H.W1

1) $y' = x\sin 3x$

$u = x \qquad dv = \sin 3x$

$du = dx \qquad v = -\frac{1}{3}\cos 3x$

$y = -\frac{1}{3}x\cos(3x) - \int -\frac{1}{3}\cos(3x)\,d$

$\boxed{= \frac{1}{9}\sin(3x) - \frac{1}{3}\cos(3x) + C}$

2) $\frac{dy}{dt} = e^{-t}\cos(4t)$; $y(0)=1$

$dy = \frac{\cos(4t)}{e^t}\,dt$

$y = \int \frac{\cos(4t)}{e^t}\,dt$

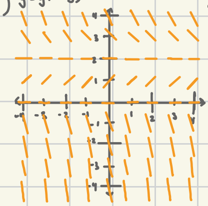
$y = e^{-t}\left(\frac{-\cos(4t) + 4\sin(4t)}{17}\right) + C$

$1 = e^0\left(\frac{-\cos(0) + 4\sin(0)}{17}\right) + C$

$C = \frac{18}{17} \Rightarrow y = \boxed{\frac{-\cos(4t) + 4\sin(4t) + 18e^t}{17e^t}}$

3)

a) $y' = y(2-y)$



b)

- $y=0 \Rightarrow y'(x,0) = 0(2-0) = 0$
- $y=2 \Rightarrow y'(x,2) = 2(2-2) = 0$
- $y = \frac{2e^{2x}}{e^{2x}+C} \Rightarrow y'\left(x, \frac{2e^{2x}}{e^{2x}+C}\right)$

$= \frac{2e^{2x}}{e^{2x}+C}\left(2 - \frac{2e^{2x}}{e^{2x}+C}\right) = \frac{4e^{2x}C}{(e^{2x}+C)^2}$

c) $y(x) = \frac{2}{1+Ce^{-2x}} = \frac{2e^{2x}}{e^{2x}+C}$

- $y(0) = 0 \Rightarrow y = 0$
- $y(0) = 2 \Rightarrow y = 2$

not dependent on x

$y(0) = 1 \Rightarrow \frac{2}{1+C} = C = 1 \Rightarrow y = \frac{2e^{2x}}{e^{2x}+1}$

4) $y' = \frac{2xy+2x}{x^2-1} \Rightarrow \frac{dy}{dx} = \frac{2x(y+1)}{x^2-1}$

$\Rightarrow \frac{dy}{y+1} = \frac{2x\,dx}{x^2-1} \Rightarrow \ln|y+1| = \ln|x^2-1| + C$

Let $A = \pm + C$ $\qquad y+1 = \pm x^2 - 1 + C$

$\Rightarrow \boxed{y = A(x^2-1) - 1}$

5) $y\frac{dy}{dt} = -t(1+y^2)$; $y(0) = -1$

$\frac{dy}{dt} = -t(1+y^2)$

$\frac{y}{1+y^2}dy = -t\,dt$

$\frac{1}{2}\ln|1+y^2| = -\frac{1}{2}t^2 + C$

$1+y^2 = \pm e^{-t^2}e^{C_1}$

$y = \pm\sqrt{e^{-t^2}e^{C_1} - 1} \Rightarrow \boxed{y = -\sqrt{2e^{-t^2} - 1}}$

$-1 = \pm\sqrt{e^0 e^{C_1} - 1}$

$-1 = \pm\sqrt{e^{C_1} - 1}$

$e^{C_1} = 2$

$C_1 = \ln(2)$

I of E: $2e^{-t^2} - 1 \geq 0 \Rightarrow \boxed{-\sqrt{\ln(2)} \leq t \leq \sqrt{\ln(2)}}$

$e^{-t^2} \geq \frac{1}{2}$

$t^2 \leq -\ln|\frac{1}{2}|$

$t \leq \pm\sqrt{\ln(2)}$

6) Given : $y = y(x)$

$T = y'(x)(t-x) + y$

$T(0) = -y'x + y = 2y$

$\Rightarrow y = -y'x \Rightarrow \boxed{y' = -\frac{y}{x}}$

where (1,1) are the initial values

b) $\frac{dy}{dx} = -\frac{y}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$

$\ln|y| = \ln|x^{-1}|$

$\Rightarrow \boxed{y = \frac{1}{x}}$

7) $y' + \frac{3}{x}y = \frac{\cos x}{x^2}$

$I(x) = e^{\int \frac{3}{x}dx} = x^3$

$\Rightarrow (x^3y)' = \int x\cos x\,dx$

$x^3y = x\sin x + \cos x + C$

$\boxed{y = \frac{x\sin x + \cos x + C}{x^3}}$

$\boxed{y_h = \frac{C}{x^3}}$

8) $(1+t^2)y' + 4ty = \frac{1}{(1+t^2)^2}$; $y(1) = 0$

$y' + \frac{4t}{1+t^2}y = \frac{1}{(1+t^2)^3}$

$I(x) = e^{\int \frac{4t}{1+t^2}dt} = (1+t^2)^2$

$((1+t^2)^2y)' = \int \frac{1}{1+t^2}$

$y = \frac{\tan^{-1}(t) + C}{(1+t^2)^2} \Rightarrow 0 = \frac{\tan^{-1}(1) + C}{4} \Rightarrow C = -\frac{\pi}{2}$

$\boxed{y = \frac{\tan^{-1}(t) - \frac{\pi}{2}}{(1+t^2)^2}}$