1

Equation 3 is

$$\dot{\theta}(t) = |P(jw)|Asin(\omega t + \angle P(jw))$$

let c = 22.55 and $\omega = \omega_0$

Result

$$|P(jw)| = \frac{22.55}{\sqrt{1^2 + 1}} = \frac{22.55}{\sqrt{2}} = 15.95$$

2

$$\phi(\omega) = \angle P(jw) = -tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

let $\omega = \omega_0$

Result

$$\phi(\omega) = -tan^{-1}(1) = -45^{\circ}$$

3

From lab 1 $\tau = 0.158\,\mathrm{s}$, then the break frequency is

Result

$$\omega_0 = \frac{1}{\tau} = \frac{1}{0.158} = 6.33\frac{rad}{s}$$

4

Converting to a frequency (Hz)

$$f = \frac{\omega}{2\pi} = 1.007\ \mathrm{Hz}$$

Then calculating the period

Result

$$T = \frac{1}{f} = \frac{1}{1.007} = 0.993\ \mathrm{s}$$

5

Equation 8 is

$$\phi = -\frac{T_D}{T} \times (360^o)$$

Using $\phi = -45^o$ from part 2 and $T = 0.993\,\mathrm{s}$ from part 4, solving for T_D

Result

$$T_D = -\frac{\phi T}{360^o} = -\frac{45^o(0.993)}{360^o} = 0.124\,\mathrm{s}$$

6

Equation 9 is

$$v = k_p e + k_d \dot{e}$$

$$v = (k_p + k_d)e \rightarrow C(D) = \frac{v}{c} = k_p + k_d D$$

Then the transfer function is

Result

$$T_{r\theta} = \frac{CP}{1+CP} = \frac{bk_d D + bk_p}{D^2 + bk_d D + bk_p}$$

where

Supplementary

$$b = \frac{c}{\tau} = \frac{22.55}{0.158} = 142.7$$

7

$$\omega_n^2 = bk_p \rightarrow \omega_n = \sqrt{bk_p}$$

Let $b = 142.7$ and $k_p = \{4, 50\}$

$k_p = 4:$

Result

$$\omega_n = 23.9\frac{rad}{s}$$

$k_p = 50:$

Result

$$\omega_n = 84.5\frac{rad}{s}$$

8

From the characteristic equation of a 2nd order system, the poles are the roots of the denominator

$$-bk_d \pm \sqrt{(bk_d)^2 - 4bk_p} \rightarrow (bk_d)^2 - 4bk_p = 0$$

then the equation for k_d is

$$k_d = 2\sqrt{\frac{k_p}{b}}$$

Let $b = 142.7$

$k_p = 4:$

Result

$$k_d = 2\sqrt{\frac{4}{142.7}} = 0.335$$

$k_p = 50:$

Result

$$k_d = 2\sqrt{\frac{50}{142.7}} = 1.184$$

9

The transfer function is

$$T_{r\theta} = \frac{bk_d D + bk_p}{D^2 + bk_d D + bk_p}$$

Then the magnitude is

Result

$$T_{r\theta}(j\omega) = \frac{\mid bk_p + jk_d\omega \mid}{\mid -\omega^2 + jbk_d\omega + bk_p \mid} = \frac{\sqrt{(bk_p)^2 + (k_d\omega b)^2}}{\sqrt{(bk_p)^2 + (-\omega^2)^2 + (bk_d\omega)^2}}$$

and the phase shift is

Result

$$\angle T_{r\theta}(jw) = tn^{-1}\frac{bk_p + jk_d\omega b}{-\omega^2 + jbk_d\omega + bk_p} = tn^{-1}\left(\frac{k_d\omega b}{bk_p}\right) - tn^{-1}\frac{k_d\omega b}{bk_p - \omega^2}$$

10

Letting $\omega = 0$

Result

$$T_{r\theta}(0) = \frac{bk_p + jk_d(0)}{-(0)^2 + jbk_d(0) + bk_p} = \frac{bk_p}{bk_p} = 1$$

Letting $k_p = 4$, $\omega = 23.9$, $b = 142.7$, and $k_d = 0.335$

Then the magnitude is

$$T_{r\theta}(j\omega) = \frac{\sqrt{(142.7 \times 4)^2 + (0.335 \times 23.9 \times 142.7)^2}}{\sqrt{(142.7 \times 4)^2 + ((23.9)^2)^2 + (0.335 \times 23.9 \times 142.7)^2}}$$

Result

$$T_{r\theta}(j\omega) = \frac{1261.5}{1384.6} = 0.91$$

and the phase shift is

$$\angle T_{r\theta}(j\omega_n) = tan^{-1}\left(\frac{0.33 \times 23.9 \times 142.7}{142.7 \times 4}\right) - tan^{-1}\left(\frac{0.33 \times 23.9 \times 142.7}{142.7 \times 4 - 23.9^2}\right)$$

Result

$$\angle T_{r\theta}(j\omega_n) = 63.1^o - 89.99^o = -26.89^o$$