TOTAL /22

6 Exercise 1 :

$$\begin{cases} a_x = 0 \\ a_y = -g \end{cases} \qquad \begin{cases} v_x = V_0 \cos\alpha \\ v_y = -gt + V_0 \sin\alpha \end{cases} \qquad \begin{cases} x = V_0 \cos\alpha \; t \\ y = -g\frac{t^2}{2} + V_0 \sin\alpha \, t \end{cases}$$ 1

we want to find the time t^* for which

$$\vec{V}_0 = \begin{bmatrix} V_0 \cos\alpha \\ -\cancel{X} + V_0 \sin\alpha \end{bmatrix} \; ; \; \vec{V}_* = \begin{bmatrix} V_0 \cos\alpha \\ -gt^* + V_0 \sin\alpha \end{bmatrix}$$ are perpendicu-lar

$$\vec{V}_0 \cdot \vec{V}_* = 0$$

$$V_0^2 \cos^2\alpha + V_0^2 \sin^2\alpha - g t^* V_0 \sin\alpha = 0$$

$$\boxed{t^* = \frac{V_0}{g \sin\alpha}}$$ 2

the corresponding positions are :

$$\boxed{\begin{aligned} x^* &= \frac{V_0^2}{g \tan\alpha} \\ y^* &= -\frac{g}{2}\frac{V_0^2}{g^2 \sin^2\alpha} + \frac{V_0^2}{g} = \frac{V_0^2}{g}\left(1 - \frac{1}{2\sin^2\alpha}\right) \end{aligned}}$$ 2

/8 Exercise 2 :

$$r = b \qquad \dot{\theta} = kt$$
$$\dot{r} = 0 \qquad z = \frac{L}{2\pi}\theta + z_0$$
$$\ddot{r} = 0$$

$$\dot{z} = \frac{L}{2\pi}\dot{\theta} = \frac{kL}{2\pi}t$$

$$\ddot{\theta} = k$$

$$\ddot{z} = \frac{kL}{2\pi}$$

1,5

1- $\vec{V}(A) = \dot{b}\,\hat{e}_r + b\dot{\theta}\,\hat{e}_\theta + \dot{z}\,\hat{e}_z = bkt\,\hat{e}_\theta + \frac{kL}{2\pi}t\,\hat{e}_z$

$$\boxed{\vec{V}(A) = kt\left[b\hat{e}_\theta + \frac{L}{2\pi}\hat{e}_z\right]}$$

$$\vec{A}(A) = (\ddot{b} - b\dot{\theta}^2)\hat{e}_r + (b\ddot{\theta} + 2\dot{b}\dot{\theta})\hat{e}_\theta + \ddot{z}\hat{e}_z$$

1,5 $$\boxed{\vec{A}(A) = -bk^2t^2\,\hat{e}_r + bk\,\hat{e}_\theta + \frac{kL}{2\pi}\hat{e}_z}$$

2- $$\|\vec{V}(A)\| = kt\sqrt{b^2 + \frac{L^2}{4\pi^2}}$$

2,5 $$\boxed{\hat{e}_t = \frac{\vec{V}(A)}{\|\vec{V}(A)\|} = \frac{1}{\sqrt{b^2 + \frac{L^2}{4\pi^2}}}\left[b\hat{e}_\theta + \frac{L}{2\pi}\hat{e}_z\right]}$$ constant

helix

3- $\vec{A}(A) = \dot{v}\,\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$

$$= k\cancel{t}\,\cancel{\sqrt{b^2+\frac{L^2}{4\pi^2}}}\left[b\,\hat{e}_\theta + \frac{L}{2\pi}\hat{e}_z\right] + \frac{k^2t^2\left(b^2+\frac{L^2}{4\pi^2}\right)}{\rho}\hat{e}_n$$

identify

$$= -bk^2t^2\,\hat{e}_r + bk\,\hat{e}_\theta + \frac{kL}{2\pi}\hat{e}_z$$

$\hat{e}_n = -\hat{e}_r$

$$+b\cancel{k^2}\cancel{t^2} = \frac{\cancel{k^2}\cancel{t^2}(b^2+L^2/4\pi^2)}{\rho}$$

$$\boxed{\rho = b\left(1+\frac{L^2}{4\pi^2b^2}\right) \quad \hat{e}_n = -\hat{e}_r}$$

2,5

/8 **Exercise 3 :**

1- $-kv^2ds = v\,dv$

$-kv\,ds = dv$

$$k\int_0^D ds = -\int_{v_0}^{v_0/2}\frac{1}{v}dv$$

3

$$kD = -\left[\ln v\right]_{v_0}^{v_0/2} = -\ln\frac{1}{2} = \ln 2$$

$$\boxed{D = \frac{1}{k}\ln 2}$$

2- $\frac{dv}{dt} = -kv^2 \longmapsto -k\,dt = \frac{1}{v^2}dv$

$$-k\int_0^T dt = \int_{v_0}^{v_0/2}\frac{1}{v^2}dv \qquad \boxed{T = \frac{1}{kv_0}} \qquad 3$$

$$-kT = -\left[\frac{1}{v}\right]_{v_0}^{v_0/2} \qquad T = \frac{1}{k}\left(\frac{2}{v_0}-\frac{1}{v_0}\right)$$

$$k \int_0^{D_\infty} ds = - \int_{v_0}^{0} \frac{1}{v}\, dv$$

not defined

(2) $$k D_\infty = - \left[\ln v \right]_{v_0}^{0} \Rightarrow k D_\infty = \ln v_0 - \ln(0)$$

as speed is reduced the influence of gravity and buoyancy need to taken → the trajectory is certainly not straigth anylonger. complex...