

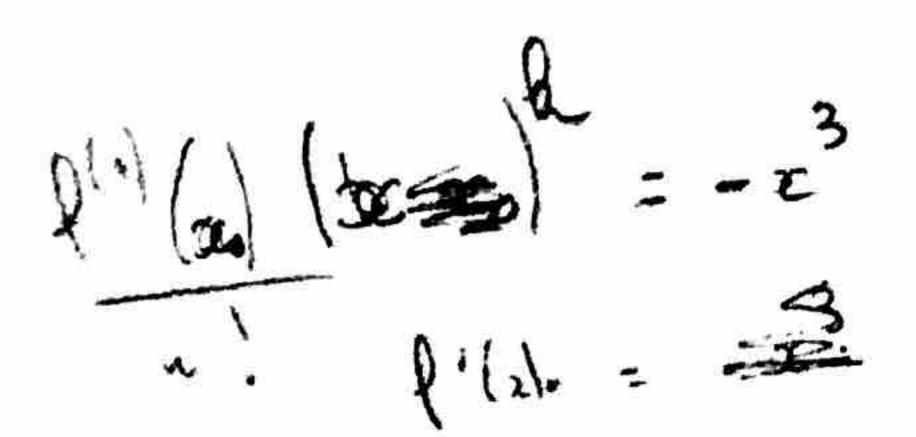
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Exercise 1. Let f be a function 3 times differentiable at 0 such that

$$f(x) = 1 - x + 5x^2 - x^3 + o(x^3).$$

Determine the value of  $f^{(k)}(0)$ , for  $k \in \{0, 1, 2, 3\}$ :





$$f(0) = \mathbf{1}$$

$$f''(0) = \frac{5}{4} \quad 5 \times 2$$

$$f'(0) = \mathcal{F} - \Lambda$$

$$f'''(0) = \mathcal{J}_{X} - \mathcal{J}_{X} \mathcal{E}$$

Exercise 2. For which value of  $\alpha \in \mathbb{R}$  is the following equality true?

$$\int_0^1 \frac{1}{1+x^3} \, \mathrm{d}x = \lim_{n \to +\infty} n^{\alpha} \sum_{k=1}^n \frac{1}{n^3+k^3}.$$

No justifications required.

2

$$\alpha =$$

Exercise 3. Fill in the blanks with the Taylor-Young expansions at the specified order:



$$\frac{\ln(1+X)}{X} = \frac{1}{X} - \frac{2}{1} + \frac{2}{1} +$$

$$\frac{\ln(1+\sinh(x))}{\sinh(x)} \underset{x\to 0}{=} \text{ Ginklet Feat } 1 - \frac{\sinh(x)}{2} + \frac{\sinh(x)^2}{3} - \frac{\sinh(x)^3}{4} + \frac{\sinh(x)^4}{5}$$

Exercise 4. Let  $f:[0,1]\to\mathbb{R}$  be a continuous function and let  $T=((x_0,x_1,x_2,x_3),(t_1,t_2,t_3))$  be the tagged subdivision of [0,1] where

$$x_0 = 0$$

$$x_1 = 0.3$$

$$x_2 = 0.8$$

$$x_3 = 1$$

$$t_1 = 0.2$$

$$t_2 = 0.65$$

$$t_3 = 0.9$$

Write the Riemann sum R(f,T) of f associated to the tagged subdivision T:

 $P(f,T) = \{0,66-0,2\} \times \{0,27-24\} + \frac{1}{3}\{1-0,8\} = \{0,2,20,3\} + \{0,65\} \times \{0,65\} \times \{0,9\} \times \{1-0,8\} = \{0,2\} \times \{0,65\} \times \{0,65\} \times \{0,9\} \times \{0,9\} \times \{1-0,8\} = \{0,2\} \times \{0,65\} \times \{0,65\} \times \{0,9\} \times \{0$ 

$$= \sum_{k=0}^{\infty} f(k) \times (x_k - x_{k-1})$$