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Question 1.

a) to have BIBO Stable system

$$\sum_{n=-\infty}^{\infty} h[n] < \infty$$

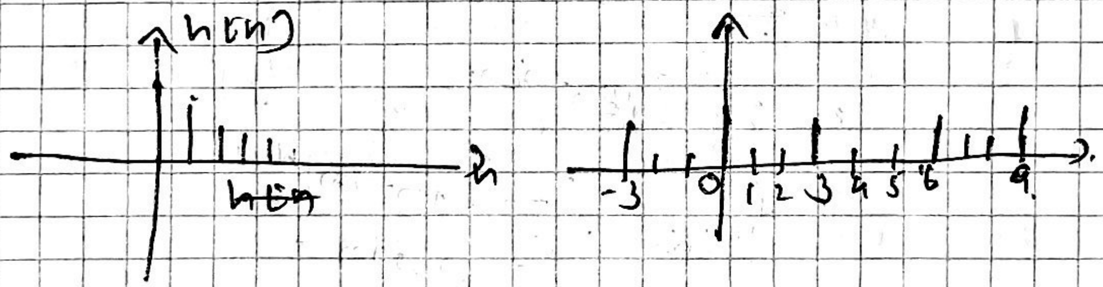
$$\sum_{n=0}^{\infty} a^n < \infty$$

$$\lim_{n \to \infty} \frac{(1 - a^{n+1})}{1-a} < \infty$$

$\therefore |a| < 1.$

b) $x_3[n] = \begin{cases} 1 & n = r \times 3 \quad r \in \mathbb{Z} \\ 0 & \text{otherwise.} \end{cases}$

$y_3[n] = h[n] * x_3[n].$



the calculation of $y_3[n]$ have 3 scenarios.

n mod 3 = 0
n mod 3 = 1
n mod 3 = 2

1/ n mod 3 = 0.

$$y_3[n] = \sum_{i=0}^{\infty} a^{3i}$$

$$= \frac{1}{1 - a^3}$$

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$n \bmod 3 = 1$

$$y_3[n] = \sum_{i=0}^{\infty} a^{3i+1}$$

$$= \frac{a}{1-a^3}$$

$n \bmod 3 = 2$

$\cancel{y_3[n] =}$

$$y_3[n] = \sum_{i=0}^{\infty} a^{3i+2}$$

$$= \frac{a^2}{1-a^3}$$

$$\therefore y_3[n] = \frac{a^{n \bmod 3}}{1-a^3} \quad //$$

c). $H(e^{j\omega}) = \sum_{-\infty}^{\infty} h[n] \cdot e^{-j\omega n}$

$$= \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$= \lim_{k \to \infty} \frac{1-\left(ae^{-j\omega}\right)^{k+1}}{1-ae^{-j\omega}}$$

$$= \frac{1}{1-ae^{-j\omega}} \quad //$$

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$$W(e^{j\omega}) = \cancel{h[n]}\ H(e^{j\omega}) * X_f(e^{j\omega})$$

$$= H(e^{j\omega}) * \sum_{k=-\infty}^{\infty} \frac{2\pi}{M}\delta\left(\omega - \frac{2\pi k}{M}\right)$$

$$= \frac{2\pi}{M}\sum_{k=-\infty}^{k=\infty} H(e^{j\omega}) * \delta\left(\omega - \frac{2\pi k}{M}\right)$$

$$= \frac{1}{M}\sum_{k=-\infty}^{\infty} H(e^{j(\omega - \frac{2\pi k}{M})})$$

$$\therefore Y_w(e^{j\omega}) = H(e^{j\omega}) \cdot W(e^{j\omega})$$

$$= \frac{H(e^{j\omega})}{M}\sum_{k=-\infty}^{\infty} H(e^{j(\omega - \frac{2\pi k}{M})})$$

Question 2

a) $T_s = \cancel{\frac{1}{2\pi}}\ \frac{2\pi}{100\pi} = \frac{1}{50}$ s.

i) $x_1[n] = 2\cos\left[30\pi \times \frac{n}{50}\right]$

$$= 2\cos\left[\frac{3}{5}\pi n\right]$$

$$\omega_1 = \Omega_1 T_s = \frac{30\pi}{50} = \frac{3}{5}\pi$$

$$x_2[n] = 2\cos\left[70\pi \times \frac{n}{50}\right] = 2\cos\left[\frac{7}{5}\pi n\right]$$

$$\omega_2 = \Omega_2 T_s = \frac{70\pi}{50} = \frac{7}{5}\pi$$

ii) $\Omega_s = 100\pi$
$\Omega_1 = 30\pi$
$\Omega_2 = 70\pi$

Since $\Omega_s > 60\pi = 2\Omega_1$ $x_1(t)$ ~~is~~ can be reconstruct according to nyquist's conditions

Since $f_s < 140 = 2\Omega_2$ $x_2(t)$ can't be reconstruct according to nyquist's conditions//

b)

i) $x[n] = a^n u[n]$.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= \frac{1-(az^{-1})^{n+1}}{1-(az^{-1})}$$

this is converging if $|az^{-1}| < 1$

$|a| < |z|$.

$\therefore X(z) = \frac{1}{1-az^{-1}}$ ROC $|z| > |a|$.

ii) $x[n] = a^{|n|}$

$$= \begin{cases} a^{-n} & n < 0 \\ a^n & n \geq 0 \end{cases}$$

$$X(z) = \underbrace{\sum_{n=-\infty}^{-1} (a^{-1}z^{-1})^n}_{\text{replace } n \text{ by } -m} + \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= \sum^{\infty} a^m \cancel{z^{-1}}$$

$$= \sum_{m=1}^{\infty} (az^{-1})^m + \sum_{n=0}^{\infty} (az^{-1})^n.$$

$$= \lim_{m\to\infty} \frac{az^{-1}(1-(az^{-1})^{m})}{1-(az^{-1})} + \lim_{n\to\infty} \frac{1-(az^{-1})^{n}}{1-(az^{-1})}$$

This also converges if $|z| > |a|$

$$\therefore X(z) = \cancel{\frac{1}{1-az^{-1}}} \frac{az^{-1}}{1-az^{-1}} + \frac{1}{1-az^{-1}}$$

$$= \frac{1+az^{-1}}{1-az^{-1}} \quad ROC\ |z| > |a|.$$

c) $x[n] = \left(\frac{1}{2}\right)^n u[n]$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}.$$

~~Let~~ $\cancel{\therefore\ Y[n] = H\ z}$

$$\therefore Y(z) = X(z) \cdot H(z)$$

$$= \frac{1}{\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{5}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

$$= \frac{A}{\left(1+\frac{1}{3}z^{-1}\right)} + \frac{B}{\left(1-\frac{1}{5}z^{-1}\right)} + \frac{C}{\left(1-\frac{1}{2}z^{-1}\right)}$$

$$= \frac{A\left(1-\frac{1}{5}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right) + B\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right) + C\left[1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{5}z^{-1}\right]}{\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{5}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right).}$$

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Hence we obtain

$$1 = \left[1 - \frac{7}{10}z^{-1} + \frac{1}{10}z^{-2}\right]A + \left[1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}\right]B$$
$$+ \left[1 + \frac{2}{15}z^{-1} - \frac{1}{15}z^{-2}\right]C.$$

$A = \frac{1}{4}$, $B = -\frac{1}{4}$, $C = 1$

$$\therefore Y(z) = \frac{1}{4}\left[\frac{1}{1+\frac{1}{3}z^{-1}}\right] - \frac{1}{4}\left[\frac{1}{1-\frac{1}{5}z^{-1}}\right]$$
$$+ \left[\frac{1}{1-\frac{1}{2}z^{-1}}\right] \qquad |z| > \frac{1}{2}.$$

$$\therefore y[n] = \frac{1}{4}\left(-\frac{1}{3}\right)^n u[n] - \frac{1}{4}\left(\frac{1}{5}\right)^n u[n]$$
$$+ \left(\frac{1}{2}\right)^n u[n].$$

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Question 3

a) $$H(z) = \frac{a_2 z^2 + a_1 z + a_0}{z^2 + b_1 z + b_0}$$

$D(z) = z^2 + b_1 z + b_0$

i) $D(1) > 0$

$1 + b_1 + b_0 > 0$

ii) $(-1)^2 \, [1 - b_1 + b_0] > 0$

$1 - b_1 + b_0 > 0$

1	b_0	b_1	1
	1	b_1	b_0

$~~2 \quad b_0^2 - 1 \quad b_1 b_0 - b_1~~$

$C_0 = \left|\begin{matrix} b_0 & 1 \\ 1 & b_0 \end{matrix}\right| = b_0^2 - 1$

$C_1 = \left|\begin{matrix} b_1 & 1 \\ b_1 & b_0 \end{matrix}\right| = b_1 b_0 - b_1$

$~~|b_0| > |1|~~$

$\therefore b_0 > 1$

$\therefore 1 + b_1 + b_0 > 0$
$1 - b_1 + b_0 > 0$

$b_0 > 1$ should be satisfied to be a stable system.

furthermore

$2 + b_1 > 0$ which leads $|b_1| < 2$.
$2 - b_1 > 0$

$-2 < b_1 < 2$

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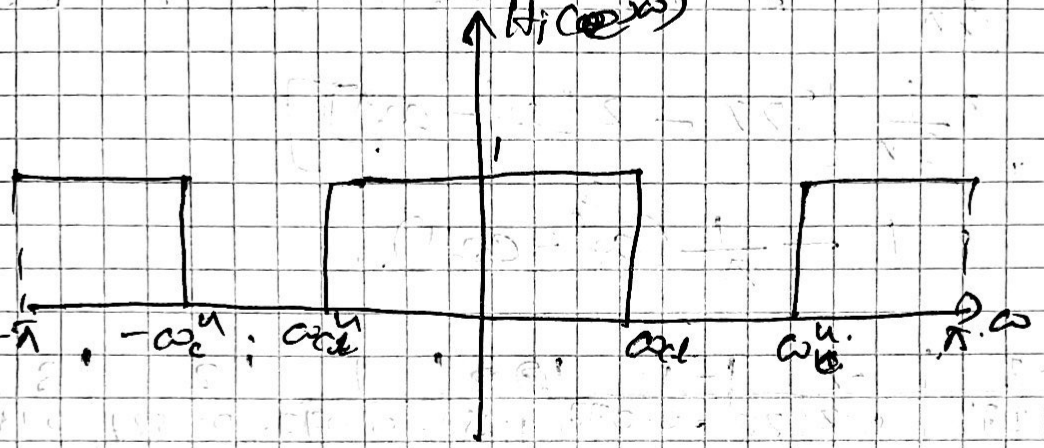
$\therefore$ $b_0 > 1$, $|b_0| < 2$ are the conditions to be satisfied to obtain a stable system.

b). $H_2(e^{j\omega}) = \begin{cases} 1 & 0 < |\omega| < \omega_c^l \text{ and } \omega_c^u < |\omega| < \pi \\ 0 & \omega_c^l < |\omega| < \omega_c^u \end{cases}$



i) $$h_2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_2(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c^u} e^{j\omega n}\, d\omega + \int_{-\omega_c^l}^{\omega_c^l} e^{j\omega n}\, d\omega + \int_{\omega_c^u}^{\pi} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\,\frac{\left[e^{-j\omega_c^u n} - e^{-j\pi n} + e^{j\omega_c^l n} - e^{-j\omega_c^l} + e^{j\pi n} - e^{j\omega_c^u}\right]}{jn}$$

$$= \frac{1}{n\pi}\left[\sin \omega_c^l n - \sin \omega_c^u n\right]$$

-3 -2 -1 0 1 2 3

~~h[n]~~

$$h[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, d\omega$$

$$= \frac{1}{2\pi}\left(-\omega_c^u + \pi\right) + \left(\omega_c^l + \omega_c^l\right) + \pi - \omega_c^u\right).$$

$$= \frac{1}{2\pi}\left[2\pi - 2\left[\omega_c^l - \omega_c^u\right]\right]$$

$$= 1 - \frac{1}{\pi}\left(\omega_c^l - \omega_c^u\right).$$

	-3	-2	-1	0	1	2	3
$h_d[n]$	0.1373	0.2527	-0.077	1.35	-0.077	0.2527	0.1373
$\omega[n]$	0.08	0.31	0.77	1	0.77	0.31	0.08
$h[n]$	0.0109	0.0783	-0.0592	1.35	-0.0592	0.0783	0.0109

iii). by ~~h_c~~ Shifting $h[n]$ to the right by 3 samples.

$$h_c[n] = h[n-3]$$

iv) $H_c(e^{j\omega}) = e^{-j\omega 3} H(e^{j\omega})$

$$|H_c(e^{j\omega})| = \underbrace{|e^{-j3\omega}|}_{1}\,|H(e^{j\omega})|$$

$$|H_c(e^{j\omega})| = |H(e^{j\omega})|$$ $\therefore$ Amplitude response won't change.

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Question 4

a)i) $S = \frac{2}{T}\left(\frac{Z-1}{Z+1}\right)$

$\frac{TS}{2}[Z+1] = Z-1$

$Z\left[\frac{TS}{2}-1\right] = -1 - \frac{TS}{2}$

$$Z = \frac{1+\frac{TS}{2}}{1-\frac{TS}{2}}$$

Substitute $S = \sigma + j\Omega$.

$$Z = \frac{1+\frac{T\sigma}{2}+\frac{j\Omega T}{2}}{1-\frac{T\sigma}{2}-\frac{j\Omega T}{2}}$$

$$|Z| = \frac{\sqrt{\left(1+\frac{T\sigma}{2}\right)^2+\frac{\Omega^2 T^2}{4}}}{\sqrt{\left(1-\frac{T\sigma}{2}\right)^2+\frac{\Omega^2 T^2}{4}}}$$ same.

$\therefore$ if $\sigma \geq 0$ $|Z| \geq 1$

if $\sigma < 0$ $|Z| < 1$.

$\therefore$ Causal stability of continuous domain map into the causal stability of discrete domain.

~~if causal causal signals~~

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$\therefore H(s)$ is stable $\Rightarrow H(z)$ is stable.

b) $$H_c(s) = \frac{0.07(s^2+2.58)}{(s+0.38)(s^2+0.31s+0.51)}$$

i) $$H(z) = \cancel{H(s)}\ H_c\left[\frac{2}{T}\left(\frac{z-1}{z+1}\right)\right]$$

$$H(z) = \frac{0.07\left(\frac{4}{T^2}\left[\frac{(z-1)^2}{(z+1)^2}\right]+2.58\right]}{\left[\frac{2}{T}\left(\frac{z-1}{z+1}\right)+0.38\right]\left[\frac{4}{T^2}\left[\frac{(z-1)^2}{(z+1)^2}\right]+0.31\frac{\times 2}{T}\left[\frac{z-1}{z+1}\right]+0.51\right]}$$

$T = \frac{2\pi}{10} = 0.2$

$$= \frac{0.07\left[100[z-1]^2+2.58[z+1]^2\right]}{10(z^2-1)+0.38[z+1]^2\left]\left[100(z-1)^2+3.1[z^2-1]+0.51(z+1)^2\right]}$$

$$= \frac{0.07\left[102.58z^2-194.84z+102.58\right]}{10\left[1.38z^2+0.76z+0.62\right]\left[103.61z^2-198.98z+100.2\right]}$$

ii) Substitute $z = e^{j\omega}$ and $s = j\Omega$

$$e^{j\omega} = \frac{1+j0.71\times 0.1}{1-j0.71\times 0.1}$$

$= 1\angle 0.1417$.

$\therefore \omega = 0.1417$ rad/sample or

$\omega = 0.7085$ rad/s.

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iii) $H(z) = \dfrac{7.1806z^2 - 13.6388z + 7.1806}{[13.8z^2+7.6z+6.2][103.61z^2-198.98z+100.3]}$

$$H_1(z) = \frac{(7.1806z^2 - 13.6388z + 7.1806)}{[13.8z^2+7.6z+6.2]}$$

$$= \frac{0.52z^2 - 0.9883z + 0.52}{z^2 + 0.5507z + 0.4492}$$

$$= \frac{0.52 - 0.9883z^{-1} + 0.52z^{-2}}{1 + 0.5507z^{-1} + 0.4492z^{-2}}$$

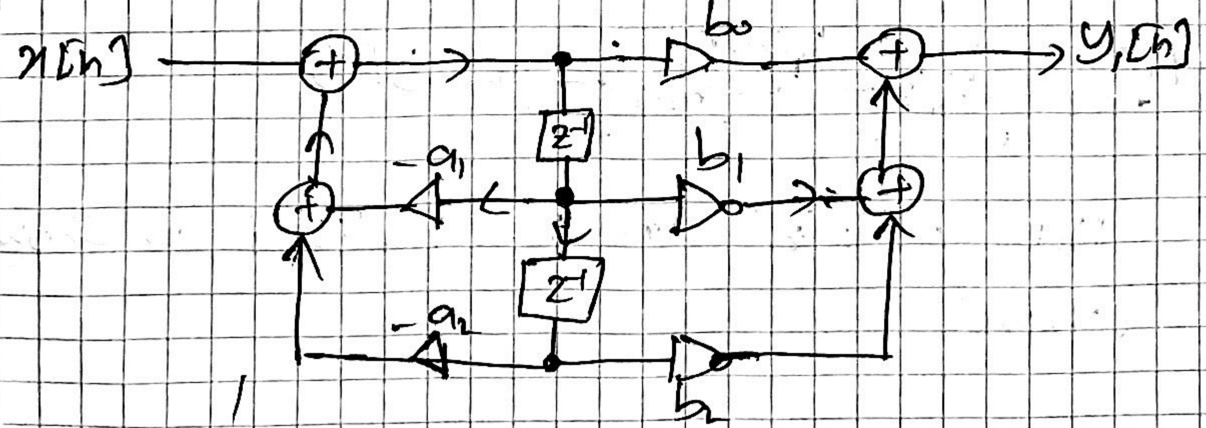
$\therefore$ $a_1 = 0.5507$

$a_2 = 0.4492$

$b_0 = 0.52$

$b_1 = -0.9883$

$b_2 = 0.52$



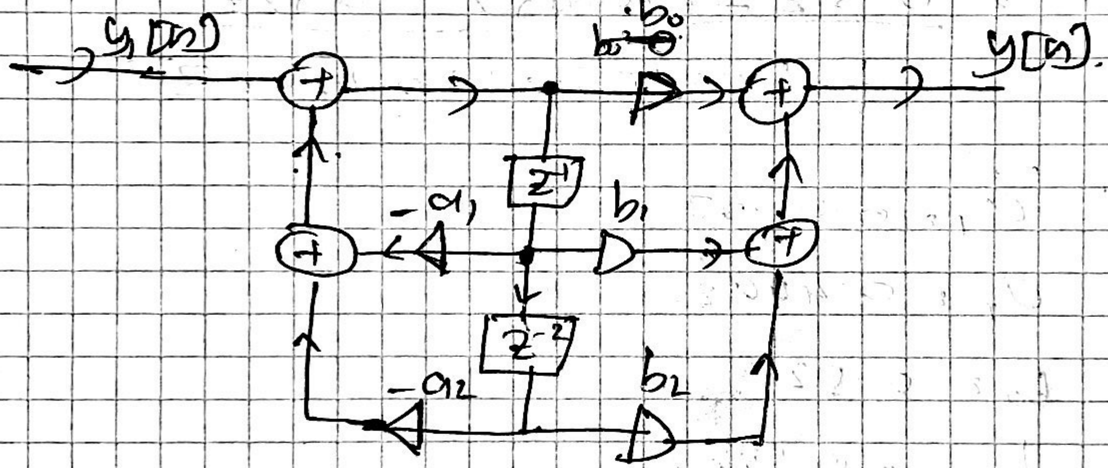
$$H_2(z) = \frac{1}{103.61z^2 - 198.98z + 100.3}$$

$$= \frac{0.009\,z^{-2}}{1 - 1.911z^{-1} + 0.9680\,z^{-2}}$$

$\therefore a_1 = -1.911$ $\quad$ $b_0 = 0$

$a_2 = 0.9680$ $\quad$ $b_1 = 0$

$b_2 = 0.009$



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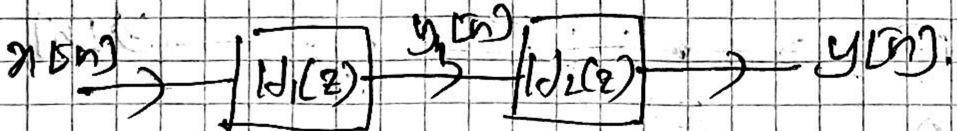

$H_1(z)$, $H_2(z)$ can be realized as shown above