# Computer Vision Spring 2020 Test 1 Solutions

# Question 1 Part A

| | | | |
|---|---|---|---|
| 10 | 10 | 10 | 10 |
| 10 | 10 | 10 | 10 |
| 20 | 20 | 20 | 30 |
| 30 | 40 | 50 | 60 |

**1.**

What is the picture histogram?

| pixel value | # pixels |
|---|---|
| 10 | 8 |
| 20 | 3 |
| 30 | 2 |
| 40 | 1 |
| 50 | 1 |
| 60 | 1 |

**2.**

An optimality criterion for thresholding was described in class. According to that criterion, which of the following values is a better threshold value? Choose ONE value.

1 15 (25) 45 130

$$
\begin{array}{lcll}
t = 15 & \rightarrow & q_1 = 10,\ q_2 = 33.75, & E_{15} = 1587.5 \\
t = 25 & \rightarrow & q_1 = 12.7,\ q_2 = 42, & E_{25} = 898.2 \\
t = 45 & \rightarrow & q_1 = 17.2,\ q_2 = 55, & E_{45} = 1335.7
\end{array}
$$

Apply your selection of threshold to the picture. The picture after the threshold is applied is:

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

**3.**

What picture is obtained by linearly scaling the pixel values OF THE ORIGINAL PICTURE to the $0-255$ range?

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 51 | 51 | 51 | 102 |
| 102 | 153 | 204 | 255 |

**4.**

What picture is obtained by histogram equalization OF THE ORIGINAL PICTURE to the $0-255$ range?

| $i$ | $h(i)$ | $f(i)$ | $\frac{f(i-1)+f(i)}{2} \cdot \frac{256}{16}$ | floor |
|---|---|---|---|---|
| 10 | 8 | 8 | 64 | 64 |
| 20 | 3 | 11 | 152 | 152 |
| 30 | 2 | 13 | 192 | 192 |
| 40 | 1 | 14 | 216 | 216 |
| 50 | 1 | 15 | 232 | 232 |
| 60 | 1 | 16 | 248 | 248 |

| | | | |
|---|---|---|---|
| 64 | 64 | 64 | 64 |
| 64 | 64 | 64 | 64 |
| 152 | 152 | 152 | 192 |
| 192 | 216 | 232 | 248 |

## Question 1 Part B

Let $P$ be an image.

**1** Let $P_1$ be the result of applying histogram equalization to $P$.

**2** Let $P_2$ be the result of applying histogram stretching (linear scaling) to $P$.

**3** Let $P_3$ be the result of applying histogram equalization to $P2$.

We wish to compare $P_1$ with $P_3$.

**a.** Which image is typically brighter? (Select most appropriate answer).

**Answer:**

$a_1$. $P_1$ is typically brighter than $P_3$.

$a_2$. $P_3$ is typically brighter than $P_1$.

$\boxed{a_3.}$ $P_1$ and $P_3$ typically have similar brightness.

$a_4$. Impossible to answer without additional information.

**b.** Which image typically has more contrast? (Select most appropriate answer).

**Answer:**

$b_1$. $P_1$ typically has more contrast than $P_3$.

$b_2$. $P_3$ typically has more contrast than $P_1$.

$\boxed{b_3.}$ $P_1$ and $P_3$ typically have similar contrast.

$b_4$. Impossible to answer without additional information.

## Question 2 Part A

Recall that in one dimension the second derivative is the derivative of the first derivative, the third derivative is the derivative of the second derivative, the fourth derivative is the derivative of the third derivative, etc.

It was proposed in class that cross-correlation of the mask $(-1, 1)$ with an image can be used as an approximation of its first derivative in the horizontal direction, and cross-correlation of the mask $(1, -2, 1)$ with an image can be used as an approximation of its second derivative in the horizontal direction.

### A

Propose a cross-correlation mask that can be used to approximate the third derivative of an image in the horizontal direction.

### Answer:

(-1, 3, -3, 1)

$$
\begin{aligned}
&(-1, 1) \otimes ((1, -2, 1) \otimes P) \\
&\qquad = (1, -1) * ((1, -2, 1) * P) \\
&\qquad = ((1, -1) * (1, -2, 1)) * P \\
&\qquad = ((-1, 1) \otimes (1, -2, 1)) * P \\
&\qquad = (1, -3, 3, -1) * P \\
&\qquad = (-1, 3, -3, 1) \otimes P
\end{aligned}
$$

### B

Propose a cross-correlation mask that can be used to approximate the fourth derivative of an image in the horizontal direction.

### Answer:

(1,-4,6,-4,1)

$$
\begin{aligned}
&(-1, 1) \otimes ((-1, 3, -3, , 1) \otimes P) \\
&\qquad = (1, -1) * ((1, -3, 3, -1) * P) \\
&\qquad = ((1, -1) * (1, -3, 3, -1)) * P \\
&\qquad = ((-1, 1) \otimes (1, -3, 3, -1)) * P \\
&\qquad = (1, -4, 6, -4, 1) * P \\
&\qquad = (1, -4, 6, -4, 1) \otimes P
\end{aligned}
$$

## Question 2 Part B

The pixels of the noisy image $P$ are shown below, where space indicates the value of “0”.

$P =$

| | $x$=0 | $x$=1 | $x$=2 | $x$=3 | $x$=4 | $x$=5 | $x$=6 | $x$=7 | $x$=8 | $x$=9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$=0 | | | | | | | | | | |
| $y$=1 | | | | | | | | | | |
| $y$=2 | | | 100 | 100 | | 100 | 100 | 100 | | |
| $y$=3 | | 100 | | | | | 100 | 100 | | |
| $y$=4 | | | | | | | | | 100 | |
| $y$=5 | | | | 100 | | 100 | 100 | | | |
| $y$=6 | | | | | | | | | | |
| $y$=7 | | | | | 100 | 100 | 100 | | | |
| $y$=8 | | | | | 100 | 100 | | | | |
| $y$=9 | | | | | | | | | | |

What image is obtained if noise cleaning is applied to $P$, using the technique of smoothing with the mask shown below. The mask is of size $21 \times 21$. Each mask value is $1/21^2$. Explain your answer.

$\frac{1}{21^2}$

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

## Answer:

$P =$

| | $x$=0 | $x$=1 | $x$=2 | $x$=3 | $x$=4 | $x$=5 | $x$=6 | $x$=7 | $x$=8 | $x$=9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$=0 | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ |
| $y$=1 | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ |
| $y$=2 | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ |
| $y$=3 | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ |
| $y$=4 | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ |
| $y$=5 | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ |
| $y$=6 | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ |
| $y$=7 | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ |
| $y$=8 | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ |
| $y$=9 | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ | $z$ |

with

$$z = \frac{17 \cdot 100}{21 \cdot 21} \approx 3.8548752 \to 4$$

Explanation:

## Question 3

| | $x=0$ | $x=1$ | $x=2$ | $x=3$ | $x=4$ | $x=5$ | $x=6$ | $x=7$ |
|---|---|---|---|---|---|---|---|---|
| $y=0$ | 10 | 20 | 30 | 40 | 50 | 0 | 70 | 80 |
| $y=1$ | 40 | 50 | 60 | 30 | 50 | 0 | 60 | 70 |
| $y=2$ | 70 | 80 | 90 | 20 | 50 | 0 | 50 | 60 |
| $y=3$ | 100 | 110 | 120 | 10 | 50 | 0 | 40 | 50 |
| $y=4$ | 130 | 140 | 150 | 0 | 50 | 0 | 30 | 40 |
| $y=5$ | 160 | 170 | 180 | 0 | 50 | 0 | 20 | 30 |
| $y=6$ | 190 | 200 | 210 | 0 | 50 | 0 | 10 | 20 |

The above picture is transformed by a geometric transformation. The (forward) description of this transformation is:

> The pixel at coordinate $(x, y)$ in the original picture moves to the location $(6-3y, 6-2x)$ in the new picture.

Inverse transformation:

$$\alpha'(x,y) = \frac{6-y}{2} \qquad \beta'(x,y) = \frac{6-x}{3}$$

Where each pixel is coming from:

| | $x=0$ | $x=1$ |
|---|---|---|
| $y=1$ | (5/2,2) | (5/2,5/3) |

### A.

Compute the transformed image using Nearest-Neighbor interpolation over the window specified below

| | $x=0$ | $x=1$ |
|---|---|---|
| $y=1$ | 90 | 90 |

or

| | $x=0$ | $x=1$ |
|---|---|---|
| $y=1$ | 20 | 20 |

### B.

Compute the transformed image using Bilinear interpolation over the window specified below

| | $x=0$ | $x=1$ |
|---|---|---|
| $y=1$ | 55 | 52 |

$$\begin{aligned}
P(5/2,2) &= \frac{1}{2} * P(2,2) + \frac{1}{2} * P(3,2) = \frac{1}{2} * 90 + \frac{1}{2} * 20 = 55 \\
P(5/2,5/3) &= \frac{1}{2} * \frac{1}{3} * P(2,1) + \frac{1}{2} * \frac{2}{3} * P(2,2) + \frac{1}{2} * \frac{1}{3} * P(3,1) + \frac{1}{2} * \frac{2}{3} * P(3,2) \\
&= \frac{1}{2} * \frac{1}{3} * 60 + \frac{1}{2} * \frac{2}{3} * 90 + \frac{1}{2} * \frac{1}{3} * 30 + \frac{1}{2} * \frac{2}{3} * 20 \\
&= 51.67
\end{aligned}$$

## Question 4

You are given the following template $\lambda$ and the image $A$:

$$\lambda = \begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & 1 & 1 \\ \hline 0 & 2 & 0 \\ \hline \end{array}$$

$A =$

| | x=0 | x=1 | x=2 | x=3 | x=4 | x=5 | x=6 |
|---|---|---|---|---|---|---|---|
| y=0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| y=1 | 3 | 3 | 3 | 0 | 1 | 1 | 1 |
| y=2 | 6 | 6 | 6 | 0 | 2 | 2 | 2 |
| y=3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| y=4 | 9 | 9 | 9 | 0 | 0 | 1 | 0 |
| y=5 | 0 | 9 | 0 | 0 | 1 | 1 | 1 |
| y=6 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| y=7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Consider the following locations in $A$:

$$P_1 = (x = 1, y = 1)$$
$$P_2 = (x = 5, y = 1)$$
$$P_3 = (x = 1, y = 4)$$
$$P_4 = (x = 5, y = 5)$$

### I

Among the locations specified above, what is the best match for the **center** of $\lambda$ according to non-normalized cross correlations matching? If there are several *best* solutions specify all of them.

Show your computations.

**Answer:**

| matching at | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|
| value | 21 | 7 | 45 | 6 |

$$P_1 \;/\; P_2 \;/\; \boxed{P_3} \;/\; P_4$$

### II

Among the locations specified above, what is the best match for the **center** of $\lambda$ according to normalized cross correlations matching? If there are several *best* solutions specify all of them.

Show your computations.

**Answer:**

| matching at | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|
| value | $\frac{21}{\sqrt{135}} = 1.81$ | $\frac{7}{\sqrt{15}} = 1.81$ | $\frac{45}{\sqrt{324}} = 2.5$ | $\frac{6}{\sqrt{5}} = 2.68$ |

$$P_1 \;/\; P_2 \;/\; P_3 \;/\; \boxed{P_4}$$

## Question 5

Consider a black box tool that can evaluate color and brightness of images. The tool requires the images to be represented in nonlinear sRGB. We assume that the tool produces the same result as a human observer. Now consider the 4 images $A, B, C, D$. Applying the tool to these images produces the following result:

| | |
|---|---|
| $A$ | looks more red than the other 3 images. |
| $B$ | looks more green than the other 3 images. |
| $C$ | looks more blue than the other 3 images. |
| $D$ | looks brighter than the other 3 images. |

### A

Consider the following processing:
The $R$ component in all 4 images is linearly stretched to the $0-1$ range.
The tool is applied again to the 4 images after the above processing. What would be the expected result? In each case mark the most appropriate answer.

**The most red image is:** A / B / C / D / $\boxed{\text{impossible-to-tell}}$

**The most green image is:** A / $\boxed{\text{B}}$ / C / D / impossible-to-tell

**The most blue image is:** A / B / $\boxed{\text{C}}$ / D / impossible-to-tell

**The brightest image is:** A / B / C / D / $\boxed{\text{impossible-to-tell}}$

### B

Consider the following processing:
Each image is converted to the Luv color space. The L values are linearly stretched to the $0-100$ range. The result is then converted back to nonlinear sRGB.
The tool is applied again to the 4 images after the above processing. What would be the expected result? In each case mark the most appropriate answer.

**The most red image is:** $\boxed{\text{A}}$ / B / C / D / impossible-to-tell

**The most green image is:** A / $\boxed{\text{B}}$ / C / D / impossible-to-tell

**The most blue image is:** A / B / $\boxed{\text{C}}$ / D / impossible-to-tell

**The brightest image is:** A / B / C / D / $\boxed{\text{impossible-to-tell}}$

### C

Consider the following processing:
Each image is converted to the XYZ color space. The Y values are linearly stretched to the $0-1$ range. The result is then converted back to nonlinear sRGB.
The tool is applied again to the 4 images after the above processing. What would be the expected result? In each case mark the most appropriate answer.

**The most red image is:** $\boxed{\text{A}}$ / B / C / D / impossible-to-tell

**The most green image is:** A / $\boxed{\text{B}}$ / C / D / impossible-to-tell

**The most blue image is:** A / B / $\boxed{\text{C}}$ / D / impossible-to-tell

**The brightest image is:** A / B / C / D / $\boxed{\text{impossible-to-tell}}$

## D

Consider the following processing:
Each image is converted to the Luv color space. The u values are linearly stretched to the $0-10$ range. The result is then converted back to nonlinear sRGB.
The tool is applied again to the 4 images after the above processing. What would be the expected result? In each case mark the most appropriate answer.

**The most red image is:** A / B / C / D / $\boxed{\text{impossible-to-tell}}$

**The most green image is:** A / B / C / D / $\boxed{\text{impossible-to-tell}}$

**The most blue image is:** A / B / C / D / $\boxed{\text{impossible-to-tell}}$

**The brightest image is:** A / B / C / $\boxed{\text{D}}$ / impossible-to-tell