ENME 585 - Solutions to Assignment 2

1. $\frac{x}{f} = \frac{1}{mD^2+bD+k} = \frac{1}{2D^2+4D+10} = \frac{1}{10}\frac{5}{D^2+2D+5} = \frac{1}{10}\frac{5}{(D+1)^2+2^2}$

$\therefore\ \sigma = 1,\ \omega_d = 2$, and the step response (from notes) is:

$$x = \frac{1}{10}\frac{5}{D((D+1)^2+2^2)} = \frac{1}{10}\left(1 - e^{-t}\left(\cos 2t + \frac{1}{2}\sin 2t\right)\right)$$

Max x occurs at $t_p = \frac{\pi}{\omega_d} = \underline{\frac{\pi}{2}} \Rightarrow x_p = \frac{1}{10}\left(1 + e^{-\pi/2}\right) = \frac{1}{10}(1.2) = \underline{0.121}$

2. $\frac{x}{f} = \frac{1}{2D^2+4D+k} = \frac{0.5}{D^2+2D+\frac{k}{2}} = \frac{0.5}{(D+1)^2}$ for critical damping (no overshoot)

$\therefore\ \frac{k}{2} = 1$, so $k=2$ is the largest k before overshoot.

3. $T_{re} = \frac{1}{1+CP} = \frac{1}{1+\frac{2(4)}{2D+1}} = \frac{2D+1}{2D+9}$

For $r = h = \frac{1}{D}$, $e_{ss} = T_{re}(0) = \underline{1/9}$

4. $T_{we} = \frac{-P}{1+PC} = -P\,T_{re} = \frac{-4}{2D+9} \Rightarrow e_{ss} = \underline{-4/9}$

If $C=0$, $T_{we} = \frac{-P}{1+0P} = -P = \frac{-4}{2D+1} \Rightarrow \underline{e_{ss} = -4}$

$\therefore$ Feedback makes e_{ss} (nine times) smaller.

5. $T_{we} = \frac{-4}{2D+9} = \frac{-4/9}{\frac{2}{9}D+1} \Rightarrow \underline{\tau = 2/9}$

The time constant of $P = \frac{4}{2D+1}$ is 2, so the closed-loop system is (nine times) faster than $P(D)$.

6. $T_{re}(D) = \frac{1}{1+CP} = \frac{1}{1+\frac{4k_I}{D(2D+1)}} = \frac{D(2D+1)}{2D^2+D+4k_I}$

$T_{we}(D) = -P\,T_{re}(D) = \frac{-4D}{2D^2+4D+4k_I}$. T_{re} and T_{we} are both stable for all $k_I > 0$

$e = r\,T_{re} + w\,T_{we} = \frac{1}{D}T_{re} + \frac{1}{D}T_{we} = \frac{2D+1-4}{2D^2+D+4k_I}$

$e_{ss} = \left(D\,e(D)\right)_{D=0} = \underline{\underline{0}}.$

7. $e = r\,T_{re} = \frac{1}{D^2}\cdot\frac{D(2D+1)}{2D^2+D+4k_I}$

$e_{ss} = \left(D\,e(D)\right)_{D=0} = \frac{1}{4k_I} = 0.1 \Rightarrow k_I = \underline{2.5}$.

8. $C(D) = \frac{k_p D + k_I}{D} \Rightarrow T_{we} = \frac{-P}{1+PC} = \frac{-4D}{2D^2+(1+4k_p)D+4k_I}$,

which has repeated real poles when $(1+4k_p)^2 = 4(2)4k_I = 80$

$\therefore\ k_p = \frac{\sqrt{80}-1}{4} = \underline{1.99}$ for critical damping.