Homework-3 Solutions

Question 1

Derive equations for computing the optimal threshold if the error of moving a pixel of value x to a pixel of value q is $|x-q|$ and not $(x-q)^2$ as was used in class.

Solution:

Let h be the picture histogram, so that $h(x)$ is the number of pixels of value x, for $x = 0, \ldots, M$. We are looking for a threshold value t and two values q_1, q_2, such that all pixels in the range $0 \leq x < t$ are replaced with q_1, and all pixels in the range $t \leq x \leq M$ are replaced with q_2. Measuring the error as the absolute value of the difference we have the following expression for the total error:

$$e(t, q_1, q_2) = \sum_{x=0}^{t-1} |x - q_1| h(x) + \sum_{x=t}^{M} |x - q_2| h(x).$$

Notice that if $y = |x|$ then

$$y' = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

The value of y' is undefined for $x = 0$, but To simplify the computation we define it to be 0.
Therefore, taking the derivative of e with respect to q_1 we have:

$$\frac{\partial e}{\partial q_1} = \sum_{x=0}^{q_1-1} h(x) - \sum_{x=q_1+1}^{t-1} h(x).$$

The requirement that $\frac{\partial e}{\partial q_1} = 0$ gives:

$$\sum_{x=0}^{q_1-1} h(x) = \sum_{x=q_1+1}^{t-1} h(x). \tag{1}$$

Thus, q_1 is the *median* of the histogram values in the range $0, \cdots, t-1$. Similarly, the equation for q_2 is:

$$\sum_{x=t}^{q_2-1} h(x) = \sum_{x=q_2+1}^{M} h(x), \tag{2}$$

so that q_2 is the *median* of the histogram values in the range $t, \cdots, M$.

Question 2

You are given the following image:

6	6	6	10
6	6	6	10
17	17	17	17
17	17	17	88

1.

What is the image histogram?

Answer:

Pixel value	0	⋯	6	⋯	10	⋯	17	⋯	88	⋯	255
# pixels	0	⋯	6	⋯	2	⋯	7	⋯	1	⋯	0

2.

What would be the result of applying the optimal thresholding algorithm that was discussed in class to this image?

Answer:

The threshold value is $t = 18$. ($q_1 = 11.7$, $q_2 = 88$.) The picture after the threshold is applied is:

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	255

3.

What image is obtained by linearly scaling the pixel values to the $0 - 255$ range?

$$x \to (x - 6) * 255/82$$

0	0	0	12
0	0	0	12
34	34	34	34
34	34	34	255

4.

What image is obtained by histogram equalization to the $0 - 255$ range?

48	48	48	112
48	48	48	112
184	184	184	184
184	184	184	248

Question 3

You are given the following color image, where the pixel values are in sRGB. The value of each pixel is given as the triplet (r, g, b).

(0,0,0)	(0,0,0)	(0, 0, 0)	(0, 0, 0)
(255, 0, 0)	(255, 0, 0)	(255, 0, 0)	(255, 0, 0)
(100, 100, 100)	(100, 100, 100)	(100, 100, 100)	(100, 100, 100)
(0, 100, 100)	(0, 100, 100)	(0, 100, 100)	(0, 100, 100)

A

Convert the above image to nonlinear RGB.

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0,0,0)
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)
(0.392, 0.392, 0.392)	(0.392, 0.392, 0.392)	(0.392, 0.392, 0.392)	(0.392, 0.392, 0.392)
(0, 0.392, 0.392)	(0, 0.392, 0.392)	(0, 0.392, 0.392)	(0, 0.392, 0.392)

B

Convert the above image to linear RGB.

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0,0,0)
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)
(0.127,0.127,0.127)	(0.127,0.127,0.127)	(0.127,0.127,0.127)	(0.127,0.127,0.127)
(0,0.127,0.127)	(0,0.127,0.127)	(0,0.127,0.127)	(0,0.127,0.127)

C

Convert the above image to XYZ.

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0,0,0)
(0.412, 0.212, 0.019)	(0.412, 0.212, 0.019)	(0.412, 0.212, 0.019)	(0.412, 0.212, 0.019)
(0.121,0.127,0.138)	(0.121,0.127,0.138)	(0.121,0.127,0.138)	(0.121,0.127,0.138)
(0.07,0.1,0.136)	(0.07,0.1,0.136)	(0.07,0.1,0.136)	(0.07,0.1,0.136)

D

Convert the above image to xyY.

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0,0,0)
(0.64, 0.33, 0.212)	(0.64, 0.33, 0.212)	(0.64, 0.33, 0.212)	(0.64, 0.33, 0.212)
(0.3127,0.3129,0.127)	(0.3127,0.3129,0.127)	(0.3127,0.3129,0.127)	(0.3127,0.3129,0.127)
(0.2247,0.3288,0.1)	(0.2247,0.3288,0.1)	(0.2247,0.3288,0.1)	(0.2247,0.3288,0.1)

E

Convert the above image to Luv.

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0,0,0)
(53.2, 175.0, 37,7)	(53.2, 175.0, 37,7)	(53.2, 175.0, 37,7)	(53.2, 175.0, 37,7)
(42.37, 0.0, 0.0)	(42.37, 0.0, 0.0)	(42.37, 0.0, 0.0)	(42.37, 0.0, 0.0)
(37.9, -29.29, -6.339)	(37.9, -29.29, -6.339)	(37.9, -29.29, -6.339)	(37.9, -29.29, -6.339)

F

Compute linear illumination stretching in the Luv domain, and convert the result back to sRGB.

The minimum value of L is 0, and the maximum value is 53.2 Multiply each L value by 100/53.2. This gives the following image in Luv (only first column is shown):

(0,0, 0)
(100, 175.0, 37.7)
(79.64, 0, 0)
(71.2, -29.29, -6.339)

Converting to XYZ:

(0,0, 0)
(1.5, 1, 0.53)
(0.533, 0.56, 0.61)
(0.344, 0.425, 0.523)

Converting to linear RGB:

(0,0, 0)
(1, 0.69, 0.69)
(0.56, 0.56, 0.56)
(0.2, 0.485, 0.485)

Converting to nonlinear RGB in bytes:

(0,0, 0)
(255, 177, 177)
(197, 197, 197)
(124, 185, 185)