The Hough Transform

Input: A set of points, p_1, p_2, $\ldots$,. The points are given by their coordinates: $p_i = (x_i, y_i)$.

Output: A set of patterns in parametric representation.

Patterns: Defined by a set of w parameters $\phi = (\phi_1, \ldots, \phi_w)$, typically by an equation of the form:

$$q(x, y, \phi_1, \ldots, \phi_w) = 0. \tag{1}$$

We write the arguments of Equation (1) in a vector form as $q(p, \phi) = 0$.

A pattern q_ϕ is the set of points p such that $q(p, \phi) = 0$ for all $p \in q_\phi$. The *size* of a pattern in the input data is the number of input points belonging to the pattern.
The Hough transform measures the pattern size:

$$H(\phi) = \text{number of picture points on the pattern } q_\phi. \tag{2}$$

Discretization: The continuous parameter space is divided into (rectangular) cells. Let Γ be the set of all cells.

The Hough Transform Algorithm:

- initialize accumulators: $H(\gamma) = 0$, for all cells $\gamma \in \Gamma$.
- for each input points p, increment $H(\gamma)$ for all cells γ that contain a parameter vector for which $q(p, \phi)$ is "small".
- produce as likely patterns the parameters of the cells γ for which $H(\gamma)$ is large.

Practical issues: The critical points in applying the algorithm are:

- determining the parametrization
- determining the discretization
- the implementation of the second and third step of the algorithm.

We always assume that the second step of the algorithm is implemented by the following procedure (even though it may not always be possible). Rewrite Equation 1 as:

$$\phi_w = q'(x, y, \phi_1, \ldots, \phi_{w-1})$$

and for each point (x, y) go over all possible values of $\phi_1, \ldots, \phi_{w-1}$ and increment the cell specified by $\phi_1, \ldots, \phi_{w-1}, \phi_w$, where ϕ_w is specified by the above equation.