Name: Chaoran Li
Student ID: (UTD ID) 2021489307; (NET ID) cxl190012
Course Number: CS 6364.002

Homework 2 Writeup Part

Q1: In at least one page (single space, Font 10), describe in your own words the difference between artificial intelligence and machine learning based on your readings of chapter 1 in the AI textbook and chapter 1 in the PRML textbook.

Machine learning is one subset of artificial intelligence.

Artificial intelligence attempts to understand human intelligence and try to build intelligent entities. Or in my words, it is a multi-field interdisciplinary subject which tries to simulate human intelligence by machines. It related to philosophy, mathematics, economics, neuroscience, psychology, computer engineering, control theory and cybernetics, linguistics, etc. From the gestation of artificial intelligence in 1943 by Walter Pitts, it has achieved tremendous accomplishments including but not limited to robotic vehicles, speech recognition, autonomous planning and scheduling, game playing, spam fighting, logistics planning, robotics, machine translation. Most of these words become popular recently, but artificial intelligence has a way longer history and prepares a lot before they exploded.

There are four approaches to artificial intelligence which are thinking humanly, thinking rationally, acting humanly, and acting rationally. From these four, we can better understand artificial intelligence. For acting humanly, the famous Turing Test cannot be ignored. This test worked on distinguished computer from human based on their test result. Many people worked on designing a computer to pass the total Turing Test and some declared that they had pass. For now, we are still not sure about whether they had really passed all Turing Test, but their work had advanced artificial intelligence a lot.

Many of the early successes of artificial intelligence such as IBM’s Deep Blue took place in formal and circumscribed environment which has a big gap with reality. Because the rules and knowledge are all coded by programmers, and they did not require to learn from real world. To solve more complicated problems, we need computers themselves to have learning ability which generates machine learning.

Machine learning is a method of data analysis which focuses on using data and algorithms to simulate human learning. it tries to allow computers to acknowledge the real world, make decisions and make improvement based on extracting patterns from raw data or experience. Especially nowadays, huge amount of data is stored in electronic form which makes it necessary to find an automated method for data analysis. Probability theory is applied widely in machine learning field. The probability is the most powerful weapon to find out the best model to explain some data or the best prediction about future. To be noted, even though machine learning is kind of thrived by big data. The effective number of data points for certain cases is usually quite small which is known as the long tail.

Machine learning is usually divided into two main types: supervised learning and unsupervised learning. Supervised learning will have labeled training set and we are interested in learning a mapping from inputs to outputs. We can do classification, regression by supervised learning. For example, we can use naïve bayes to separate legitimate e-mails from spam e-mails. We can detect and recognize face within an image. Unsupervised learning is different, since we are only given inputs and our goal is to find interesting patterns from data. We can find clusters, latent factors, graph structure by it. Finally, we have reinforcement learning which find the optimal outputs by a process of trial and error.

Artificial intelligence's subject does not limit to computer. It also worked on learning and understanding human intelligence and tried to apply them in real life by building intelligent entities. For example, natural language processing, knowledge representation, expert system, computer vision, etc.

Artificial intelligence works on a bigger concept which focus on understanding and simulating human intelligence. While as machine learning is one way in artificial intelligence which can allow computer to learn and improve based on data without being programmed explicitly. Or in another word, machine leaning is a method to extract knowledge from raw data from real world. These knowledges are usually narrow and specifical for it design intention and mostly not suitable for others situation even for some quite similar ones. For me, I will treat artificial intelligence as a comprehensive review while as machine learning as a popular paper which is cited in artificial intelligence's review.

To sum up, machine learning is one subset of artificial intelligence. Machine learning and its sub-field deep learning might be the hottest keywords along with artificial intelligence, but they cannot represent all artificial intelligence. Artificial intelligence has many other fields which are also important and meaningful.

Q2: In at least one page (single space, Font 10), describe in your own words the difference between artificial intelligence and deep learning based on your readings of chapter 1 in the AI textbook and chapter 1 in the DL textbook.

Deep learning is one subset of artificial intelligence. To be clear, deep learning is one subset of machine learning which is one subset of artificial intelligence.

Artificial intelligence attempts to understand human intelligence and try to build intelligent entities. Or in my words, it is a multi-field interdisciplinary subject which tries to simulate human intelligence by machines. It related to philosophy, mathematics, economics, neuroscience, psychology, computer engineering, control theory and cybernetics, linguistics, etc. From the gestation of artificial intelligence in 1943 by Walter Pitts, it has achieved tremendous accomplishments including but not limited to robotic vehicles, speech recognition, autonomous planning and scheduling, game playing, spam fighting, logistics planning, robotics, machine translation. Most of these words become popular recently, but artificial intelligence has a way longer history and prepares a lot before they exploded.

There are four approaches to artificial intelligence which are thinking humanly, thinking rationally, acting humanly, and acting rationally. From these four, we can better understand artificial intelligence. For acting humanly, the famous Turing Test cannot be ignored. This test worked on distinguished computer from human based on their test result. Many people worked on designing a computer to pass the total Turing Test and some declared that they had pass. For now, we are still not sure about whether they had really passed all Turing Test, but their work had advanced artificial intelligence a lot.

Many of the early successes of artificial intelligence such as IBM's Deep Blue took place in formal and circumscribed environment which has a big gap with reality. Because the rules and knowledge are all coded by programmers, and they did not require to learn from real world. To solve more complicated problems, we need computers themselves to have learning ability which generates machine learning.

However, for many tasks, it is difficult to know what features should be extracted. One solution is to use machine learning to discover not only mapping from representations to outputs but also representations themselves. This is known as representation learning. But it still be difficult since many of the factors of variation influence the result in real life. Deep learning solves this problem.

Deep learning is a particular kind of machine learning which can help build artificial intelligence systems in complicated, real-world environments. It is built based on artificial neural networks. Deep learning gains great power and flexibility by learning to represent the world as a nested hierarchy of concepts with each concept defined in relation to simpler concepts, and more abstract representations computed in terms of less abstract ones. The introduction of additional layers and more abstract features brings more possibility for it to solve more complicated problems. In recent years, it has seen tremendous growth and became popular due to more powerful computers, larger datasets, and some new techniques to train deeper networks. Increasing dataset sizes, model sizes, accuracy, complexity, and real-world impact makes deep learning appear in more and more fields and problems in real-world.

In traditional machine learning, most of the features need to be identified by a domain expert to reduce the complexity of the data. Deep learning can eliminate the need of domain expertise and hard-core feature extraction. Deep learning allows computer to build complex concepts out of simpler concepts by neural network structure. This can make computer learn the right representation for the data. There is also another perspective on deep learning that depth allows the computer to learn a multi-step computer program. By this, later instructions can refer to earlier instructions. The representation also stores state information which helps to execute a program that can make sense of the input. The state information did nothing to the content of input, but it could help the model to organize its processing.

To sum up, deep learning is one sub-field of machine learning which is one sub-field of artificial intelligence. Deep learning is a type of machine learning which allows computer to learn and improve from experience and data. While machine learning helps build artificial intelligence systems in complicated, real-world environments. The subset comes to solve some the difficulty that its parent set faces and promotes the development of its parent set by its own huge breakthrough. Today, deep learning is a pop star in artificial intelligence field.

Q3: In at least one page (single space, Font 10), describe in your own words the difference between machine learning and deep learning based on your readings of chapter 1 in the PRML textbook and chapter 1 in the DL textbook.

Deep learning is one subset of machine learning. To be clear, deep learning is one subset of neural networks which is one subset of machine learning.

Machine learning is a method of data analysis which focuses on using data and algorithms to simulate human learning. it tries to allow computers to acknowledge the real world, make decisions and make improvement based on extracting patterns from raw data or experience. Especially nowadays, huge amount of data is stored in electronic form which makes it necessary to find an automated method for data analysis. Probability theory is applied widely in machine learning field. The probability is the most powerful weapon to find out the best model to explain some data or the best prediction about future. To be noted, even though machine learning is kind of thrived by big data. The effective number of data points for certain cases is usually quite small which is known as the long tail.

Machine learning is usually divided into two main types: supervised learning and unsupervised learning. Supervised learning will have labeled training set and we are interested in learning a mapping from inputs to outputs. We can do classification, regression by supervised learning. For example, we can use naïve bayes to separate legitimate e-mails from spam e-mails. We can detect and recognize face within an image. Unsupervised learning is different, since we are only given inputs and our goal is to find interesting patterns from data. We can find clusters, latent factors, graph structure by it. Finally, we have reinforcement learning which find the optimal outputs by a process of trial and error.

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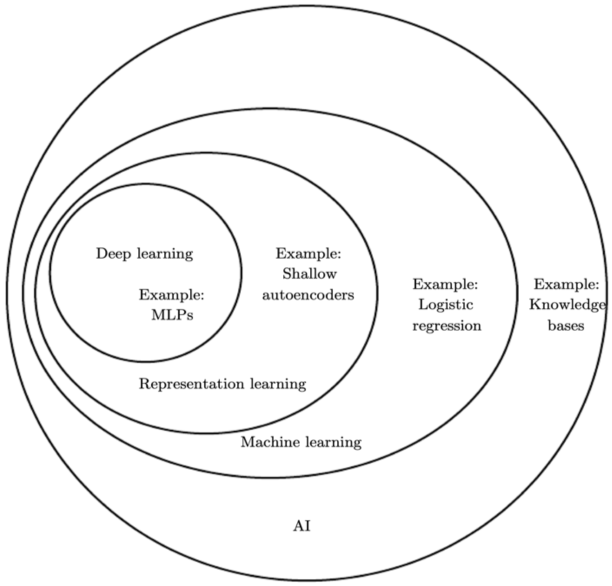
In traditional machine learning, most of the features need to be identified by a domain expert to reduce the complexity of the data. Deep learning can eliminate the need of domain expertise and hard-core feature extraction. Deep learning allows computer to build complex concepts out of simpler concepts by neural network structure. This can make computer learn the right representation for the data. There is also another perspective on deep learning that depth allows the computer to learn a multi-step computer program. By this, later instructions can refer to earlier instructions. The representation also stores state information which helps to execute a program that can make sense of the input. The state information did nothing to the content of input, but it could help the model to organize its processing.

Besides, since deep learning can build complicated representation, it can directly get a mapping from input to output without intermedia step. Like a multiple object detection problem, deep learning techniques can directly use images as inputs and give the location and names of objects as outputs. But in traditional machine learning, you need to split the task into several steps. Such as firstly identify all possible objects and then recognize each object. Usually, deep learning algorithm takes a long time and more resource to train, but it usually has better performance than traditional one.

To sum up, deep learning is one sub-field of machine learning. When the data size is large, or the domain expertise and feature extraction is hard or expensive, deep learning performs better than traditional machine learning. It solved the difficulty of traditional machine learning technique and thrived because of more powerful computers, larger datasets, and some new techniques to train deeper networks.

Attach a diagram from AI textbook after Q1-3.



Q4: State the condition under which the Binomial distribution arises?
1) The number of trials is identified. (Like given 5 trials)
2) 2 and only 2 possible outcomes for multiple trial. (Like True or False)
3) Trials are independent from each other.
4) The probability for two outcomes should be the same for each trial and sum up to 1. (P(True) + P(False) = 1)

Q5: If X ~ Bin (12, 0.7) calculate
Solution:
n = 12, p = 0.7, q = 1- 0.7 = 0.3
(a) P (X = 9) =

$$\frac{12!}{3! * 9!} * 0.7^9 * \ 0.3^{12-9} = 0.2397$$

(b) P (X > 10) = P (X = 11) + P (X = 12) =

$$\frac{12!}{1! * 11!} * 0.7^{11} * \ 0.3^1 + \frac{12!}{0! * 12!} * 0.7^{12} * \ 0.3^0 = 0.0850$$

(c) P (X <= 11) = 1 – P (X = 12) =

$$1 - \frac{12!}{0! * 12!} * 0.7^{12} * \ 0.3^0 = 0.9862$$

Q6: A lecturer uses a laptop to give a series of 8 lectures. There is a 5% chance that the laptop causes problems in any given lecture. What is the probability of observing 2 out of 8 lectures in which the laptop exhibits 'technical difficulties'?
Solution:
X ~ Bin (8, 0.05), in which n = 8, p = 0.05, q = 1 - 0.05 = 0.95
P (X = 2) =

$$\frac{8!}{6! * 2!} * 0.05^2 * \ 0.95^6 = 0.0515$$

Q7: Two groups of twelve children are taught two different methods of arithmetic. (Assume that a child in group one is matched in terms of their arithmetic ability with a child in group 2 before the start of the study). What is the probability that at least 9 children from one of the groups will obtain higher scores than the other group? What other assumptions have you made?
Solution:
Assumptions:
Denote 'win' as the child in group one obtains higher score than the matched child in group two. Denote 'lose' if not 'win' which means the child in group one obtains lower or equal score than the matched child in group two.
P (win) = P (lose) = 0.5 for each child (P (win) + P (lose) = 1)
X ~ Bin (12, 0.5), in which n = 12, p = 0.5 and q = 1 – 0.5 = 0.5
P (at least 9 children win) = P (0) + P (1) + P (2) + P (3) + P (9) + P (10) + P (11) + P (12)
= 2 * (P (9) + P (10) + P (11) + P (12)) =

$$2 * \left(\frac{12!}{3! * 9!} * 0.5^9 * 0.5^3 + \frac{12!}{2! * 10!} * 0.5^{10} * 0.5^2 + \frac{12!}{1! * 11!} * 0.5^{11} * 0.5^1 + \frac{12!}{0! * 12!} * 0.5^{12} * 0.5^0\right)$$
$$= 0.1460$$

Q8: Two teams, A and B, play a series of games. If team A has probability 0.4 of winning each game, is to it's advantage to play the best three out of five games or the best four out of seven? Assume outcomes of successive games are independent.
Solution:
Best 3 out of 5:
X_1 ~ Bin (5, 0.4), in which n_1 = 5, p_1 = 0.4 and q_1 = 1 - 0.4 = 0.6
P (X_1 >= 3) = P (X_1 = 3) + P (X_1 = 4) + P (X_1 = 5)

$$\frac{5!}{2! * 3!} * 0.4^3 * 0.6^2 + \frac{5!}{1! * 4!} * 0.4^4 * 0.6^1 + \frac{5!}{0! * 5!} * 0.4^5 * 0.6^0 = 0.3174$$

X_2 ~ Bin (7, 0.4), in which n_2 = 5, p_2 = 0.4 and q_2 = 1 - 0.4 = 0.6
Best 4 out of 7:
P (X_2 >= 4) = P (X_2 = 4) + P (X_2 = 5) + P (X_2 = 6) + P (X_2 = 7)

$$\frac{7!}{3! * 4!} * 0.4^4 * 0.6^3 + \frac{7!}{2! * 5!} * 0.4^5 * 0.6^2 + \frac{7!}{1! * 6!} * 0.4^6 * 0.6^1 + \frac{7!}{0! * 7!} * 0.4^7 * 0.6^0 = 0.2898$$

0.3174 > 0.2898
The probability to play best 3 out of 5 is higher.
The result is obvious since team A has lower probability to win. The smaller the games number is, the higher chance it shall win. Because in small game number condition, accident happens.

Q9: State the conditions under which the Poisson distribution can be used to approximate a Binomial distribution? If X ~ Bin (200, 0.01), calculate P (X <= 2)
(i) using a Binomial distribution
(ii) using a Poisson approximation to the Binomial
Solution:
For X ~ Bin (n, p) and Y ~ Po (ƛ)
If n is large (i.e., n > 50) and p is small (i.e., p < 0.1), then X ~ Bin (n, p) can be approximated with a Po (ƛ) where ƛ = np.
N = 200, p = 0.01, q = 0.99, ƛ = 200 * 0.01 = 2
(i) P (X <= 2) = P (X = 0) + P (X = 1) + P (X = 2) =

$$\frac{200!}{2! * 198!} * 0.01^2 * 0.99^{198} + \frac{200!}{1! * 199!} * 0.01^1 * 0.99^{199} + \frac{200!}{0! * 200!} * 0.01^0 * 0.99^{200} = 0.6767$$

(ii) P (X <= 2) = P (X = 0) + P (X = 1) + P (X = 2) =

$$e^{-2} * \frac{2^2}{2!} + e^{-2} * \frac{2^1}{1!} + e^{-2} * \frac{2^0}{0!} = 0.6767$$

This result shows that the approximation works.

Q10: The mean number of bacteria per millimeter of a liquid is known to be 4. Assuming that the number of bacteria follows a Poisson distribution, find the probability that, in 1 ml of liquid, there will be
(a) no bacteria
(b) 4 bacteria
(c) less than 3 bacteria
Find the probability that
(i) in 3 ml of liquid there will be less than 2 bacteria
(ii) in 0.5 ml of liquid there will be more than 2 bacteria
Solution:
In 1 ml, mean = 4 = ƛ
X ~ Po (4)
(a) P (X = 0) =

$$e^{-4} * \frac{4^0}{0!} = 0.0183$$

(b) P (X = 4) =

$$e^{-4} * \frac{4^4}{4!} = 0.1954$$

(c) P (X < 3) = P (X = 0) + P(X = 1) + P (X = 2) =

$$e^{-4} * \frac{4^0}{0!} + e^{-4} * \frac{4^1}{1!} + e^{-4} * \frac{4^2}{2!} = 0.2381$$

(i) In 3 ml, Y_1 ~ Po (3 * 4) = Po (12)
P (Y_1 < 2) = P (Y_1 = 0) + P (Y_1 = 1) =

$$e^{-12} * \frac{12^0}{0!} + e^{-12} * \frac{12^1}{1!} = 7.9874 \times 10^{-5}$$

(i) In 0.5 ml, Y_2 ~ Po (0.5 * 4) = Po (2)
P (Y_2 > 2) = 1 - P (Y_2 = 0) - P (Y_2 = 1) - P (Y_2 = 2) =

$$1 - e^{-2} * \frac{2^0}{0!} - e^{-2} * \frac{2^1}{1!} - e^{-2} * \frac{2^2}{2!} = 0.3233$$

Q11: Stroke patients with aphasic deficits are each given a number of straight-forward tasks in a psychometric test. The number of errors made by 123 patients are shown in the table below. Calculate the mean and variance of the number of errors per patient and comment on these values. Fit a Poisson distribution and comment on how well it fits the observed data.

Number of errors	0	1	2	3	4	5 or more
Number of patients	5	30	56	15	10	7

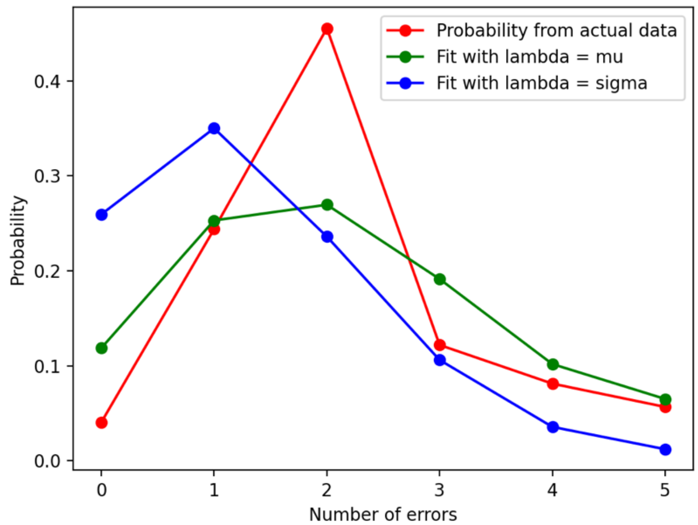
Solution:
Assume 5 or more is 5 errors for calculation.

$$\mu = \frac{5*0 + 30*1 + 56*2 + 15*3 + 10*4 + 7*5}{123} = 2.1301 = 2.13$$

$$\sigma^2 = \frac{5*-2.13^2 + 30*-1.13^2 + 56*-0.13^2 + 15*0.87^2 + 10*1.87^2 + 7*2.87^2}{123} = 1.3489$$

$$\sigma = 1.1614$$

Number of errors	0	1	2	3	4	5 or more
Probability from actual data	0.0407	0.2439	0.4553	0.1220	0.0813	0.0569
Fit with ƛ = μ	0.1188	0.2531	0.2696	0.1914	0.1019	0.0652
Fit with ƛ = σ	0.2595	0.3501	0.2361	0.1062	0.0358	0.0123



From the above diagram, Poisson distribution does not fit well. From the Statistic course I took, I guess maybe Gamma distribution can fit well.

Q12: In a large town, one person in 80, on average, has blood type X. If 200 blood donors are taken at random, find an approximation to the probability that they include at least five persons having blood type X.
How many donors must be taken at random in order that the probability of including at least one donor of type X shall be 0.9 or more?
Solution:
X ~ Po (200 * 1/80) = Po(2.5) in 200 people
P (X >= 5) = 1 – P (X = 0) – P (X = 1) – P (X = 2) – P (X = 3) – P (X = 4) =

$$1 - e^{-2.5} * \frac{2.5^0}{0!} - e^{-2.5} * \frac{2.5^1}{1!} - e^{-2.5} * \frac{2.5^2}{2!} - e^{-2.5} * \frac{2.5^3}{3!} - e^{-2.5} * \frac{2.5^4}{4!} = 0.1088$$

To Include at least one donor of type X be 0.9:
Y ~ Po (n * 1/80) = Po(n/80) in n people
P (X >= 1) = 1 – P (X = 0) =

$$1 - e^{-\frac{n}{80}} * \frac{\left(\frac{n}{80}\right)^0}{0!} >= 0.9$$
$$e^{-\frac{n}{80}} <= 0.1$$
$$n \geq 184.21$$

A least 184 donors should be taken.

Q13: Telephone calls reach a secretary independently and at random, internal ones at a mean rate of 2 in any 5 minute period, and external ones at a mean rate of 1 in any 5 minute period. Calculate the probability that there will be more than 2 calls in any period of 2 minutes.
Solution:

	In 5 minute period	In 2 minute period
Internal: Int ~	Po (2)	Po (2 * 0.4) = Po (0.8)
External: Ext ~	Po (1)	Po (1 * 0.4) = Po (0.4)

Total telephone calls number follows: X ~ Po (0.8 + 0.4) = Po (1.2)
P (X > 2) = 1 – P (X = 0) – P (X = 1) – P (X = 2) =

$$1 - e^{-1.2} * \frac{1.2^0}{0!} - e^{-1.2} * \frac{1.2^1}{1!} - e^{-1.2} * \frac{1.2^2}{2!} = 0.1205$$

Q14: If X ~ N (200, 20^2), calculate
(i) P (X < 210)
(ii) P (X < 195)
(iii) P (X > 215)
(iv) P (185 < X < 205)
(v) a such that P (X > a) = 0.2
Solution:
Z ~ N (0, 1^2)
(i) P (X < 210) =

$$P\left(Z < \frac{210-200}{20}\right) = P(Z < 0.5) = 0.6915$$

(ii) P (X < 196) =

$$P\left(Z < \frac{195-200}{20}\right) = P(Z < -0.25) = P(Z > 0.25) = 1 - \ P(Z < 0.25) = 1 - \ 0.5987 = 0.4013$$

(iii) P (X > 215) =

$$P\left(Z < \frac{215-200}{20}\right) = P(Z > 0.75) = 1 - P(Z < 0.75) = 1 - 0.7734 = 0.2266$$

(iv) P (185 < X < 205) =

$$P\left(\frac{185-200}{20} < Z < \frac{205-200}{20}\right) = P(-0.75 < Z < 0.25) = P(Z < 0.25) - P(Z < -0.75)$$
$$= P(Z < 0.25) - P(Z > 0.75) = P(Z < 0.25) - 1 + P(Z < 0.75) = 0.5987 - 1 + 0.7734 = 0.3721$$

(v) P (X > a) = 1 – P (X < a) = 0.2

$$P(X < a) = 0.8$$
$$P\left(Z < \frac{a-200}{20}\right) = 0.8$$
$$\frac{a-200}{20} = 0.84$$
$$a = 216.04$$

Q15: The number of accidents on a certain railway line occur at an average rate of one every 2 months. Find the probability that
(a) there are 25 or more accidents in 4 years.
(b) there are 30 or less accidents in 5 years.
Solution:
X ~ Po (1) in every 2 months
Z ~ N (0, 1^2)
(a) Y_1 ~ Po (1 * 6 * 4) = Po (24)
Since 24 > 20, Y_1 ~ N (24, 24)
P (Y_1 >= 25) = P (Y_1 > 24.5) = 1 – P (Y_1 < 24.5) =

$$1 - P\left(Z < \frac{24.5-24}{24^{0.5}}\right) = 1 - P(Z < 0.102) = 1 - (0.5398 * 0.8 + 0.5438 * 0.2) = 0.4594$$

(b) Y_2 ~ Po (1 * 6 * 5) = Po (30)
Since 30 > 20, Y ~ N (30, 30)
P (Y_2 <= 30) = P (Y < 30.5) =

$$P\left(Z < \frac{30.5-30}{30^{0.5}}\right) = P(Z < 0.091) = 0.5359 * 0.9 + 0.5398 * 0.1 = 0.5363$$

Q16: In a certain country the heights of adult males have mean 170 cm and stand deviation 10 cm, and the heights of adult females have mean 160 cm and standard deviation 8 cm; for each sex the distribution of heights approximates closely to a normal probability model. One the hypothesis that height is not a factor in selecting a male, calculate the probability that
(i) a husband and wife selected at random are both taller than 164 cm.
(ii) in a randomly selected husband and wife, the wife is taller than the husband.
(iii) the average height of a random couple is greater than 168 cm.

Solution:
M ~ N (170, 10^2)
F ~ N (160, 8^2)
Z ~ N (0, 1^2)
(i) P (M > 164) * P (F > 164) = (1 – P(M < 164)) * (1 – P(F < 164)) =

$$\left(1 - P\left(Z < \frac{164-170}{10}\right)\right) * \left(1 - P(Z < \frac{164-160}{8}\right) = \left(1 - P(Z < -0.6)\right) * \left(1 - P(Z < 0.5)\right)$$

$$= \left(1 - P(Z > 0.6)\right)\left(1 - P(Z < 0.5)\right) = P(Z < 0.6) * \left(1 - P(Z < 0.5)\right) = 0.7257 * (1 - 0.6915) = 0.2239$$

(ii) D = M – F ~ N (170 – 160, 10^2 + 8^2) = N (10, 164)
P (D < 0) =

$$P\left(Z < \frac{0-10}{164^{0.5}}\right) = P(Z < -0.781) = P(Z > 0.781) = 1 - P(Z < 0.781)$$

$$= 1 - (0.7823 * 0.9 + 0.7852 * 0.1) = 0.2174$$

(iii) A = M/2 + F/2 ~ N(170/2 + 160/2, 10^2/4 + 8^2/4) = N (165, 41)
P (A > 168) =

$$P\left(Z > \frac{168-165}{41^{0.5}}\right) = P(Z > 0.469) = 1 - P(Z < 0.469) = 1 - (0.6772 * 0.1 + 0.6808 * 0.9) = 0.3196$$

Q17: 10% of chocolates produced in a factory are mis-shapes. In a sample of 1000 chocolates find the probability that the number of mis-shapes is
(a) less than 80
(b) between 90 and 115 inclusive
(c) 120 or more
Solution:
X ~ Bin (1000, 0.1)
1000 > 30
X ~ N (1000 * 0.1, 1000 * 0.1 * 0.9) = N (100, 90)
Z ~ N (0, 1^2)
(a) P (X < 80) =

$$P\left(Z < \frac{80-100}{90^{0.5}}\right) = P(Z < -2.108) = P(Z > 2.108) = 1 - P(Z < 2.108)$$

$$= 1 - (0.9821 * 0.2 + 0.9826 * 0.8) = 0.0175$$

(b) P (90 < X < 115) =

$$P\left(\frac{90-100}{90^{0.5}} < Z < \frac{115-100}{90^{0.5}}\right) = P(-1.054 < Z < 1.581)$$

$$= P(Z < 1.581) - P(Z < -1.054) = P(Z < 1.581) - P(Z > 1.054)$$

$$= P(Z < 1.581) - \left(1 - P(Z < 1.054)\right) = P(Z < 1.581) - 1 + P(1.054)$$

$$= (0.9429 * 0.9 + 0.9441 * 0.1) - 1 + (0.8531 * 0.6 + 0.8554 * 0.4) = 0.7970$$

(c) P (X > 120) =

$$P\left(Z > \frac{120-100}{90^{0.5}}\right) = P(Z > 2.108) = 1 - P(Z < 2.108)$$

$$= 1 - (0.9821 * 0.2 + 0.9826 * 0.8) = 0.0175$$

Actually, P (X > 120) = P (X < 80).

Q18: A sample of 100 apples is taken from a load. The apples have the following distribution of sizes

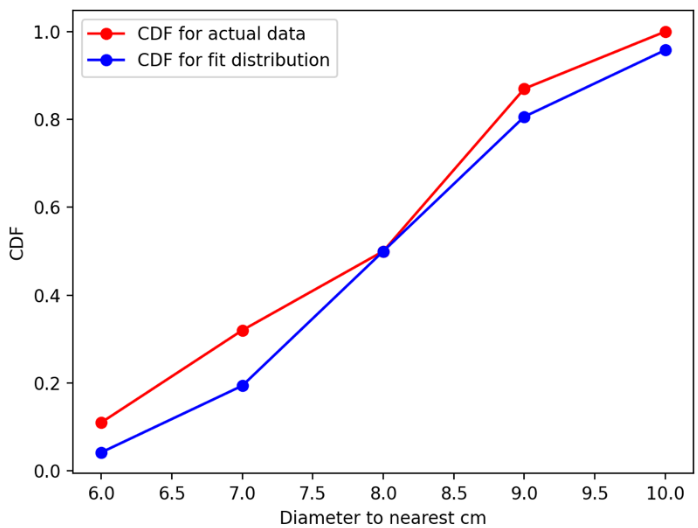
Diameter to nearest cm	6	7	8	9	10
Frequency	11	21	38	17	13

Determine the mean and standard deviation of these diameters.
Assuming that the distribution is approximately normal with the estimated mean and standard deviation find the range of size of apples for packing, if 5% are to be rejected as too small and 5% are to be rejected as too large.
Solution:

$$\mu = \frac{11*6+21*7+38*8+17*9+13*10}{100} = 8$$
$$\sigma^2 = \frac{11*4+21*1+38*0+17*1+13*4}{100} = 1.34$$
$$\sigma = 1.1576$$

X ~ N (8, 1.34)

Diameter to nearest cm	6	7	8	9	10
CDF for actual data	0.11	0.32	0.70	0.87	1.00
CDF for fit distribution	0.0420	0.1938	0.5000	0.8062	0.9580



From the above diagram, Normal distribution fits well