# INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON



## Département du Premier Cycle - SCAN1 MTES - Semester 1

2017-2018

### TEST 1 17/10/17 DURATION 1H

### Warnings and advices

- Documents, dictionaries, phones, and calculators are FORBIDDEN.
- The marking scheme is only given as an indication.

### EXERCISE 1

Solve in C the equation  $z^2 + (3 + 4i)z - 1 + 5i = 0$ .

# EXERCISE 2 (6 pts)

Let 
$$\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$$
,  $n \in \mathbb{N}$  and  $z = (1 + ie^{i\theta})^n$ .

- 1. Find a complex number u such that |u| = 1 and  $z = 2^n \cos^n \left(\frac{\theta}{2} + \frac{\pi}{4}\right) u$ .
- 2. Determine the modulus and an argument of z if n is even.
- 3. Determine the modulus and an argument of z if n is odd.

#### EXERCISE 3 (2 pts)

Consider 2 weighted points  $(A, m_A)$  and  $(B, m_B)$  in  $\mathbb{R}^3$  with  $A \neq B$  and  $m_A m_B \neq 0$ .

- 1. Give the definition of a barycenter G of  $(A, m_A)$  and  $(B, m_B)$ .
- 2. Give a necessary and sufficient condition for the existence of G.
- 3. Let  $C \in \mathbb{R}^3$  such that  $\overrightarrow{AC} = \frac{m_B}{m_A + m_B} \overrightarrow{AB}$ . What can you say about the point C?

#### EXERCISE 4 (2.5 pts)

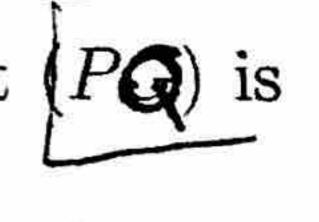
Consider the points A(0, 2, 3) and B(3, 2, 0) in  $\mathbb{R}^3$ .

- 1. Compute the coordinates of the barycenter G of (A, 1) and (B, 2).
- 2. Compute the volume of the parallelepiped generated by the vectors  $\overrightarrow{OA}, \overrightarrow{OB}$  and  $\overrightarrow{OG}$ .

# EXERCISE 5 (6.5 pts)

Consider in  $\mathbb{R}^3$  the points A(0,0,0), D(0,0,1), C(1,0,1) and F(0,1,0).

We are looking for the coordinates of the points  $P \in (AC)$  and  $Q \in (FD)$  such that PQ is orthogonal to (AC) and (FD).



- 1. Justify that there exists  $u, v \in \mathbb{R}$  such that  $\overrightarrow{AP} = \overrightarrow{uAC}$  and  $\overrightarrow{DQ} = \overrightarrow{vDF}$ .
- 2. Using Chasles property show that  $\overrightarrow{PQ} \cdot \overrightarrow{AC} = -2u + 1 v$  and  $\overrightarrow{PQ} \cdot \overrightarrow{DF} = u 1 + 2v$ .
- 3. Deduce from it that the coordinates of P and Q are respectively  $\left(\frac{1}{3}, 0, \frac{1}{3}\right)$  and  $\left(0, \frac{1}{3}, \frac{2}{3}\right)$ .
- 4. Compute the area  $\mathcal{A}$  of the triangle APQ.