Solutions

Name:
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Section:

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MAT 254 -01-02 Fundamentals of Linear Algebra Final
June 13, 2019

Note: You have 120 minutes.

1-) Let $W = span\{(1,-1,4),(3,-1,4),(1,1,-4),(4,-2,8)\}$.

a) Find $\dim W$. **(10 pt.)**

b) Find an orthogonal basis for the subspace W. **(10 pt.)**

First way

a)
$$\begin{bmatrix} 1 & -1 & 4 \\ 3 & -1 & 4 \\ 1 & 1 & -4 \\ 4 & -2 & 8 \end{bmatrix} \xrightarrow[\text{operations}]{\text{some row}} \begin{bmatrix} 1 & -1 & 4 \\ 0 & 2 & -8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$W = span\{(1,-1,4),(0,2,-8)\}$

so a basis is $\{(1,-1,4),(0,2,-8)\}$

second way

$$\begin{bmatrix} 1 & 3 & 1 & 4 \\ -1 & -1 & 1 & -2 \\ 4 & 4 & -4 & 8 \end{bmatrix} \xrightarrow[\text{opration}]{\text{Some row}} \begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$W = span\{(1,-1,4),(3,-1,4)\}$

so a basis $\{(1,-1,4),(3,-1,4)\}$

b) Let's take $\{(1,-1,4),(0,2,-8)\}$

By using Gram-Schmidt ortagonalization process to ortagonalize this set

$\alpha_1 = (1,-1,4)$

$$\alpha_2 = (0,2,-8) - \frac{(0,2,-8).(1,-1,4)}{(1,-1,4).(1,-1,4)}(1,-1,4) = (0,2,-8) + \frac{34}{18}(1,-1,4)$$

$$= \left(\frac{17}{9}, \frac{1}{9}, \frac{-4}{9}\right)$$

So $\{(1,-1,4),(\frac{17}{9},\frac{1}{9},\frac{-4}{9})\}$ is an orthogonal basis of W.

2-) Find the inverse of the matrix (You can use any method) (**10 pt.**)

$$\begin{bmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{bmatrix}.$$

$$\left[\begin{array}{ccc:ccc} 0 & -3 & -2 & 1 & 0 & 0 \\ 1 & -4 & -2 & 0 & 1 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_2 \leftrightarrow r_1} \left[\begin{array}{ccc:ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_3 \to r_3 + 3r_1} \left[\begin{array}{ccc:ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ 0 & -8 & -5 & 0 & 3 & 1 \end{array}\right]$$

$$\xrightarrow{r_3 \to r_3 - 3r_2} \left[\begin{array}{ccc:ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -3 & 3 & 1 \end{array}\right] \xrightarrow{r_2 \leftrightarrow r_3} \left[\begin{array}{ccc:ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & -3 & 3 & 1 \\ 0 & -3 & -2 & 1 & 0 & 0 \end{array}\right] \xrightarrow{r_3 \to r_3 + 3r_2}$$

$$\left[\begin{array}{ccc:ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & -3 & 3 & 1 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array}\right] \xrightarrow[r_1 \to r_1 + 2r_3]{r_2 \to r_2 - r_3} \left[\begin{array}{ccc:ccc} 1 & -4 & 0 & -16 & 19 & 6 \\ 0 & 1 & 0 & 5 & -6 & -2 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array}\right]$$

$$\xrightarrow{r_1 \to r_1 + 4r_2} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 4 & 5 & 2 \\ 0 & 1 & 0 & 5 & -6 & -2 \\ 0 & 0 & 1 & \underbrace{-8 \quad 9 \quad 3}_{A^{-1}} \end{array}\right]$$

You can use $A^{-1} = \frac{1}{\det A} \operatorname{Adj} A$ or Cayley Hamilton Theorem, as well.

3-) Let $A = \begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 3 \\ 1 & 2 & -3 \end{bmatrix}$ be the matrix represantation of the linear transformation $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ relative to the standart bases. Find $L(2,-3,1)$. **(10 pt.)**

$e_1 = (1,0,0)$
$e_2 = (0,1,0)$
$e_3 = (0,0,1)$

$$L(1,0,0) = 0.e_1 + (-2)e_2 + 1.e_3 = (0,-2,1)$$

$$L(0,1,0) = (-1)e_1 + 1e_2 + 2e_3 = (-1,1,2)$$

$$L(0,0,1) = 2e_1 + 3e_2 - 3e_3 = (2,3,-3)$$

$$(2,-3,1) = 2.e_1 + (-3)e_2 + 1e_3 \Rightarrow$$

$$L(2,-3,1) = L(2e_1 + (-3)e_2 + e_3) = 2.L(1,0,0) - 3L(0,1,0) + L(0,0,1)$$

$$= 2.(0,-2,1) - 3(-1,1,2) + (2,3,-3) = (5,-4,-7)$$

4-) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ such that $T(a,b,c) = (a+c, b-c)$.
a) Find a bases for $\ker T$. **(10 pt.)**
b) Find a generator set for imT. **(10 pt.)**

a) $\ker T = \{ (a,b,c) : T(a,b,c) = 0 \}$

$T(a,b,c) = 0 \Rightarrow (a+c, b-c) = (0,0) \Rightarrow$ $a+c=0$, $b-c=0$

$c = t = b$

$a = -t$

Then

$\ker T = \{ (-t,t,t) : t \in \mathbb{R} \} = \langle (-1,1,1) \rangle$ so $\{(-1,1,1)\}$ is a basis for $\ker T$.

b) $im\, T = \{ T(a,b,c) : (a,b,c) \in \mathbb{R}^3 \}$

$= \{ (a+c, b-c) ; a,b,c \in \mathbb{R} \}$

$= \{ a(1,0) + b(0,1) + c(1,-1) ; a,b,c \in \mathbb{R} \}$

$= \langle (1,0), (0,1), (1,-1) \rangle$

so $\{(1,0), (0,1), (1,-1)\}$ is a generator set.

5-) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$.

a) Find the characteristic polynomial of A. (**5 pt.**)

b) Find the eigenvalues and eigenvectors of A. (**15 pt.**)

c) Is A diagonalizable? If so, determine the invertible matrix P and diagonal matrix D such that $P^{-1}AP = D$. (**10 pt.**)

a) $|xI - A| = \begin{vmatrix} x-1 & -2 & 1 \\ -1 & x & -1 \\ -4 & 4 & x-5 \end{vmatrix} = (x-1)\begin{vmatrix} x & -1 \\ 4 & x-5 \end{vmatrix} + 2\begin{vmatrix} -1 & -1 \\ -4 & x-5 \end{vmatrix} + \begin{vmatrix} -1 & x \\ -4 & 4 \end{vmatrix}$

$= (x-1)(x^2-5x+4) + 2(1-x) + 4(x-1)$

$= (x-1)(x^2-5x+4-2+4) = (x-1)(x^2-5x+6)$

$= (x-1)(x-2)(x-3)$

b) $\lambda_1 = 1$
$\lambda_2 = 2$
$\lambda_3 = 3$

For $\lambda_1 = 1$ $\quad A - I = \begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 1 \\ 4 & -4 & 4 \end{bmatrix} \xrightarrow{r_3 \to r_3 - 4r_2} \begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow (A-I)x = 0 \Rightarrow 2x_2 - x_3 = 0 \qquad x_2 = t \quad x_3 = 2t$

$x_1 - x_2 + x_3 = 0 \qquad x_1 = -t$

$\Rightarrow$ eigenvector $= t \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

For $\lambda_2 = 2$

$A - 2I = \begin{bmatrix} -1 & 2 & -1 \\ 1 & -2 & 1 \\ 4 & -4 & 3 \end{bmatrix} \xrightarrow[r_3 \to r_3 + 4r_1]{r_2 \to r_2 - r_1} \begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 4 & -1 \end{bmatrix} \Rightarrow (A-2I)x = 0$

$4x_2 - x_3 = 0$

$-x_1 + 2x_1 - x_3 = 0$

$x_2 = t \quad x_3 = 4t$

$\Rightarrow$ eigenvector $= t \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}$

$x_1 = -2t$

For $\lambda_3 = 3$

$$(A-3I) = \begin{bmatrix} -2 & 2 & -1 \\ 1 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix} \xrightarrow{r_3 \to r_3 + 2r_1} \begin{bmatrix} -2 & 2 & -1 \\ 1 & -3 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{r_1 \to r_1 + 2r_2} \begin{bmatrix} 0 & -4 & 1 \\ 1 & -3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow (A-3I)x = 0 \Rightarrow$ $4x_2 - x_3 = 0$ $\quad x_2 = t \quad x_3 = 4t$

$x_1 - 3x_2 + x_3 = 0 \quad x_1 = -t$

$$\Rightarrow \text{eigenvector} = t\begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$$

c) Every eigenvalue is distinct then A is diagonalizable

$$\begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \text{ eigenvectors} \\ P = \begin{bmatrix} -1 & -2 & -1 \\ 1 & 1 & 1 \\ 2 & 4 & 4 \end{bmatrix} \end{array} \qquad \begin{array}{c} D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \\ \uparrow \uparrow \uparrow \text{ eigen values with same order} \end{array}$$

6-) True or False. If true, you should prove the statement. If false, you should provide a counterexample (Undisclosed answers will not be evaluated).

a) The set $W = \{f \in P_3(x) : \deg f = 3\}$ is a subspace of $P_3(x)$. **(5 pt.)**

b) The set of $n \times n$ skew-symmetric matrices is closed under the matrix addition. **(5 pt.)**

c) Let $A = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 4 & 0 \end{bmatrix}$. Then $\det A = 4$. **(5 pt.)**

d) Let A be an $n \times n$ diagonalizable matrix with eigenvalues only 1's and -1's. Then $A^2 = I$. **(5 pt.)**

Note that a and b were also your midterm questions.

a) (F) $x_3 \in W$ $-x^3 \in W$ but $x_3 + (-x_3) = 0 \notin W$

so it is not closed under addition

b) (T) Let A, B be skew symmetric matrices $(A^T = -A,\ B^T = -B)$

$A+B$ is skew-symmetric?

$(A+B)^T = A^T + B^T = -(A+B)$ so $A+B$ skew symetric

c) (F) determinants of nonsquare matrices are not defined.

d) (T) Since A is diagonalizable, we can write $A = PDP^{-1}$

where all diagonal entries of D consist 1's and (-1)'s

since $A^2 = PD^2P^{-1}$ and $1^2 = 1$, $(-1)^2 = 1$, $D^2 = I$ and $A^2 = PIP^{-1} = I$

GOOD LUCK

Talha Arıkan (01) - H. Melis Tekin Akçin (02)

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MAT 254 -01-02 Fundamentals of Linear Algebra Midterm

April 25, 2019

1-) a) Let the set $\{\alpha_1, \alpha_2, \alpha_3\}$ be linearly independent. Determine whether the set

$$\{\alpha_1 + 3\alpha_2, \alpha_1 + 5\alpha_2 - 2\alpha_3, \alpha_2 - \alpha_3\}$$

is linearly independent or not. **(10 pt.)**

b) Find k if $(k, 2k+1, 3k-1, k^2+3k-1)$ is linear combination of the vectors $(1,1,1,1)$, $(-1,-1,-2,-1)$, $(1,2,3,2)$ and $(1,0,4,0)$. **(10 pt.)**

2-) For $A = \begin{bmatrix} -2 & 6 & 2 & -2 \\ 1 & -1 & 0 & 1 \end{bmatrix}$, find an invertible matrix P and a row reduced echelon matrix R such that $R = PA$. **(10 pt.)**

3-) Find the values of a for which the system

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + \left(a^2 - 14\right) z &= a + 2 \end{aligned}$$

has *i)* a unique solution, *ii)* infinitely many solution, *iii)* no solution. **(20 pt.)**

4-) Show that **(10 pt.)**

$$\begin{vmatrix} 1 & x & x^2 & x^3 \\ a & 1 & x & x^2 \\ p & b & 1 & x \\ q & r & c & 1 \end{vmatrix} = (1 - ax)(1 - bx)(1 - cx).$$

5-) Let $H = \{(t + 2s, -t + s, 3t - s) : t, s \in \mathbb{R}\}$.

a) Is H a subspace of $\mathbb{R}^3$? **(10 pt.)**

b) If so, find a generator set for H. **(10 pt.)**

6-) True or False. If true, you should prove the statement. If false, you should provide a counterexample.

a) The set $W = \{f \in P_3(x) : \deg f = 3\}$ is a subspace of $P_3(x)$. **(5 pt.)**

b) The set of $n \times n$ skew-symmetric matrices is closed under matrix addition. **(5 pt.)**

c) Let A and B be 3×3 matrices and $\det A = -1$ and $\det B = 2$ then $\det(-AB^3) = 8$. **(5 pt.)**

d) Let A be a square matrix. If $A^2 + 2A + 3I = 0$, then A is invertible. **(5 pt.)**

BONUS: Write your expected grade $\boxed{100}$. If your guess is between your grade-5 and your grade$+5$, you will get 5 extra point.

Note: You have 120 minutes.

GOOD LUCK

Talha Arıkan (01) - H. Melis Tekin Akçin (02)

1) a) $c_1(\alpha_1+3\alpha_2)+c_2(\alpha_1+5\alpha_2-2\alpha_3)+c_3(\alpha_2-\alpha_3)=0$

$\Rightarrow (c_1+c_2)\alpha_1+(3c_1+5c_2+c_3)\alpha_2+(-2c_2-c_3)\alpha_3=0$

Since $\{\alpha_1,\alpha_2,\alpha_3\}$ linearly independent, . $c_1+c_2=0$

$3c_1+5c_2+c_3=0$

$-2c_2-c_3=0$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 3 & 5 & 1 \\ 0 & -2 & -1 \end{bmatrix} \xrightarrow{r_2\to r_2-3r_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -2 & -1 \end{bmatrix} \xrightarrow{r_3\to r_3+r_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow \begin{matrix} c_1+c_2=0 \\ 2c_2+c_3=0 \end{matrix} \quad \Rightarrow \begin{matrix} c_2=t \\ c_3=-2t \end{matrix} \quad \Rightarrow c_1=-t$

there is a solution where c_1,c_2,c_3 are nonzero

so the corresponding set linearly dependent.

b) $c_1(1,1,1,1)+c_2(-1,-1,-2,-1)+c_3(1,2,3,2)+c_4(1,0,4,0)$

$=(k,2k+1,3k-1,k^2+3k-1)$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & k \\ 1 & -1 & 2 & 0 & 2k+1 \\ 1 & -2 & 3 & 4 & 3k-1 \\ 1 & -1 & 2 & 0 & k^2+3k-1 \end{array}\right] \xrightarrow[\substack{r_3\to r_3-r_1 \\ r_4\to r_4-r_1}]{r_2\to r_2-r_1} \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & k \\ 0 & 0 & 1 & -1 & k+1 \\ 0 & -1 & 2 & 3 & 2k-1 \\ 0 & 0 & 1 & -1 & k^2+2k-1 \end{array}\right] \xrightarrow{r_4\to r_4-r_2}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & k \\ 0 & 0 & 1 & -1 & k+1 \\ 0 & -1 & 2 & 3 & 2k-1 \\ 0 & 0 & 0 & 0 & k^2+k-2 \end{array}\right] \Rightarrow k^2+k-2=0$$

$$\Rightarrow (k+2)(k-1)=0$$

$$\Rightarrow k=-2 \text{ or } k=1$$

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2) $$\left[\begin{array}{cccc|cc} -2 & 6 & 2 & -2 & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 & 1 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_2} \left[\begin{array}{cccc|cc} 1 & -1 & 0 & 1 & 0 & 1 \\ -2 & 6 & 2 & -2 & 1 & 0 \end{array}\right] \xrightarrow{r_2 \to r_2 + 2r_1}$$

$$\left[\begin{array}{cccc|cc} 1 & -1 & 0 & 1 & 0 & 1 \\ 0 & 4 & 2 & 0 & 1 & 2 \end{array}\right] \xrightarrow{r_2 \to \frac{1}{4} r_2} \left[\begin{array}{cccc|cc} 1 & -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1/2 & 0 & 1/4 & 1/2 \end{array}\right] \xrightarrow{r_1 \to r_1 + r_2} \left[\begin{array}{cccc|cc} 1 & 0 & 1/2 & 1 & 1/4 & 3/2 \\ 0 & 1 & 1/2 & 0 & 1/4 & 1/2 \end{array}\right]$$

$$R = \begin{bmatrix} 1 & 0 & 1/2 & 1 \\ 0 & 1 & 1/2 & 0 \end{bmatrix} \qquad P = \begin{bmatrix} 1/4 & 3/2 \\ 1/4 & 1/2 \end{bmatrix}$$

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3) $$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{array}\right] \xrightarrow[r_3 \to r_3 - 4r_1]{r_2 \to r_2 - 3r_1} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{array}\right]$$

$$\xrightarrow{r_3 \to r_3 - r_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2-16 & a-4 \end{array}\right] \Rightarrow$$

$a=-4 \Rightarrow \quad 0 \ 0 \ 0 : -8$

no solution

$a=4 \Rightarrow \quad 0 \ 0 \ 0 : 0$

infinitely many solutions

$a \neq \mp 4 \Rightarrow \quad 0 \ 0 \ \overset{\neq 0}{\downarrow} : \overset{\neq 0}{\downarrow}$

unique solution

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4) $\begin{vmatrix} 1 & x & x^2 & x^3 \\ a & 1 & x & x^2 \\ p & b & 1 & x \\ q & r & c & 1 \end{vmatrix} \overset{C_4 \to C_4 - xC_3}{=} \begin{vmatrix} 1 & x & x^2 & 0 \\ a & 1 & x & 0 \\ p & b & 1 & 0 \\ q & r & c & 1-cx \end{vmatrix}$

$\overset{C_3 \to C_3 - xC_2}{=} \begin{vmatrix} 1 & x & 0 & 0 \\ a & 1 & 0 & 0 \\ p & b & 1-bx & 0 \\ q & r & c-rx & 1-cx \end{vmatrix} \overset{C_2 \to C_2 - xC_1}{=} \begin{vmatrix} 1 & 0 & 0 & 0 \\ a & 1-ax & 0 & 0 \\ p & b-px & 1-bx & 0 \\ q & r-qx & c-rx & 1-cx \end{vmatrix}$

$= 1.(1-ax).(1-bx).(1-cx)$

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5) a) $u, v \in H$ $\left(\begin{array}{l} u = (t_1+2s_1, -t_1+s_1, 3t_1-s_1) \quad \exists\, t_1, s_1 \in \mathbb{R} \\ v = (t_2+2s_2, -t_2+s_2, 3t_2-s_2) \quad \exists\, t_2, s_2 \in \mathbb{R} \end{array} \right.$

$\Rightarrow$ i) $u+v = \big(\underbrace{(t_1+t_2)}_{\in\mathbb{R}} + 2\underbrace{(s_1+s_2)}_{\in\mathbb{R}}, -(t_1+t_2)+(s_1+s_2), 3(t_1+t_2)-(s_1+s_2) \big)$

$= (t_3+2s_3, -t_3+s_3, 3t_3-s_3)$ s.t $t_3 = t_1+t_2 \in \mathbb{R}$
$s_3 = s_1+s_2 \in \mathbb{R}$

$\Rightarrow u+v \in H$

ii) $cu = (ct_1+2cs_1, -ct_1+cs_1, 3ct_1-cs_1)$

$= (t_4+2s_4, -t_4+s_4, 3t_4-s_4)$ s.t $t_4 = ct_1 \in \mathbb{R}$
$s_4 = cs_1 \in \mathbb{R}$

$\Rightarrow cu \in H$ $\Rightarrow H$ is a subspace of $\mathbb{R}^3$

b) $H = \{(t+2s, -t+s, 3t-s) \mid s,t \in \mathbb{R}\}$

$= \{t(1,-1,3) + s(2,1,-1) \mid s,t \in \mathbb{R}\}$

$= \langle (1,-1,3), (2,1,-1) \rangle = \text{span}((1,-1,3),(2,1,-1))$

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6) a) (F) Because $0 \notin W$, or $-x^3 \in W$
$x^3+1 \in W$
on the other hand
$(-x^3) + (x^3+1) = 1 \notin W$
not closed under addition

b) (T) Let A and B be skew-symmetric matrices.
Is A+B a skew-symetric matrix?

$(A+B)^T = A^T + B^T = -A-B$ (Since A and B are skew-symmetric)

$= -(A+B) \Rightarrow A+B$ is skewsymmetric ✓

c) (T) $\det(-AB^3) = (-1)^3 \det A \cdot (\det B)^3 = (-1).(-1).2^3 = 8$

d) (T) $A^2 + 2A = -3I \Rightarrow A.(A+2I) = -3I$

$\Rightarrow A\left(-\frac{1}{3}A - \frac{2}{3}I\right) = I$

$\Rightarrow A$ is invertible and $A^{-1} = -\frac{1}{3}A - \frac{2}{3}I$

or,
$A.(A+2I) = -3I$
$\det A . (\det(A+2I)) = (-3)^n$
$\Rightarrow \det A \neq 0$
$\Rightarrow A$ invertible