

## Thresholding by quantization

Let  $p$  be the picture histogram, so that  $p(x)$  is the number of pixels of value  $x$ , for  $x = 0, \dots, M$ . We are looking for a threshold value  $t$  and two values  $q_1, q_2$ , such that all pixels in the range  $0 \leq x < t$  are replaced with  $q_1$ , and all pixels in the range  $t \leq x \leq M$  are replaced with  $q_2$ . Define the following expression as the total error:

$$E(t, q_1, q_2) = \sum_{x=0}^{t-1} (x - q_1)^2 p(x) + \sum_{x=t}^M (x - q_2)^2 p(x).$$

For each  $t$  we can compute the minimum of  $E$  by choosing the “best possible” values for  $q_1, q_2$ . These are computed by taking the derivatives of  $E$  with respect to  $q_1, q_2$ .

Taking the derivative of  $e$  with respect to  $q_1$  we have:

$$\frac{\partial e}{\partial q_1} = 2 \sum_{x=0}^{t-1} xp(x) - 2q_1 \sum_{x=0}^{t-1} p(x).$$

The requirement that  $\frac{\partial e}{\partial q_1} = 0$  gives:

$$q_1 = \frac{\sum_{x=0}^{t-1} xp(x)}{\sum_{x=0}^{t-1} p(x)}$$

and similarly:

$$q_2 = \frac{\sum_{x=t}^M xp(x)}{\sum_{x=t}^M p(x)}$$

Therefore, we can compute the value of  $E$  for any given value of  $t$  by first computing  $q_1, q_2$  and then substituting their values in the above expression for  $E$ . Since there are only 255 possible values for  $t$  the minimizer of  $t$  can be determined by examining all values of  $E(t)$  for  $t = 1..255$ .