1

The equation for the linear servo base unit without a pole is

$$m_{ec}\ddot{x} + b_{ec}\dot{x} = \eta v$$

where the time constant is

$$\tau = \frac{m_{ec}}{b_{ec}}$$

and the dc gain is

$$c = \frac{\eta}{b_{ec}}$$

✓ Time Constant & DC Gain

$$\tau = \frac{m_{ec}}{b_{ec}} = \frac{0.500}{11.27} = 0.0444\ s$$

$$c = \frac{\eta}{b_{ec}} = \frac{1.072}{11.27} = 0.0951$$

2

The relationship is

$$b_{ec} = 2(\zeta\omega)$$

where ζ is equal to 1.0 since it is a critically damped system.

Then

$$b_{ec} = 2\omega = 11.27$$

$$\omega = \frac{b_{ec}}{2} = \frac{11.27}{2}$$

The proportional control gain relationship is

$$k_p = \frac{\omega^2\tau}{c} = \frac{(11.27)^2(0.0444)}{0.0951} = 59.299$$

✓ Proportional Gain

$$k_p = 59.3$$

3

The parameters given are

$$m_p = 0.127\ kg$$

$$l_p = 0.1778\ m$$

$$J_p = 1.20 \times 10^{-3}\ kgm^2$$

$$b_p = 0.0024\ \frac{Nm}{s}$$

Equation 6 with $up = 0$ and a constant x such that $\ddot{x} = 0$ is

$$(J_p + m_p l_p^2)\ddot{\alpha} + m_p l_p g \alpha = 0$$

Subbing in the given parameters results in

$$((1.20 \times 10^{-3}) + (0.127)(0.1778)^2)\ddot{\alpha} + (0.127)(0.1778)(9.81)\alpha = 0$$

The equation simplifies to

$$5.2148 \times 10^{-3}\ddot{\alpha} + 0.22152\alpha = 0$$

then the roots are

$$r_1 = 6.575i$$

$$r_2 = -6.575i$$

then

$$\alpha = c_1 cos(6.575t) + c_2 sin(6.575t)$$

✓ Frequency with up = 0

$$\omega = 6.575\ \frac{rad}{s}$$

4

Running the script for the gantry system (up = 0) yields

✓ Gantry System

$$k_{px} = 200.1736$$

$$k_{dx} = 64.0046$$

$$k_{p\alpha} = -85.4782$$

$$k_{d\alpha} = 9.6389$$

Running the script for the inverted pendulum system (up = 1) yields

✓ Inverted Pendulum System

$$k_{px} = -12.9285$$

$$k_{dx} = -29.5742$$

$$k_{p\alpha} = 64.1429$$

$$k_{d\alpha} = 9.0588$$