Supplementary Material:

Robust Feature Selection for Continuous BP Estimation in Multiple Populations: Towards Cuffless Ambulatory BP Monitoring

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Feature Space from PPG signals

The generated features can be divided into four main groups: time domain (TF), frequency-based (FF), statistical (SF), and demographic features (DF)

1) Time-domain Features (TF)

The proposed time-domain features can be divided into six subcategories regarding intensity, time, area, slope, branch width, and others. Several of these features are computed using one or two fiducial points. The fiducial points include the onset (O), valley (V), systolic peak (S), maximum derivative point (MD), diastolic peak (D), inflection point (IP), dicrotic notch (DN), point a, point b, and intersection point (IT).

| Abbreviation | Name | Definition | | |
|------------------------------|---|---|--|--|
| I - Intensity-based features | | | | |
| i_{x1} | Intensity | Absolute Intensity or amplitude of point x1 | | |
| i _{x1-x2} | Relative intensity | Relative intensity or amplitude between point $x1$ and $x2$ $I_{X1-X2}=I_{x1}-I_{x2}$ | | |
| ir _{X1-X2} | Intensity ratio | Intensity ratio of points $x1$ and $x2$ $RI_{X1-X2}=I_{x1}/I_{x2}$ | | |
| ni_{X} | Normalized intensity | Normalized intensity within the range of 0 to 1 | | |
| ni _{X1-X2} | Relative normalized intensity | Relative normalized intensity of amplitude between point x1 and x2 $ni_{X1-X2}=ni_{X1-}ni_{X2}$ | | |
| id_{x1} | Intensity of the first derivative | Absolute intensity or amplitude of the first derivative at point x1. | | |
| id _{x1-x2} | Relative intensity of the first derivative | Relative intensity or amplitude between point x1 and x2 in the first derivative | | |
| $id2_{x1}$ | Intensity of the second derivative | Absolute intensity or amplitude of the second derivative at point x1. | | |
| ird2x1-x2 | Relative intensity of the second derivative | Relative intensity or amplitude between point x1 and x2 in the first derivative | | |
| Im | Mean intensity | Mean intensity of the waveform | | |
| mean_irx1 | Mean intensity ratio | Mean intensity ratio when dividing the waveform into two segments at point x1. | | |
| | | T - Time-based features | | |
| t_{x1} | Relative time | Time between key-point x1 and v Note that t _{v1} is the total duration of the pulse waveform | | |
| t _{x1-x2} | Time difference | Time difference between points x1 and x2 $t_{x_1-x_2} = t_{x_1} - t_{x_2}$ | | |

| tn_{x1} | NT 1' 1.' | Normalized time from the onset to the x1 point with respect to the total |
|--|---|---|
| | Normalized time | duration of the pulse waveform. |
| | | $nt_{x1} = t_{x1}/t_{v1}$ |
| tr_{x1-x2} | | Ratio between the time from the onset to x1 and the time from the onset |
| | Time ratio | to x2. |
| | | $tr_{x_1-x_2} = t_{x_1} / t_{x_2}$ |
| trn _{x1-x2} | Normalized time | Normalized time ratio between points |
| | ratio | $trn_{x1-x2} = t_{x1-x2} / t_{v1}$ |
| | | A - Area-Related features |
| A_{x1-x2} | Area | Area under the curve defined by points x_1 and x_2 |
| ART_{x1-x2} | | Total area ratio between S and D. Ratio of the area under the curve |
| | Total area ratio | between x_1 and x_2 to the total area under the pulse waveform. |
| | | $ART_{x1-x2} = A_{x1-x2}/A_{v-v1}$ |
| ARs_x | | Break the area under PPG into two parts based on the key point x, where |
| | Ratio of areas | A_1 and A_2 are the areas of the first and second parts. |
| | | $RA = A_{V-x}/A_{x-v1}$ |
| | | SLP - Slope |
| SLP _{X1} | Clono | Slope from v to key-point x1 |
| | Slope | $SLP_{X1} = (I_{x1}-I_V) / t_{x1}$ |
| SLP _{x1-n} | NT 1' 1 1 | Slope from v to key-point x1 |
| A. I. | Normalized slope | $nSLP_{X1} = (I_{x1}-I_{V}) / tn_{x1}$ |
| | | BW - Branch-width related features |
| | | 0%, 25%, 33%, 50%, 66%, 75% and 90% |
| SBW_x | Systolic Branch | Systolic branch width at x% of pulse height of PPG |
| | Width | |
| DBW_X | Diastolic Branch | Diastolic branch width at x% of pulse height of PPG |
| ·· A | Width | |
| BW_x | | Branch width at x% of the pulse height of PPG |
| D W X | Branch Width | BW _x =SBW _X +DBW _X |
| BWR_x | Branch Width | Branch width ratio at x% of the pulse height of PPG |
| $\mathbf{D} \mathbf{W} \mathbf{K}_{\lambda}$ | | |
| | I Ratio | $I RWR_{-} = DRW_{V}/SRW_{V}$ |
| | Ratio | $BWR_x = DBW_X/SBW_X$ O - Others |
| corr | | O - Others |
| corr | Autocorrelation | O - Others Autocorrelation |
| centr | Autocorrelation Centroid | O - Others Autocorrelation Centroid |
| centr min_p | Autocorrelation Centroid Minimum peaks | O - Others Autocorrelation Centroid Number of local minima of the signal |
| centr | Autocorrelation Centroid Minimum peaks Maximum peaks | O - Others Autocorrelation Centroid |
| centr min_p | Autocorrelation Centroid Minimum peaks Maximum peaks Mean absolute | O - Others Autocorrelation Centroid Number of local minima of the signal Number of local maxima of the signal |
| centr min_p max_p | Autocorrelation Centroid Minimum peaks Maximum peaks Mean absolute differences | O - Others Autocorrelation Centroid Number of local minima of the signal |
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| $coef_x$ | Polynomial | The coefficients of a 15th-order polynomial are employed to accurately |
|----------|------------------|--|
| | coefficient x | fit the PPG pulse waveform. |
| RI or AI | Reflexion index | $RI = I_{IP}/I_{S}$ |
| LASI | Large Artery | LASI = $h/\Delta T_{S-I}$, where H is the height of the subject (H \approx 1). |
| | Stiffness Index | |
| mNPV | Normalized Pulse | mNPV= I_{ac} / (I_{ac} + I_{dc}), where I_{ac} is the peak-to-peak amplitude and I_{dc} is |
| | Volume | the average of the pulse |
| PPGK | characteristic | $PPGK = (I_m - I_v)/(I_s - I_v)$ |
| | value or K value | respect to the baseline |

2) Frequency-based features (FT)

These features were derived from the beat-to-beat PPG pulse waveforms and PPG segments, which were constructed exclusively using validated PPG pulses (i.e., removing invalid pulses from the original 30-second segment during the signal processing). Features labeled with an asterisk (*) denote extraction from PPG segments, whereas those without an asterisk were derived from pulse waveforms.

| Ab | Code Name | Name | Definition |
|-------------------|----------------|--------------|--|
| | | Relative | Ratio between two frequency bands. The frequency |
| | | | bandwidth for the ratios' numerator is from 1 to 2.25 Hz |
| fsqi | fsqi | power | and ratios' denominator is from 0 to 8 Hz |
| | | Relative | Ratio between two frequency bands. The frequency |
| | | power at the | bandwidth for the ratios' numerator is from 0 to frequency |
| | | first | of first harmonic and ratios' denominator is the whole |
| fsqi1 | fsqi1 | harmonic | spectrum |
| | | Relative | Ratio between two frequency bands. The frequency |
| | | power at the | bandwidth for the ratios' numerator is from 0 to frequency |
| | | second | of second harmonic and ratios' denominator is the whole |
| fsqi2 | fsqi2 | harmonic | spectrum |
| | | Relative | Ratio between two frequency bands. The frequency |
| | | power at the | bandwidth for the ratios' numerator is from 0 to frequency |
| | | third | of third harmonic and ratios' denominator is the whole |
| fsqi3 | fsqi3 | harmonic | spectrum |
| f1 | | Frequency | |
| | | of the first | Predominant frequency of the signal |
| | f1 | harmonic | |
| mag_{f1} | | Magnitude | |
| | | of the first | |
| | mag_f1 | harmonic | Magnitude of the predominant frequency of the signal |
| f2 | | Frequency | |
| | | of the | Frequency of the second harmonic of the signal |
| | | second | Trequency of the second number of the signal |
| | f2 | harmonic | |
| mag_{f2} | | Magnitude | |
| | | of the | |
| | | second | |
| | mag_f2 | harmonic | Magnitude of the second harmonic of the signal |
| f3 | | Frequency | |
| | | of the third | Frequency of the third harmonic of the signal |
| | f3 | harmonic | |
| mag _{f3} | | Magnitude | |
| | | of the third | |
| | mag_f3 | harmonic | Magnitude of the third harmonic of the signal |
| Sp. distance | spectral_dista | Spectral | Distance of the signal's cumulative sum of the FFT |
| | nce | distance | elements to the respective linear regression. |
| Fundamental | fundamental_ | Fundamenta | Predominant frequency of the signal |
| f | frequency | 1 frequency | |
| Max PS | | Maximum | |
| | max_power_s | power | Maximum value of the power spectrum density |
| | pectrum | spectrum | |

| Max f | max_frequen | Maximum frequency | Maximum frequency of the signal. |
|------------------|------------------------|---|--|
| Med f | median_frequ | Median frequency | Median frequency of the signal. |
| Sp. centroid | spectral_centr oid | Spectral centroid | Barycenter of the spectrum |
| Sp. decrease | spectral_decr ease | Spectral decrease | Represents the amount of decreasing of the spectra amplitude. |
| Sp. K | spectral_kurt osis | Spectral kurtosis | Measures the flatness of a distribution around its mean value. |
| Sp. S | spectral_skew ness | Spectral skewness | Measures the asymmetry of a distribution around its mean value. |
| Sp. spread | spectral_spre ad | Spectral spread | Measures the spread of the spectrum around its mean value. |
| Sp. SLP | spectral_slop e | Spectral slope | Computes the spectral slope. |
| Sp. variation | spectral_varia | Spectral variation | Computes the amount of variation of the spectrum along time. Spectral variation is computed from the normalized cross-correlation between two consecutive amplitude spectra. |
| Sp. Max Peaks | spectral_max peaks | Spectral maxpeaks | Number of maximum spectral peaks of the signal. |
| Sp. Roll-off | spectral_roll_ | Spectral Roll-off | The spectral roll-off corresponds to the frequency where 95% of the signal magnitude is contained below of this value. |
| Sp. Roll-on | spectral_roll_ on | Spectral Roll-on | The spectral roll-on corresponds to the frequency where 5% of the signal magnitude is contained below of this value. |
| HRER | human_range _energy | Human range energy ratio | The human range energy ratio is given by the ratio between the energy in frequency 0.6-2.5Hz and the whole energy band. |
| PW | power_bandw | Power spectrum density bandwidth | It corresponds to the width of the frequency band in which 95% of its power is located. |
| SE | spectral_entro | Spectral entropy | Computes the spectral entropy of the signal based on Fourier transform. |
| WE | wavelet_entro | Wavelet entropy | Computes CWT Shannon entropy of the signal. |

3) Statistical features (SF)

| Ab. | Name/Definition |
|--------|---------------------------|
| s | Skewness |
| k | Kurtosis |
| mav | Mean Absolute Value |
| median | Median |
| mad | Mean Absolute Deviation |
| med_ad | Median Absolute Deviation |
| rms | Root-Mean-Square |
| sd | Standard Deviation |
| sf | Shape Factor |
| if | Impulse Factor |
| cf | Crest Factor |

| v | Variance |
|-----|---------------------|
| irq | Interquartile range |
| p | Perfusion |

4) Demographic features (DF)

| Abbreviation | Name/Definition |
|--------------|----------------------------------|
| age | Age |
| weight | Weight |
| bmi | Body Mass Index |
| baseline_sbp | Systolic Blood Pressure at rest |
| baseline_dbp | Diastolic Blood pressure at rest |