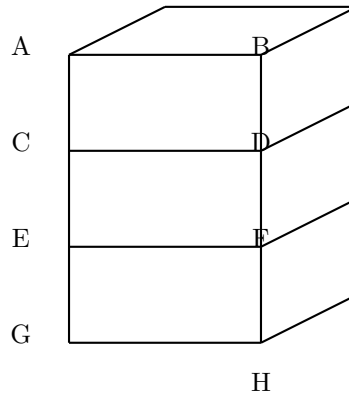


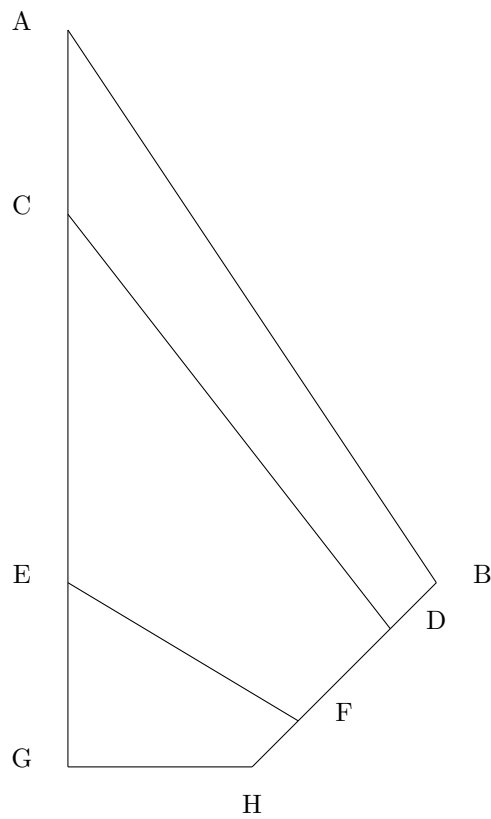
Q/persp2.stex

% f95 hw05f ho06 ho06f h07 ho07f ho08 ho09 ho12 ho11 ho10 ho12f ho13 ho16 ho17 ho18 ho19

Three rectangular bricks of identical size are put one on top of the other, as illustrated in the following figure:



A picture is taken with the camera positioned along the line A-G. The camera constant is $f = 0.5$.



You are given the location in the picture of the points A,B, C, G, H.

$A = (0, 0)$, $B = (2, 3)$, $C = (0, 1)$, $G = (0, 4)$, $H = (1, 4)$.

Compute the exact location of the camera in the picture. (Notice that it is known to be along the line A-G.)

Answer:

The intersection of the line AG and the line BH is at the point $u_1 = 0, v_1 = 5$.

The intersection of the line GH and the line AB is at the point $u_2 = 8/3, v_2 = 4$.

These points satisfy the following equation:

$$(u_1 - u_0)(u_2 - u_0) + (v_1 - v_0)(v_2 - v_0) + f^2 = 0$$

In our case this simplifies to:

$$(5 - v_0)(4 - v_0) + 1/4 = 0 \quad \Leftrightarrow \quad v_0^2 - 9v_0 + 20 + 1/4 = 0$$

The solution gives $v_0 = 4.5$.

Therefore, the camera location in the picture is $u = 0, v = 4.5$.

2.

Compute the direction of the line A-G in the 3D camera coordinates.

Answer:

$$\begin{pmatrix} u_1 - u_0 \\ v_1 - v_0 \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ 5 - 4.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.5 \\ 0.5 \end{pmatrix}$$

3.

Compute the exact location of the point E in the picture.

Answer:

The Cross Ratio in 3D gives:

$$\text{CR} = \frac{|AC| \cdot |EG|}{|AE| \cdot |CG|} = \frac{1}{4}$$

In the picture: write $|AE| = x$. We have: $|AC| = 1, |EG| = 4 - x, |AE| = x, |CG| = 3$. Therefore in the image the Cross Ratio gives:

$$\text{CR} = \frac{1 \cdot (4 - x)}{x \cdot 3} = \frac{1}{4}$$

The solution is $x = 16/7$.

The location in the picture of the point E is $u = 0, v = 16/7$.