

$$f_{i+1} = f_i + \Delta x f'_i + \frac{\Delta x^2}{2} f''_i + \frac{\Delta x^3}{6} f'''_i + \dots$$

$$f_{i+2} = f_i + 2\Delta x f'_i + 2\Delta x^2 f''_i + \frac{4}{3} \Delta x^3 f'''_i + \dots$$

$$\textcircled{1} \rightarrow \frac{f_{i+1} - f_i}{\Delta x} = f'_i + \frac{\Delta x}{2} f''_i + \frac{\Delta x^2}{6} f'''_i + \dots$$

$$\textcircled{2} \rightarrow \frac{f_{i+2} - f_{i+1}}{\Delta x} = f'_i + \frac{3\Delta x}{2} f''_i + \frac{4}{6} \Delta x^2 f'''_i + \dots$$

$$a \textcircled{1} + b \textcircled{2} = f'_i + 0 \Delta x f''_i + \dots$$

$$\Rightarrow a + b = 1 \quad \& \quad \frac{a}{2} + \frac{3b}{2} = 0$$

$$\Rightarrow -2b = 1 \Rightarrow \boxed{b = -\frac{1}{2}} \\ \boxed{a = \frac{3}{2}}$$

$$\Rightarrow \frac{3}{2} \left( \frac{f_{i+1} - f_i}{\Delta x} \right) - \frac{1}{2} \left( \frac{f_{i+2} - f_{i+1}}{\Delta x} \right) = f'_i$$

$$\Rightarrow \boxed{\frac{4f_{i+1} - 3f_i - f_{i+2}}{2\Delta x} = f'_i}$$

is the formula of 2nd order