

## Geometric-Transformation-Example-Solutions

### Question 1

You are given the following (forward) geometric transformation:

The pixel at coordinate  $(x, y)$  in the original picture moves to the location  $(\frac{2x+y}{3}, x)$  in the new picture.

Describe the result of applying this transformation to the following picture. Compute the transformed image only in a  $4 \times 3$  window, where the values of  $x$  are 2, 3, 4, and the values of  $y$  are 1, 2, 3, 4.

	$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$
$y = 0$	0	1	2	3	4
$y = 1$	1	2	3	4	5
$y = 2$	2	3	4	5	6
$y = 3$	3	4	5	6	7
$y = 4$	4	5	6	7	8
$y = 5$	5	6	7	8	9
$y = 6$	6	7	8	9	9
$y = 7$	7	8	9	9	9
$y = 8$	8	9	9	9	9

### Answer

Write the forward transformation as:

$$\frac{2x + y}{3} = X, \quad x = Y$$

Solving for  $x, y$  we get:

$$x = Y, \quad y = 3X - 2Y$$

Therefore, the inverse transformation is:

$$X = \text{new } x = y, \quad Y = \text{new } y = 3x - 2y$$

The following table shows where each pixel is coming:

	$X = 2$	$X = 3$	$X = 4$
$Y = 1$	$x = 1, y = 4$	$x = 1, y = 7$	$x = 1, y = 10$
$Y = 2$	$x = 2, y = 2$	$x = 2, y = 5$	$x = 2, y = 8$
$Y = 3$	$x = 3, y = 0$	$x = 3, y = 3$	$x = 3, y = 6$
$Y = 4$	$x = 4, y = -2$	$x = 4, y = 1$	$x = 4, y = 4$

Since all locations are integer coordinates, the result of nearest neighbor is the same as the result of bilinear interpolation, and it is given by:

	$x = 2$	$x = 3$	$x = 4$
$y = 1$	5	8	0*
$y = 2$	4	7	9
$y = 3$	3	6	9
$y = 4$	0*	5	8

The two pixels marked with a \* are out of the range of the original image.