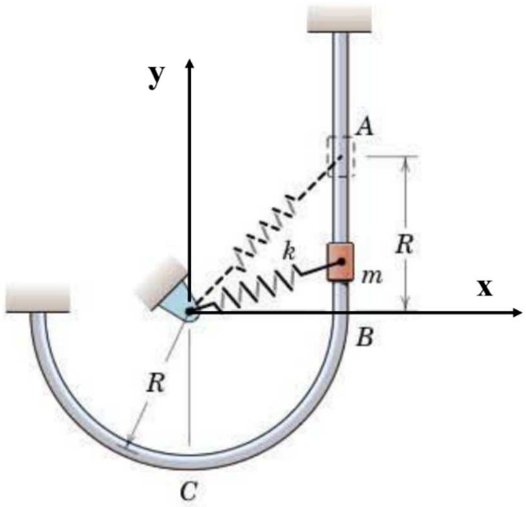
Mechanics – Physics

Elements of correction and marking scheme (*in red*)– Formative test 2

Exercise 1: /5.5



1 - Conservation of energy:

point	Potential Energy	Kinetic Energy
A	• Gravity mgR 0.25 • Spring $\frac{1}{2}k(OA-R)^2$ $=\frac{1}{2}k\left(1-\sqrt{2}\right)^2 R^2$ 0.25	0
B	0	$\frac{1}{2}mv_B^2$ 0.25
C	• Gravity $-mgR$ 0.25 • Spring 0	$\frac{1}{2}mv_C^2$ 0.25

Nb: the constants have been omitted (same constant for the spring which cancels out in the energy conservation principle) and datum for gravity is y=0 (give marks if a different datum was used for potential energy and if the results are correct!)

a) The speed at B is derived from:

$$\frac{1}{2}mv_B^2 = mgR + \frac{1}{2}k\left(1-\sqrt{2}\right)^2 R^2$$

1

$$v_B = \sqrt{2gR + \frac{k}{m}\left(1-\sqrt{2}\right)^2 R^2}$$

b) The speed at C

$$\frac{1}{2}mv_C^2 - mgR = mgR + \frac{1}{2}k\left(1-\sqrt{2}\right)^2 R^2$$

1

$$v_C = \sqrt{4gR + \frac{k}{m}\left(1-\sqrt{2}\right)^2 R^2}$$

2 – Newton's second law when the mass is at C:

Using the projection in the normal direction (towards the centre of the circular part) gives:

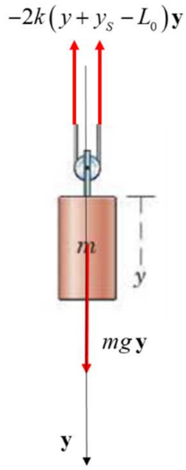
$$m\frac{v_C^2}{R} = N - mg$$

2

$$N = m\left(g + \frac{v_C^2}{R}\right) = m\left(5g + \frac{k}{m}\left(1-\sqrt{2}\right)^2 R\right)$$

NB: the spring generates no force at C as it is un-stretched at this point

Exercise 2: /5.5

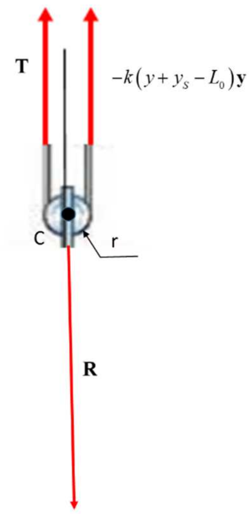


1 – Differential equation

Denoting y_S the static deflection, the spring force amplitude is expressed as:

$$k(L-L_0)$$

with $L = y + y_S$: actual length
L_0 : un-strechted length



The moment balance about C, centre of the pulley, leads to:

$$r\,k\left(y+y_S-L_0\right)-r\,T=0$$

then

$$T=k\left(y+y_S-L_0\right)$$

The total force amplitude on the block is therefore $2k\left(y+y_S-L_0\right)$, which corresponds to a single spring of stiffness $2k$

NB: because of the moment balance of the pulley, the spring force on the mass is doubled so the equivalent stiffness is 2k 0.5

Newton's 2nd law:

$$m\ddot{y}=-2k\left(y+y_S-L_0\right)+mg \qquad 1$$

which can be simplified by the static equation (equilibrium)

$$0 = -2k\left(y_S - L_0\right) + mg \qquad 1$$

Leading to:

$$m\ddot{y} + 2ky = 0 \qquad 1$$

The natural circular frequency is deduced as:

$$\omega_n = \sqrt{\frac{2k}{m}} \qquad 0.5$$

2 –Immersed in an oil bath:

Using the logarithmic decrement, one obtains:

$$\delta = \ln(4) = \frac{2\pi\zeta}{\sqrt{}}$$

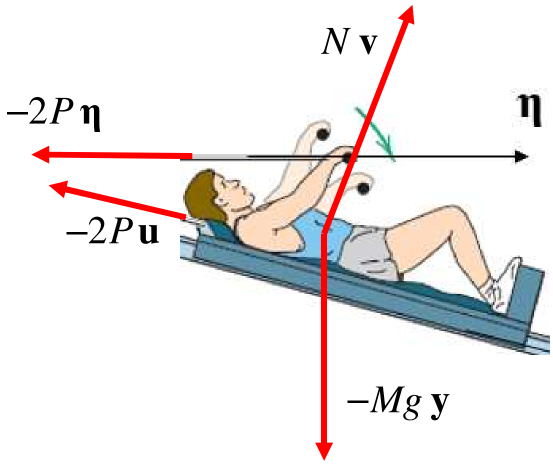
$$\left(1-\zeta^2\right)\left[\ln(4)\right]^2 = 4\pi^2\zeta^2 \qquad 1.5$$

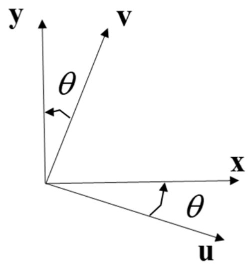
$$\zeta = \sqrt{\frac{\left[\ln(4)\right]^2}{\left[4\pi^2 + \left[\ln(4)\right]^2\right]}} = 0.215$$

The use of the approximation $\zeta^2 << \zeta$ leads to $\zeta \cong \frac{\delta}{2\pi} = 0.22$ hence acceptable.

The damping coefficient is derived as $c = 2\zeta m\omega_n$.

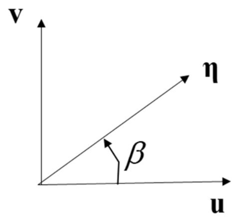
Exercise 3: /5





FBD: sketch or list of external forces expressed

as vectors 1



Force equilibrium

$$-2P\,\boldsymbol{\eta}-2P\,\mathbf{u}-Mg\,\mathbf{y}+N\,\mathbf{v}=\mathbf{0}$$

$$/\mathbf{u}: \quad -2P\cos\beta-2P+Mg\sin\theta=0 \qquad 2$$

$$/\mathbf{v}: \quad -2P\sin\beta-Mg\cos\theta+N=0$$

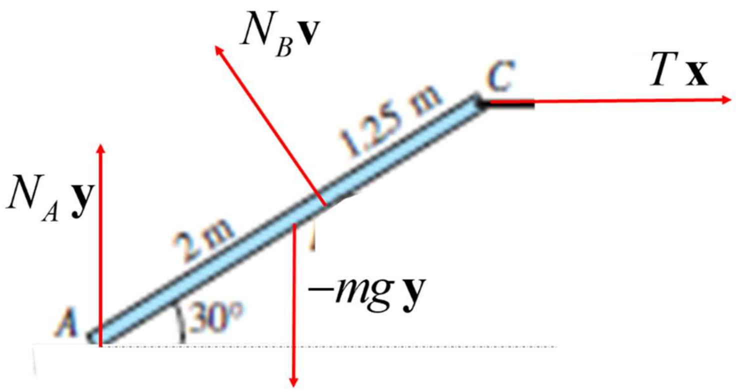
From which, one can derive:

$$P=Mg\frac{\sin\theta}{2(1+\cos\beta)}$$

$$N=Mg\left(\cos\theta+\frac{\sin\theta\sin\beta}{(1+\cos\beta)}\right) \qquad 1.5$$

Numerical applications: $P\simeq 52.3\,N$ $N\simeq 793.9\,N$ 0.5

Exercise 4: **/4.5 or 5.5 (with the extra mark)**



Free-body diagram 0.5

Force balance:

$$N_A\,\mathbf{y} + N_B\mathbf{v} + T\,\mathbf{x} - mg\,\mathbf{y} = \mathbf{0}$$

$$/\mathbf{x}: \qquad -N_B \sin 30° + T = 0 \qquad\qquad 1.5$$

$$/\mathbf{y}: \qquad N_A + N_B \cos 30° - mg = 0$$

Moment balance about point A:

$$-(\frac{2+1.25}{2})\cos 30°\, mg + 2N_B - (2+1.25)\sin 30° T = 0 \qquad\qquad 2$$

Solving

$$N_B = 1.185\,mg$$
$$T = 0.5925\,mg \qquad\qquad 0.5$$
$$N_A = -0.026\,mg \;\; < 0$$

The reaction force at point A cannot be negative so there is loss of contact and we have to re-start with the condition $N_A = 0$ (*note that the moment equilibrium is unchanged*), thus:

$/\mathbf{x}: \qquad -N_B \sin 30° + T = 0$

$/\mathbf{y}: \qquad N_B \cos 30° - mg = 0$

extra mark 1

$N_A = 0$
$N_B = 1.155\, mg = 435.2\, N$
$T = 0.577\, mg = 226.6\, N$

NB: one can verify that the moment balance is also satisfied with these forces