Thesis Proposal: Stochastic Volatility within the KNW-Model

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1 Introduction

The KNW-model is a financial model developed by [4] as an extension on [1]. In an academic setting, the model is used to investigate the importance of time-varying bond risk premia in a consumption and portfolio choice problem for a life-cycle investor.

Outside of academia, the model is used in the Netherlands for the yearly feasibility test for pensionfunds (jaarlijkse haalbaarheids toets). In this situation, the model is used to generate a economic scenario set which are used for ALM studies by the Dutch pensionfunds.

2 A Limitation of the model

The model is written as a multivariate Ornstein-Uhlenbeck process

$$d\begin{bmatrix} X \\ \ln \Pi \\ \ln S \\ \ln P^{F0} \\ \ln P^{F_t} \end{bmatrix} = \left(\begin{bmatrix} \mu \\ \delta_{0\pi} - \frac{1}{2}\sigma_{\Pi}^{'}\sigma_{\Pi} \\ R_0 + \eta_S - \frac{1}{2}\sigma_S^{'}\sigma_S \\ R_0 \\ R_0 + B^N(\tau)^{'}\Sigma_X^{'}\Lambda_0 - \frac{1}{2}B^{N'}\Sigma_X^{'}\Sigma_X B^N \end{bmatrix} + \begin{bmatrix} -K & 0 \\ \delta_{1\pi}^{'} & 0 \\ R_1^{'} & 0 \\ R_1^{'} & 0 \\ R_1^{'} + B^N(\tau)^{'}\Sigma_X^{'}\Lambda_1 & 0 \end{bmatrix} \begin{bmatrix} X \\ \ln \Pi \\ \ln S \\ \ln P^{F0} \\ \ln P^{F_t} \end{bmatrix} \right) dt + \begin{bmatrix} \Sigma_X^{'} \\ \sigma_{\Pi}^{'} \\ \sigma_{\Pi}^{'} \\ \sigma_S^{'} \\ B^{N'}(\tau)\Sigma_X^{'} \end{bmatrix} dZ_t. \tag{1}$$

The model makes various assumptions about the financial markets. One of these assumptions is about the stock index process S. Within the model the stock index develops according to

$$\frac{dS_t}{S_t} = (R_t + \eta_s)dt + \sigma_s^{'} dZ_t, \tag{2}$$

where $\sigma_s \in \mathcal{R}^4$, R is the nominal instantaneous interest rate and η_S the equity risk premium. Note that the vector $\sigma_s^{'}$ which is used to determine the variance of the stock consists of four constants. The assumption of constant variance / volatility / drift may not be correct. Three different approaches to determine if this assumption is correct are outlined bellow. A theoretical argument, results from the stochastic volatility literature and lastly empirical evidence.

The theoretical argument posed is based on [6] who made the observation that stock price series tend to show too many outliers to be from a constant-variance log-normal distribution. Hence he posed that a stock price may consist of a normal vibration in price due to various changes (supply and demand, new information) which may be captured by a geometric Brownian motion. However, the outliers that are not from a constant-variance log-normal distribution may be explained by the arrival of important new information about the stock that has more than a marginal effect on price. [6] modeled this component by a "jump" process reflecting the new information. However, this may also be due to other sources.

An other approach which may show that stock prices do not follow a generalized Brownian motion with constant drift and volatility is by comparing forecasting methods and model fits. If a forecasting method that assumes stochastic volatility yields a better estimate than a method with constant volatility, there may be evidence that rejects the constant variance assumption. The same logic may be used for models which attempt to fit stock market data.

A recent example of forecasting with stochastic volatility is [3] who found that the model fits a monthly US stock dataset better than alternatives that do not include these features. A overview of stochastic volatility models and non volatile models can be found in [2].

Lastly, the last approach that will investigate the constant variance assumption is by investigating empirical data. A convenience sample is taken from the monthly close position of S&P500 and the Dow Jones Index between 1990 and 2022. Assuming that the stock returns are from the Log-normal distribution, the log difference of S_{t+1} and S_t is investigated. Respective plots can be seen in Fig. 1. Notice that these plots do appear to be normally distributed. However, they do appear to have more kurtosis than a normal distribution. Running Kolmogorov-Smirnov tests indicates that these distributions are not from the normal distributions ($p < 0.05$).

Splitting the time series 5 year parts and running the Kolmogorov-Smirnov tests shows for each sample $p > 0.05$ which indicates that we fail to reject the null hypothesis. Hence we find no evidence that the data is from a different distribution. Computing the variance of each unique sample returns a different variance for each sample. So within these two samples we find evidence that the S&P500 and DJ index may not have constant variance.

3 Thesis Proposal

Having briefly outlined that there is evidence that assuming constant volatility may not be a correct assumption for the development of a stock-price index, this master thesis would investigate the effects of introducing stochastic volatility in the KNW-Model. This would be done by rewriting $\sigma_s^{'}$ as a function that governs volatility within a stock price index.

Various approaches could be considered with the optimal approach being determined through literature review and simulation. An example of this would to rewrite the stock process S by introducing the stochastic variance term v_t which follows a mean-reverting process akin to the Heston dynamics [5]. Hence,

$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dW_t. \tag{3}$$

By adding an equation to this within the model as described in Eqn. 1, the exercise of calibrating the new model would mean rewriting the system of measurement equations and redoing the simulated annealing to find the global optimum.

After stochastic volatility has been introduced in the stock price index, the effects of stochastic volatility on the inflation index may be considered as there is evidence that assuming constant variance for inflation may not be correct [7].

The relevance of this master thesis is twofold. Firstly, the KNW-model and similar models are used to find optimal life-cycles for investors which may be unrealistic due to the constant variance assumption. The life-cycles generated by the extended model could be compared to the original life-cycles to investigate the changes of portfolio composition. The second part of relevance is that the model is currently facing scrutiny as it is used by Dutch pension funds to preform certain ALM-studies. The critique is often that the model assumes normality and does not have tail risks. Through the addition of stochastic volatility tails risks may be introduced as done by other papers.

Having identified a possible issue with a computational model by considering relevant literature and data and having determined that there is relevancy in studying this problem a research question can be formulated. Note that this is a preliminary research question and may be adjusted if the final research proposal. There are two preliminary research questions that this thesis would considered. First, "Does a stochastic volatility component within the stock index processes of the KNW model capture the outliers in stock market data that are not able to be explained by a constant-variance lognormal distribution?". The second research question would be "What is the effect of stochastic volatility within the stock price index process of the KNW model on the optimal portfolio composition of a life-cycle investor?".

3.1 Thesis Objectives

Lastly, this research project must satisfy the objectives of the master thesis for computational science students. There are a total of 6 objectives of which one will not be considered as it deals with presentation of the report which is beyond the scope of this proposal.

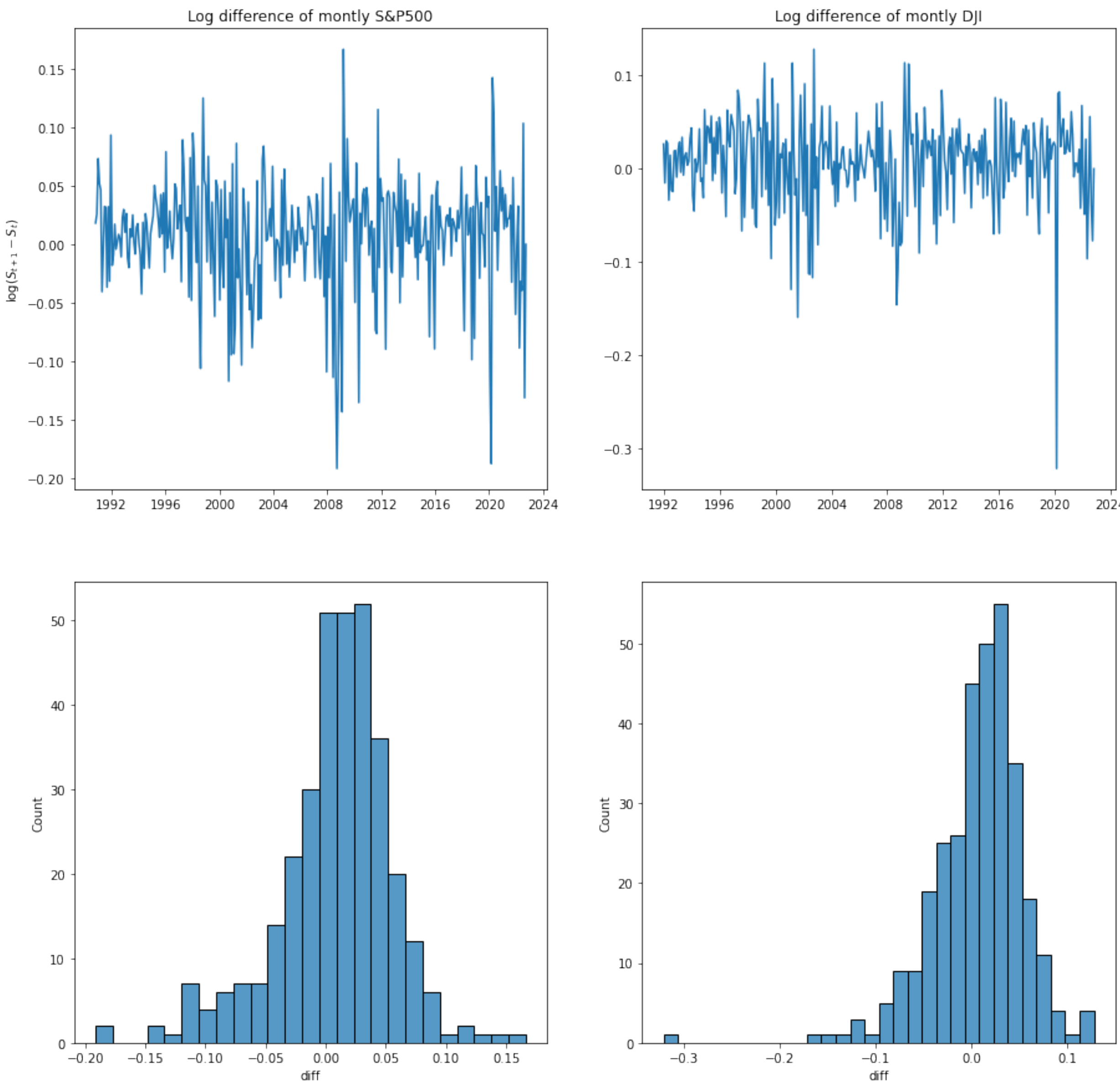

Figure 1: Time series and histograms of the log of S&P 500 and DJI

The first objective is to "formulate and develop their own research question". This objective is met as a limitation within an academic model is identified. Based on this observation academic literature is consulted to see if the assumption is correct. As the assumption is not deemed correct a research question is proposed to investigate an alternative.

The second is to "demonstrate a deep knowledge of their chosen topic". This would be done by evaluating the literature on computational financial models that take in account stochastic volatility and appraise their merit. Furthermore, to extend computational model knowledge of relevant literature is required which would demonstrate a deep knowledge of the chosen topic.

The third is to "demonstrate the ability to take a holistic view of a research problem and develop novel solutions". The ability to take a holistic view of a research problem is done by considering relevant literature and literature from other computational fields that could be used to solve the problem. Novel solutions will be developed as stochastic volatility has not been considered for this model.

The fourth objective is to "develop and demonstrate the ability to plan, analyse and execute long term goals and objectives". This objective will be met as this is part of handing the in research draft proposal by the end of the year.

The fifth objective is to "demonstrate the capability to think both critically and systematically with regard to scientific data and experimentation". This criteria will be met as through questioning the underlying assumption of the model in question possible research questions are developed.

References

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