MATHEMATICS IN ENGINEERING, SCIENCE AND AEROSPACE
MESA - www.journalmesa.com
Vol. 3, No. 2, pp. 151-158, 2012

Two independent approaches used for estimating 2d contamination distribution on the ground level- based on air monitoring information

Dubi Chen[1], Yaar Ilan[1], and Mark Shlomo[2,3]*

[1] Nuclear Research Center Negev,
P.O. Box 9001, Beer-Sheva, 84190, Israel

[2] Negev Monte Carlo Research Center (NMCRC),
Sami Shamoon College of Engineering (SCE),
Bialik/Basel Sts. Beer Sheva, 84100, Israel

[3] Software Engineering Department,
Sami Shamoon College of Engineering (SCE),
J'abotinski 84 Ashdod 77245 Israel

* *Corresponding Author.* E-mail: marks@sce.ac.il

Abstract. One of the main uses of radiation aerial monitoring is in the mapping of an area contaminated by radioactive isotopes. In this work, we will introduce two independent approaches, redundancy and complementary methods, to estimate the radiation activity level of a contaminated area, using air monitoring information. Due to mathematical limitations, aerial monitoring of a radioactive field can not provide a 3d distribution of the contamination within the radioactive cloud. However, a reasonably good estimation of the 2d contamination distribution on the ground can be obtained by measuring the radioactive field from a constant height of 100 to 500 meters. Two complementary independent approaches have to be used in order to solve this problem: the elementary matrix solution after a conditional simulation and the application based on Stein equation.

1 Introduction and Calculation principles

One of the principal uses of radiation aerial monitoring is in the mapping of an area contaminated by radioactive isotopes. The input data is composed of the radioactive contamination distribution found

2010 **Mathematics Subject Classification:**
Keywords: Air monitoring, matrix solution, Stein equation, scientific software development.

on a square ground patch obtained by a helicopter equipped with a γ radiation detector performing a scan flight over the patch. An imaginary N by N grid is placed on this patch, dividing it into N^2 individual squares. These square's flank lengths are the step lengths of the grid and these square's centers are the grid points. Each square (cell) of the grid is then considered to be of a homogeneous and isotropic activity. As such, each cell may be considered to have a "center of contamination", which will be treated in the calculations as a point source located at the cell's center. The relation between the measured radiation field and the contaminated area is given by a set of linear equations, in which the contamination is modeled as a lumped parameter, concentrated at the center of each square. The response function of a detector D to a radiation of unit of intensity emitted from a point source at a distance R from the detector is given by:

$$D = C(1+kR)e^{-\mu R}/R^2 \tag{1.1}$$

Where:
c = proportion coefficient of the monitor [cps m^2/gamma]
k = radiation build up factor in air [m^{-1}]
μ = radiation absorption coefficient in air [m^{-1}]
R = source to detector distance [m]

As shown in Figure 1. , the sampling area is divided into a (N x N) grid and the detector moves at a height h above the monitored area. The measurement obtained by the detector is the sum of contributions from all the contaminated area points, modeled by N^2 single point sources located at the center of the grid cells. Where the distance from the detector above point i to any given point j on the grid is given by:

$$R^2_{i,j} = (X_i - X_j)^2 + (Y_i - Y_j)^2 + Z^2 \tag{1.2}$$

where X,Y,Z are Cartesian coordinates (m).

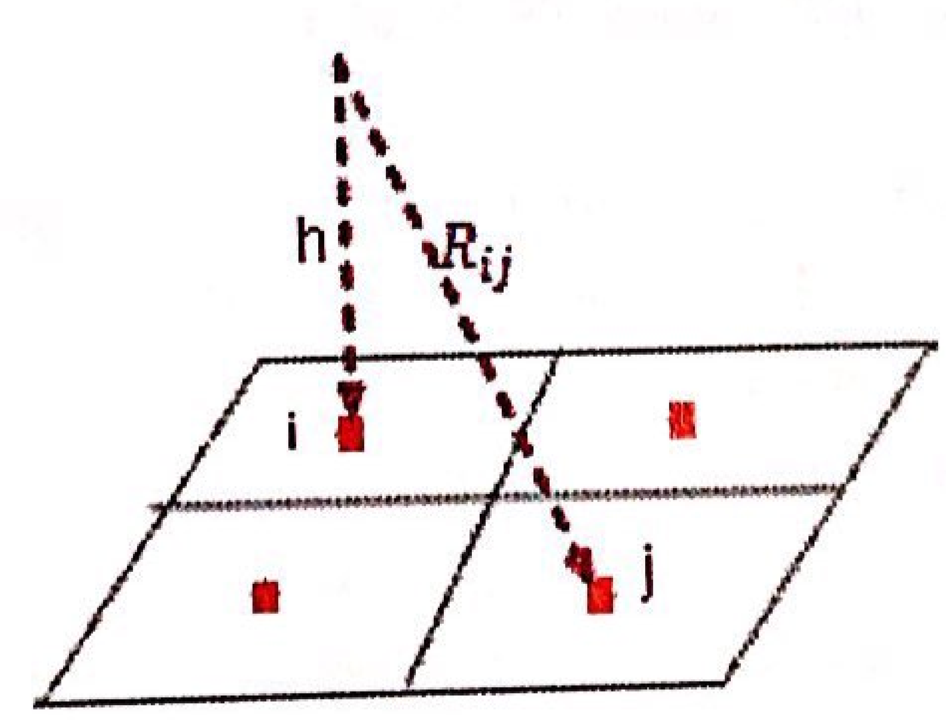

Fig. 1 Sensor location and distances with respect to cells i and j.

Substituting Equation (2) into Equation (1), provides
-the detector's response function from cell j when the sensor is located in height h exactly above cell i:

$$D_{i,j} = \frac{c\,(1+k\cdot R_{i,h})\,e^{-\mu R_{i,j}}}{R^2_{i,j}} \tag{1.3}$$

We denote the amount of contamination concentrated at point j as C_j, being roughly equal to the total contamination within the respective square. The detector reading above cell i is given by:

$$M_i = \sum_j R_{i,j} C_j \tag{1.4}$$

Where j is summed over all of the grid points.

This produces N^2 linear equations that link the unknown contamination at cell j (C_j) with the measurements M_i. In a matrix notation:

$$\mathbf{D}\,\mathbf{C} = \mathbf{M} \tag{1.5}$$

Whose solution is,

$$\mathbf{C} = \mathbf{D}^{-1}\mathbf{M} \tag{1.6}$$

Where,

D - Matrix coefficient from Eq. 3.

C - Unknown vector of intensities

M - The measured values vector

The matrix solution of Eq. 6 would appear to provide a general, consistent method for calculating the contamination field distribution from the measurements of radiation done by the helicopter. A number of issues, however, rise when the method is applied. Eq. 6 is solvable only for certain parameters, i.e. it is satisfied only when the inverse matrix $\mathbf{D}^{-1}$ exists, so that the relation presented in Eq. 7 is satisfied;

$$\mathbf{D}^{-1}\mathbf{D} = \mathbf{I} \tag{1.7}$$

The numerical values of the components of D (Eq. 3) vary over a range of several orders of magnitude within the matrix, so that even if D^{-1} exists, it may occasionally be non-conditioned, making the inverse matrix incomputable and inhibiting the solution of Eq. 6. One can see from Eq. 3 and Eq. 4 that while the matrix depends on three parameters: the detector response, the height of the measurements R, and the grid size it is completely independent of the level of contamination in the field. Computing D^{-1} enables us to map the space of two-dimensional parameters (detector height, step size) and find the area which satisfies Eq. 6. We can then find the solution of any level of contamination on this area. Computing the contamination field requires the discretization of a continuous contamination distribution to a finite number of point sources, with the contamination of each square concentrated at a single point at its exact center. These are the unknowns of Eq. 6, and they have the same number of points as the measured points. They are computed by measurements of the continuous field taken from above each point of the grid, and are solved using Eq. 6. In practice, readings taken from a helicopter may disrupt the distribution of contaminating particles in an area, when performed within a certain distance range; i.e. downdrafts from the helicopter blades may cause a radioactive particle resuspension, if the helicopter comes too close. However, if it hovers at too great a height, individual readings from sample cells will converge, and nullify efforts to differentiate point source activity levels. In addition, increasing grid dimensions (respectively shrinking cell size) to enhance accuracy will result in increased readings due to the cumulative effect of point sources, as well as in an increased complexity of analysis. In fact, the lumped parameter assumption of all radiation centered at a discrete number of points can be made only for certain parameters. Only above a certain number of cells, N^2, does the lumped discretization result in small enough round-off errors.

Actually, there exists an optimum value (or range of values) for each parameter, so that the prior assumptions remain valid (mainly that each grid cell is of a homogeneous and isotropic activity), while observer interference and analysis complexity are minimized. In order for the matrix in Eq. 5 to be solvable, this optimum must be found.

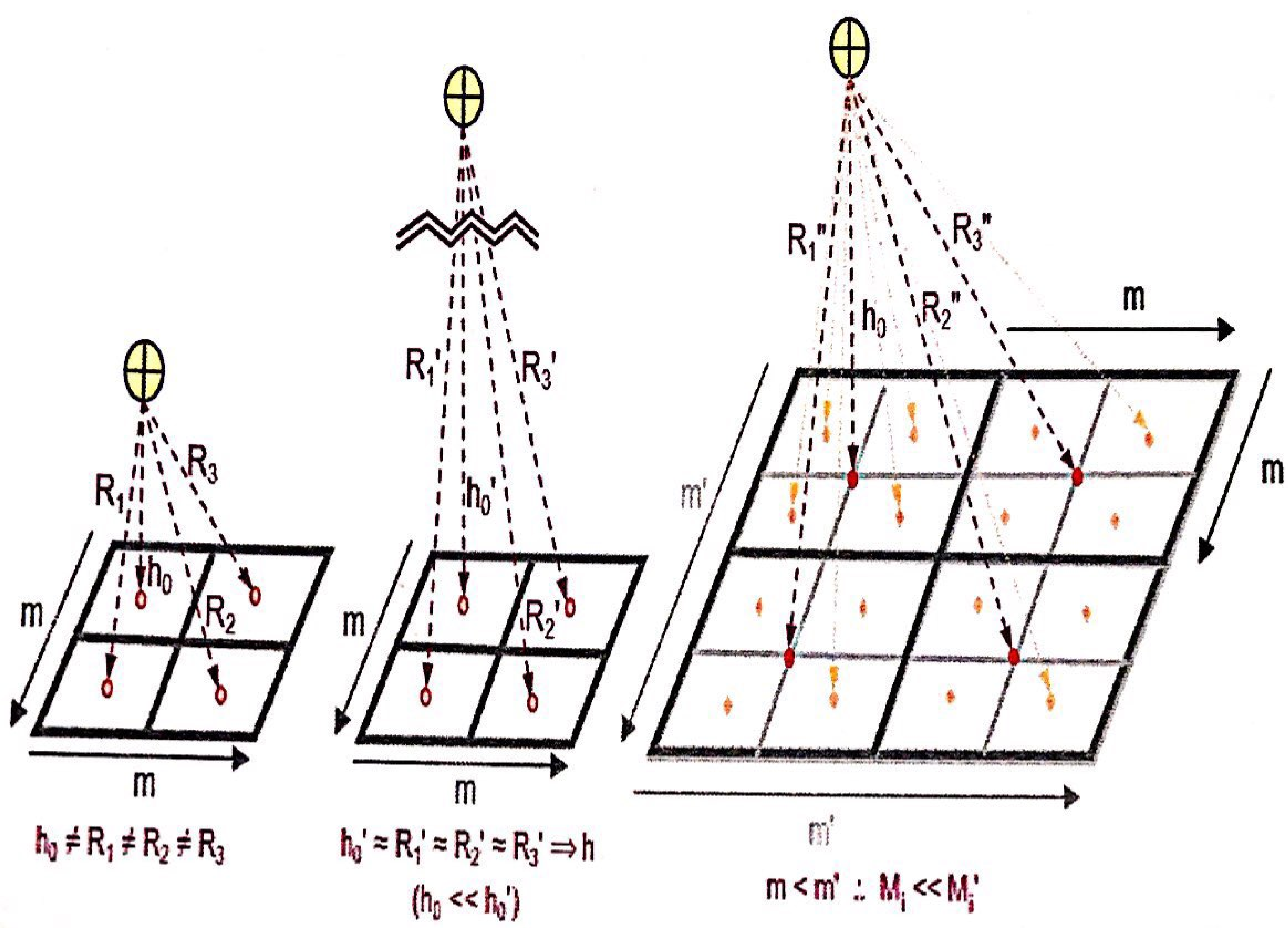

Fig. 2 Complications arising from different measurement configurations

2 Software development considerations

One of the main issues in software development is the motivation to improve the development of the software's scientific part. Mostly,, the simulated process is complex, large, confusing, highly sensitive to parameter changes and expensive. Many articles [1, 9-11] concentrate on understanding the factors that effect scientific software development. Most of them, however, focus mainly on the production process and lifecycle of the software. The software that should be developed in order to find the real 2d ground contamination concentration based on the helicopter reading, is highly sensitive to parameter changes. Add to that the fact that faults may arise from a variety of sources, including: lack of precise understanding of the process, flaws in the mathematical model as an abstraction of the process, difficulties in translating the mathematical model into a computable form, program error, logical flaws and imprecision in computer arithmetic and you get a very sensitivity system without a proven process. Our system belongs to the category of real time computing tools, in which the parameters that influence the system and effect the ability to improve the results, derive from the detection action and directly depend on it (matrix D). This limitation is essentially important especially when the tool to improve the results dose not exist.

The answer in the case of a multitude of sources of failure is often based simply on a comparison between computer output and physical observations, a comparison between different software approaches and especially a comparison between different independent approaches; redundancy and complementary [6].

This article presents the case study of aerial radiation monitoring; the case requires the solution of a linear system of equations, where the matrix might be very ill-conditioned. This problem is a so-called ill-posed problem. There are available methods for solving this type of problems [2,3] , Tikhonov regularization is one approach, another possibility is using an iterative method and terminating the iterations sufficiently early.

In this article we want to solve this case study from the view point of software engineers, which need to assure the user (technician or physicist) that the results he obtains are validated and verified.The comparison independent approaches were found to be the most suitable methods for improving the results. The two complementary independent approaches that we choose to establish here are the elementary matrix solution based on conditional simulation and the application based on the Stein equation.

3 The elementary matrix solution after a conditional simulation

As earlier mentioned, in order to solve Eq. 6 for a given detector, we have to find the optimum values (or range of values) for the height of the measurements R, and the number of grid points N^2, that will cause the matrix D not to be ill-conditioned, or require much of the computer memory to compute the inverse $\mathbf{D}^{-1}$. Once these values are found, Eq. 6 can be solved with any elementary methods.