Q/conv4.stex
% ho06 ho06f ho05 ho07 ho08 ho07f ho09 ho12 ho11 ho10 ho12f ho13 ho16 ho17 ho18 ho19

1	−1

A

1	0

B

1
1

C

1	0
3	2

D

Cross correlation of a picture with the mask in A, then cross correlation of the result with the mask in B, then cross correlation of the result with the mask in C, then cross correlation of the result with the mask in D, is the same as cross correlation of the same picture with the following mask:

1	-1		
4	-2	-2	
3	-1	-2	

$$
\begin{aligned}
D \otimes (C \otimes (B \otimes (A \otimes P))) &= D' \star (C' \star (B' \star (A' \star P))) \\
&= (D' \star C' \star B' \star A') \star P \\
&= (D' \star (C' \star (B' \star A'))) \star P \\
&= (D \otimes (C \otimes (B \otimes A'))) \star P \\
&= (D \otimes (C \otimes ((1,0) \otimes (-1,1)))) \star P \\
&= (D \otimes (C \otimes ((-1,1)))) \star P \\
&= (D \otimes (\begin{pmatrix}1 & 1\end{pmatrix} \otimes ((-1,1)))) \star P \\
&= (D \otimes \begin{pmatrix}-1 & 1 \\ -1 & 1\end{pmatrix}) \star P \\
&= \begin{pmatrix}-2 & -1 & 3 \\ -2 & -2 & 4 \\ 0 & -1 & 1\end{pmatrix} \star P
\end{aligned}
$$