Competitive Programming Week 6

Introduction to graphs

Membership sign up:

JUNIOR EXEC APPLICATIONS OPEN

Go to link below or scan QR code to apply.
The deadline is OCTOBER 31st at midnight.

https://forms.gle/8dkGw1Zezfm4BVnRA

SCAN ME

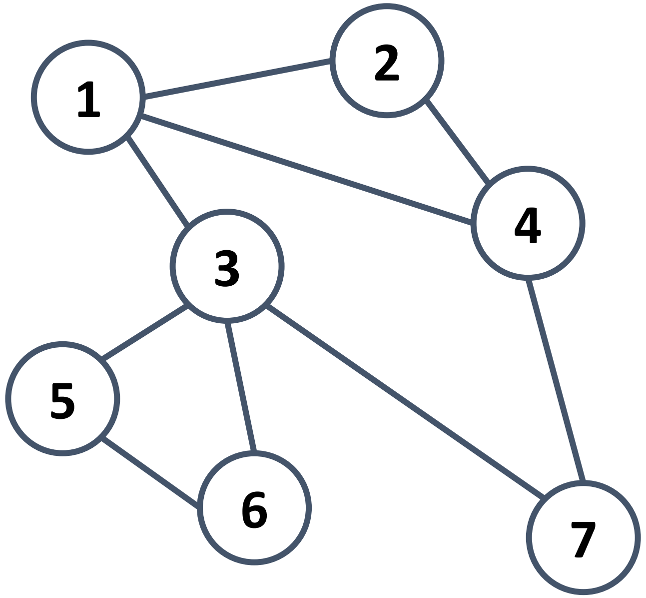
Alberta Collegiate Programming Contest (ACPC):

This year ACPC will take place on November 25th
We will be hosting a location here at U of C
Sign up begins next week! November 1st

Practice Contests Leaderboard

- Tied for first place with 37 problems solved:
  - **Alex Chen, Martin L**
- In third place with 35 problems solved:
  - **Quwsar Ohi**
- In fourth place with 34 problems solved:
  - **Nathan Weiss**
- In fifth place with 32 problems solved:
  - **Jimmy Xu**
- In sixth place with 30 problems solved:
  - **Max McEvoy**
- In seventh place with 29 problems solved:
  - **Mikhail Singh**
- Tied for eighth place with 22 problems solved:
  - **Anh Nguyen, Khoa Nguyen, Hy Huynh, Nguyễn Bá Khánh Tùng**

Graphs



A Graph consists of a set of nodes/vertices and a set of edges connecting said nodes. Mathematically written as G = (V, E) where V is the vertex set and E is the edge set

A graph can be directed or undirected (default):

- In an **undirected** graph edges are bidirectional i.e. you can go from vertex i to j if there exists an edge (i, j) or (j, i) in E.
- In a **directed** graph edges are directional/“one way” i.e. you can go from vertex i to j iff there is an edge (i, j) in E.

A **path** from vertex i to vertex j is a sequence of edges
$(i, k_1), (k_1, k_2), \dots, (k_n, j)$

A graph is **connected** iff there exists a path from any vertex to any other vertex in the graph

A subset U of V is a **component** iff there exists a path from any vertex to any other vertex in U

Graphs

A path is a **cycle** if it starts at a vertex v and ends back at v

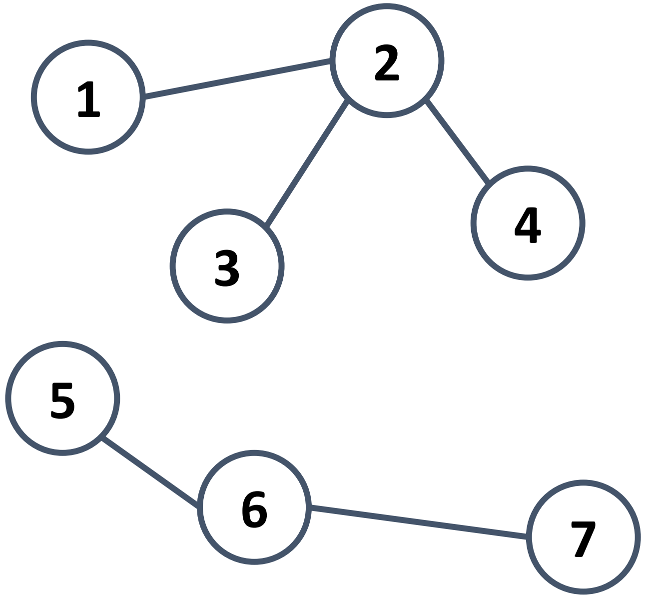
A graph is **acyclic** if there are no cycles in it

A **tree** is a connected acyclic graph

A graph is a **forest** iff every component in it is a tree

A graph can be **weighted** i.e. each edge has a cost associated with it

The **degree** of a vertex v is the number of vertices that connect to it



Graph Representations

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Graph adjacency list

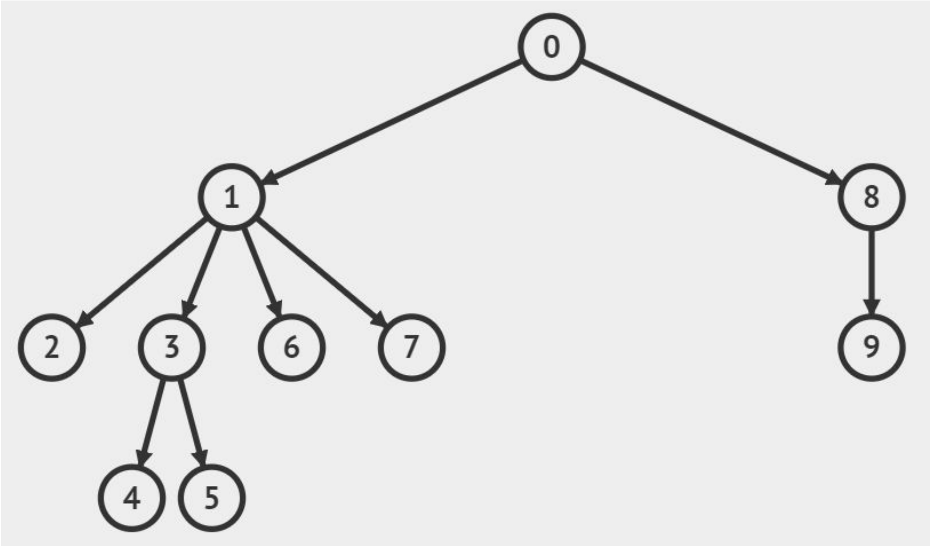
- For each node store each its neighbours in a list called an "adjacency list"
  - List of lists in Python, or vector of vectors in C++

Pros:

- We can easily and efficiently traverse each node's children by traversing its list of neighbours
- Very space efficient. Storage of the vertex and edge data is minimal

Cons:

- Checking if an edge exists is worst case O(n)
- Inverting a directed graph is expensive
- For weighted graphs, the weights will have to be stored elsewhere



Adjacency List				
0:	1	8		
1:	2	3	6	7
2:				
3:	4	5		
4:				
5:				
6:				
7:				
8:	9			
9:				

Graph adjacency matrix

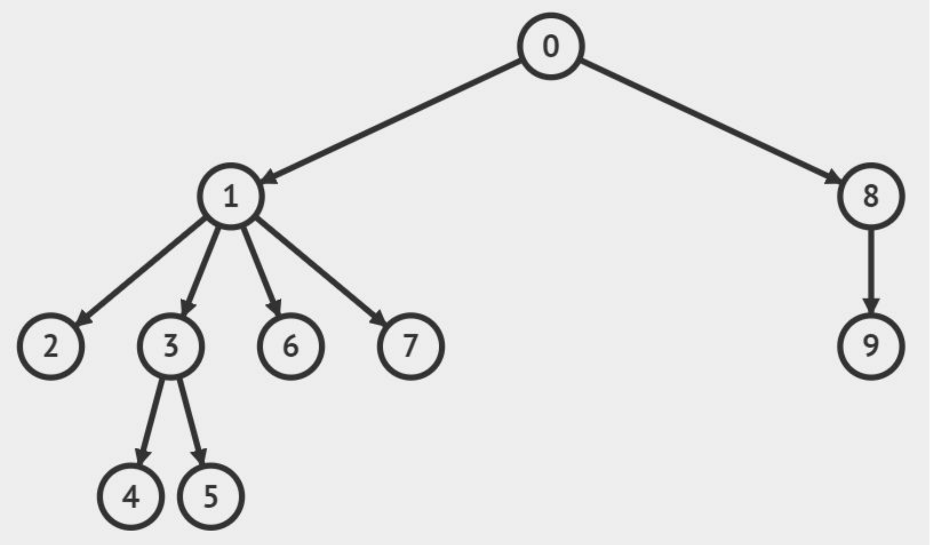
- Define a nxn array A where A[i][j] = 1 if there is an edge from i to j, 0 otherwise.
    - List of lists in Python, or vector of vectors in C++

Pros:

- Inverting directed graphs is easy (just swap the indices)
- You can store weights right in the matrix (replace 1 with the edge weight be careful if weights can be 0)
- Checking if an edge exists is O(1)

Cons:

- Traversals are slower since we need to check if there is an edge between the current node and every other node
- For sparse graphs (graphs with few edges) a lot of extra information is stored



	0	1	2	3	4	5	6	7	8	9
0	0	1	0	0	0	0	0	0	1	0
1	0	0	1	1	0	0	1	1	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	1	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0	0

Graphs as pointers

- Define a class/struct etc. representing a node with a list of pointers to neighbouring nodes. For undirected connected graphs storing a reference to any node is sufficient

Pros:

- Can be useful for small trees

Cons:

- Hard/slow to create
- Nodes take a lot of space + time to instantiate
- Can only represent connected graphs

Binary Tree:

```
struct Node {
	int val;
	Node* left;
	Node* right;
};
```

Graph Traversals

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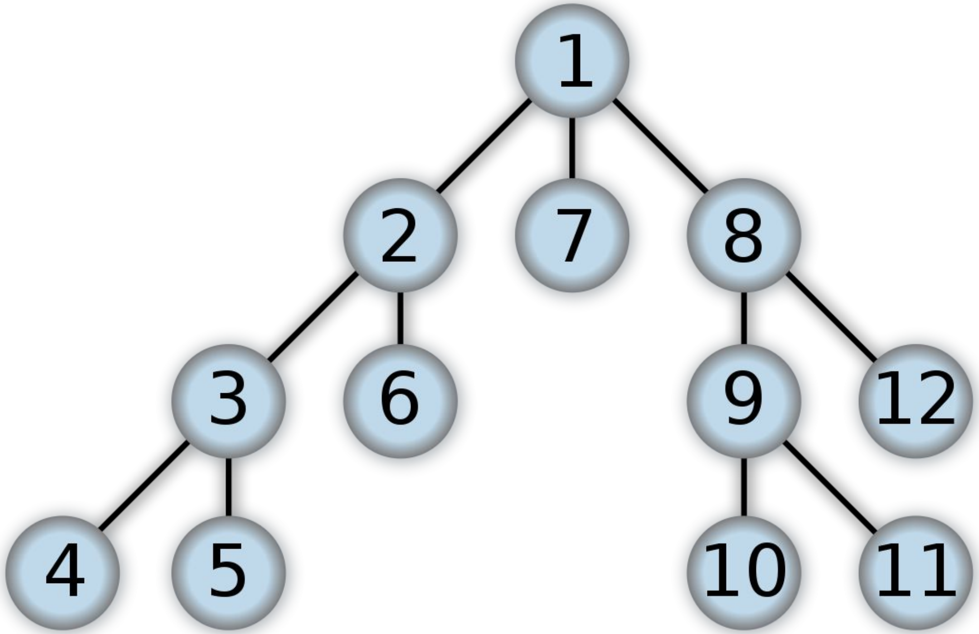
Graph Traversals

A traversal is a way of going from node to node in a connected graph/component. Very useful for problems where you have to visit every node or need to search for a node



Binary Tree Traversals

Traversals over binary trees have very simple recursive implementations.
There are 3 traversal patterns:

- **Pre-order:** visit the node, then its left child, then its right child
- **In-order:** visit the node's left child, then it, then its right child
- **Post-order:** visit the node's left child, then its right child, then it

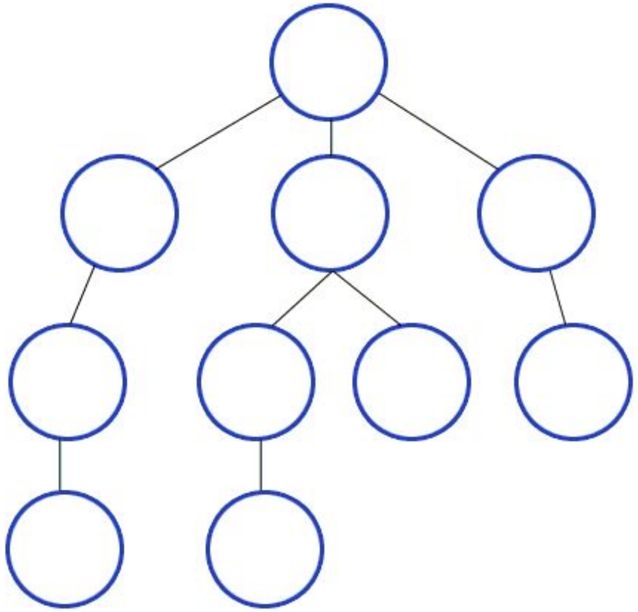
```
def preOrderTraversal(node)
    visit(node)
    preOrderTraversal(node.left)
    preOrderTraversal(node.right)
```

```
def inOrderTraversal(node)
    inOrderTraversal(node.left)
    visit(node)
    inOrderTraversal(node.right)
```

```
def postOrderTraversal(node)
    postOrderTraversal(node.left)
    postOrderTraversal(node.right)
    visit(node)
```

Say you have a binary search tree and want to print the values in sorted order.
What traversal should you use?

Depth-first search



Depth-first search (DFS) is a graph traversal algorithm that acts on an arbitrary connected graph prioritizing depth (i.e. fully explore a path before backtracking)

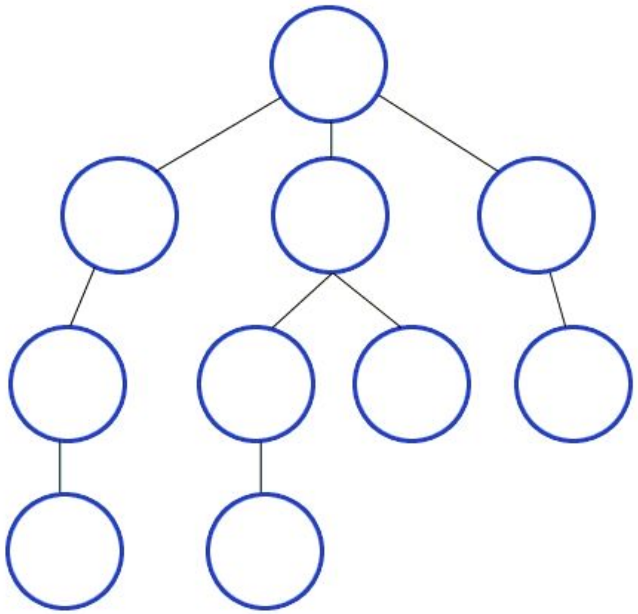
DFS has a simple recursive description:

```
def dfs(node:Node, seen:Set<Node>)
    if (seen.contains(node))
        return
    seen.add(node)
    for (Node n : node.neighbours)
        dfs(n, seen)
    return
```

Alternate Iterative implementation

```
def dfs(start:Node)
    Set<Node> seen
    Stack<Node> toVisit
    toVisit.push(start)
    while (!toVisit.isEmpty())
        Node next = toVisit.pop()
        if (seen.contains(next))
                continue;
        seen.add(next)
        for (node n : next.neighbours)
                toVisit.push(n)
```

Breadth-first search



Breadth-first search (BFS) is a graph traversal algorithm that acts on an arbitrary connected graph prioritizing breadth (i.e. explore paths simultaneously)

```
def bfs(start:Node)
    Set<Node> seen
    Queue<Node> toVisit
    toVisit.enqueue(start)
    while (!toVisit.isEmpty())
        Node next = toVisit.dequeue()
        if (seen.contains(next))
                continue;
        seen.add(next)
        for (node n : next.neighbours)
                toVisit.enqueue(n)
```

Note 1: The only difference in implementation is using a queue instead of a stack
Note 2: The first time we reach a node it will the the shortest path from the starting vertex to it

Today’s Contest:

https://open.kattis.com/contests/c74hx9/

(or look up “CPC Fall 2023 Practice Contest Week 6” in the Kattis contest list)

Feel free to ask questions until 7pm, and then throughout the week on Discord!

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