Year: 2016-2017

Mid-term Test - PHYSICS

Friday, January 13th 2017 | Duration: 2 h 00

- No documents allowed. No mobile phone allowed. The use of not-programmable calculator is allowed.

- The marks will account for the justifications, the critical analysis of your results, as well as for the writing and the general clarity and cleanness of your documents.

- The conventions and the physical quantities of interest will always have to be defined and specified on your schemes.

TWO SEPARATED DOCUMENTS ARE REQUIRED FOR THE OPTICS AND THE ELECTRODYNAMICS ASSIGNEMENTS.

I. OPTICAL GEOMETRY: ABBE REFRACTOMETER (≈ /8 PTS)

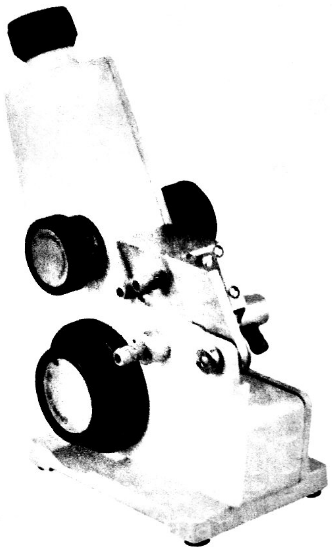
A refractometer is an optical instrument used to measure substances' refraction index. Liquids are most commonly characterized. For instance, refractometry can be used to measure the level of sucrose in fruit juices. They can also be used to measure gases and solids, such gas glass and gemstones.

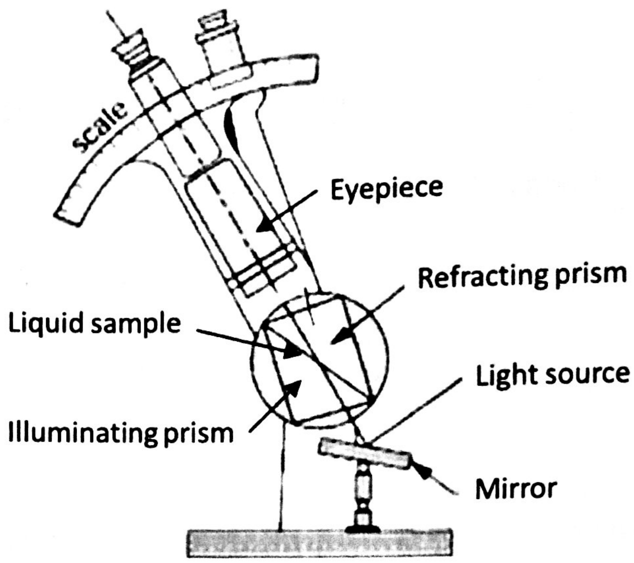
Ernst Abbe, a German researcher working for Carl Zeiss, designed the first refractometer in 1871. Although the usefulness and precision of refractometer has improved, the principle of operation has changed very little.

The liquid sample, of index n, is placed between two prisms (the illuminating prism and the refracting one). The bottom surface of the illuminating prism is ground (i.e. roughened) so each point of this surface can be thought as generating light rays travelling in all directions. As a consequence, rays propagate into the liquid and come onto the interface with the refracting prism with all possible incidence angles. The refracting prism is made of glass with a refractive index *N* higher than that of the liquid. The whole device is placed in air, of index $n_0 = 1$. A picture of the Abbe refractometer is given in Figure 1a and a scheme is displayed in Figure 1b.

1) Schematically represent the path of the rays in Figure 2 (see last page).

2) One considers the interface between the liquid and the refracting prism.
a. Is it possible to have total reflection on this interface? Why?
b. Knowing that $0 \leq i \leq 90°$, express the condition on the refraction angle r.





a) b)

Figure 1: a) picture and b) scheme of an Abbe refractometer.

3) One considers now the interface between the refracting prism and the air.

a. Prove that the angle of incidence r' satisfies the relation $r' = \frac{\pi}{2} - A - r$.

b. Is it possible to have total reflection on this interface? Why?

c. i' is defined as the angle between the normal to the interface and the ray that emerges from the refracting prism. Show that

$$N.\cos\left(A + sin^{-1}\left(\frac{n}{N}\right)\right) \leq \sin(i') \leq N.\cos(A)$$

4) Numerical application: $n = 1.33$; $N = 1.50$ and $A = 45°$.

a. Compute the values of the minimum and maximum values of $\sin(i')$.

b. Compute the uncertainty on $N.\cos(A)$, when taking into account that :

- The relative uncertainty on n is equal to 5%;
- The absolute uncertainty on N is equal to 0.05;
- The absolute uncertainty on A is equal to 4°30'.

c. Is $N.\cos(A)$ significantly higher than 1?

II. ELECTRODYNAMICS (≈ /12 PTS)

The electrodynamics problem is composed of two independent parts (A and B).

A. Saving Mr. Thévenin

On a Friday the 13th, Mr. Thévenin was wandering in the corridors of Louis Neel Building, when he heard a terrifying scream. He rushed into the second floor and what he saw caused him a cardiac arrest. The first aid post was called directly by a SCAN student passing by and an automated external defibrillator (AED) could be used directly to try to save Mr. Thévenin.

An automated external defibrillator (AED) is a portable electronic device that automatically diagnoses the life-threatening ventricular fibrillation of the heart in a patient, and is able to treat them through defibrillation, by delivering a therapeutic dose of electrical energy with the goal of depolarizing the heart muscle completely, allowing the heart to reestablish an effective rhythm.

We are going to study the defibrillator, which was used on that day to save Mr. Thévenin. A defibrillator can be modeled as depicted on Figure 3 with a DC power supply, a charging resistor of resistance R_{ch}, a capacitor C, an inductor L of internal resistor r_L, and a set of electrodes positioned on the person's chest, modeled by a resistor to account for the torso resistance R_{body}.

During a first stage, the capacitor is charged to store the necessary energy, which will be delivered to the heart in a second stage. The current delivered to the heart must last for several milliseconds in order for the heart to completely depolarize. However, the current of a discharging capacitor decreases exponentially. This is why an inductor is needed to prolong the duration of the current flow.

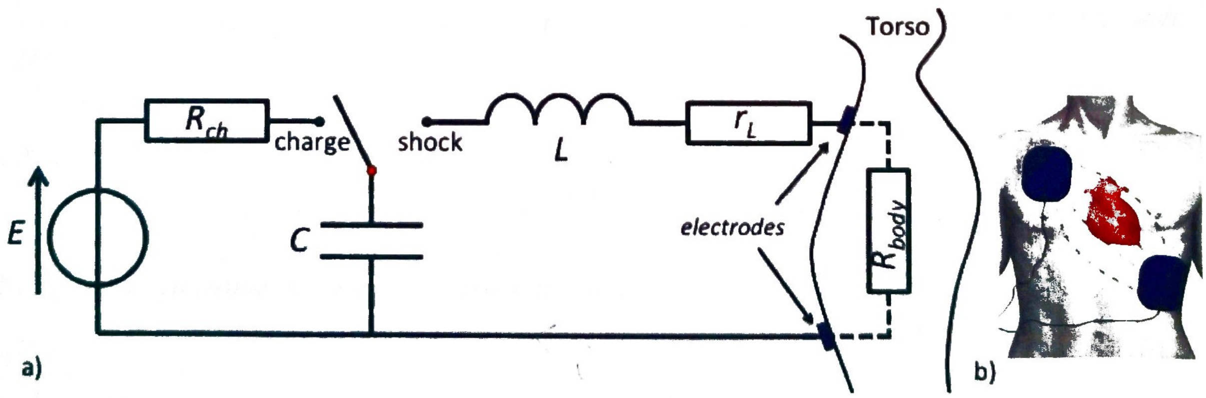


Figure 3 a) Simplified circuit of a defibrillator b) View of electrodes position and placement.

The defibrillator we are studying has the following characteristics:
R_{ch}=10Ω, E=4kV, C=50μF, L=100mH, r_L=20Ω;
and the discharge resistance, which models the patient's torso, is R_{body} = 130Ω (for a typical electrode size of 80 cm^2).

I. After placing the electrodes on the patient's torso, the capacitor needs to be charged by flipping the switch on the "charge" position.
 1. Give the voltage across the capacitor when it is completely charged. We will denote this voltage U_0.
 2. Compute the amount of energy stored in the capacitor when it is completely charged.

II. Once the capacitor is fully charged, the switch is flipped from "charge" to "shock", at time t_{shock}=0, to deliver electrical energy through the electrodes to the body.
 3. Justify (without calculation) the values of the intensities of the current through the body i_{shock}, right after the shock and i_{∞}, after a few minutes.
 4. Give the expression of the intensity of the current flowing through the patient's body (as a function of U_0, R_{body}, r_L, L, C and the time t).
 5. The inductor is inserted into the circuit to slow down the discharge from the capacitor. However, any real inductor has its own resistance. Explain, thanks to the study of the energy balance between t_{shock} and t_{∞}, what is the disadvantage of using an inductor with a large internal resistance r_L.

BONUS QUESTIONS:

6. *What did Mr. Thévenin see to cause him a heart attack?* [1]
7. *Where are the AED located at INSA?*

B. Alternating Current

The sinusoidal voltage across a real coil of inductance L = 30 mH and internal resistance R_L reads $u_D(t) = U_D\sqrt{2}\cos(\omega t)$, with U_D = 220V and f = 50 Hz. The coil cannot stand too high a current and is therefore protected by a fuse, which would blow if the RMS intensity of the current through it becomes higher than $I_{MAX} = 16\,A$. The power line can be modeled by a resistor of total resistance R_0 =1.2 Ω (fuse included). Also, a shunt resistor of resistance R_S= 2.8 Ω has been purposely added in the circuit by the experimenter Mr. Norton, who wanted to measure the intensity of the current in the circuit: $i_D(t) = I_D\sqrt{2}\cos(\omega t + \varphi)$.

1. Express $\underline{Z_D}$ the complex impedance of the real coil and Z_D, its modulus.

2. Using a phasor diagram, express the power factor of the real coil. Then, show that the active power it absorbs can be written $P_D = \frac{U_D^2}{Z_D^2} R_L$.

3. Mr. Norton has measured P_D = 2.5 kW. Compute the two possible values R_{L_1} and R_{L_2} of the resistance of the coil.

4. Compute the RMS intensities I_{D_1} and I_{D_2} running through the real coil for each case (R_{L_1} and R_{L_2} respectively). Taking into account the presence of the fuse, conclude on your results.

[1] Hint: it happened in room C12 or C13

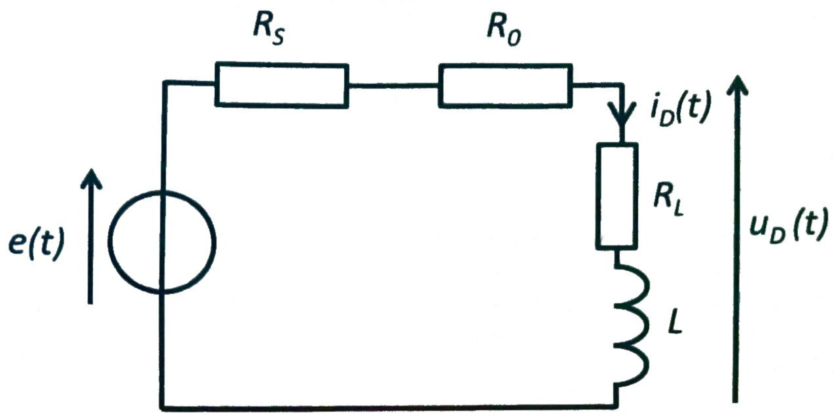


Figure 4

Mr. Norton now decides to add a capacitor of capacitance 129 µF connected in parallel with the real coil. For the following, we will take $R_L' = 12\ \Omega$ (L remains equal to 30 mH). The new circuit he sets up is schemed in Figure 5 and the oscillogram he obtains is shown in Figure 6.

5. <u>From the oscillogram</u>, and explaining your method, find the new value for P_D', the active power received by the real coil.

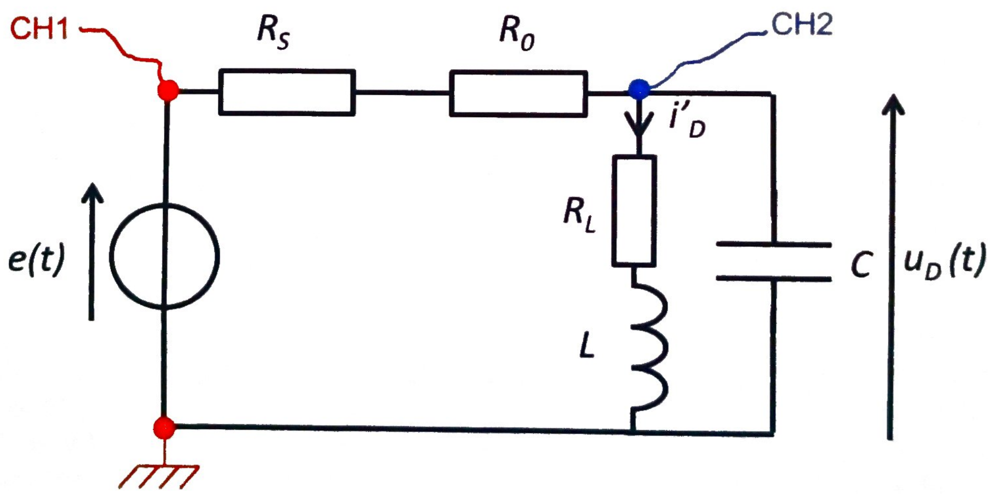


Figure 5

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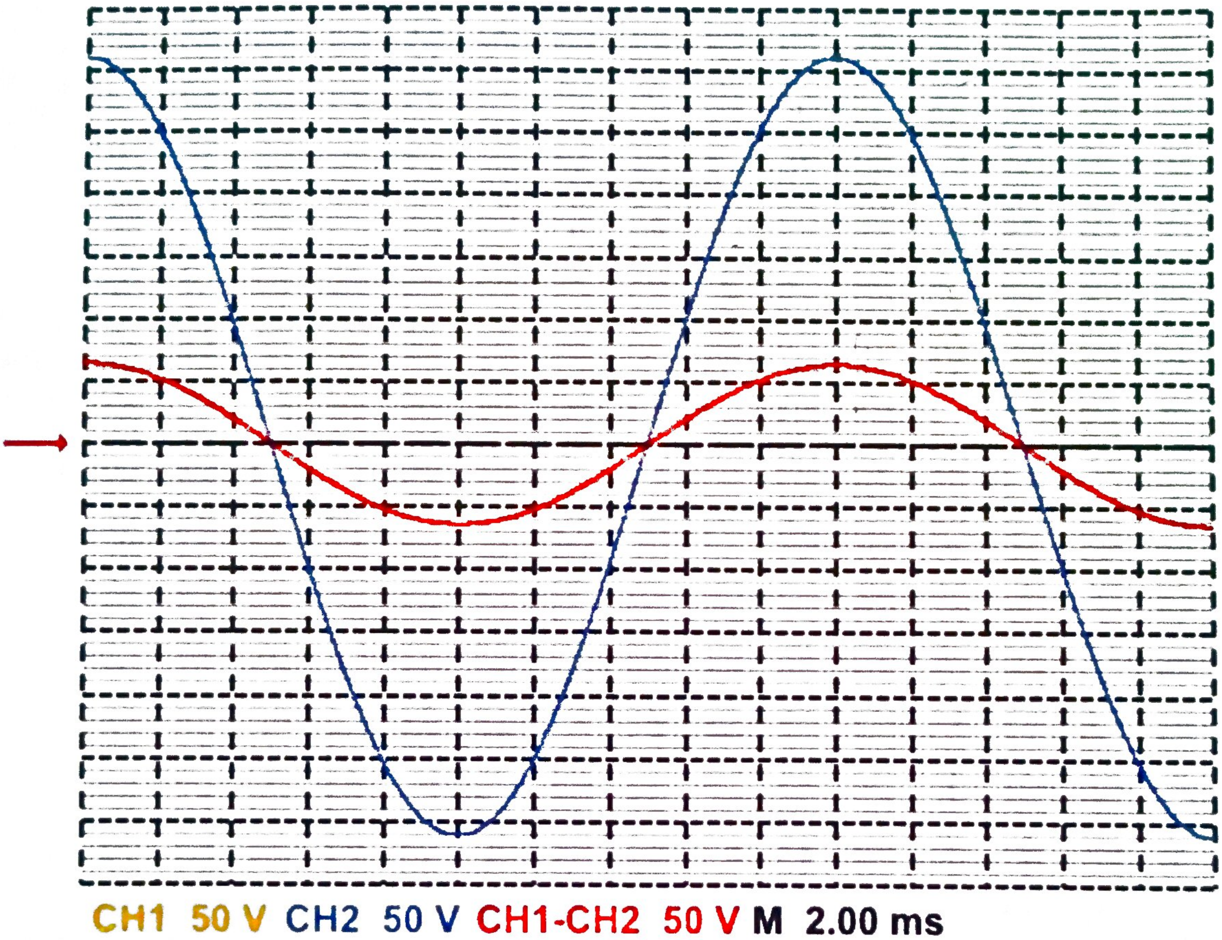
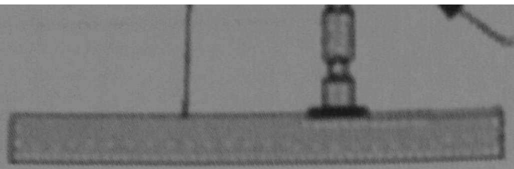


Figure 6: Ocillogram displayed by Mr.Norton's oscilloscope



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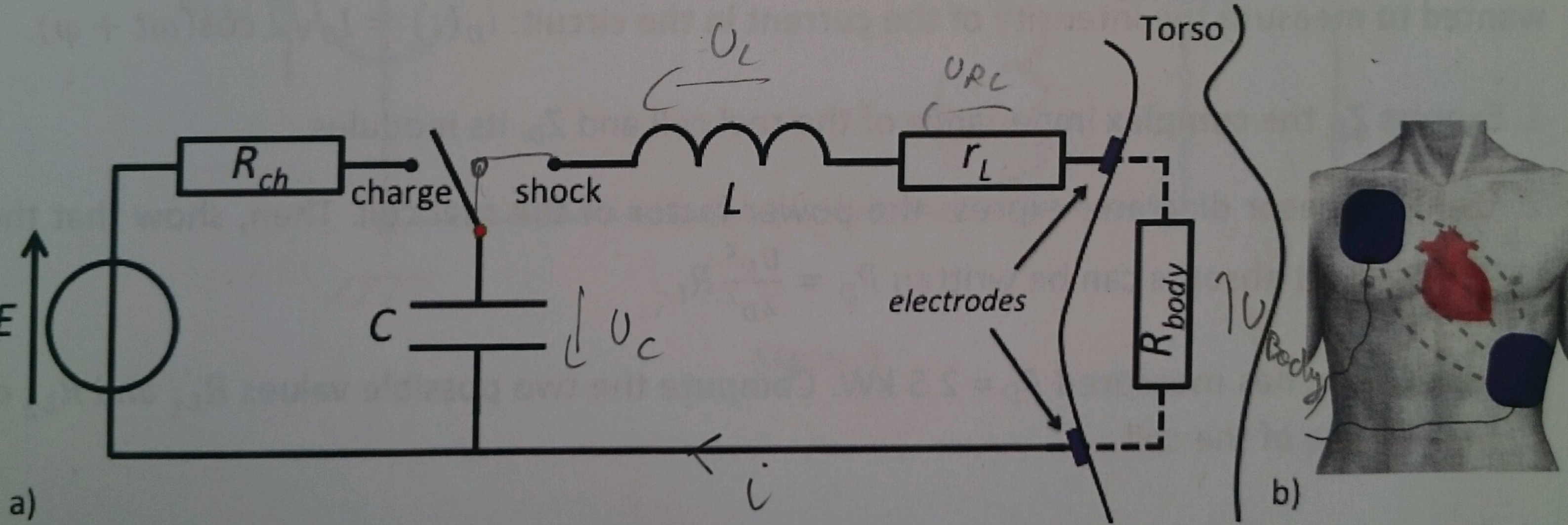

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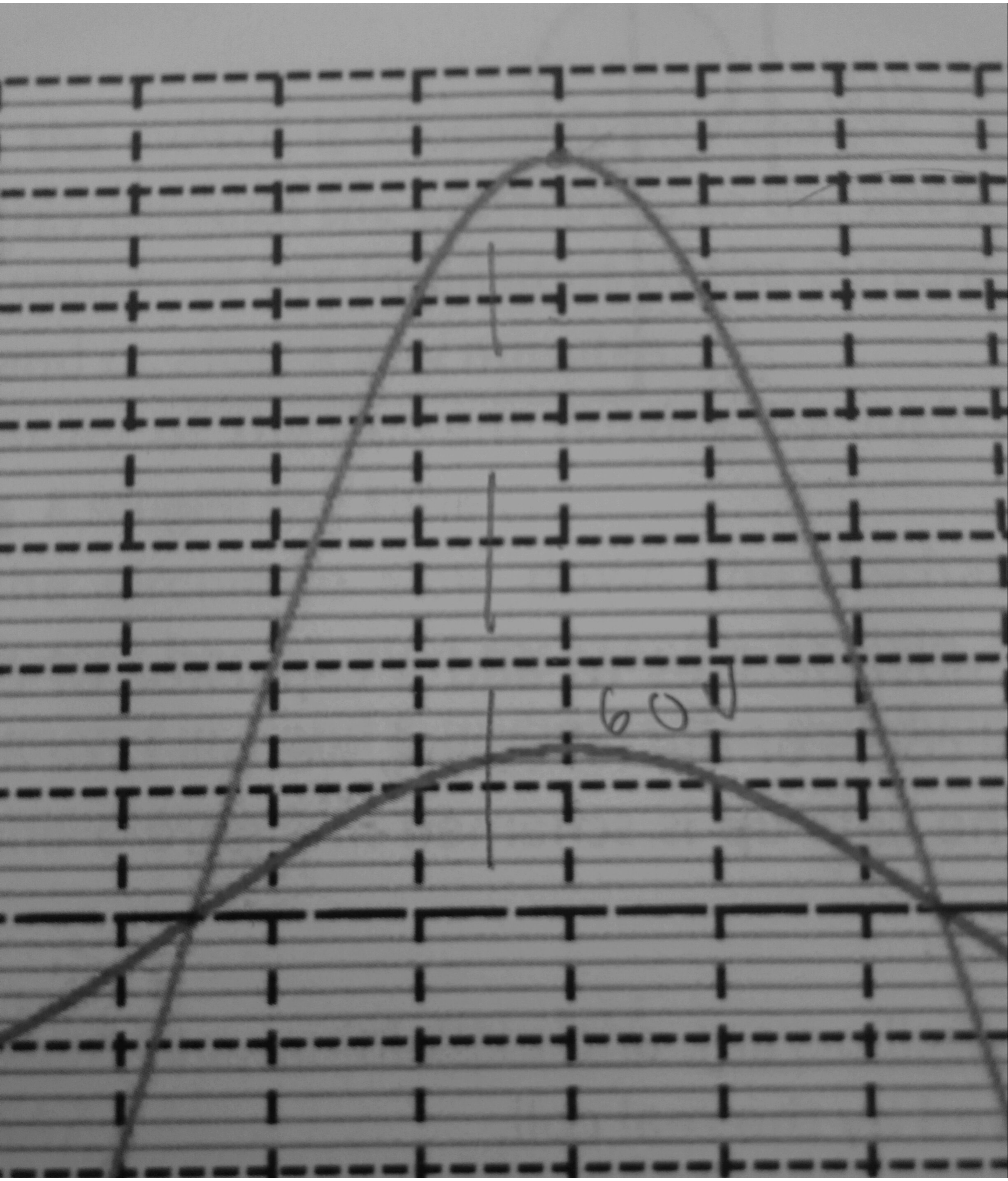
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