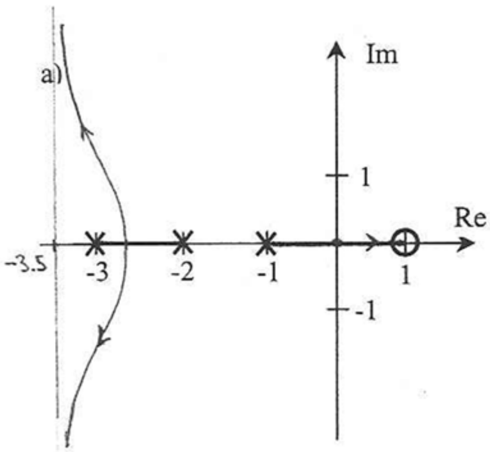
University of Calgary
Department of Mechanical & Manufacturing Engineering

ENME 585 – Control Systems – Assignment 3

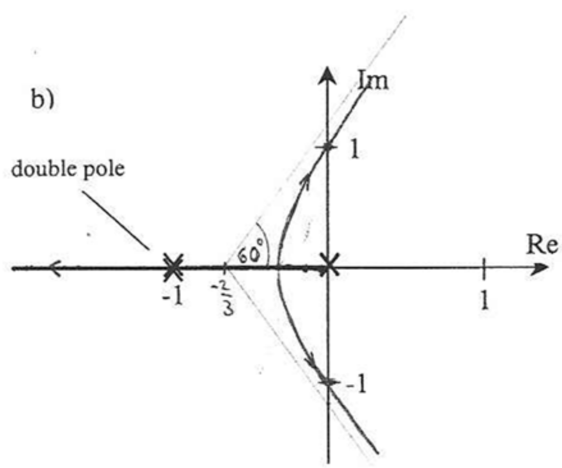
Each of the following plots shows the poles (marked as 'X') and zeros (marked as 'O') of a rational transfer function $L(s)$, whose numerator and denominator polynomials are monic in s. For each plot (i.e. each $L(s)$) sketch the root loci of the characteristic equation $1+KL(s)=0$ as K varies from 0 to $+\infty$. Lightly sketch any asymptotes, and label their angles and intersection point on the real axis. Label angles of departure (from poles of $L(s)$) and arrival (at zeros of $L(s)$) with their calculated value, unless they equal 0, 90, 180, or 270 degrees. Clearly label any crossings of the imaginary axis with the corresponding value of the (purely imaginary) root. For each system, give the range K for which the roots of $1+KL(s)=0$ are stable.



$\alpha = \frac{-7}{2} = -3.5$

$s=0$ when $K = \frac{1\cdot 2\cdot 3}{1} = 6$

$\therefore$ Stable for $\underline{\underline{0 \leq K < 6}}$



asymptotes at ± 60, ~~120~~ 180°

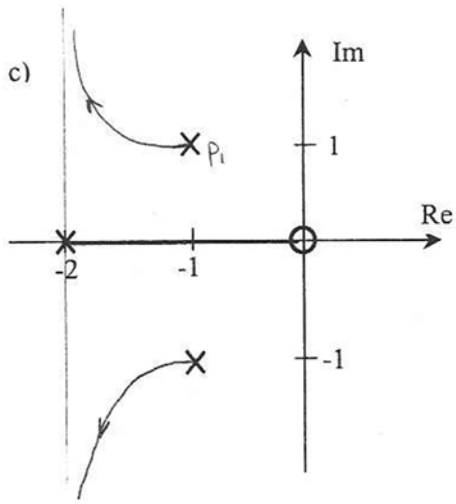
$\alpha = -2/3$

$s = \pm j$ is on loci since the angles from poles of $L(s)$ to $s=j$ total

$90° + 2(45°) = 180°$

At $s=j$, $K = 1\cdot\sqrt{2}\cdot\sqrt{2} = 2$

$\therefore$ Stable for $\underline{\underline{0 < K < 2}}$



asymptotes at $\pm 90°$

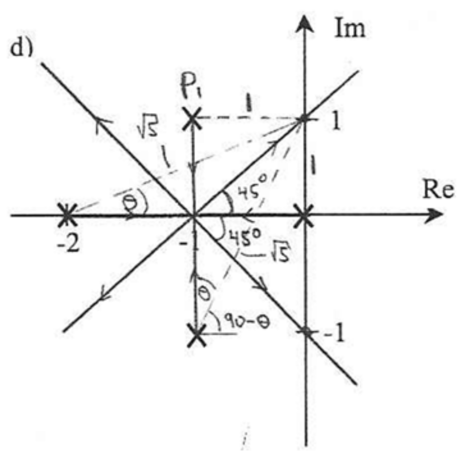
$$\alpha = \frac{-2-1-1-0}{2} = -2$$

dep: $\psi_1 - \phi_1 - \phi_2 - \phi_3 = \pm 180 \pm 360i$

$$135 - \phi_1 - 90 - 45 = \pm 180 \pm 360i$$

$$\phi_1 = 180$$

Stable for all $K \geq 0°$



asymptotes at $\pm 45°, \pm 135°$

$$\alpha = \frac{-2-1-1+0}{4} = -1$$

$s = \pm j$ is on loci since the angles from the poles of $L(s)$ to $s = j$ total

$$90° + 0° + \theta + (90 - \theta) = 180°$$

At $s = j$,

$$K = 1 \cdot 1 \cdot \sqrt{2^2+1^2} \cdot \sqrt{2^2+1^2}$$
$$= 5$$

$\therefore$ Stable for $0 < K < 5$

dep: $\phi_1 + 135 + 45 + 90 = \pm 180 \pm 360i$

$$\phi_1 = -90$$