1

Equation 3 is

$$\dot{\theta}(t) = |P(jw)|Asin(\omega t + \angle P(jw))$$

let c = 22.55 and $\omega = \omega_0$

> Result
>
> $$|P(jw)| = \frac{22.55}{\sqrt{1^2+1}} = \frac{22.55}{\sqrt{2}} = 15.95$$

2

\phi(\omega) = \angle P(jw)= -tan^{-1}\left(\frac{\omega}{\omega_{0}} \right)

$$let\$\omega = \omega_0\$ > [!tip]Result >$$

\phi(\omega) = -tan^{-1}(1) = -45 \degree

3 From lab 1 $\tau = 0.158 \text{ s}$, then the break frequency is >[!tip] Result >

\omega_{0} = \frac{1}{\tau} = \frac{1}{0.158} = 6.33 \frac{rad}{s}

4 Converting to a frequency (Hz)

f = \frac{\omega}{2\pi} = 1.007 \text{ Hz}

$$Thencalculatingtheperiod > [!tip]Result >$$

T = \frac{1}{f} = \frac{1}{1.007} = 0.993 \text{ s}

5 Equation 8 is

\phi = -\frac{T_{D}}{T}\times(360^{o})

$$Using\$\phi = -45^o\$frompart2and\$T = 0.993\text{ s}\$frompart4, solvingfor\$T_D\$ > [!tip]Result >$$

T_{D} = -\frac{\phi T}{360^{o}} = - \frac{45^{o}(0.993)}{360^{o}} = 0.124 \text{ s}

6 Equation 9 is

v = k*{p}e + k*{d}\dot{e}

v = (k*{p}+k*{d})e \to C(D) = \frac{v}{c} = k*{p} + k*{d}D

$$Thenthetransferfunctionis > [!tip]Result >$$

T*{r\theta} = \frac{CP}{1+CP} = \frac{bk*{d}D + bk*{p}}{D^{2} + bk*{d}D + bk_{p}}

$$where > [!tip]Supplementary >$$

```
b = \frac{c}{\tau}=\frac{22.55}{0.158}=142.7
```

7

```
\omega{n}^{2} = bk{p} \to \omega{n} = \sqrt{ bk{p} }
```

$$Let\$b = 142.7\$and\$k_p = \{4, 50\}\$\$k_p = 4 : \$ > [!tip] Result >$$

```
\omega_{n} = 23.9 \frac{rad}{s}
```

$$\$k_p = 50 : \$ > [!tip] Result >$$

```
\omega_{n} = 84.5 \frac{rad}{s}
```

8 From the characteristic equation of a 2nd order system, the poles are the roots of the denominator

```
-bk{d} \pm \sqrt{ (bk{d})^{2} - 4bk{p} } \to (bk{d})^{2} - 4bk_{p} = 0
```

$$then the equation for \$k_d\$ is$$

```
k{d} = 2\sqrt{ \frac{k{p}}{b} }
```

$$Let\$b = 142.7\$\$k_p = 4 : \$ > [!tip] Result >$$

```
k_{d} = 2\sqrt{ \frac{4}{142.7} } = 0.335
```

$$\$k_p = 50 : \$ > [!tip] Result >$$

```
k_{d} = 2\sqrt{ \frac{50}{142.7} } = 1.184
```

9 The transfer function is

```
T{r\theta} = \frac{bk{d}D + bk{p}}{D^{2}+bk{d}D+bk_{p}}
```

$$Then the magnitude is > [!tip] Result >$$

```
T{r\theta}(j\omega) = \frac{\mid bk{p}+jk{d}\omega\mid}{\mid-\omega^{2} +
jbk{d}\omega + bk{p} \mid} = \frac{\sqrt{ (bk{p})^{2} + (k{d}\omega b)^{2} }}
{\sqrt{ (bk{p})^{2}+(-\omega^{2})^{2}+(bk_{d}\omega)^{2} }}
```

$$and the phase shift is > [!tip] Result >$$

```
\angle T{r\theta}(jw) = tn^{-1} \frac{bk{p} + jk{d}\omega b}{-
\omega^{2}+jbk{d}\omega+bk{p}} = tn^{-1}\left( \frac{k{d}\omega b}{bk{p}} \right)
- tn^{-1} \frac{k{d}\omega b}{bk_{p}-\omega^{2}}
```

10 Letting $\omega = 0$ >[!tip] Result >

```
T{r\theta}(0) = \frac{bk{p} + jk{d}(0)}{-(0)^{2} + jbk{d}(0) + bk{p}} =
\frac{bk{p}}{bk_{p}} = 1
```

$$Letting\$k_p = 4\$, \$\omega = 23.9\$, \$b = 142.7\$, and\$k_d = 0.335\$Then the magnitude is$$

```
T_{r\theta}(j\omega) = \frac{\sqrt{ (142.7 \times 4)^{2} + (0.335 \times 23.9
\times 142.7)^{2} }}{\sqrt{ (142.7 \times 4)^{2} + ((23.9)^{2})^{2} + (0.335
\times 23.9 \times 142.7)^{2}}}
```

$$> [!tip] Result >$$

T_{r\theta}(j\omega) = \frac{1261.5}{1384.6} = 0.91

$andthephaseshiftis$

\angle T*{r\theta}(j\omega*{n}) = tan^{-1}\left(\frac{0.33 \times 23.9 \times 142.7}{142.7\times4} \right)- tan^{-1}\left(\frac{0.33\times23.9\times142.7}{142.7\times4 - 23.9^{2}} \right)

$> [!tip] Result >$

\angle T*{r\theta}(j\omega*{n}) = 63.1^{o} - 89.99^{o} = -26.89^{o}