Physics Test 2 SCAN 1

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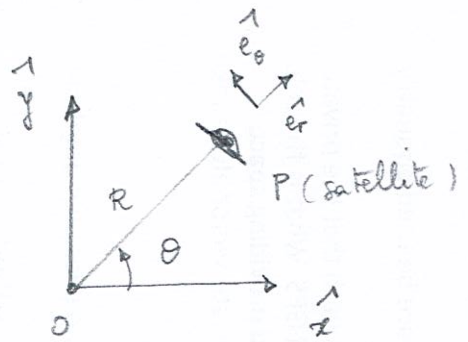
Exercise 1



1°) Newton's 2ND law :

$$m_S (R\ddot{\theta}\,\hat{e}_\theta - R\dot{\theta}^2\,\hat{e}_r) = - G\, m_S \frac{M_e}{R^2}\,\hat{e}_r$$

Projections

$/\hat{e}_\theta$: $\ddot{\theta} = 0 \mapsto \dot{\theta} = \omega = \text{constant}$

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then $\vec{V(P)} = R\dot{\theta}\,\hat{e}_r$ is constant

$/\hat{e}_r$: $R\dot{\theta}^2 = G\frac{M_e}{R^2}$

$\rightarrow \omega = \sqrt{\frac{GM_e}{R^3}}$

2°) The time for one revolution is

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$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM_e}}$$

3°) Geostationary orbit $T = 24 \times 60 \times 60$ s (one day)

a) expression of GM_e using the particular case of the weight

$$\frac{GM_e\, m}{r_E^2} = mg \mapsto GM_e = g\, r_E^2$$

b) $T = 2\pi\sqrt{\frac{R^3}{g\, r_E^2}} = 2\pi\sqrt{\frac{(r_E + h)^3}{g\, r_E^2}}$

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$\rightarrow h + r_E = \sqrt[3]{\frac{T^2}{4\pi^2}\, g\, r_E^2}$

$h = 35\,855$ km

Exercise 2

Conservation of total energy:

	initial at A	final at B
T	0	$\frac{1}{2} m v_B^2$
U	$U_G = 0$ $U_S = \frac{1}{2} k (OA - L_0)^2$	$U_G = mg(\vec{AB} \cdot \hat{y})$ $U_S = \frac{1}{2} k (OB - L_0)^2$

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$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= (R-r)\hat{y} - \{\vec{OC} + \vec{CA}\}$$

$$= (R-r)\hat{y} - \{-r\hat{y} + R\hat{\eta}\}$$

$$R(-\sin\alpha\, \hat{x} + \cos\alpha\, \hat{y})$$

$$= \begin{bmatrix} R\sin\alpha \\ R - R\cos\alpha \end{bmatrix}$$

$$OA^2 = (-r\hat{y} + R\hat{\eta})^2 = r^2 + R^2 - 2rR\cos\alpha$$

$$OB = R - r$$

$$\frac{1}{2} k \left(\overbrace{\sqrt{R^2 + r^2 - 2rR\cos\alpha}}^{OA} - L_0 \right)^2 =$$

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$$\frac{1}{2} m v_B^2 + mgR(1-\cos\alpha)$$

$$+ \frac{1}{2} k (R - r - L_0)^2$$

$OA = 0.2479$ m

$OB = 0.2$ m

$$1.55 = 0.3 v_B^2 + 0.3154 + 0.792$$

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$$v_B = 1.21 \text{ m/s}$$

Exercise 3

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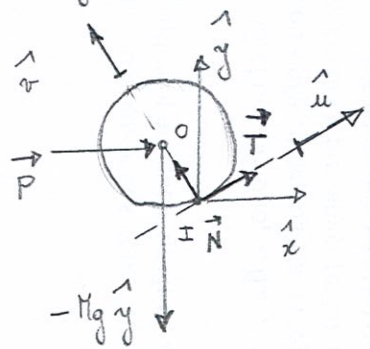
Free-body diagram



External forces

- $P\,\hat{x}$
- $-Mg\,\hat{y}$
- $N\,\hat{v}$
- $T\,\hat{u}$ $(\pm)$ friction

(NB $\vec{M}_{EXT}(o) = \vec{0} \rightarrow T = 0$!)

1 $\vec{M}_{EXT}(I) = \vec{0}$ $\quad \vec{I_o} \times (P\,\hat{x} - Mg\,\hat{y}) = \vec{0}$

$Mg\,R \sin 35^\circ - P \cos 35^\circ = 0$

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$P = Mg \tan 35^\circ = 206.1\,N$

The same result can be obtained by $\Sigma \vec{F}_{EXT} = \vec{0}$

$(T = 0)$

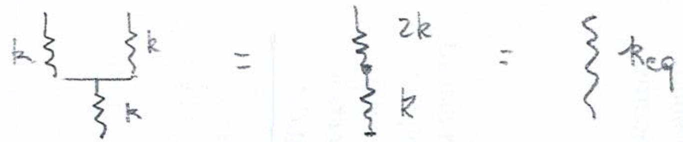
$/\hat{x}$: $P - N \sin 35^\circ = 0$

$/\hat{y}$: $-Mg + N \cos 35^\circ = 0$

etc.

<u>Exercise 4 :</u>

1°)



(1) $\frac{1}{k_{eq}} = \frac{1}{2k} + \frac{1}{k} \rightarrow k_{eq} = \frac{2}{3} k$

2°) + 3°)

(1) $\delta_{st} = \frac{mg}{k_{eq}} = \frac{3\,mg}{2k} = 3.36 \text{ mm}$

(1) $\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{2k}{3m}} = 54 \text{ rad/s}$

(1) $f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}} = 8.59 \text{ Hz}$

$T_n = \frac{1}{f_n} = 0.116 \text{ s}$

4°) Initial conditions

$x = 0 \qquad \dot{x} = 0 \quad \rightarrow$ no transient

$$\delta_{MAX} = X + \delta_{st} = \frac{F_0/k}{1-(\omega/\omega_n)^2} + \underbrace{\frac{3g}{2\omega_n^2}}_{\delta_{st}}$$

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