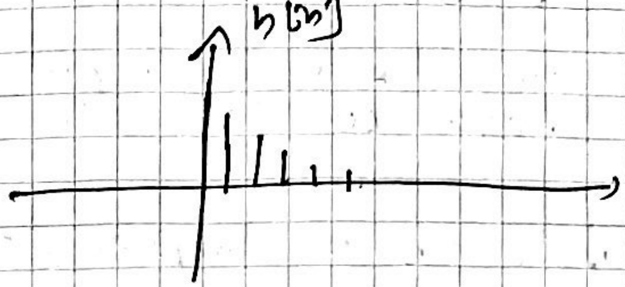
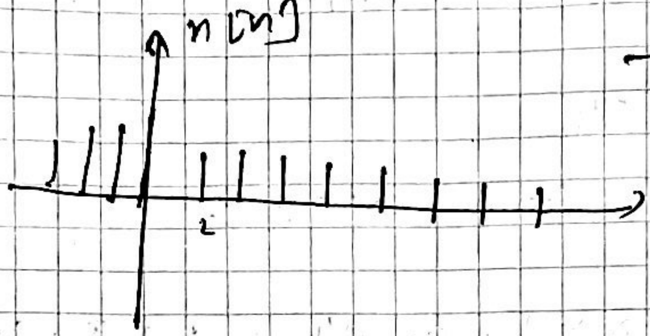
Question 1

a). $y[n] = h[n] * x[n]$





$y[n] = 0$ for $n < 2$.

$$y[n] = \sum_{i=0}^{n-2} a^i$$

$$= \frac{(1 - a^{n-1})}{1-a} \text{ //}$$

b). $X[0] = 3.1$
$X(2) = 2.5 + j4$
$X(4) = -1.7 + j5.2$
$X(6) = 9.4 + j6.3$
$X(8) = 5.5 - j8$

$X(2) = X(-2)$, $X(3) = X(-3)$, $X(-4) = X(4)$,
$X(-6) = X(6)$, $X(-8) = X(8)$.

the $X[n]$ should be even since $x[n]$ is real.
also $X[k]$ should periodic with $N = 9$

$\therefore X[1] = X[-8]$, $X[3] = X[-6]$, $X[5] = X[-4]$,
$X[7] = X[-2]$

$\therefore X[1] = 5.5 - j8$
$X[3] = 9.4 + j6.3$
$X[5] = -1.7 + j5.2$
$X[7] = 2.5 + j4$.

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c) No. of Samples in $x[n]$

$2 \times 80 = 160$.

So we have to put 96 samples which are equal to zero to get 256 samples.

i) the no. of samples $\leq N$

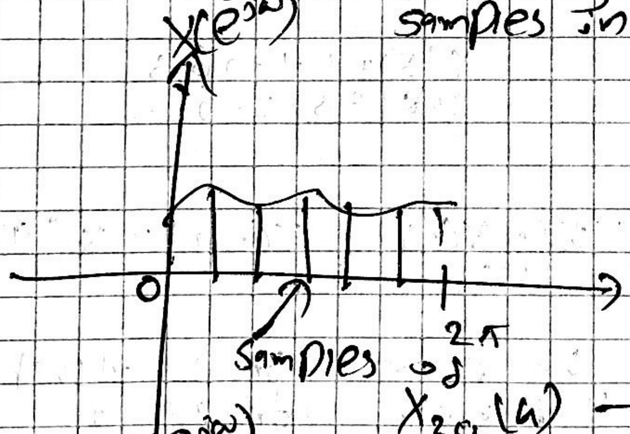
$160 \leq N$, to free from aliasing.

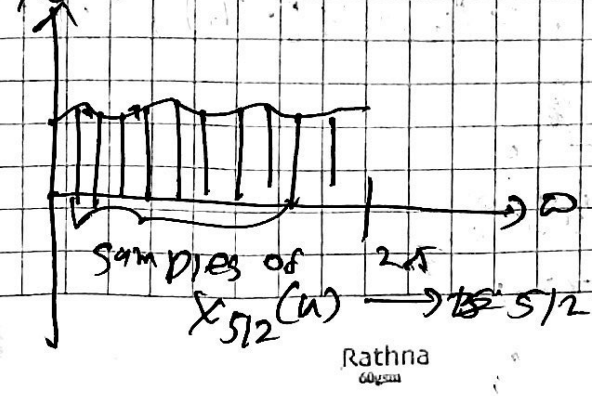
iii) we can obtain the DFT by sampling the DTFT.

$$X_{256}(k) = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{256}}$$

$$X_{512}(k) = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{512}}$$

∴ both $X_{256}(k)$ and $X_{512}(k)$ have same envelope but $X_{512}(k)$ have large no. of samples within the interval $0 - 2\pi$ (actually twice than the no. of samples in $X_{256}(k)$).





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Question 2

a) i) $x[n] = a^n u[n]$.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= \lim_{N \to \infty} \frac{1-(az^{-1})^{N+1}}{1-az^{-1}}$$

converge iff $|az^{-1}| < 1$

$\therefore |a| < |z|$

$$\therefore X(z) = \frac{1}{1-az^{-1}} \quad \text{Roc } |z| > |a|$$

ii) $y[n] = -a^n u[-n-1]$.

$$Y(z) = \sum_{n=-\infty}^{\infty} y[n] z^{-n}$$

$$= \sum_{n=-\infty}^{-1} -(az^{-1})^n$$

Substitute $m = -n$

$$= -\sum_{m=1}^{\infty} (a^{-1}z)^{m}$$

$$= \lim_{M \to \infty} \frac{az^{-1}[1-(az^{-1})^M]}{1-(az^{-1})}$$

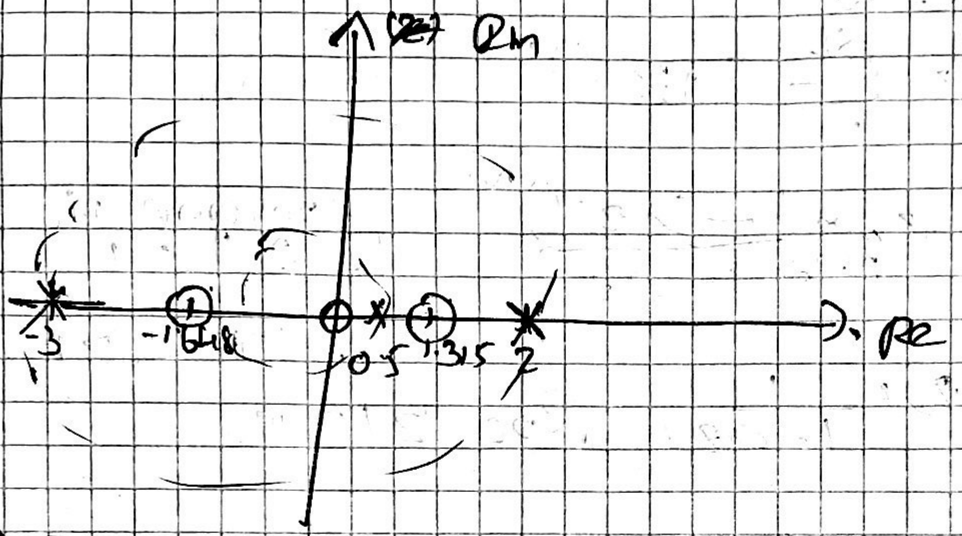
converge if $|z^{-1}a| > 1$.

$|z| < |a|$.

$$\therefore Y(z) = \frac{az^{-1}}{1-az^{-1}} \quad \text{Roc } |z| < |a|$$

b). $H(z) = \dfrac{z(z-1.315)(z+1.648)}{(z-0.5)(z-2)(z+3)}$

i)



ii) ROC can't contain any poles.

3 possible ROCs.

$2 < |z| < 3$

$|z| < 0.5$

$0.5 < |z| < 2$

to system to be stable unit circle must contain the Roc.

$\therefore$ ROC $= 0.5 < |z| < 2$.

iii) No. to system to be causal ROC should be outwards from the outermost pole//.

$H(z) = \dfrac{z^4}{4z^4+2z^3+bz^2+2z+1}$

$\cancel{D(z) = 4z^4+2z^3+bz^2+1}$

$D(z) = 4z^4+2z^3+bz^2+2z+1.$

$\cancel{D(1) =}$

$D(1) = b+9>0$

$b>-9$

$D(-1) = b+1>0$

$b>-1$

1.	4	2	b	2	1
2	1	2	b	2	4
3.	15	6	3b	6	
4.	6	3b	6	15	
5.	189	90-18b.	45b-36		

$4>1 \therefore b_0>b_4$

$|15|>|6|$

$|C_0|>|C_4|$

$189>|45b-36|$

$-189<45b-36<189$

$-153<45b<225$

$-3.4<b<5$

$\therefore b \therefore -1<b<5$ //

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Question 3.

a)

i) $H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] \cdot e^{-j\omega n}$

$= e^{-j\omega} + e^{j\omega} + 3$

$= 3 + 2\cos(\omega).$

ii) Group delay $= \frac{N-1}{2}$ samples.

$= \frac{7-1}{2} = 3$ samples.

b). $h_2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, d\omega$ — for $n \neq 0$.

$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_{c3}} e^{j\omega n}\, d\omega + \int_{-\omega_{c2}}^{-\omega_{c1}} e^{j\omega n} d\omega + \int_{\omega_{c1}}^{\omega_{c2}} e^{j\omega n}\, d\omega + \int_{\omega_{c3}}^{\pi} e^{j\omega n} d\omega\right]$

$= \frac{1}{2\pi j\omega n}\left[-e^{-j\pi n} + e^{-j\omega_{c3} n} + e^{-j\omega_{c1} n} - e^{-j\omega_{c2} n} + e^{j\omega_{c2} n} - e^{j\omega_{c1} n} + e^{j\pi n} - e^{j\omega_{c3} n}\right]$

$= \frac{1}{2\cancel{j}\omega n}\left[2\sin\omega_{c2}n - 2\sin\omega_{c1} n - \sin\omega_{c3} n\right)$

$= \frac{1}{\pi\omega n}\left[\sin\omega_{c2}n - \sin\omega_{c1}n - \sin\omega_{c3}n\right)$

for $n = 0$

$h_2[0] = \frac{1}{\cancel{2}\pi}\left[\cancel{2}\left[\pi - \omega_{c3} + \omega_{c2} - \omega_{c1}\right]\right]$

~~$= \frac{1}{\pi}\left[\omega_{c3} + \omega_{c2} - \omega_{c1}\right]$~~

$= 1 + \frac{\boxed{\omega_{c1} - \omega_{c2} + \omega_{c3}}}{\pi}$

	-3	-2	-1	0	1	2	3
$h_d[n]$	-0.2704	-0.0569	-0.0977	0.5	-0.0977	~~-0.0569~~ -0.0569	-0.2704
$w[n]$	-0.5	-0.12	0.88	1.5	0.88	-0.12	-0.5
$h[n]$	0.1352	0.0068	-0.085	0.75	-0.0085	~~0.0068~~ -0.0068	0.1352

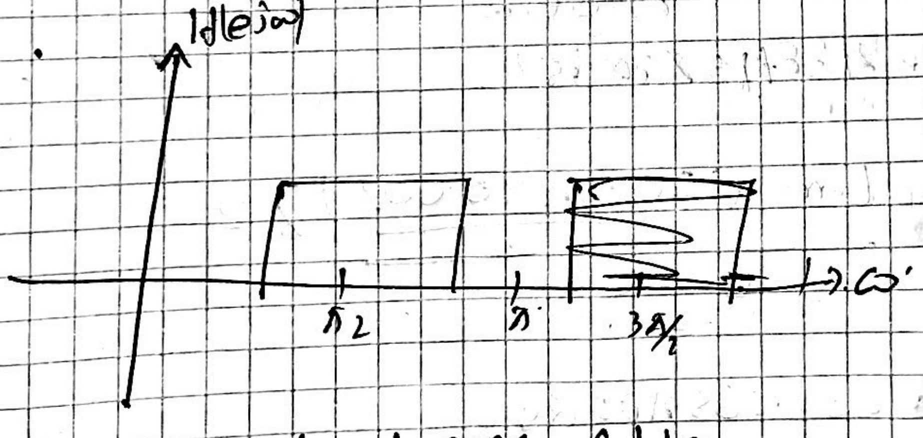
Question 4

a)

i) for a digital filter all of the poles should located at $Z=0$ to be ~~the~~ filter to be an ~~FIR filter~~ IIR.

∴ this filter is ~~FIR~~ IIR filter.

ii) we can approximate this filter as



this is a band pass filter.

b) $\cancel{Z}S = \frac{2}{T}\left(\frac{Z-1}{Z+1}\right)$

$$Z = \frac{1+\frac{TS}{2}}{1-\frac{TS}{2}}.$$

Substitute $z = e^{j\omega}$ and $s = j\Omega$.

$$e^{j\omega} = \frac{1 + \frac{j\Omega T}{2}}{1 - \frac{j\Omega T}{2}}$$

$$\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right).$$

$$T = \frac{2\pi}{1000\pi} = 0.002$$

$\therefore$ ~~$\Omega_c =$~~

$$\Omega = \frac{\tan\left(\frac{\omega}{2}\right)}{T} \times 2.$$

$$\Omega_p = \frac{\tan\left(\frac{300\pi \times 0.002}{2}\right)}{0.002} \times 2$$

$= 438.115$ rads^{-1}

$$\Omega_s = \frac{\tan\left(\frac{200\pi \times 0.002}{2}\right)}{0.002} \times 2.$$

$= 231.265$ rads^{-1}

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c) $H_a(s) = \dfrac{6}{(s^2+0.8s+4)(s^2+1.8s+1.5)}$

$s = \dfrac{2}{T}\left(\dfrac{z-1}{z+1}\right)$ $T = \dfrac{2\pi}{4\pi} = 0.5$

i). $\therefore$ $\cancel{H_a(z)}$ s $= 4\left(\dfrac{z-1}{z+1}\right)$

$H_a(z) = \dfrac{6}{\left(\dfrac{16(z-1)^2}{(z+1)^2}+0.8\times4\left(\dfrac{(z-1)}{(z+1)}\right)+4\right)\left(16\left(\dfrac{z-1}{z+1}\right)^2+1.8\times4\left(\dfrac{z-1}{z+1}\right)+1.5\right)}$

$H_a(z) = \dfrac{6(z+1)^2}{[16(z-1)^2+3.2(z^2-1)+4(z+1)^2][16(z-1)^2+7.2(z^2-1)+1.5(z+1)^2]}$

$= \dfrac{6[z^2+2z+1]}{[23.2z^2-24z+16.8][24.7z^2-29z+10.3]}$

$\cancel{H(z) = \dfrac{6z^{-2}+6+4z\,z^{-1}+6z^{-2}}{}}$

$H(z) = \underbrace{\dfrac{0.2586+0.5172z^{-1}+0.2586z^{-2}}{[1-1.03z^{-1}+0.724]}}_{H_1(z)} \times \underbrace{\left[\dfrac{0.0413\ \cancel{24.2}\,z^{-2}}{1-1.174z^{-1}+0.417z^{-2}}\right]}_{H_2(z)}$

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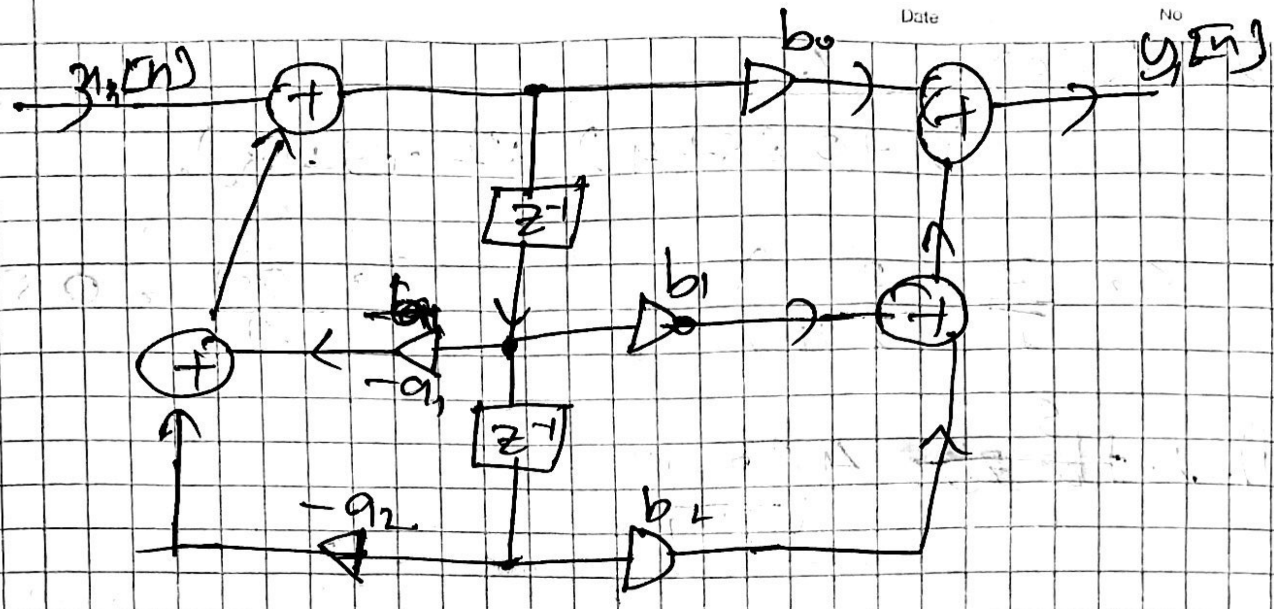
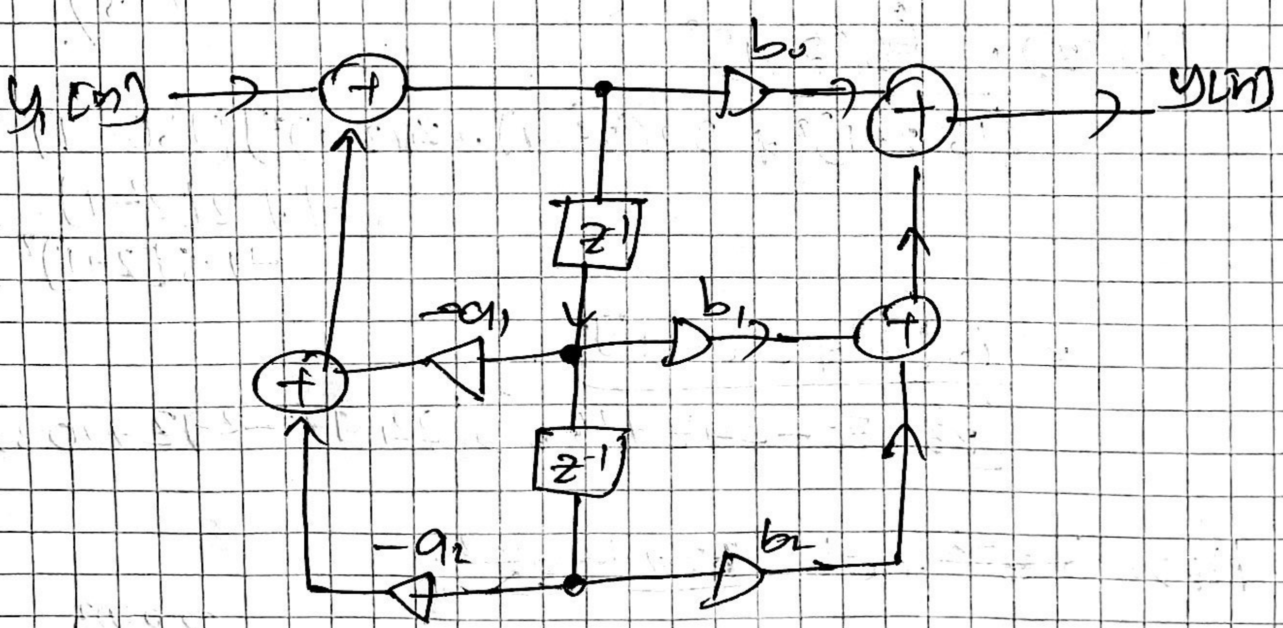


$b_0 = 0.2586$

$a_1 = -1.03$ $b_1 = 0.5172$

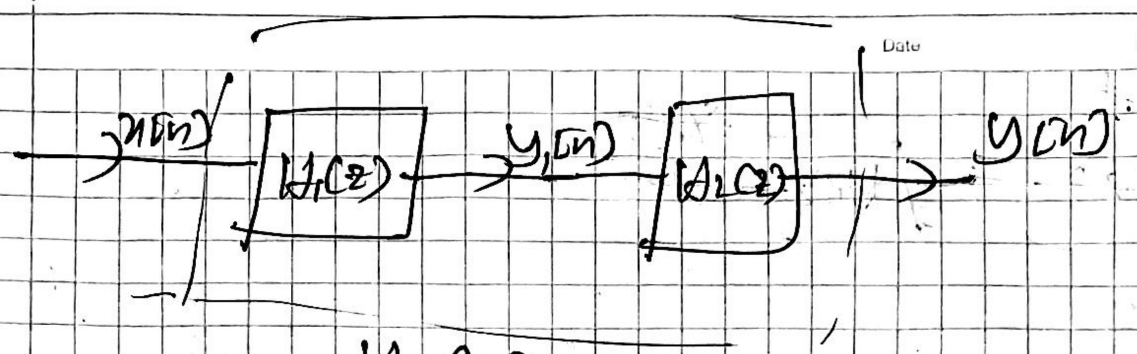
$a_2 = 0.7241$ $b_2 = 0.2586$



$a_1 = -1.174$ $b_0 = 0$

$a_2 = 0.417$ $b_1 = 0$

$b_2 = 0.0413$



$H_a(z)$

We can realize $H_a(z)$ by two cascade second order filters $H_1(z)$, $H_2(z)$ //.