

Decibels

Logarithmic scales, ratios, and reference levels

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Winter Semester 2024/25

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I. Logarithmic Scales in Acoustics

The human hearing range

Human hearing range

The human ear **perceives** sound pressures from **20 μPa** (threshold of hearing) to **20 Pa** (pain threshold), a ratio of **1 : 1,000,000**.

$$\frac{20}{20 \times 10^{-6}} = 1 \times 10^6$$

The pain threshold is about a million times louder than the weakest audible sound.

Scaling human hearing

The human hearing is too large for a linear scale, so the logarithmic decibel scale is used, matching human hearing.

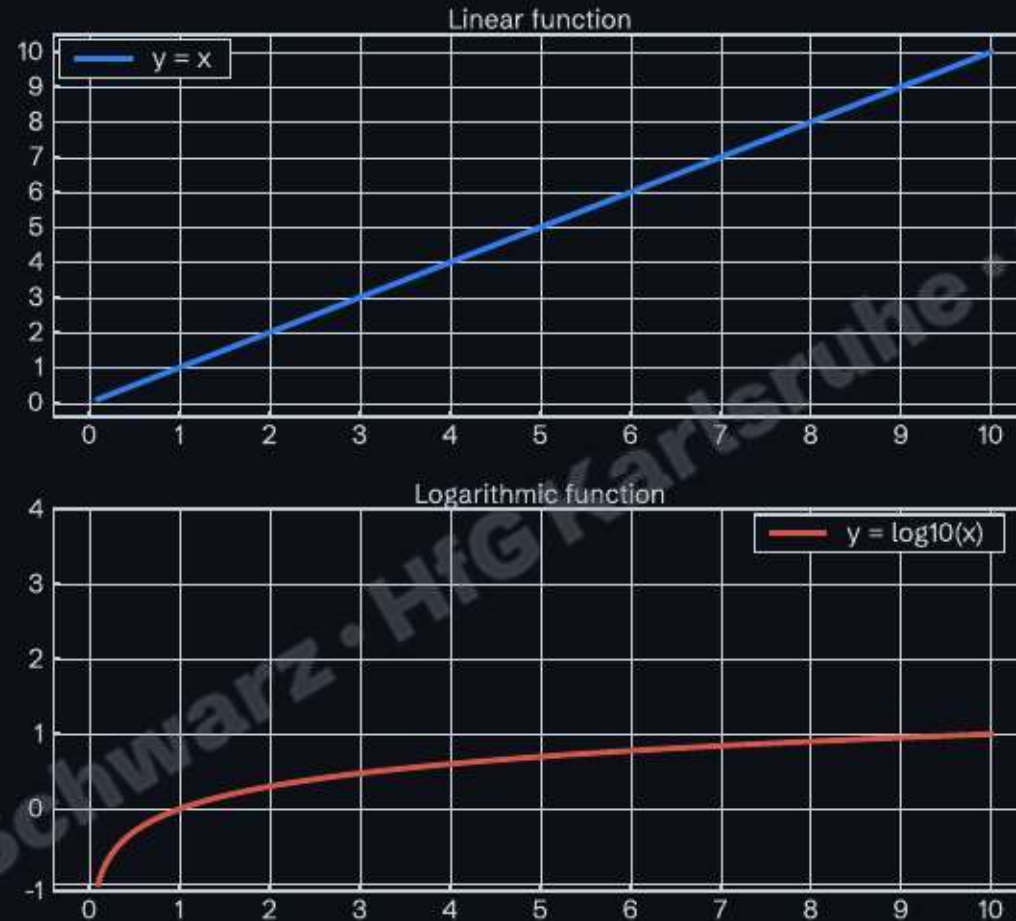
$$(0.00002 \rightarrow 20Pa)$$



$$(0 \rightarrow 120dB SPL)$$

Examples in air at standard atmospheric pressure

Sound source	Distance	Pa	dB _{SPL}
Eruption of Krakatoa	165 km	—	172
Jet engine	1 m	632	150
Trumpet	0.5 m	63.2	130
Threshold of pain	At ear	20-200	120-140
Risk of instantaneous noise-induced hearing loss	At ear	20.0	120
Jet engine	30-100 m	6.32-200	110-140
Traffic on a busy roadway	10 m	0.20-0.63	80-90
Hearing damage (long-term exposure)	At ear	0.36	85
TV (home level)	1 m	6.32×10^{-3} -0.02	50-60
Normal conversation	1 m	2×10^{-3} -0.02	40-60
Light leaf rustling, calm breathing	Ambient	6.32×10^{-5}	10
Hearing threshold	—	20×10^{-6}	0



Logarithmic compression makes the enormous range of human hearing manageable.

II. Logarithmic Principles

Decibels and ratios

Decibel

A decibel is one-tenth of a Bel, a unit named after Alexander Graham Bell, expressing power ratios logarithmically.

- Expresses **relative change** (ratio between two physical quantities)
- A **logarithmic unit** (compresses large ranges into manageable numbers)

Logarithm

The logarithm is the inverse function of exponentiation:

$$b^x = a \implies x = \log_b(a)$$

Example:

$$2^5 = 32 \implies 5 = \log_2(32)$$

Note: $\log_x(1) = 0 \implies$ ratio of 1 gives 0 dB

Properties of logarithmic scales

Human perception of intensity follows an approximately logarithmic relationship.

Benefits:

- Matches nonlinear human perception
 - Represent ratios ($\times 2$, $\times 10$, etc.) consistently
 - Display data spanning many orders of magnitude on one scale
 - Reveal details at both low and high ends
- Used for frequency and magnitude displays

Linear and logarithmic scales

With a logarithmic scale, the values of the tick marks increase by the same factor over equal distances (e.g., a base value of 10 raised to the powers 0, 1, 2, 3, etc.)



With a linear scale, the values of the tick marks increase by the same amount over equal distances.

Decibel relationships

Change (dB)	Power	Amplitude	Perception
+1 dB	$\sim 1.26\times$	$\sim 1.12\times$	Barely noticeable
+3 dB	$\sim 2\times$	$\sim 1.41\times$	Clearly noticeable
+6 dB	$\sim 4\times$	$\sim 2\times$	Noticeably louder
+10 dB	$10\times$	$\sim 3.16\times$	About $2\times$ as loud
-1 dB	$\sim 0.79\times$	$\sim 0.89\times$	Barely noticeable
-3 dB	$\sim \frac{1}{2}$	$\sim 0.71\times$	Clearly quieter
-6 dB	$\sim \frac{1}{4}$	$\sim \frac{1}{2}$	Noticeably quieter
-10 dB	$1/10$	$\sim 0.32\times$	About $\frac{1}{2}$ as loud

Audio demonstration: Decibel changes

Pink noise demonstrating relative dB changes (each compared to reference):

▶ [Play demonstration](#)

Three A/B comparisons:

1. Reference → **+3 dB** (2× power, clearly noticeable)
2. Reference → **+6 dB** (4× power, 2× amplitude, noticeably louder)
3. Reference → **+10 dB** (10× power, about 2× as loud)

Absolute and relative dB

- **With suffix** → absolute level (SPL, dBV, dBm, dBFS)
- **Without suffix** → relative ratio (input/output)

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III. Decibel Formulas

Power and magnitude ratios

Decibel formulas

Power quantities:

$$X_{dB} = 10 \log_{10} \left(\frac{X}{X_{\text{ref}}} \right)$$

Field quantities (magnitude):

$$X_{dB} = 20 \log_{10} \left(\frac{X}{X_{\text{ref}}} \right)$$

- $10 \log_{10}$ → Power, Intensity
- $20 \log_{10}$ → Amplitude, Voltage, Current, Pressure

Power and magnitude relationship

Power is proportional to the square of field quantities:

$$P = U \cdot I = R \cdot I^2 = \frac{U^2}{R}$$

When converting field quantities to decibels, this square relationship means:

$$20 \log_{10} \left(\frac{U}{U_{\text{ref}}} \right) = 10 \log_{10} \left(\frac{U^2}{U_{\text{ref}}^2} \right) = 10 \log_{10} \left(\frac{P}{P_{\text{ref}}} \right)$$

→ *The factor of 2 comes from the square relationship between power and field quantities.*

Converting between linear values and decibels

Power ratio:

$$x \text{ dB} = 10 \log_{10}(a) \iff a = 10^{\frac{x}{10}}$$

Amplitude ratio:

$$x \text{ dB} = 20 \log_{10}(a) \iff a = 10^{\frac{x}{20}}$$

Example:

Two signals whose levels differ by 6 dB have an amplitude ratio of $10^{\frac{6}{20}} \approx 2$

Example: doubling voltage and power in dB

- Voltage doubles (1 V \rightarrow 2 V):

$$20 \log_{10}(2/1) \text{ dB} \approx 20 \cdot 0.3010 \text{ dB} = +6.02 \text{ dB}$$

- Power doubles (1 W \rightarrow 2 W):

$$10 \log_{10}(2/1) \text{ dB} \approx 10 \cdot 0.3010 \text{ dB} = +3.01 \text{ dB}$$

dB conversion table

Decibels (dB)	Magnitude (ratio, $20 \cdot \log$)	Power (ratio, $10 \cdot \log$)
-20	0.1	0.01
-12	0.25	0.06
-6	0.5	0.25
-3	0.7	0.5
0	1	1
+3	1.4	2
+6	2	4
+12	4	16
+20	10	100

dB conversion graph



IV. Reference Systems

Absolute measurements

Decibel reference values

To express an **absolute value**, the suffix specifies the reference X_{ref} :

$$X_{dB} = k \cdot \log_{10} \left(\frac{X}{X_{\text{ref}}} \right), \quad k = \begin{cases} 10 & \text{power-like quantities} \\ 20 & \text{field-like quantities} \end{cases}$$

Unit	Quantity	Reference value X_{ref}
dB SPL	Sound pressure level	20 μPa
dBm	Power	1 mW
dBV	Voltage	1 V
dBu	Voltage	0.775 V
dBFS	Digital full scale	Maximum quantizing level

Example: sound pressure (SPL)

$$L_p = 20 \log_{10} \left(\frac{p_{\text{rms}}}{p_{\text{ref}}} \right) \text{ dB SPL}$$

p_{rms} is the root mean square of the measured sound pressure

p_{ref} is the standard reference sound pressure of 20 micropascals in air

$$p_{\text{ref}} = 20 \times 10^{-6} \text{ Pa} = 0.00002 \text{ Pa} = 20 \mu\text{Pa}$$

Example: calculating dB SPL

Reference: $p_{\text{ref}} = 20 \mu\text{Pa}$ (threshold of hearing)

$$L_p = 20 \log_{10} \left(\frac{p_{\text{rms}}}{p_{\text{ref}}} \right) \text{ dB SPL}$$

Example with $p_{\text{rms}} = 0.02 \text{ Pa}$:

$$\Rightarrow 20 \log_{10} \left(\frac{0.02 \text{ Pa}}{20 \times 10^{-6} \text{ Pa}} \right) \text{ dB SPL} = 20 \log_{10}(10^3) \text{ dB SPL} = 20 \cdot 3 \text{ dB SPL} = 60 \text{ dB SPL}$$

Example: dBFS (Full Scale)

Digital audio systems:

- Reference = maximum quantizing level (0 dBFS)
- Values typically negative (counted down from maximum)

16-bit system: Range = -32768 to +32767, so $X_{\text{ref}} = 32767$

Example calculation:

$$X_{dB} = 20 \log_{10} \left(\frac{16384}{32767} \right) = 20 \log_{10}(0.5) \approx -6 \text{ dBFS}$$

→ *Half maximum amplitude = -6 dBFS*

V. Practical Applications

Using decibels in practice

Example: adding gain stages

Stage →	1	2	3	4	Total
Linear Gain ×	2.0	3.0	1.5	0.5	4.5
Gain in dB +	+6 dB	+9.5 dB	+3.5 dB	-6 dB	+13 dB

→ Since dB is logarithmic, multiplying ratios in linear terms becomes addition in dB.

Adding independent sound sources

Convert decibel values to linear form, perform the summation, then reconvert to decibels.

$$L_{\text{total}} = 10 \log_{10} \left(10^{L_1/10} + 10^{L_2/10} \right)$$

Independent sources add **powers**, not pressures.

Adding two sound sources 80 and 85 dB

$$10 \log_{10} \left(10^{80/10} + 10^{85/10} \right)$$

⇓

$$10 \log_{10} \left(10^8 + 10^{8.5} \right)$$

⇓

$$10 \log_{10} (4.1623 \times 10^8)$$

⇓

86.2 dB

VI. Reference

Conversion factors and relationships

Decibel quick reference

General relationships (relative changes):

- +3 dB doubles power/intensity
- +6 dB doubles amplitude/pressure
- +10 dB doubles perceived loudness
- 0 dB = no change (ratio = 1)

Applied to sound pressure level (SPL):

- +3 dB SPL doubles intensity (since $I \propto p^2$)
- +6 dB SPL doubles pressure
- +10 dB SPL doubles perceived loudness

Common misconceptions

Adding dB SPL values directly

- Wrong: $70 \text{ dB} + 70 \text{ dB} = 140 \text{ dB}$
- Correct: Must convert to linear, add, convert back ($\approx 73 \text{ dB}$)

Confusing 10 log vs 20 log

- **20 log** for: Voltage, Current, Pressure, Amplitude
- **10 log** for: Power, Intensity

Example 1

Question: A sound measures 0.2 Pa. What is the SPL in dB?

Solution

$$L_p = 20 \log_{10} \left(\frac{0.2}{20 \times 10^{-6}} \right) = 20 \log_{10}(10^4) = 20 \times 4 = 80 \text{ dB SPL}$$

Example 2

Question: Two independent sound sources each produce 70 dB SPL. What is the total SPL when both operate?

$$\begin{aligned} L_{\text{total}} &= 10 \log_{10}(10^7 + 10^7) \\ &= 10 \log_{10}(2 \times 10^7) \\ &= 10[\log_{10}(2) + \log_{10}(10^7)] \\ &= 10[0.301 + 7] \approx 73 \text{ dB SPL} \end{aligned}$$

→ *Independent sources add powers, not dB values.*

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