System Mechanics
Final test of semester 1

Wednesday 1st February 2023 – 2h (10:00 – 12:00)

Study of a mechanism for depositing coatings

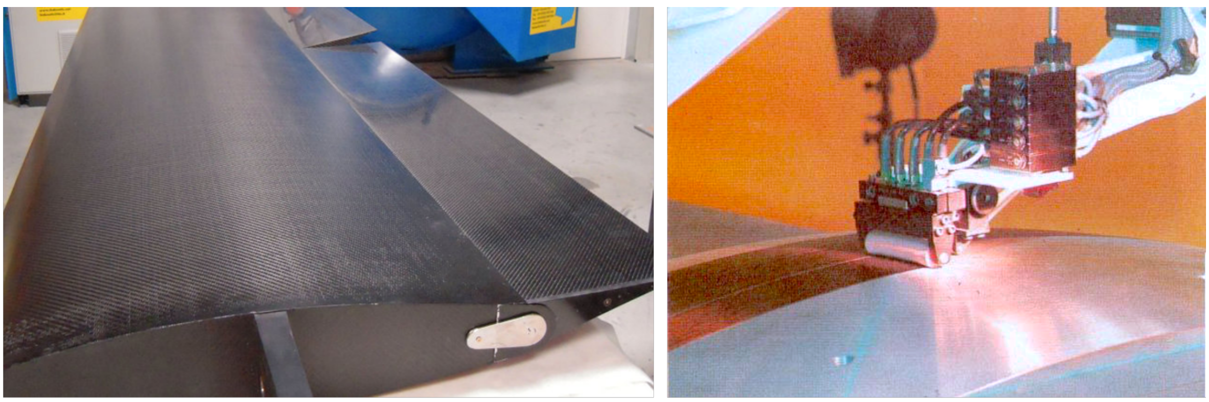
Indicative marking scheme : part 1 (/13), part 2 (/7)

Authorised documents : 2 pages A4 (1 sheet)
Table of the classic joints (without their constraints)
A calculator

System overview

The system under consideration is a mechanism for depositing coatings (varnish, paint, etc.) on the surface of curved structures such as airplane wings (figure 1a). In order to achieve this objective, a roller is pressed against the curved structure on which, it rolls without slipping (figure 1b). To guarantee the homogeneity of the deposited coating, it is necessary that the roller (not motorized and rotating freely) keeps a rotational speed as constant as possible. The mechanism is driven by a single linear actuator, which makes it possible to follow the curvature of the structure surface.

The challenge for the engineers who created this mechanism is to optimally control the cylinder motion in order to ensure the homogeneity of the coating, hence a rotational speed as constant as possible.



(a) (b)

Figure 1. (a) Airplane wind in carbon fibber, and (b) mechanism head (roller) depositing varnish on the wing surface

Parameters and coordinate systems

The present analysis relies on a **simplified planar system** whose kinematical model is shown in **Figure 2**. In this study, the **curved structure Σ** , on which coating is deposited, is a part of a **circle of centre I and radius R**.

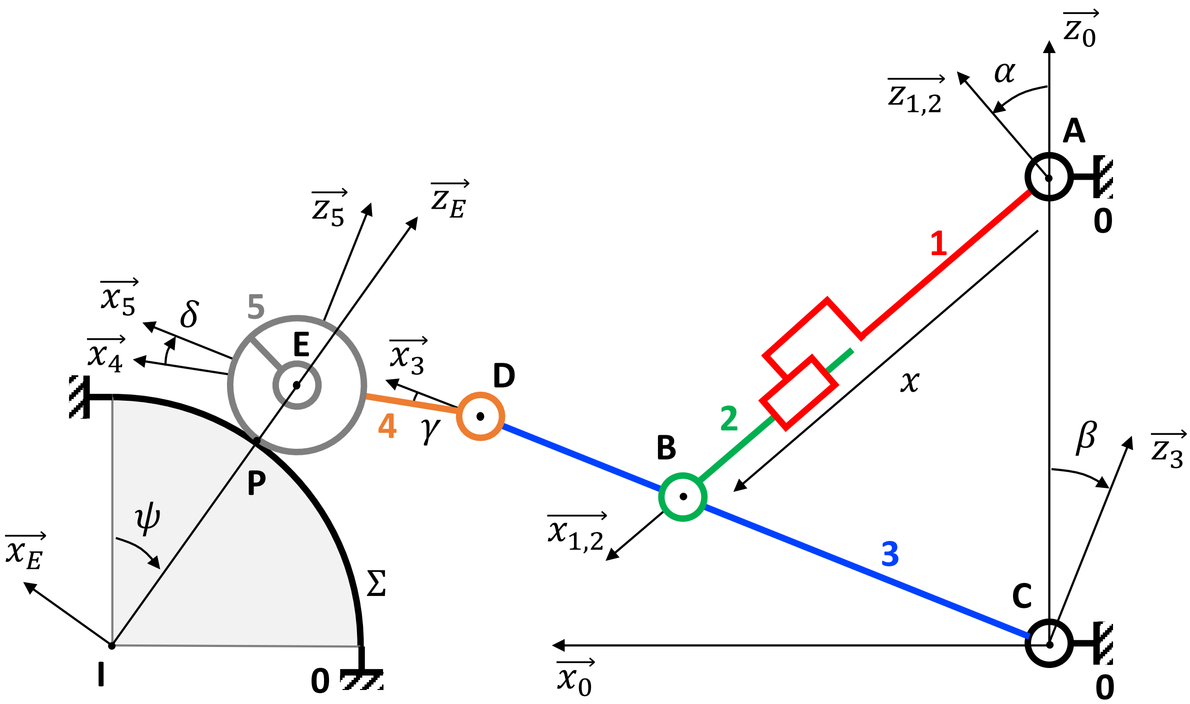
The model is made of:

- group $\boldsymbol{S_0}$, comprising the curved structure **Σ** and the mechanism casing attached to the ground, whose coordinate system is $R_0 = (C, \overrightarrow{x_0}, \overrightarrow{y_0}, \overrightarrow{z_0})$
- the jack body $\boldsymbol{S_1}$, of coordinate system $R_1 = (A, \overrightarrow{x_1}, \overrightarrow{y_1}, \overrightarrow{z_1})$, connected to the ground $\boldsymbol{S_0}$ via a revolute joint of axis $(A, \overrightarrow{y_{0,1}})$ and motion parameter 1/0 :
$$\boldsymbol{\alpha} = (\overrightarrow{x_0}, \overrightarrow{x_1}) = (\overrightarrow{z_0}, \overrightarrow{z_1})$$
- the jack piston $\boldsymbol{S_2}$, of coordinate system $R_2 = (B, \overrightarrow{x_2}, \overrightarrow{y_2}, \overrightarrow{z_2})$, connected to the jack body $\boldsymbol{S_1}$ via a prismatic joint of axis $(B, \overrightarrow{x_{1,2}})$ and motion parameter 2/1 :
$$\boldsymbol{x} = \overrightarrow{AB}.\overrightarrow{x_{1,2}}$$
- the bigger arm $\boldsymbol{S_3}$, of coordinate system $R_3 = (B, \overrightarrow{x_3}, \overrightarrow{y_3}, \overrightarrow{z_3})$, connected to :
  - group $\boldsymbol{S_0}$ via a revolute joint of axis $(C, \overrightarrow{y_{0,3}})$, and motion parameter 3/0 :
$$\boldsymbol{\beta} = (\overrightarrow{x_0}, \overrightarrow{x_3}) = (\overrightarrow{z_0}, \overrightarrow{z_3})$$
  - and, the jack piston $\boldsymbol{S_2}$ via a revolute joint of axis$(B, \overrightarrow{y_3})$ **with no parameter**
- the smaller arm $\boldsymbol{S_4}$, of coordinate system $R_4 = (D, \overrightarrow{x_4}, \overrightarrow{y_4}, \overrightarrow{z_4})$, connected to the bigger arm $\boldsymbol{S_3}$ via a revolute joint of axis $(D, \overrightarrow{y_{3,4}})$ and motion parameter 4/3 :
$$\boldsymbol{\gamma} = (\overrightarrow{x_3}, \overrightarrow{x_4}) = (\overrightarrow{z_3}, \overrightarrow{z_4})$$
- roller $\boldsymbol{S_5}$, of coordinate system $R_5 = (E, \overrightarrow{x_5}, \overrightarrow{y_5}, \overrightarrow{z_5})$, connected to:
  - the smaller arm $\boldsymbol{S_4}$ via a revolute joint of axis $(E, \overrightarrow{y_{4,5}})$ and motion parameter 5/4 :
$$\boldsymbol{\delta} = (\overrightarrow{x_4}, \overrightarrow{x_5}) = (\overrightarrow{z_4}, \overrightarrow{z_5})$$
  - and, group $\boldsymbol{S_0}$ by a condition of contact at point P **with no parameter** and associated with **a rolling with no-slipping condition**

Moreover, the position of the centre E of roller $\boldsymbol{S_5}$ is defined by using the coordinate system $R_E = (I, \overrightarrow{x_E}, \overrightarrow{y_E}, \overrightarrow{z_E})$ such that vector $\overrightarrow{IE}$ is in the direction of $\overrightarrow{z_E}$. The motion parameter for the motion of **point E with respect to** $\boldsymbol{S_0}$ is :
$$\boldsymbol{\psi} = (\overrightarrow{x_0}, \overrightarrow{x_E}) = (\overrightarrow{z_0}, \overrightarrow{z_E})$$

CAUTION : Although parameter ψ is not associated with any joint, it will be accounted for in the calculation of the degree of mobility of the mechanism. Similarly, even if it is not does not fully describe motion 5/0, it will be represented in the graph of links by a dotted line between **these two solids.**

Remark : The actual mechanism comprises a torsional spring about axis $(D, \overrightarrow{y_3})$ positioned at point D between solids $\boldsymbol{S_3}$ and $\boldsymbol{S_4}$, which imposes permanent contact at point P during the system motion. This spring, not represented here, is not considered in the purely kinematical study in this paper.



Geometrical data :

$\overrightarrow{CA} = a\overrightarrow{z_0}$, $\overrightarrow{BD} = b\overrightarrow{x_3}$, $\overrightarrow{CB} = c\overrightarrow{x_3}$, $\overrightarrow{DE} = e\overrightarrow{x_4}$, $\overrightarrow{PE} = r\overrightarrow{z_E}$, $\overrightarrow{IP} = R\overrightarrow{z_E}$, $\overrightarrow{CI} = L\overrightarrow{x_0}$

Lengths a, b, c, e, r, R *and* L are constant.

Figure 2. Kinematical model and geometrical data

Part 1 : Analytical kinematics

Question 1.1 Draw the graph of links and the change of basis diagrams.

Question 1.2 Specify the trajectory of point E with respect to R_0.

Question 1.3 Determine $\overrightarrow{V(E/0)}$ exclusively in terms of R, r and $\dot{\psi} = \dfrac{d\psi}{dt}$.

Question 1.4 Write and <u>then develop</u> the no-slipping condition at point P using the previous results and introducing parameter $\dot{\psi} = \dfrac{d\psi}{dt}$.

Question 1.5 Write and <u>then develop</u> the equation associated with the contact condition at point P. The result must be expressed in R_0.

Question 1.6 Write and <u>then develop</u> the constraint equation corresponding to the closure condition at point B. The result must be expressed in R_0.

Question 1.7 Give the degree of mobility of the mechanism. Explanations are required. Is it possible to operate the system by controlling a single parameter ? If so, which parameter would you suggest ?

Question 1.8 Express the kinematic screw for motion 2/0 at point B in terms of x, α and their first order time-derivatives only.

Question 1.9 Justify the fact that motion 2/0 is tangent to a rotation.

Question 1.10 Denoting $\overrightarrow{\mathrm{BI}_{20}} = \mathrm{X_I}\overrightarrow{\mathrm{x_1}} + \mathrm{Z_I}\overrightarrow{\mathrm{z_1}}$, express the coordinates $\mathrm{X_I}$ and $\mathrm{Y_I}$ of the instant centre of rotation I_{20} in terms of x, α and their first order time-derivatives.

Question 1.11 Express the kinematic screw for motion 5/0 at point E (moment at E) in terms of $\dot{x}$ and $\dot{\alpha}$ amongst other things

Question 1.12 Determine acceleration $\overrightarrow{\Gamma(E/0)}$ in terms of $\dot{x}$ and $\dot{\alpha}$ along with their time-derivatives amongst other things

Question 1.13 Considering the system operating principles, give the condition on $\overrightarrow{\Gamma(E/0)}$ in order to deposit a homogeneous layer. Do not develop this condition.

Part 2 : Graphical kinematics

The objective is to determine the velocity vector $\overrightarrow{V(E/0)}$ from that of the jack $\overrightarrow{V(B,2/1)}$. Each and every step in the graphical construction, which should be carried out on page 5, must be justified by a text explaining the reasoning.

Question 2.1 Specify the line support of $\overrightarrow{V(E/0)}$.

Question 2.2 By graphical construction, determine $\overrightarrow{V(B/0)}$. Justify.

Question 2.3 By graphical construction, determine $\overrightarrow{V(D/0)}$. Justify

Question 2.4 By graphical construction, determine $\overrightarrow{V(E/0)}$. Justify

Question 2.5 By graphical construction, determine the position of the instant centre of rotation for the motion of $\mathbf{S_4}$ with respect to $\mathbf{S_0}$.

To be handed in with your paper - Name: First name: Group: