

So, roughly speaking, your goal is to minimize load in some way.

Let us assume we have servers s_1, s_2, \dots, s_n , groups G_1, \dots, G_m . We know the number of members of group i , $|G_i|$ for each $i \in \{1, 2, \dots, m\}$ and the capacity $c(i)$ of server s_i . An assignment $f : \bigcup_{i=1}^m G_i \rightarrow \{1, 2, \dots, n\}$ is a function with the property that $|f^{-1}(i)| \leq c(i)$. Here $f(x) = i$ means that person x is assigned to server i , and the condition simply states that the number of people assigned to server i is no more than the capacity of server i . The notation f^{-1} should be familiar from Calculus I.

Given an assignment, we have a way to compute the load that is on the connection between s_i and s_j : I will denote this load by $\ell_{ij}(f)$.

One way to minimize load is to minimize the load that you may have on any connections between two servers is to find

$$M_1 := \min_f \max_{1 \leq i < j \leq n} \ell_{ij}(f). \quad (0.1)$$

This makes sure that on any connection the load is as small as possible. However, this may make the load on any single server large (many small loads on the connections to that server may add up to a large load on the server). So your real goal is probably to find

$$M_2 := \min_f \max_{1 \leq i \leq n} \left(\sum_{\substack{j=1 \\ j \neq i}}^n \ell_{ij}(f) \right) \quad (0.2)$$

Now, if you want to get really fancy (and also you have a way to figure out how to do it...) you may want to find the set of assignment

$$\mathcal{F} = \left\{ f : \max_{1 \leq i \leq n} \left(\sum_{\substack{j=1 \\ j \neq i}}^n \ell_{ij}(f) \right) = M_2 \right\},$$

i.e. \mathcal{F} is the set that contain those assignments that achieve the goal (0.2) and then find amongst them one is best with respect to the first goal listed, i.e. one that achieves

$$\min_{f \in \mathcal{F}} \max_{1 \leq i < j \leq n} \ell_{ij}(f).$$

But we still need to decide how to compute $\ell_{ij}(f)$. I will list a couple of ways here. To make my life easier, I will define the membership function $m : \bigcup_{i=1}^m G_i \rightarrow \{1, 2, \dots, m\}$, where $m(x) = i$ means $x \in G_i$.

1. $\ell_{ij}(f) = 1$, if $m(f^{-1}(i)) \cap m(f^{-1}(j)) \neq \emptyset$, 0 otherwise. $f^{-1}(i)$ contains the people who are assigned to server i , and $m(f^{-1}(i))$ has the indices of those groups who have elements assigned to server i , so this means that the load is 1 if servers i, j both have some members from the same group (no matter how many). This is probably way too simplistic.
2. $\ell_{ij}(f) = |m(f^{-1}(i)) \cap m(f^{-1}(j))|$. This means that the load is the number of groups that have members on both server i and server j .
3. $\ell_{ij}(f) = \sum_{p=1}^m |G_p \cap f^{-1}(i)| \cdot |G_p \cap f^{-1}(j)|$. This means that for each group you multiply the number of members in server i with the number of members in server j , and then add the resulting numbers up. This means that for every pair of the group you count a unit amount of communication, and then you have to count how many pairs the assignment cuts.
4. $\ell_{ij}(f) = \sum_{p=1}^m \frac{|G_p \cap f^{-1}(i)| \cdot |G_p \cap f^{-1}(j)|}{\binom{|G_p|}{2}}$. This means that for each group you multiply the number of members in server i with the number of members in server j , and then add the resulting numbers

up, and divide with the number of possible connections within the group.. This means that for every member of the groups you count a unit amount of communication that he is going to spend on the group, and you assume that this communication is equally divided between the groups.

I suspect you need number 3. or 4. here.