

A line between two points

Given the two points $P_1 = (u_1, v_1)$, $P_2 = (u_2, v_2)$. Find the equation of the line that passes through both.

Case I: $u_1 = u_2 = \alpha$

This is a vertical line. The line equation: $\boxed{u = \alpha}$.

Case II: $u_1 \neq u_2$

$$v - v_1 = a(u - u_1) \quad \text{or} \quad v = au + (v_1 - au_1), \quad \text{where} \quad a = \frac{v_2 - v_1}{u_2 - u_1}$$