SCAN 1st

Mechanics test 2

Wednesday 12th May 2021 – Duration: 1h30

2-page personal formula sheet authorised.

Exercise 1:

The satellite P in Figure 1 has a circular orbit of radius R (measured from the centre of the Earth). The masses of the satellite and the Earth are m_s and M_E respectively, the radius of the Earth is r_E and the universal constant of attraction if G.

Using Newton's 2nd law,

1 – Prove that the speed of the satellite is constant

2 – Determine the time for the satellite to complete one orbit around the Earth

3 – Deduce from the previous question, the altitude h_{geo} measured from the surface of the Earth for a geostationary orbit (i.e., the satellite is fixed with respect to the Earth surface). Give the numerical value in *km* knowing that $r_E = 6371 km$ and $g = 9.81 m/s^2$

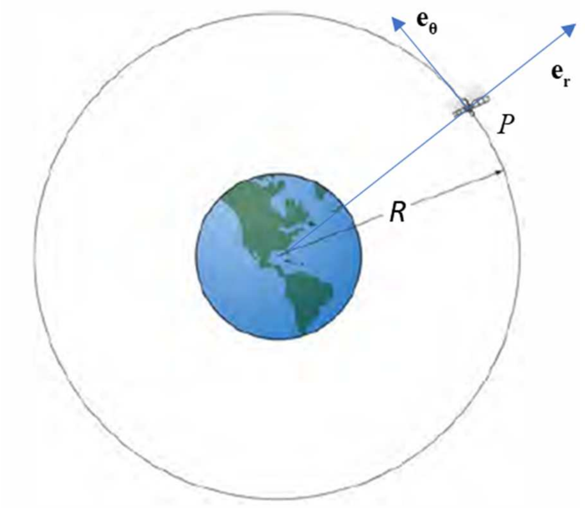

Figure 1.

Exercise 2:

Mass m slides with negligible friction on the cylindrical surface (Figure 2). The spring attached to the mass has a stiffness of k, and its free (unstretched) length is L_0.

1 - If the mass is released from rest at A, determine its speed at B in terms of the problem variables. *Hint: Use energies.*

2 - Numerical application: calculate the speed using the following data
$m = 0.6\,\text{kg}$, $k = 110\,\text{N/m}$, $L_0 = 0.08\,\text{m}$, $R = 0.4\,\text{m}$, $r = 0.2\,\text{m}$, α=30°

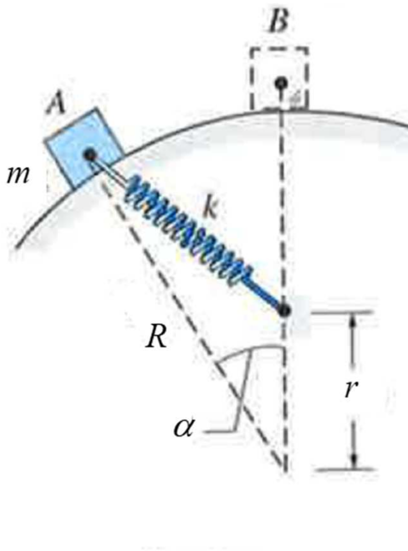

Figure 2.

Exercise 3:

Determine the horizontal force P required to keep the homogeneous 30-kg cylinder in equilibrium on the rough inclined surface in Figure 4.

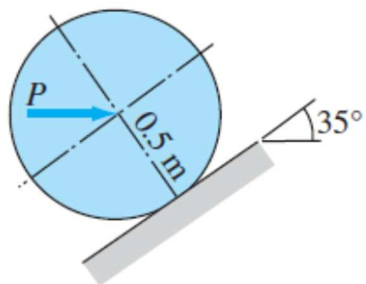

Figure 3

Exercise 4:

Three identical springs, each of stiffness k , support a block of mass m as shown in Figure 4. Bar AB is rigid.

1 Prove that the three stiffness elements can be replaced by one equivalent stiffness $k_{eq} = \dfrac{2}{3}k$

2 Find the expression of the static displacement, the circular natural frequency, the natural frequency and period of the system.

3 Numerical application: m =0.2 kg; k =875 N/m

4 A force $F = F_0 \sin(\omega t)$ is applied to mass m. Find the maximum displacement downwards in terms of m, k, F_0, ω and g.

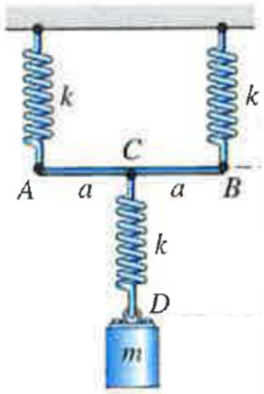

Figure 4.