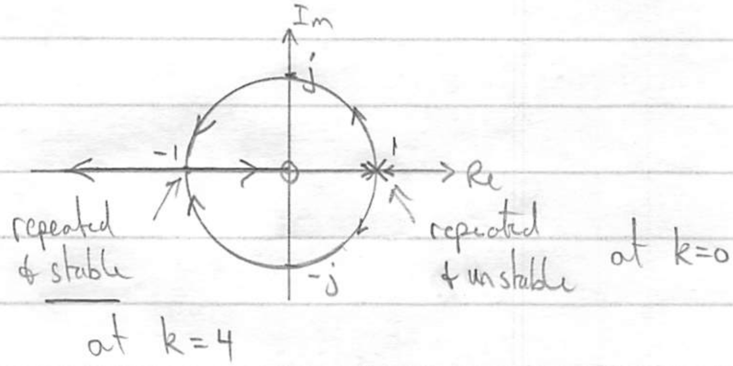
ENME 585 - Solutions to Quiz 2 W2023

1. $P(s) = \dfrac{s}{(s-1)^2}$

a)



b) $1 + kP(s) = 0$

$$1 + \frac{ks}{(s-1)^2} = 0$$

$$(s-1)^2 + ks = 0$$

$$s^2 + (k-2)s + 1 = 0 \qquad (1)$$

$s = j\omega \Rightarrow 1 - \omega^2 + j\omega(k-2) = 0 \quad \Rightarrow \quad \omega = \pm 1$ and $k = 2$

$\therefore$ R-L crosses at $\underline{\pm j}$, and $\underline{k > 2}$ for stability

c) Repeated poles at $-a$ when (1) has the form $(s+a)^2 = 0$

Setting $s^2 + (k-2)s + 1 = (s+a)^2$

$$= s^2 + 2as + a^2 \qquad (2)$$

$\Rightarrow a^2 = 1 \Rightarrow a = \pm 1 \Rightarrow a = +1$ for repeated stable poles at $s = -a = \underline{\underline{-1}}$

(2) also gives $k - 2 = 2a = 2$

$$\Rightarrow \underline{\underline{k = 4}}$$

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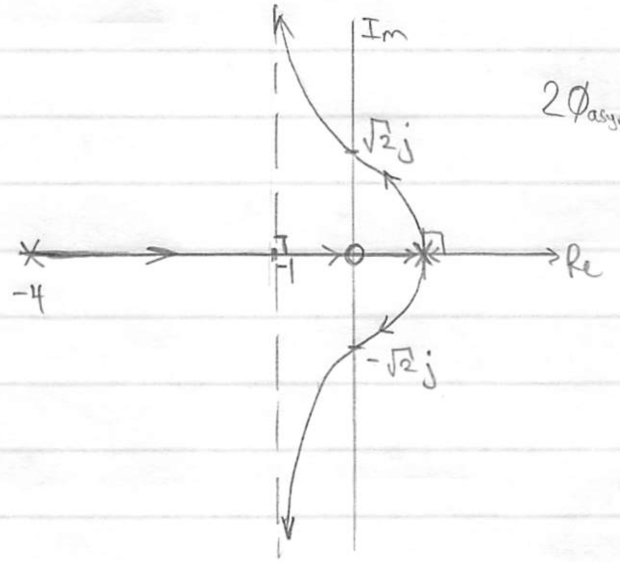
Alternatively, can see from R-L that poles repeat at $s = -1$ since R-L includes a circle.

Can then get k from magnitude condition:

$$k = \frac{|(s-1)^2|}{|s|} = \frac{(-1-1)^2}{|-1|} = 4 \checkmark$$

2. $1 + C(s)P(s) = 0 \Rightarrow 1 + kL(s) = 0$ where $L(s) = \dfrac{s}{(s+4)(s-1)^2}$

a)



$$2\phi_{asymp} = 180 \Rightarrow \phi_{asymp} = \pm 90^\circ$$

$$\alpha = \frac{-4+1+1}{3-1} = -1$$

$$2\phi_{dep} + 0 = 180^\circ$$

$$\phi_{dep} = \pm 90^\circ$$

b) $1 + kL(s) = 0$

$$(s-1)^2(s+4) + ks = 0$$

$$(s^2 - 2s + 1)(s+4) + ks = 0$$

$$s^3 + 2s^2 + (k-7)s + 4 = 0$$

$s = j\omega \Rightarrow$ $-j\omega^3 - 2\omega^2 + (k-7)j\omega + 4 = 0$

$$4 - 2\omega^2 + j\omega(k - 7 - \omega^2) = 0$$

$\Rightarrow \omega = \pm\sqrt{2}$ and $k = 7 + \omega^2 = 9$

$\therefore$ R-L crosses at $\underline{\pm\sqrt{2}j}$ and $\underline{k > 9}$ for stability

3. $C(D) = \frac{k}{D}$, $P(D) = \frac{1}{D+2}$

a) $T_{re}(D) = \frac{1}{1+PC} = \frac{1}{1+\frac{k}{D(D+2)}} = \frac{D(D+2)}{D^2+2D+k}$

b) $r = [1] = \frac{1}{D} \Rightarrow e = \frac{D+2}{D^2+2D+k} \equiv E(D)$ $e_{ss} = \lim_{s\to 0} s\,E(s) = \underline{\underline{0}}$

c) $r = [t] = \frac{1}{D^2} \Rightarrow e = \lim_{s\to 0}\left(s \frac{1}{s^2} T_{re}(s)\right) = \underline{\underline{\frac{2}{k}}}$

d) $T_{ry}(D) = \frac{PC}{1+PC} = \frac{k}{D^2+2D+k}$ from (a)

e) $r = \frac{1}{D} \Rightarrow y_{ss} = \lim_{s\to 0} s \frac{1}{s} T_{ry}(s) = \frac{k}{k} = \underline{\underline{1}}$

f) $T_{wy} = \frac{P}{1+PC} = \frac{D}{D^2+2D+k}$ from (a).

$w = \frac{1}{D} \Rightarrow y_{ss} = \lim_{s\to 0} s \frac{1}{s} T_{wy}(s) = \underline{\underline{0}}$

g) $s^2+2s+k = (s+a)^2$

$= s^2+2as+a^2$

$\Rightarrow a = 1$ and $k = a^2 = \underline{\underline{1}}$

h) If $k=2$, $s^2+2s+k = s^2+2s+2 = \underset{-\sigma}{(s+1)^2} + \underset{w_d}{1}$

$\Rightarrow \sigma = -1$ and $w_d = 1$

$t_p = \frac{\pi}{w_d} = \underline{\underline{\pi}}$

and $y_p = 1 + e^{-\sigma t_p} = 1 + e^{-\pi} = \underline{\underline{1.0432}}$