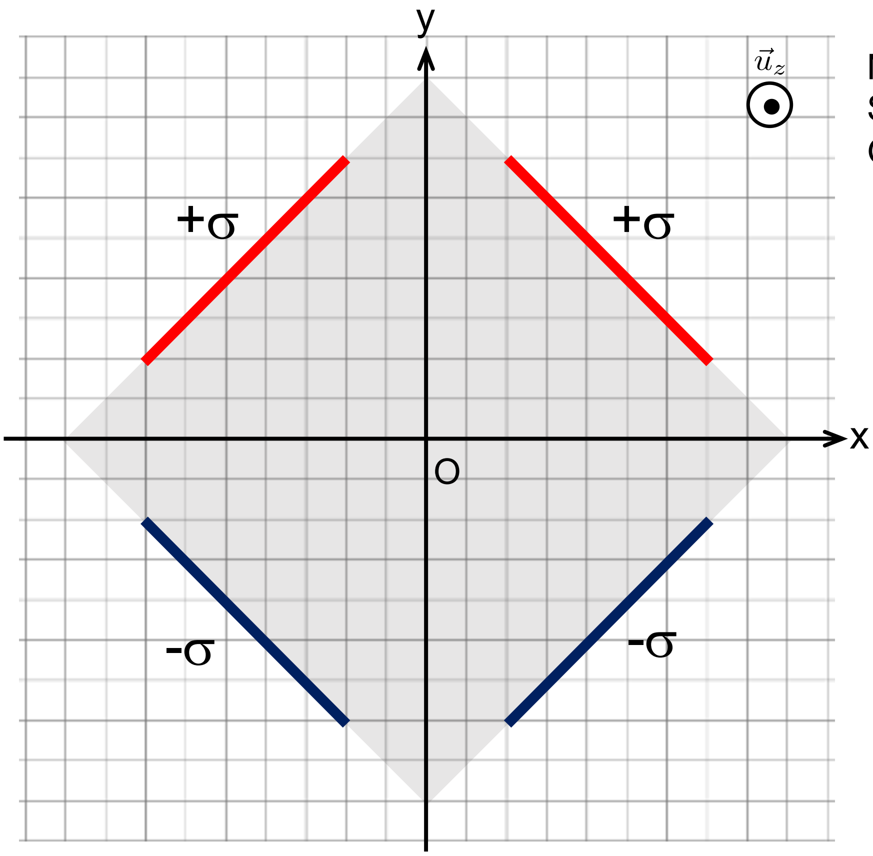
Physics test - Electromagnetism
June 14th 2021
Duration : 1h30
non programmable calculator allowed
no document allowed

The marks will account not only for the results, but also for the justifications, and the way you analyze the results. Moreover, any result must be given in its literal form involving only the data given in the text. It is also reminded that the general clarity and cleanness of your paper may also be taken into account.

Exercise 1: Maps of electric field (~ 6 pts)

Four metallic plates (see Figure 1), parallel to (Oz) are carrying a uniform charge distribution $\pm\sigma$ ($\sigma > 0$). The plates are supposed to be infinitely long along (Oz). Through a detailed analysis of the charge distribution, deduce and draw the field lines within the gray squared region of Figure 1 in the (xOy) plane. Comment on the intensity of the electric field at O.

Put your name on Figure 1 an return it back with your paper.



Name: ____________
Surname: ____________
Group: ________

Figure 1

Exercise 2: On magnetic fluxes (~ 6 pts)

As depicted in Figure 2(a), an infinite solenoid along (Oz) (circular cross-section, diameter $d_1 = 10$ cm) crosses the plane of a circuit belonging to the plane (xOy). The circuit loop holds two resistances ($4\ \Omega$ and $8\ \Omega$). Over its cross-section (represented as a dashed area), the solenoid delivers a uniform magnetic field $\overrightarrow{B_1}(t)$ of increasing magnitude over time (t): $\overrightarrow{B_1}(t) = A\,t\,\vec{u}_z$ with $A = 50$ T/s. Given the solenoid is infinite, one can neglect the magnetic field outside of the dashed area.

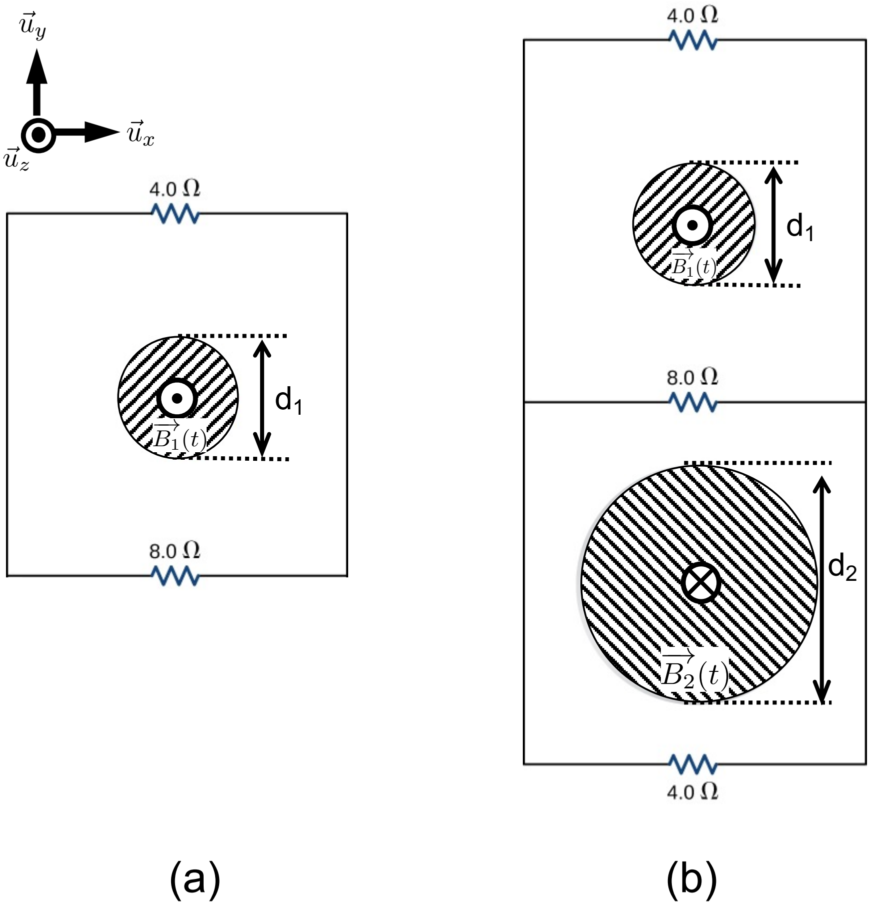


Figure 2

1. Determine the current intensity i_1 running in the loop, justifying precisely its orientation.

A second loop, still in the (xOy) plane, is connected below the first one (see Figure 2(b)) including a $4\ \Omega$ resistance. The second loop is crossed by an infinite circular solenoid of diameter $d_2 = 20$ cm. The second solenoid also delivers a uniform magnetic field $\overrightarrow{B_2}(t)$ over its cross-section: $\overrightarrow{B_2}(t) = -A\,t\,\vec{u}_z$.

2. Determine the different current intensities running through the resistances, justifiying their orientation.

Exercise 3: Barlow's wheel ($\sim$ 12 pts)

Consider a wheel placed in a vertical plane, entirely made of metal, of center O, radius a, and comprising $N = 4$ spokes. The wheel can freely rotate about a pivot axis along (Oz) without friction. The wheel's moment of inertia about the (Oz) axis is denoted J. In this exercise, the resistance of each spokes (OA, OB, OC and OD) is denoted R_0. The resistance of the contour of the wheel and of the pivot axis is supposed to be nil. The self-inductance and any friction of the wheel will be neglected.

The wheel being initially at rest is then totally immersed in a uniform magnetic field $\vec{B} = B_0\,\vec{u}_z$ ($B_0 > 0$) and connected to a real voltage source (e.m.f. E and resistance R) between a perfect sliding contact G and its center O (see Figure 3). A switch K can be used to open or close the circuit.

Throughout this exercise, we will be using the local cylindrical frame with respect to the center of wheel ($\vec{u_r}$, $\vec{u_\theta}$, $\vec{u_z}$) represented at an abritrary point M on Fig. 3.

At time $t = 0$, K is closed: a current of intensity $i(t)$ then flows in the circuit. We observe that the wheel starts to rotate. It's angular velocity increases and then saturates.

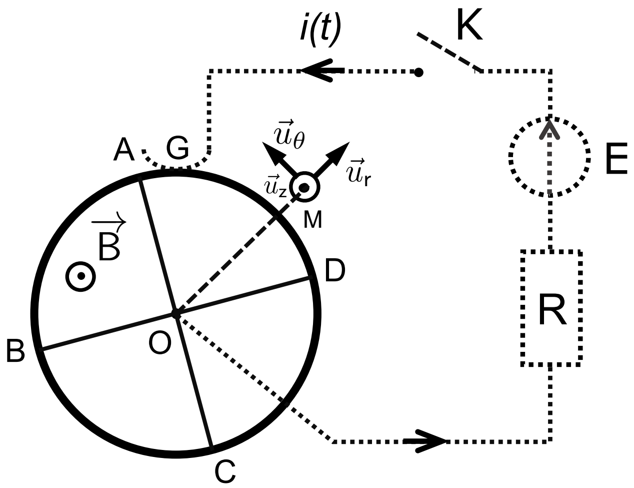


Figure 3: Barlow's wheel

Qualitative interpretation of the physical phenomena

1. Within few sentences, could you explain the observed phenomena? For instance you might identify the different successive physical phenomenon occuring on the wheel.

Mechanical equation

One aims to establish the expression of $\omega(t)$ the angular velocity of the wheel. Let's assume that the current is equally distributed and hence that each spoke carries a current of intensity $\frac{i}{N}$. This assumption could be justified by symmetry considerations, knowing that the resistance of the pivot axis and of the wheel's contour are nil.

2. Let's study a small element of a spoke (*e.g.* OA), of length dr, located at a distance r from the O. Calculate $\overrightarrow{dF}$, the Laplace force exerted on this element in terms of $i(t)$, N, dr, B_0 and one of the unit vector of the local frame.

3. Let us find the moment exerted by Laplace for on this spoke.

   a) Calculate the elementary moment of $\overrightarrow{dF}$ about (Oz) , namely $d\Gamma_z$, in terms of $i(t)$, N, r, B_0 and dr.

   b) Calculate the moment Γ_z due to Laplace force exerted on the whole spoke.

4. Deduce from the previous questions the *total moment* $\Gamma_z^{\texttt{tot}}$ about (Oz) axis due to the Laplace force of the N spokes. Show that $\Gamma_z^{\texttt{tot}} = K \cdot i$ where K is a constant to be determined.

5. Establish (but do not solve) the differential equation ruling $\omega(t)$.

Electrical equation

6. Which type of induction phenomenon are we facing? Find the expression of e, the induced electromotive force in each spoke, by computing the circulation of a vector (*i.e.* doing a line integral). Show that e can be written $e = \pm K \cdot \omega$, the $\pm$ sign depending on your orientation choice you will have to detail.

7. Draw the equivalent electrical scheme of the circuit, accounting for all resistances in the circuit and the induced electromotive forces. Deduce the electrical equation associated to this circuit in terms of $i(t)$, E, e, R and R_0. Finally, infer the expression of $i(t)$ in terms of E, R, R_0, K and $\omega(t)$. Provide then a new expression of for the differential equation ruling $\omega(t)$.

Solving the coupled equations

Solving the mechanical equation would lead to the following expression for the angular velocity:

$$\omega(t) = \omega_0 \,(1 - e^{-\frac{t}{\tau}})$$

8. By exploiting the previous question, find the expression of ω_0, the angular velocity in steady-state in terms of E and K.
   Bonus: Determine τ from Q7 (you may for instance use a dimensional analysis or exploit the expression provided for $\omega(t)$...). What is τ worth as $R_0 \ll R$?

9. Show that $i(t)$ can be expressed as $i(t) = i_0 \, e^{-t/\tau'}$ and determine i_0 and τ' in terms of E, R and K and J; note that we will assume that $R_0 \ll R$. Plot the qualitative graph of $i(t)$. Comment on the value of the intensity once the steady state has been reached.