-0)\hat{i} - (0-1)\hat{j}\right] = -\frac{k\lambda}{R}(\hat{i}+\hat{j})$
$\pi/2$	π	$-\lambda$	$-\frac{k(-\lambda)}{R}\left[(0-1)\hat{i} - (-1-0)\hat{j}\right] = \frac{k\lambda}{R}(-\hat{i}+\hat{j})$
π	2π	2λ	$-\frac{k(2\lambda)}{R}\left[(0-0)\hat{i} - (1-(-1))\hat{j}\right] = \frac{-2k\lambda}{R}(-2\hat{j}) = \frac{4k\lambda}{R}\hat{j}$

+

$$\Rightarrow \vec{E}_0 = \frac{k\lambda}{R}\left[-\hat{i}-\hat{j}-\hat{i}+\hat{j}+4\hat{j}\right] = \frac{k\lambda}{R}(-2\hat{i}+4\hat{j})$$

$$\vec{E}_0 = \frac{2k\lambda}{R}(-\hat{i}+2\hat{j})$$

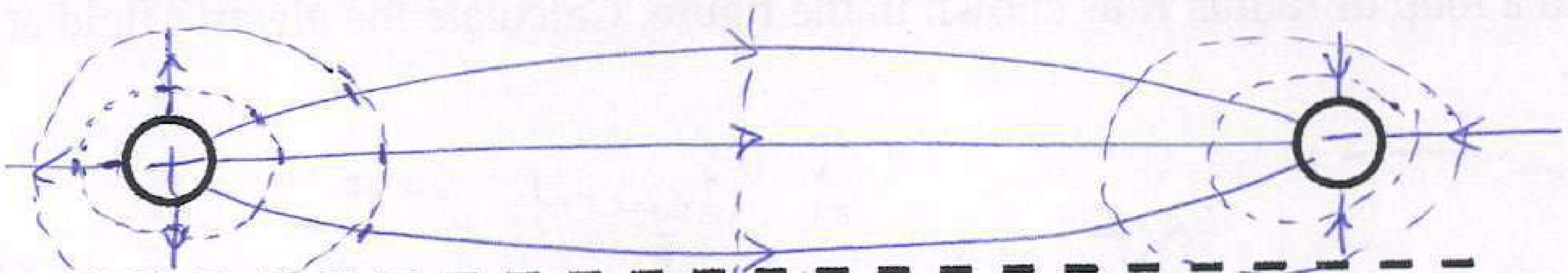


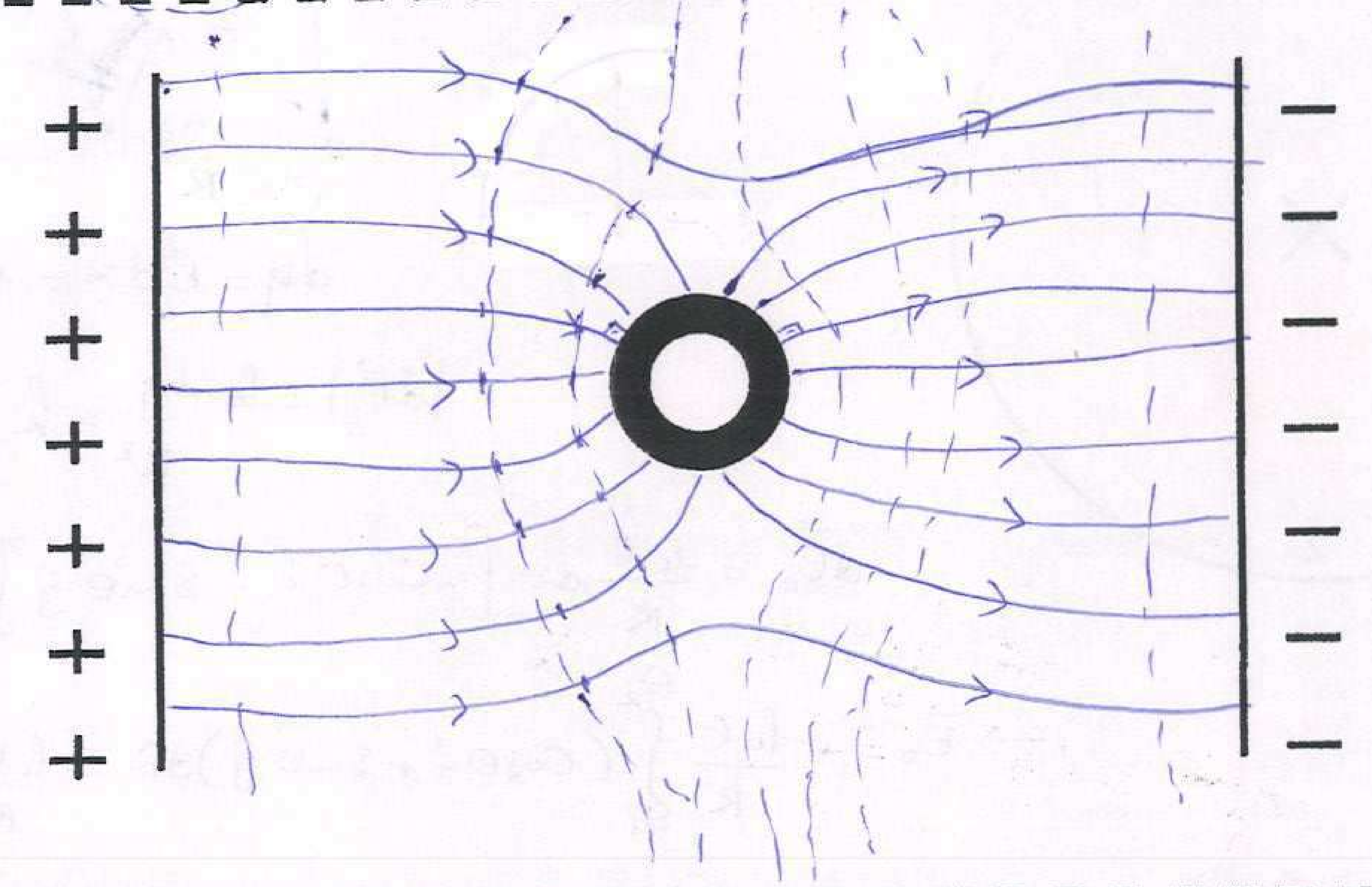
Q4) Draw the electric field lines and equipotential surfaces for the following systems:

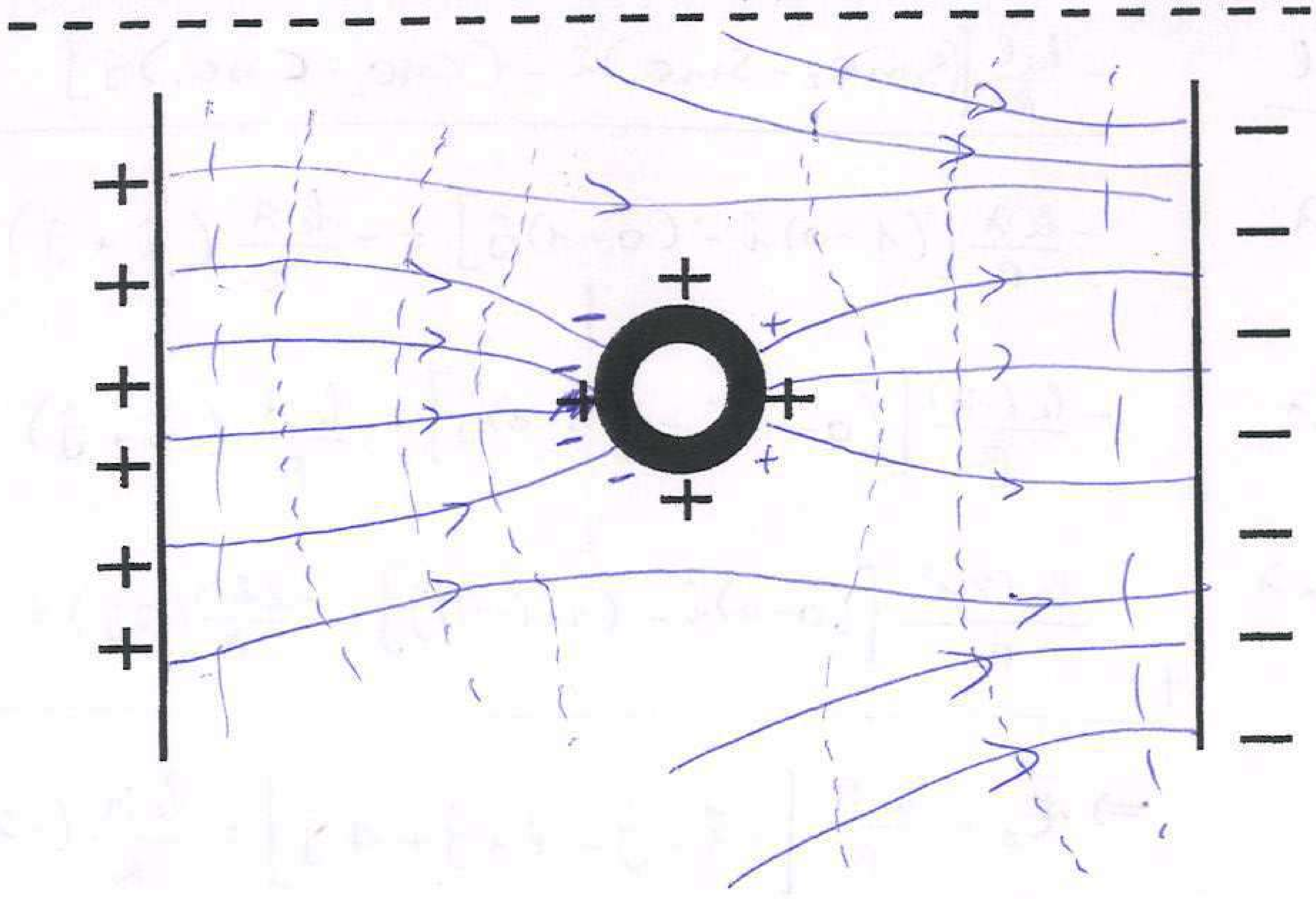
a) Two oppositely charged point-like particles separated by a distance d

b) A conducting, spherical shell with no net charge placed in the middle of the distance between the parallel plates of a charged capacitor.

c) A conducting, positively charged (total charge Q_S) spherical shell placed in the middle of the distance between the charged parallel plates of Q_A and $-Q_B$, respectively ($Q_S < Q_A < |Q_B|$).



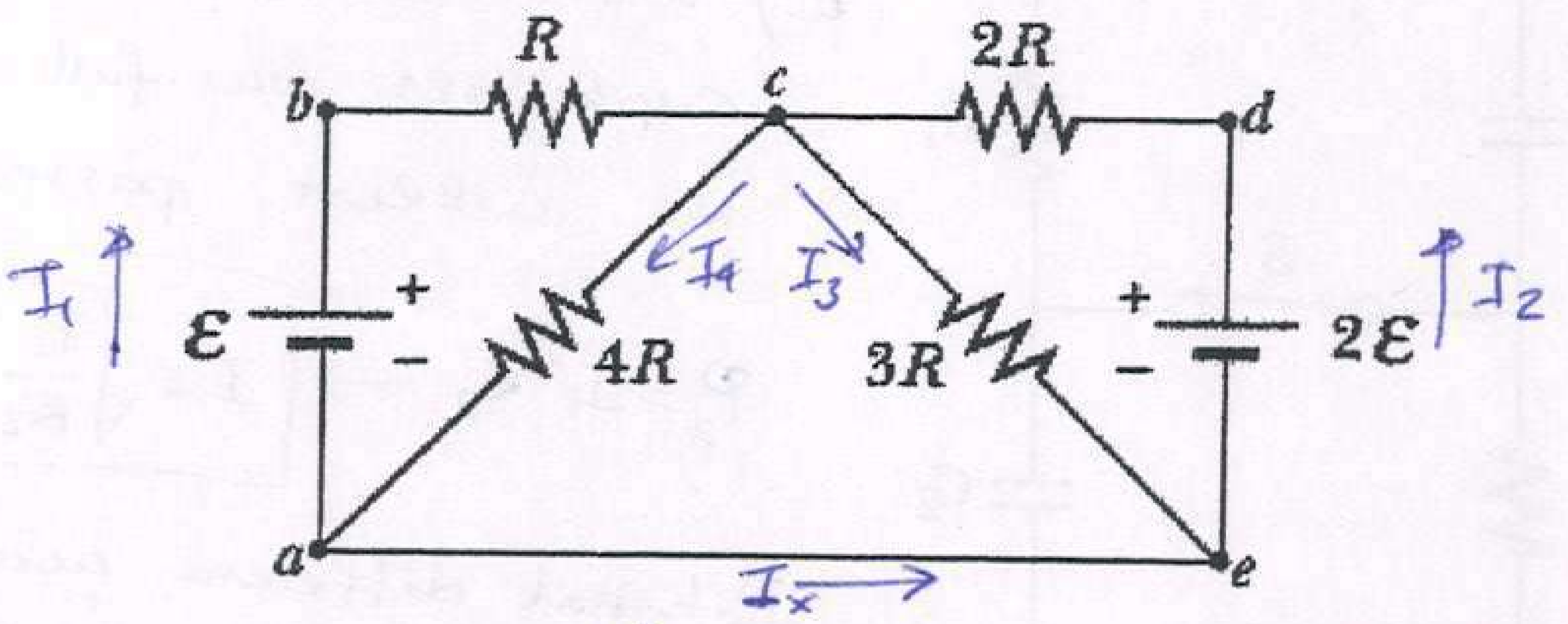




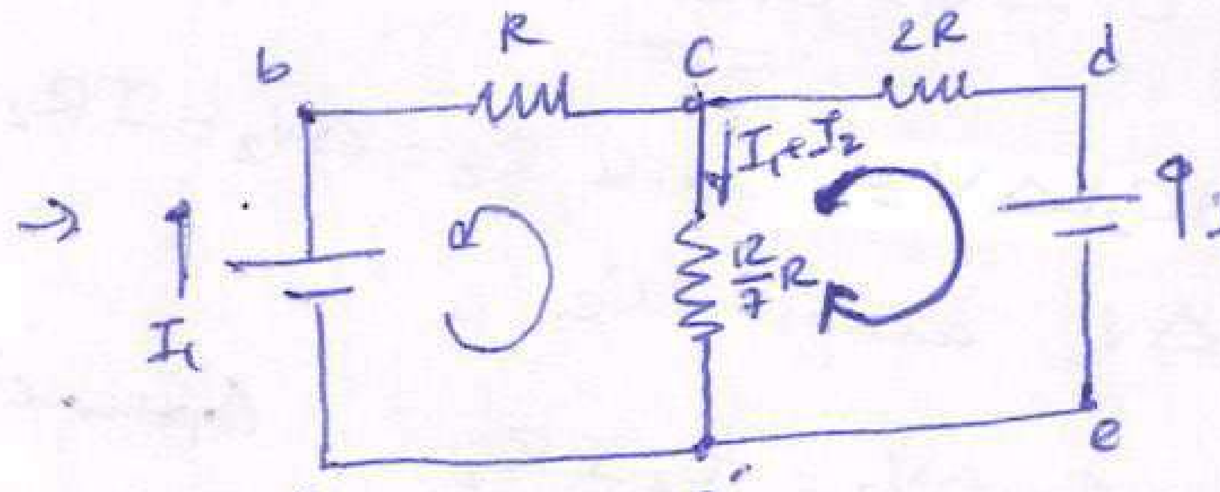
Automotive Engineering, FIZ138 1st Midterm Exam | Instructor: Emre S. Taşcı | 1 / 4 / 2016

Name: Number: Signature:

Q5) Calculate the direction and magnitude of the current in the wire between a and e in terms of R and ε.



$$4R \parallel 3R \rightarrow R_{eq} = \left(\frac{1}{4R} + \frac{1}{3R}\right)^{-1} = \frac{12}{7}R$$



bca'a: $I_1 R + (I_1 + I_2)\left(\frac{12}{7}R\right) = \mathcal{E}$

cdea'c: $(I_1 + I_2)\left(\frac{12}{7}R\right) + I_2(2R) = 2\mathcal{E}$

$$\frac{19}{7}I_1 + \frac{12}{7}I_2 = \frac{\mathcal{E}}{R}$$

$$\frac{12}{7}I_1 + \frac{26}{7}I_2 = \frac{2\mathcal{E}}{R} \quad \times\left(-\frac{12}{13}\right)$$

$$\frac{19}{7}I_1 + \frac{12}{7}I_2 = \frac{\mathcal{E}}{R}$$

$$-\frac{12\cdot 6}{13\cdot 7}I_1 - \frac{12}{13}\cdot\frac{13}{7}I_2 = -\frac{12}{13}\frac{\mathcal{E}}{R}$$

$$\frac{I_1}{7}\left(19 - \frac{12\cdot 6}{13}\right) = \frac{1}{13}\frac{\mathcal{E}}{R}$$

$$\frac{I_1}{7\cdot 13}(19\cdot 13 - 12\cdot 6) = \frac{1}{13}\frac{\mathcal{E}}{R}$$

$$\frac{175}{7}I_1 = \frac{\mathcal{E}}{R}$$

$$\boxed{I_1 = \frac{1}{25}\frac{\mathcal{E}}{R}}$$

$$\frac{19}{7}\frac{1}{25}\frac{\mathcal{E}}{R} + \frac{12}{7}I_2 = \frac{\mathcal{E}}{R}$$

$$\frac{19}{25}\frac{\mathcal{E}}{R} + 12I_2 = \frac{7\mathcal{E}}{R}$$

$$12I_2 = \left(7 - \frac{19}{25}\right)\frac{\mathcal{E}}{R}$$

$$I_2 = \frac{175 - 19}{25\cdot 12}\frac{\mathcal{E}}{R}$$

$$I_2 = \frac{156}{25\cdot 12}\frac{\mathcal{E}}{R}$$

$$\boxed{I_2 = \frac{13}{25}\frac{\mathcal{E}}{R}}$$

$$V_{ca'} = (I_1 + I_2)\cdot\frac{12}{7}R = \frac{14\mathcal{E}\,12}{25R\,7}R = \frac{24}{25}\mathcal{E}$$

$$I_4 = \frac{V_C - V_A}{4R} = \frac{\frac{24}{25}\mathcal{E}}{4R} = \frac{6}{25}\frac{\mathcal{E}}{R}$$

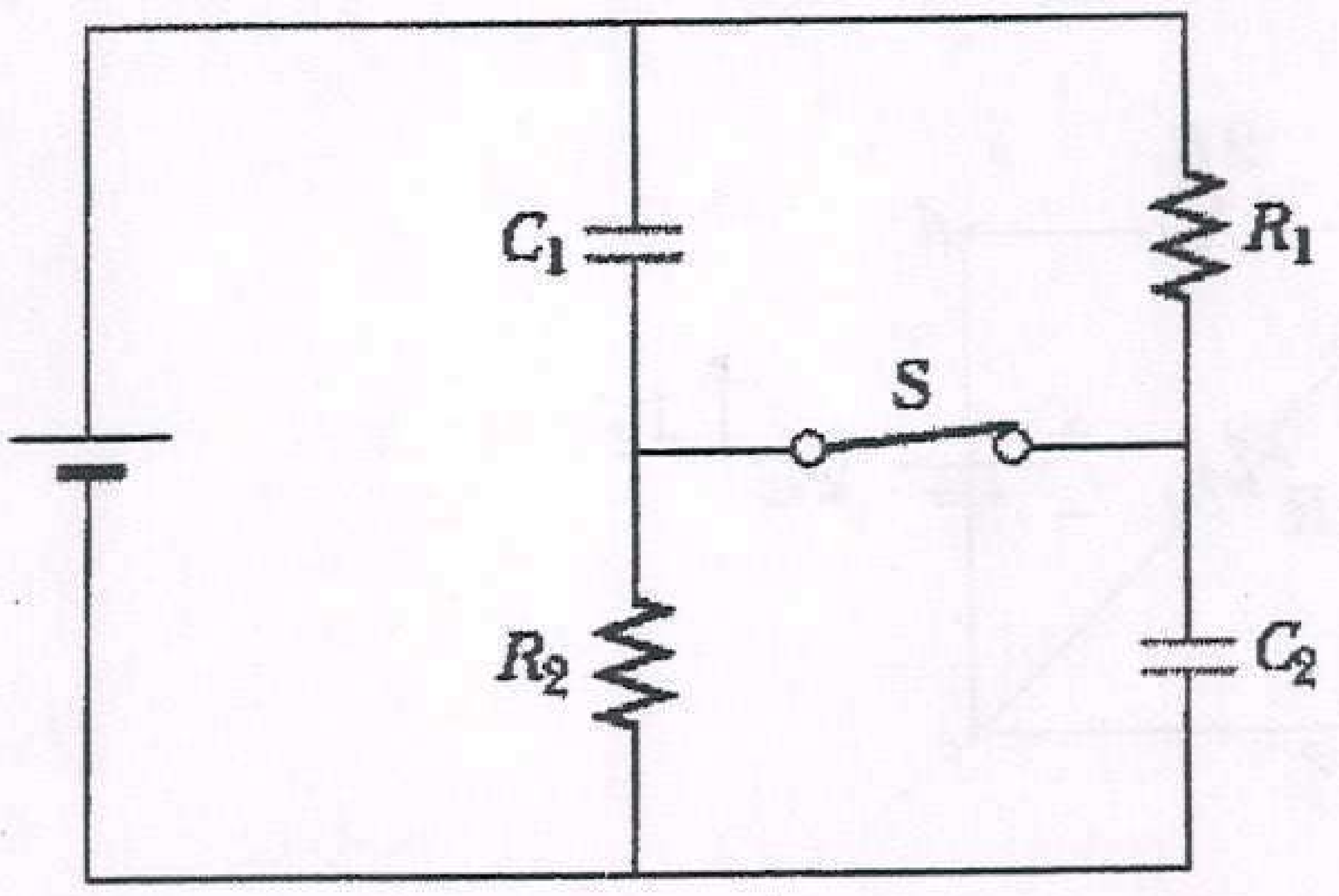
$$I_4 = I_1 + I_x$$

$$I_x = I_4 - I_1 = \frac{6}{25}\frac{\mathcal{E}}{R} - \frac{1}{25}\frac{\mathcal{E}}{R} = \frac{1}{5}\frac{\mathcal{E}}{R}$$

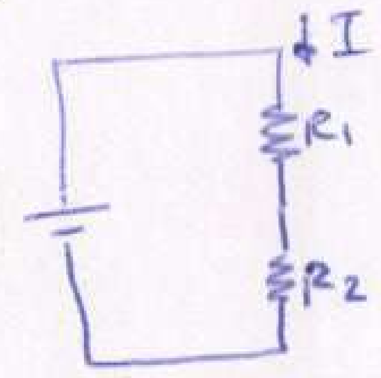
Q6) The circuit carries a constant current. The switch is closed for a long time.

a) If the power delivered to R_2 is P_2, calculate the charge on C_1.
b) After the switch is opened, and a long time has passed, calculate the charge on C_2.

(The source EMF is intentionally not given: you'll have to derive it.
Express all your results in terms of R_1, R_2, C_1, C_2 and P_2)



a) With the switch closed, the capacitors are full and thus no current passes them →.



$P_2 = I^2 R_2 \rightarrow \boxed{I = \sqrt{\frac{P_2}{R_2}}}$

Potential difference across R_1: $\Delta V_1 = I R_1$ is equal to ΔV_1 across C_1

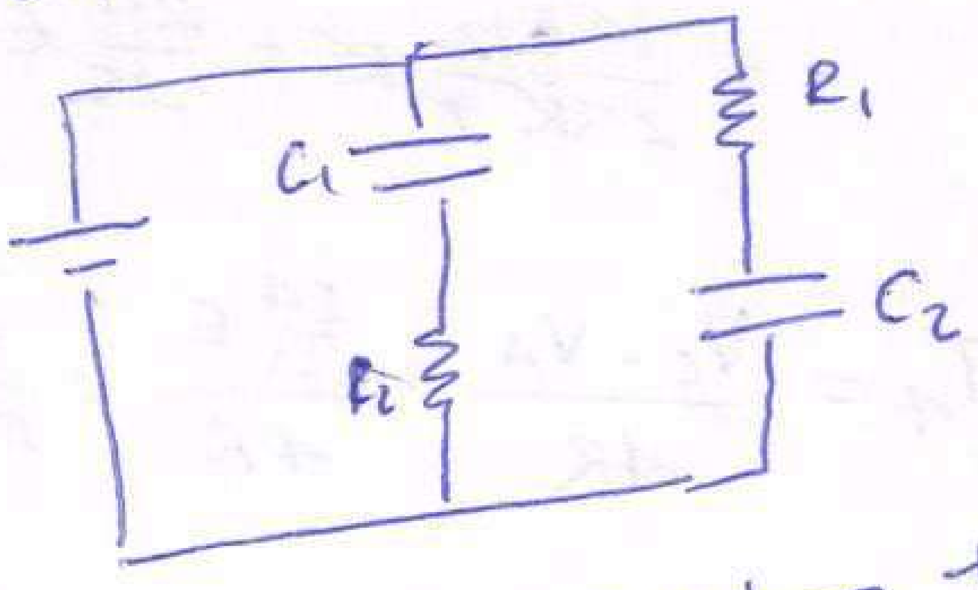
$\rightarrow Q_1 = C_1 \Delta V_1 = \underline{C_1 I R_1}$

Potential difference across R_2: $\Delta V_2 = I R_2$ is equal to ΔV_2 across C_2

$\rightarrow Q_2 = C_2 \Delta V_2 = C_2 I R_2$ (optional)

The source emf : $\mathcal{E} = I R_{eq} = I(R_1 + R_2)$

b) After the switch is opened, eventually the capacitors will once again be filled in equilibrium and they will cut the currents:

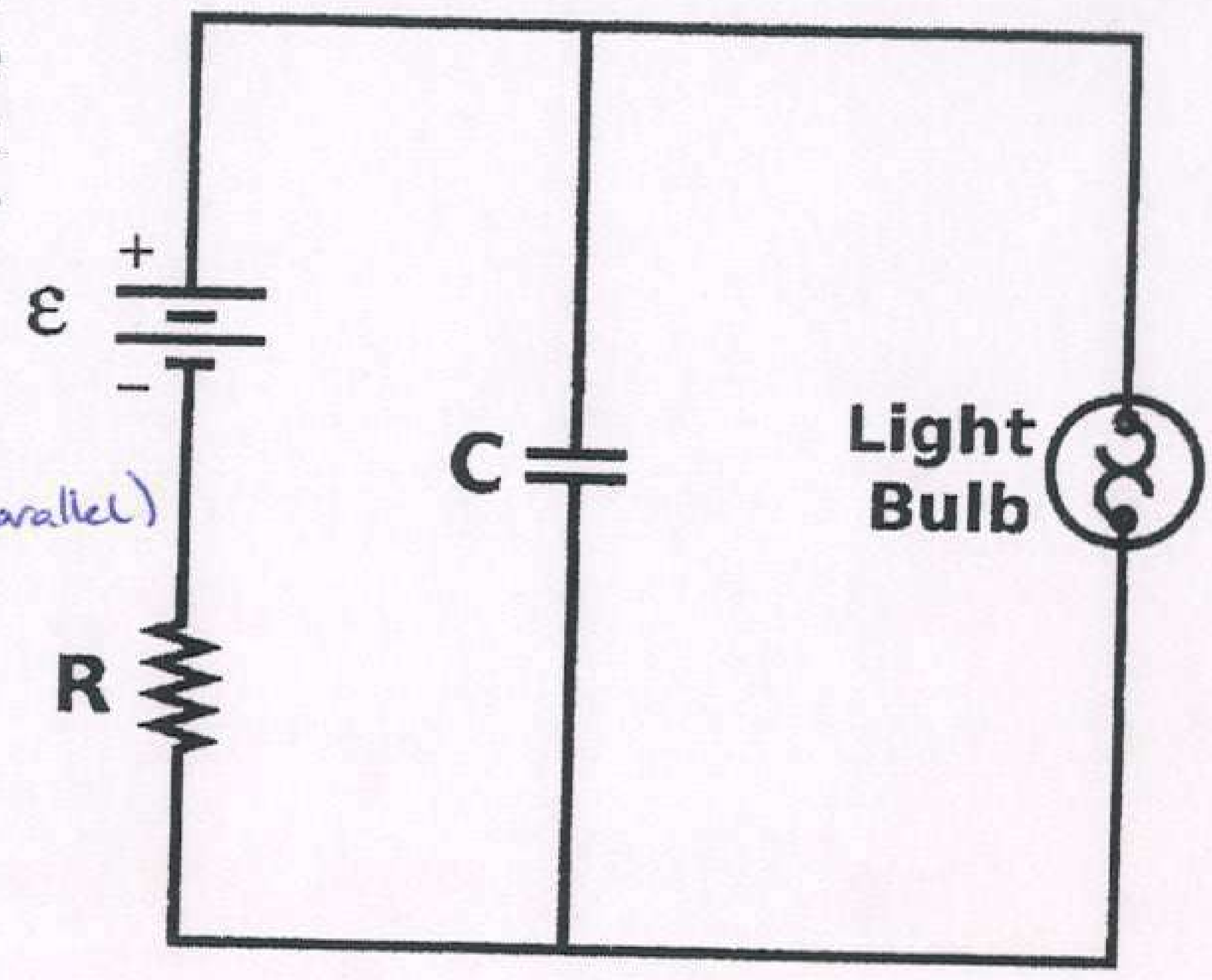


There is no current → the potential difference across each resistor is zero.
→ The potential difference across the capacitors is equal to that of the source (parallel)

$\rightarrow Q_2 = C_2 \Delta V_2' = C_2 \mathcal{E} = C_2 I (R_1 + R_2) = C_2 \sqrt{\frac{P_2}{R_2}} (R_1 + R_2)$

Name: Number: Signature:

Q7) A light bulb is connected to an RC circuit as shown in the figure. The light bulb has a voltage threshold V_L such that, below this voltage, it doesn't operate. In order to have the lamp flash n times per second, what should the value of R be in terms of n, C, ε and V_L? (Assume that the emf device is ideal, with no internal resistance)



Potential diff. across bulb = ΔV across C (parallel)

$$V_L = \mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$\rightarrow \frac{V_L}{\mathcal{E}} = 1 - e^{-t/RC}$$

$$1 - \frac{V_L}{\mathcal{E}} = e^{-t/RC}$$

$$-t/RC = \ln\left(\frac{\mathcal{E} - V_L}{\mathcal{E}}\right)$$

$$t/RC = \ln\left(\frac{\mathcal{E}}{\mathcal{E} - V_L}\right)$$

f = n times per second

$$\rightarrow \underbrace{t = \frac{1}{f} = \frac{1}{n}}$$

$$\Rightarrow R = \frac{t}{C\ln\left(\frac{\mathcal{E}}{\mathcal{E} - V_L}\right)} = \frac{1}{nC\ln\left(\frac{\mathcal{E}}{\mathcal{E} - V_L}\right)}$$