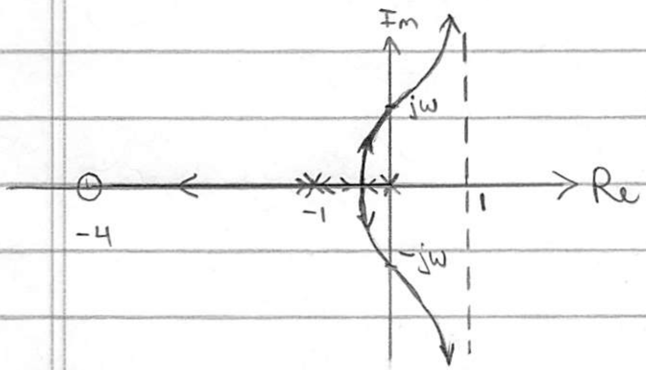
ENME 585 - Quiz 2 2022 Solutions

1.a) $u = k(e + 4he) = k\left(1 + \frac{4}{D}\right)e = k\left(\frac{D+4}{D}\right)e \Rightarrow C(D) = k\left(\frac{D+4}{D}\right)$

$(D^2 + 2D + 1)y = u \Rightarrow P(D) = \frac{y}{u} = \frac{1}{D^2+2D+1} = \frac{1}{(D+1)^2}$

R-L: $1 + C(s)P(s) = 0 \Rightarrow 1 + k\frac{s+4}{s(s+1)^2} = 0 \quad (1)$



$n - m = 3 - 1 = 2 \qquad 2\phi_a = 180$

$\phi_a = \pm 90$

$\alpha = \frac{0 - 1 - 1 - (-4)}{2} = 1$

b) $(1) \Rightarrow \quad s(s+1)^2 + k(s+4) = 0$

$s(s^2+2s+1) + ks + 4k = 0$

$s^3 + 2s^2 + (k+1)s + 4k = 0$

$s = j\omega \Rightarrow \quad j\omega(k+1-\omega^2) + 4k - 2\omega^2 = 0$

$\Rightarrow \omega^2 = 2k$ and $0 = k+1-\omega^2 = k+1-2k$

$\Rightarrow k = 1$ and $\underline{\omega = \pm\sqrt{2}}$

$\therefore$ stable for $\underline{0 < k < 1}$

c) $T_{re}(s) = \frac{1}{1 + C(s)P(s)} = \frac{s(s+1)^2}{s^3+2s^2+(k+1)s+4k}$

$E(s) = T_{re}(s)R(s) = \frac{1}{s^2}T_{re}(s) \quad \cdot \; e_{ss} = \lim_{s\to 0} sE(s) = \frac{1}{4k}$

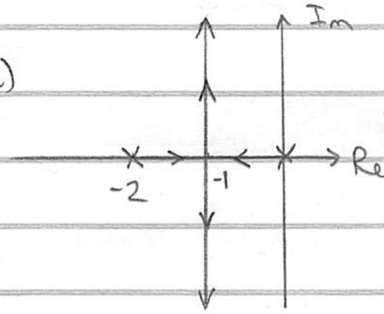
$\Rightarrow \underline{k = \frac{1}{4}}$ for $e_{ss} = 1$. Since $0 < \frac{1}{4} < 1$, the C.L.S is stable.

d) $e_{ss} = 0.1 = \frac{1}{4k}$ requires $k = 2.5$, which makes the C.L.S. unstable. Hence, $e_{ss} = 0.1$ is not possible. Since $k < 1$ is needed for stability, $e_{ss} > \frac{1}{4}$. I.e., $\frac{1}{4}$ is the greatest lower bound on the achievable e_{ss}.

2. $1 + k\,P(s) = 0 \quad , \quad P(s) = \frac{1}{s(s+2)}$

$\Rightarrow \quad s^2 + 2s + k = 0$

a)



b) $(s+1)^2 + k - 1 = 0$

$\Rightarrow \quad \underline{k = 1}$ for critically-damped closed-loop poles (at $s = -1$).

c) $T_{we} = \frac{-P}{1 + CP} = \frac{-\frac{1}{s(s+2)}}{1 + \frac{k}{s(s+2)}} = \frac{-1}{s^2 + 2s + k}$

$W(s) = \frac{1}{s} \Rightarrow$

$e_{ss} = \lim_{s \to 0} s \frac{1}{s} T_{we}(s) = T_{we}(0) = \frac{-1}{k} = \underline{\frac{-1}{5}} = -0.2$

d) $s^2 + 2s + 5 = (s+1)^2 + 2^2 \Rightarrow \sigma = 1 , \; \omega_d = 2$

$$\max_t |e(t)| = \left(1 + e^{-\frac{\sigma \pi}{\omega_d}}\right) |e_{ss}|$$
$$= \left(1 + e^{-\pi/2}\right)\left(\tfrac{1}{5}\right)$$
$$= (1.208)\,\tfrac{1}{5}$$
$$= \underline{0.242}$$