Filters

Shaping the spectrum of audio signals

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Course info

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Filters

Shaping the spectrum of audio signals

Examples of a filter

- **Vocal tract:** Dynamic filtering.
- **Human hearing (e.g., pinna, head, torso):** Acts as a direction-selective filter.
- **Musical instruments (e.g., Helmholtz resonator):** Filtering through resonances
- **Rooms:** Altering the sound waves through boundary behaviors.
- **Loudspeakers and microphones:** Shape sound through their frequency response.
- ...

Filter

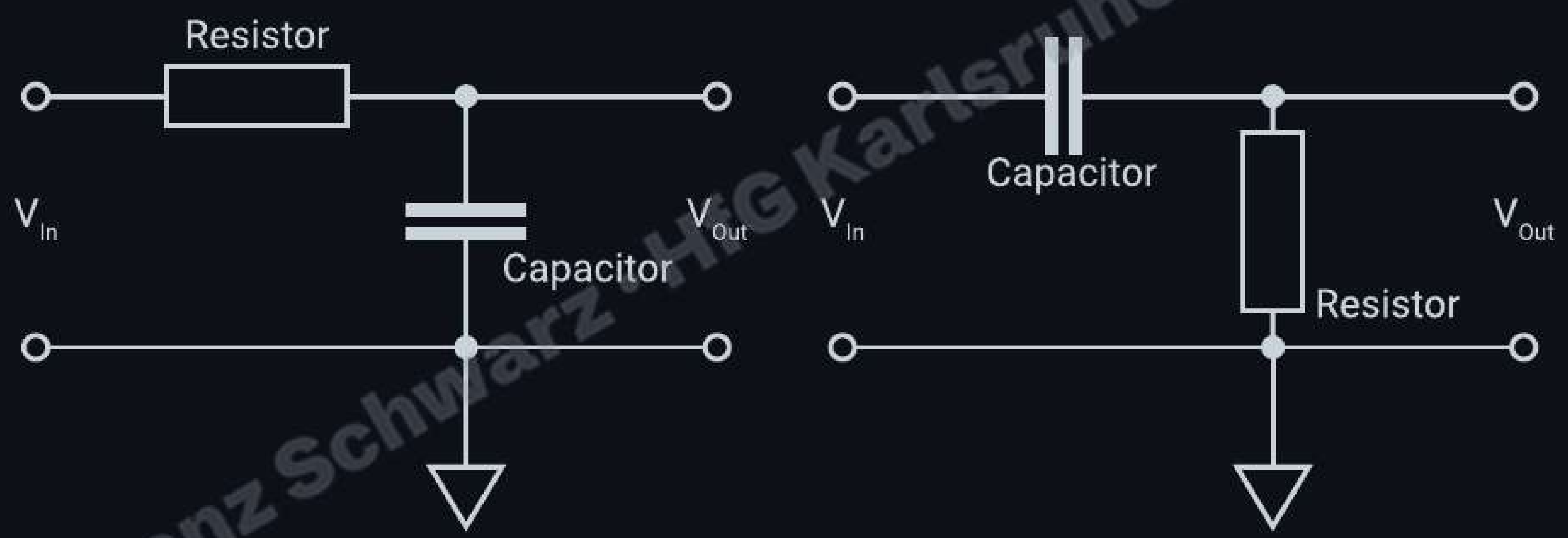
A filter selectively emphasizes or attenuates certain frequencies in a signal:

- commonly a device, electronic circuit or software algorithm
- used to shape the frequency spectrum of an audio signal

→ *Phase response is often perceptually less critical, but may be relevant in some contexts.*

Electronic filter

First order lowpass filter (left) and high pass filter (right)

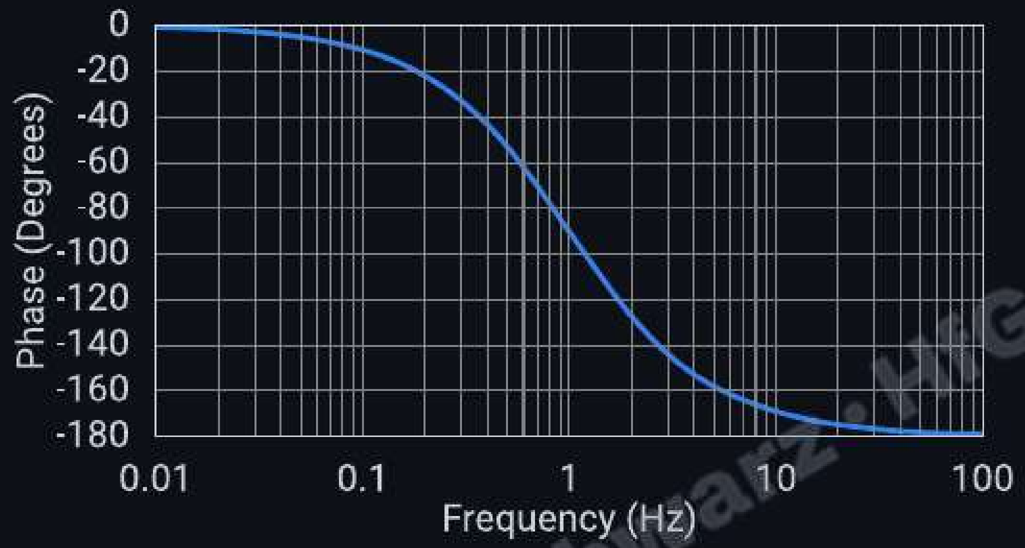


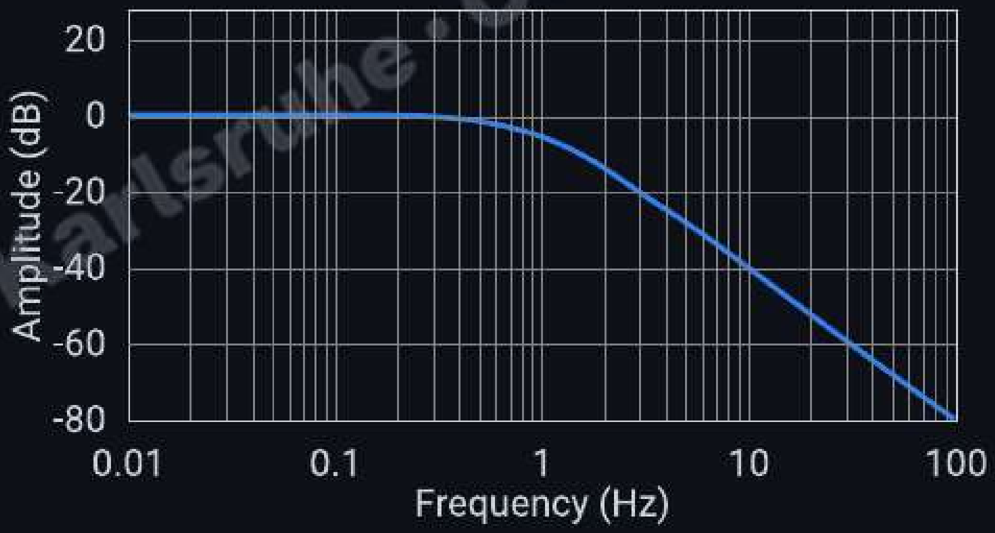
Visualization of a filters behavior

Transfer function plots graphically represent how a filter modifies input signals across frequencies.

- **Frequency (x-axis):** Plotted on a logarithmic scale (in Hz).
- **Magnitude (y-axis):** Plotted on a logarithmic scale (in dB).
- **Phase angle (y-axis):** Plotted on a linear scale (in degrees or radians).

Phase (left) and magnitude response (right) of a low-pass filter





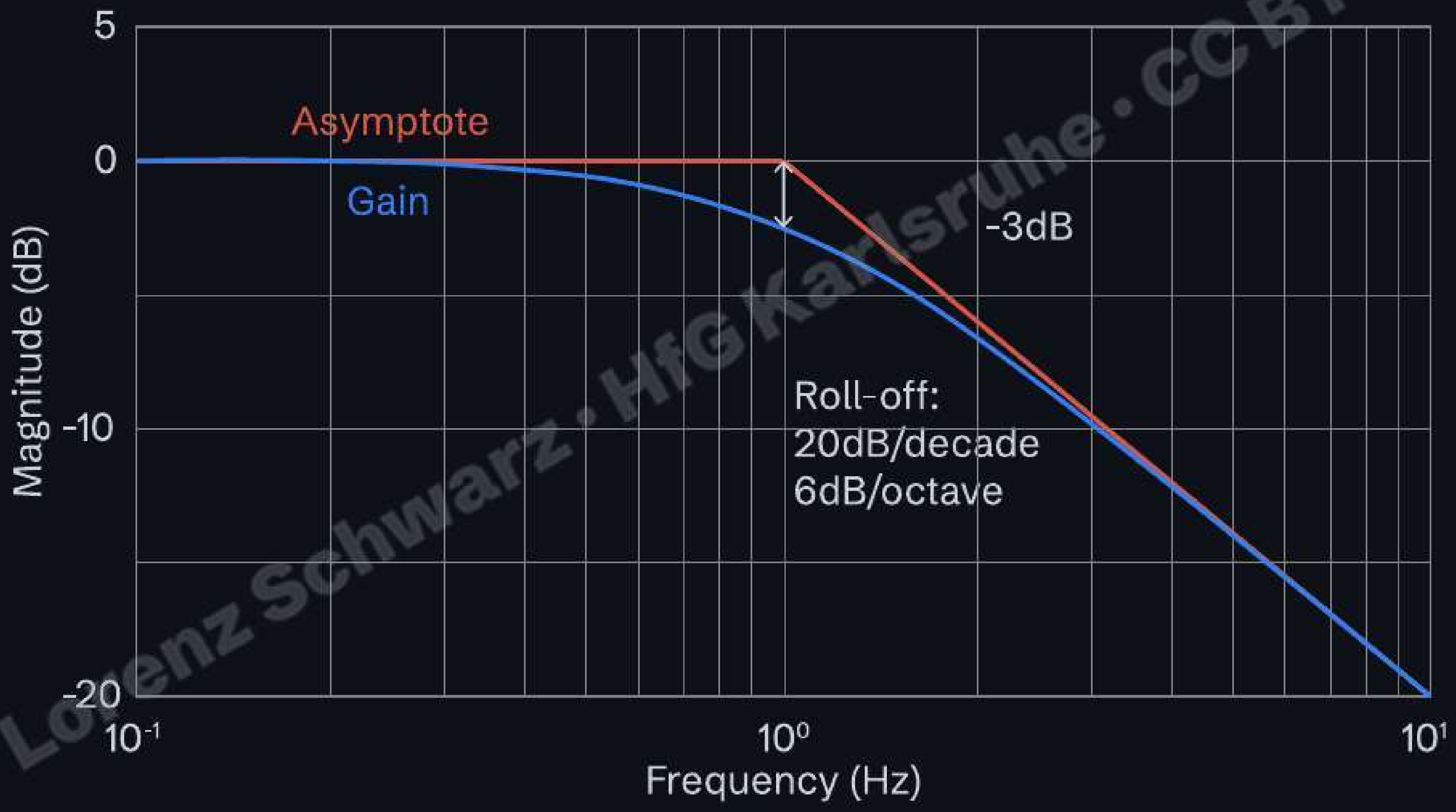
Magnitude response of a filter

The Bode magnitude response plot shows the gain (ratio of output amplitude to input amplitude) of a filter as a function of frequency, typically displayed on a logarithmic scale.

Gain versus frequency:

- **Vertical axis (magnitude):** Logarithmic scale, typically in decibels (dB).
- **Horizontal axis (frequency):** Logarithmic scale in Hertz (Hz).

Bode magnitude plot

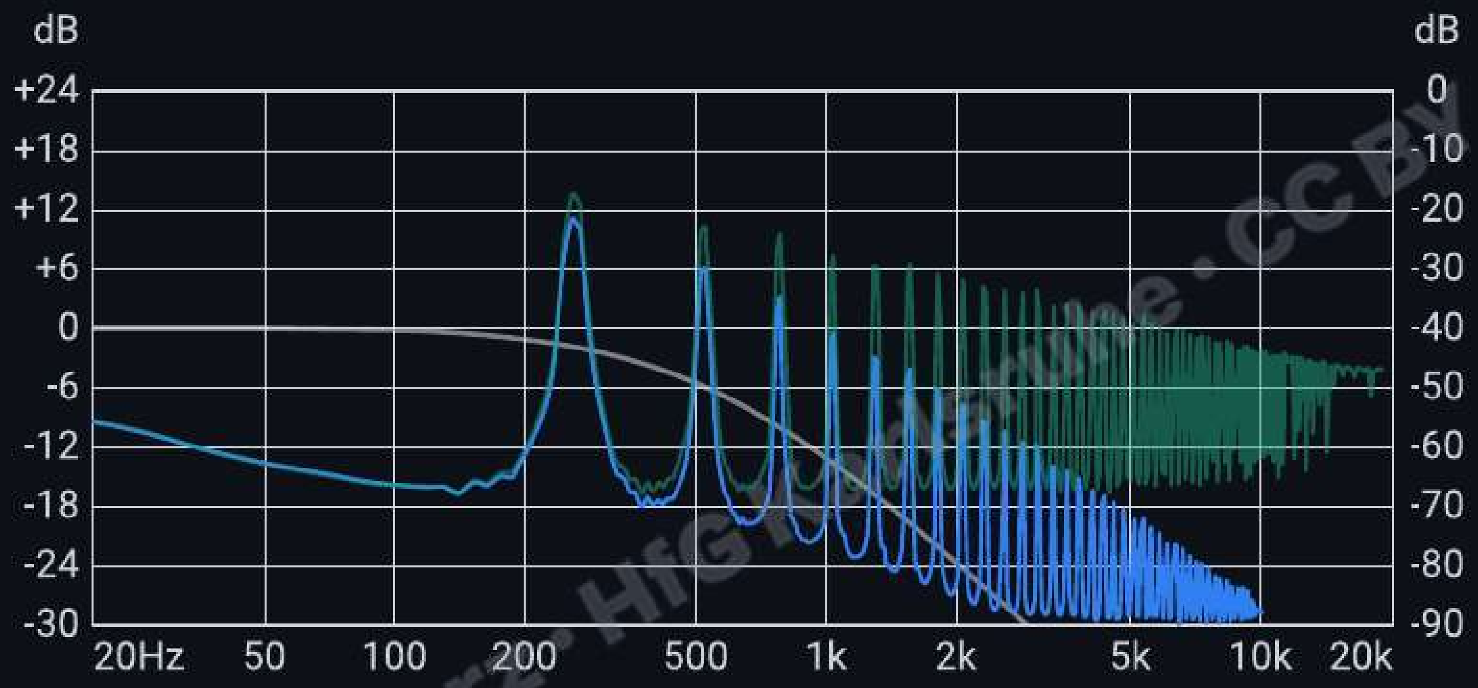


Magnitude of gains

$\left\lvert\frac{V_{out}}{V_{in}}\right\rvert$	10^{-4}	0.001	0.01	0.1	1	10	100	1000	10^{4}
Gain	-80dB	-60dB	-40dB	-20dB	0dB	20dB	40dB	60dB	80dB

Attenuation Unity Amplification

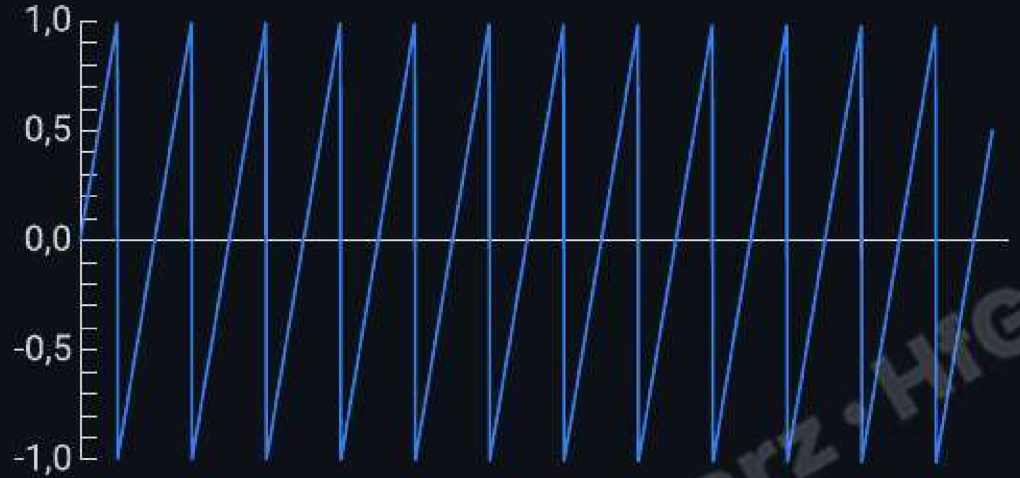
$$gain\ in\ dB = 20\log_{10}\left|\frac{V_{out}}{V_{in}}\right|$$



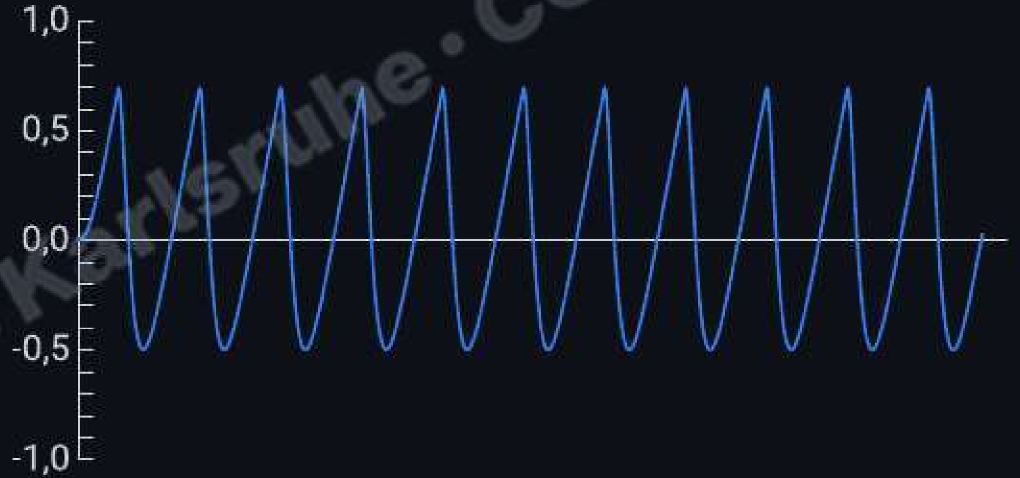
The output spectrum (blue) is the input spectrum (green) multiplied by the filter's magnitude response (grey curve).

▶ Sawtooth 260 Hz unfiltered - filtered

Filtered waveform in the time domain



Input

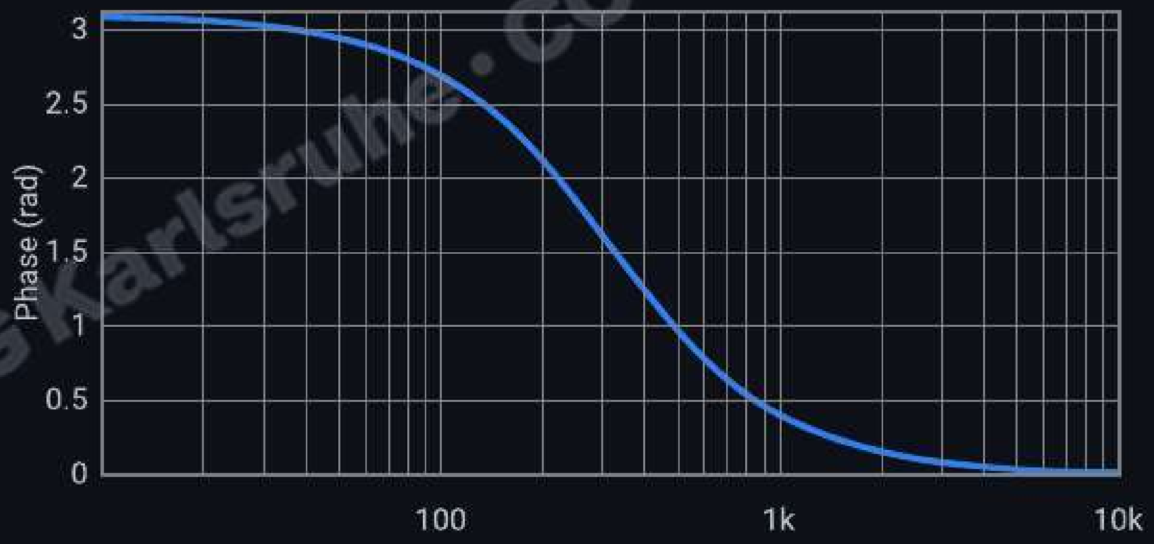


Output

Phase response of a filter

A Bode phase plot is a graph that shows the phase relationship between a sinusoidal input signal and the output signal of a filter, as a function of frequency.

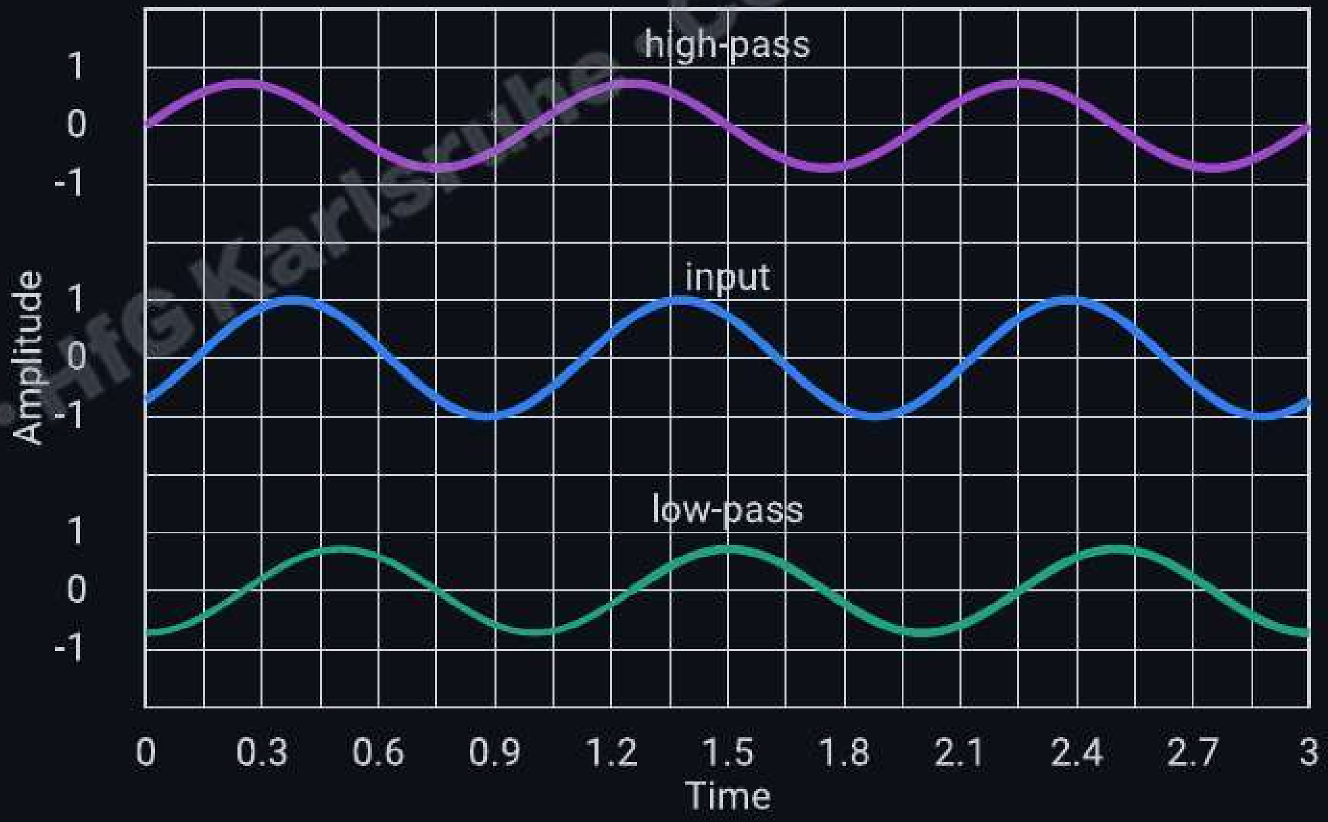
- **Vertical axis (phase):** Linear scale, typically in degrees or radians, representing the phase shift introduced by the filter.
- **Horizontal axis (frequency):** Logarithmic scale in Hertz (Hz).



Phase and time-domain filters

The following filters don't primarily remove or boost frequencies, but make use of a filters property to manipulate phase relationships of a signal to create interference patterns.

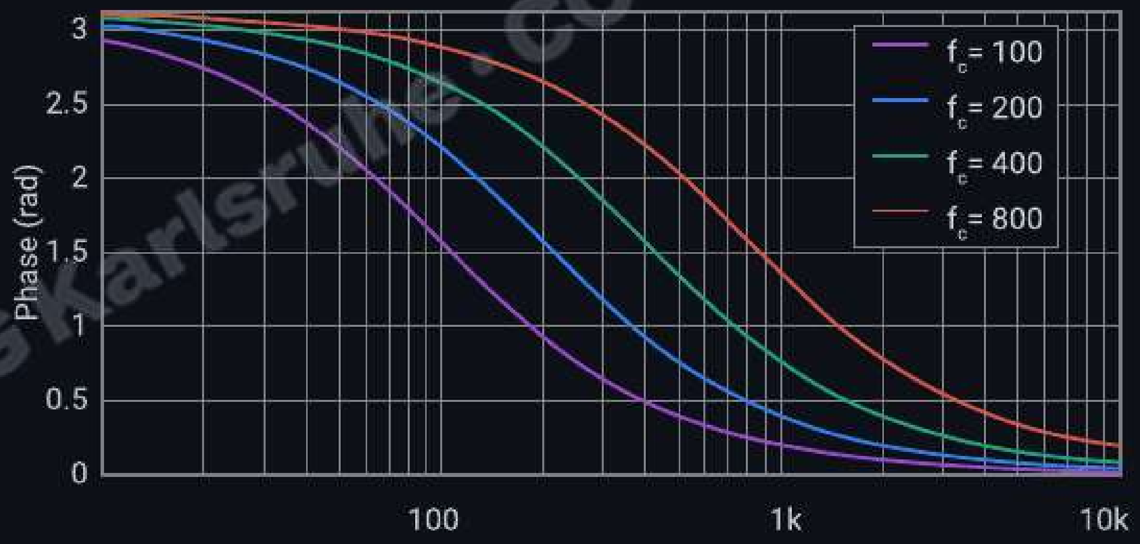
left: Input and outputs of a single-pole high-pass and low-pass filter



All-pass filter

An all-pass filter changes the phase relationship between frequencies by introducing a frequency-dependent phase shift, while allowing all frequency components to pass with equal amplitude (unity gain).

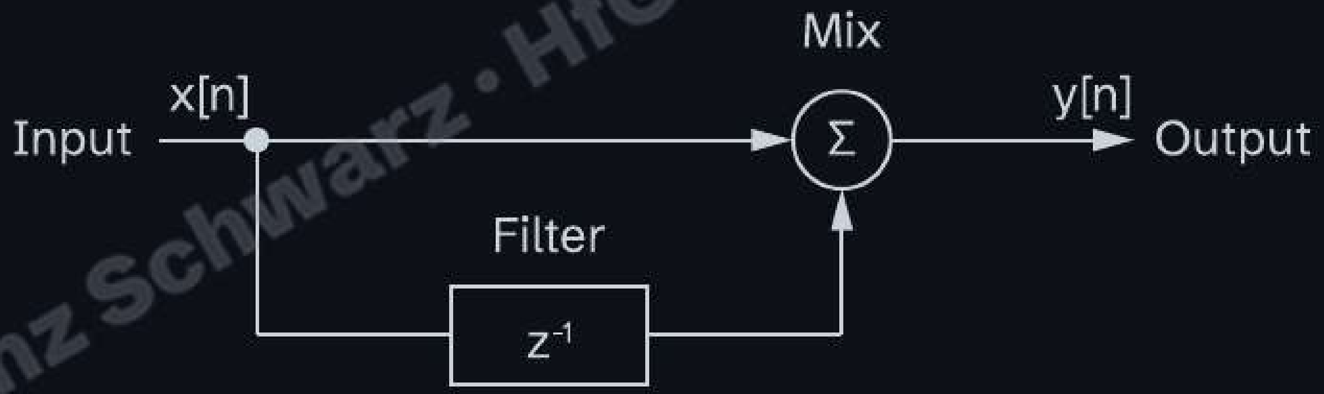
Applications: Phaser audio effect



Comb filtering in sound (phaser)

A comb filter mixes a signal with a delayed copy of itself:

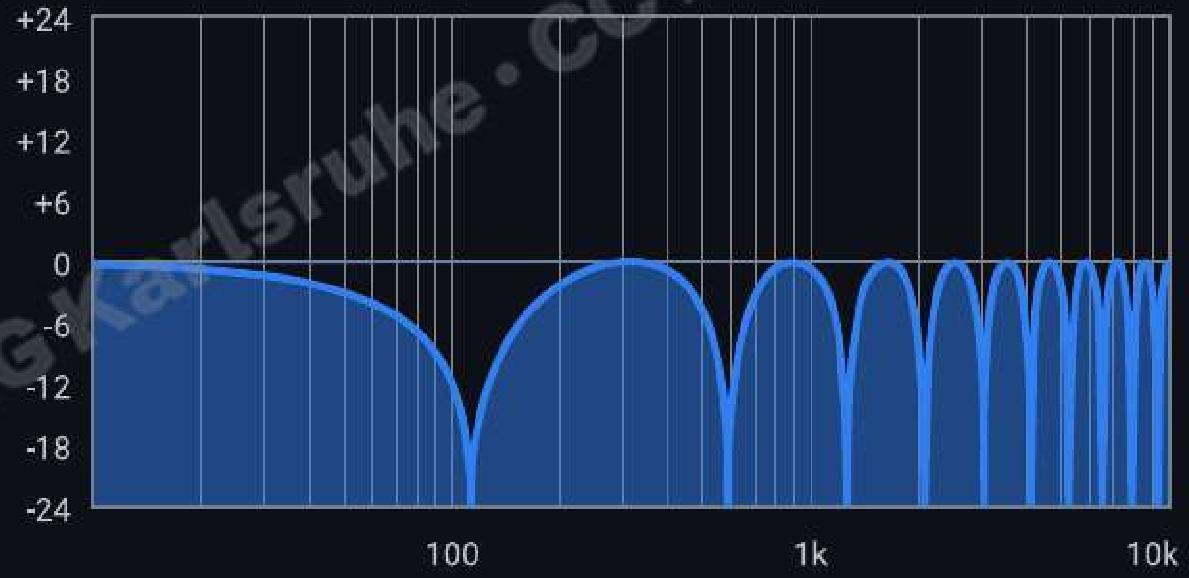
- causes constructive and destructive interference
- produces characteristic spectral notches (resemble the teeth of a comb)



▶ White noise comb filtered

Natural occurrences of comb filtering

- Early reflections from hard surfaces (floor or wall reflections)
- Multiple microphones capturing the same source at different distances

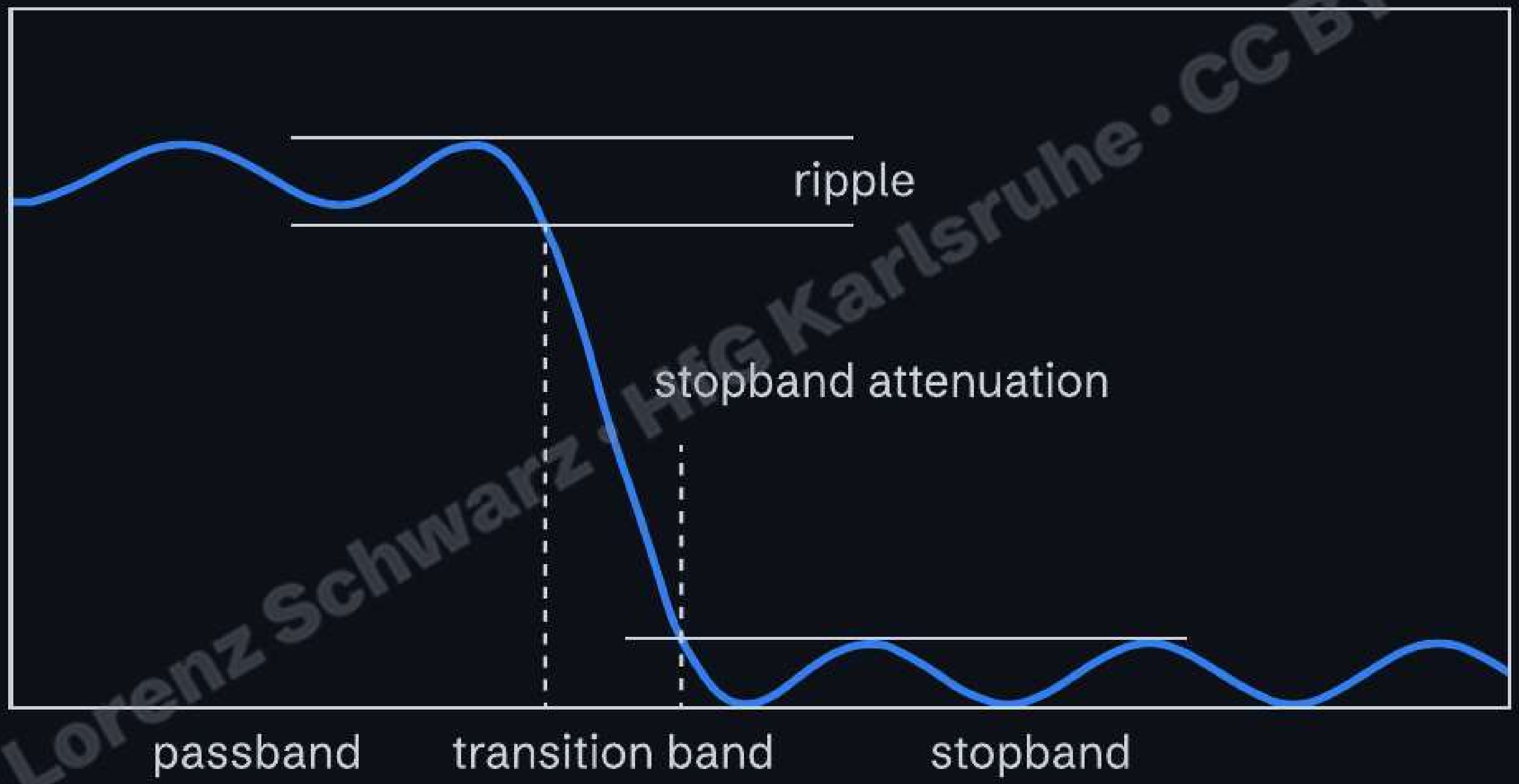


→ *Comb filtering can be an unwanted acoustic artifact or a deliberate effect.*

Passband, stopband, transition band, ripple

- **Passband:** The range of frequencies that pass through the filter with no attenuation.
- **Stopband:** The range of frequencies that are significantly attenuated or blocked by the filter.
- **Transition band:** The region between the passband and stopband where attenuation gradually increases.
- **Ripple:** Deviation from flatness in a filter's magnitude response, showing the ratio of max to min gain in the passband or stopband.

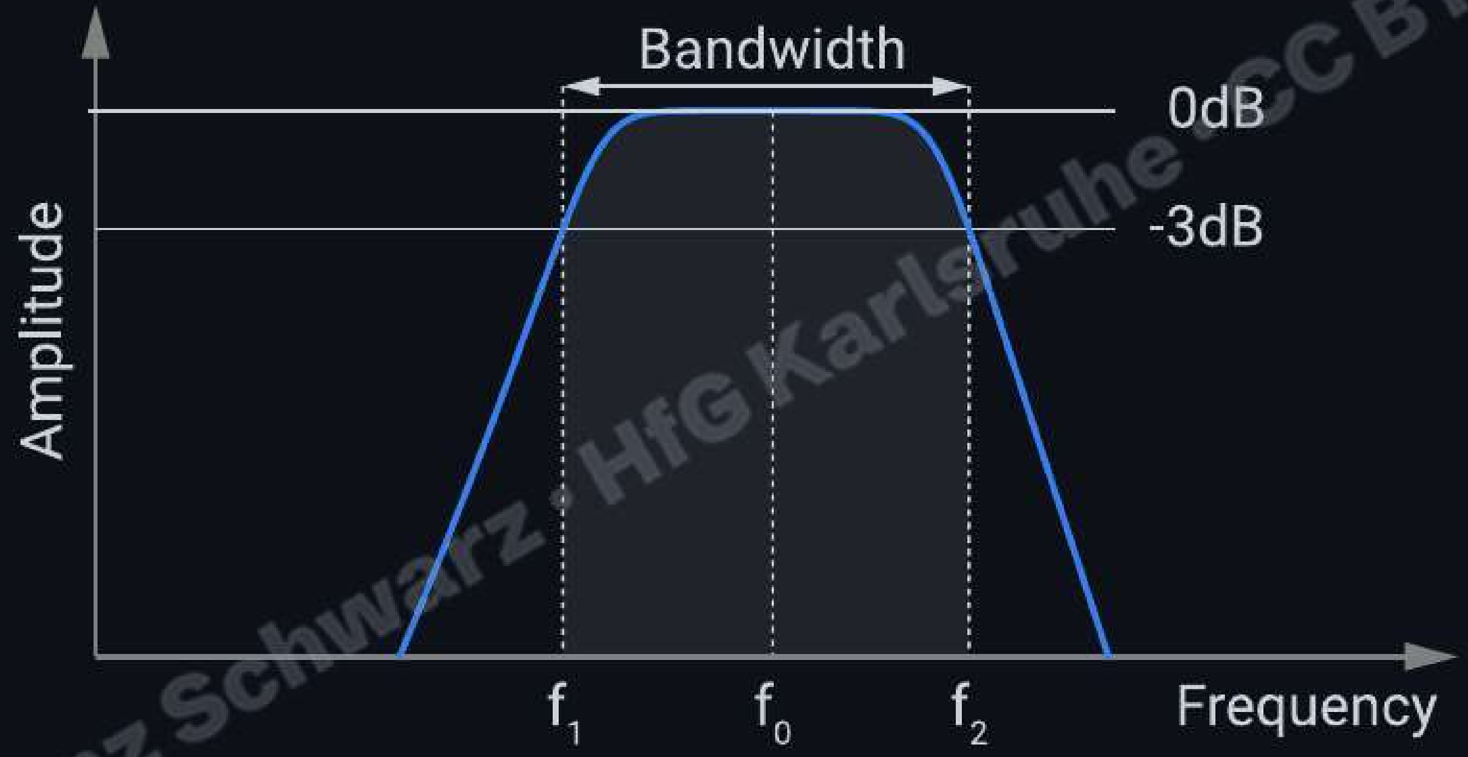
Description of the magnitude response



Bandwidth

The range of frequencies passed by a band-pass filter or attenuated by a notch filter, defined as the difference between the upper and lower cutoff frequencies. It represents the section of the frequency spectrum that the filter affects most significantly.

Bandwidth, lower and upper corner frequency



Cutoff or corner frequency

The cutoff frequency (also known as the half-power point) is the frequency at which the output voltage level decreases by 3 dB compared to the input voltage level (0 dB). Beyond this point, the output voltage progressively decreases relative to the input voltage.

The -3 dB level corresponds to the factor:

$$\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = 0.7071$$

This means the output voltage is about 0.7071 times the input voltage.

Center frequency f_0

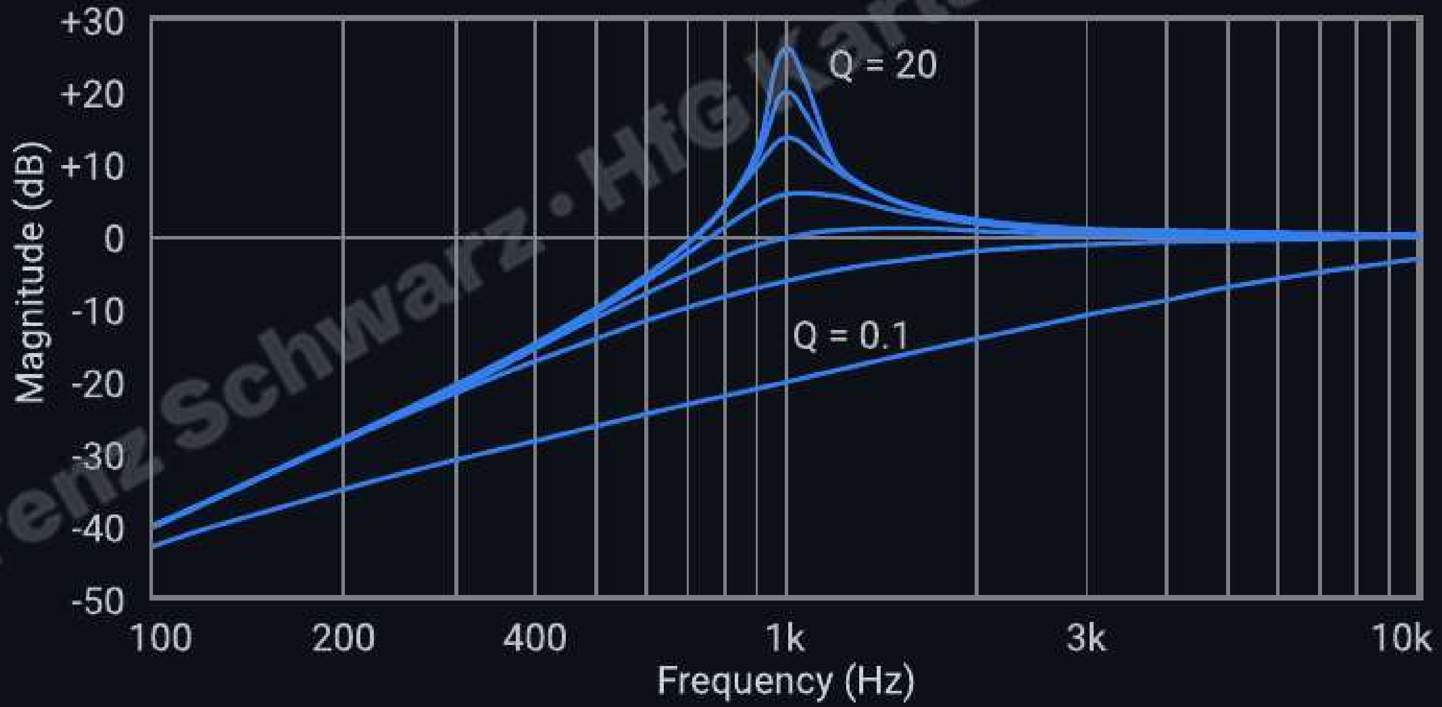
The midpoint center frequency of a band pass filter is the geometric mean between the lower cuttoff frequency and the upper cutoff frequency.

Geometric mean:

$$f_{0geo} = \sqrt{f_1 \cdot f_2}$$

Quality factor (Q-factor)

Parameter that describes the selectivity of a filter, defined as the ratio of the center frequency to the bandwidth of the filter



Filter resonance

Filter resonance refers to the phenomenon where a filter exhibits a peak in amplitude at a specific frequency, often near the cut-off frequency.

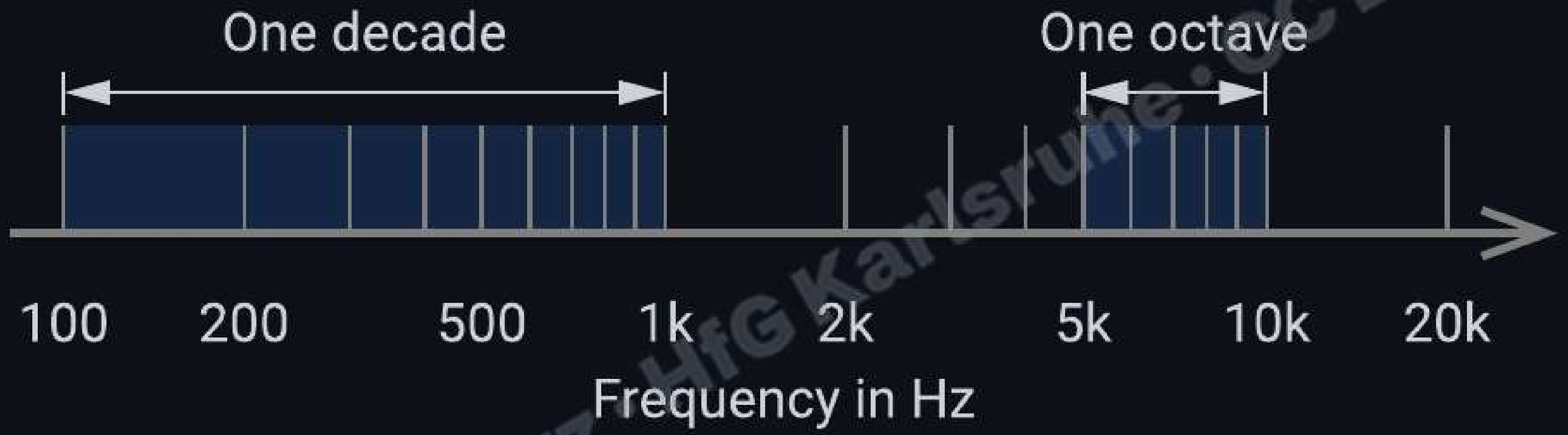
- A sufficiently high Q-factor can lead to self-oscillation even without an input signal.

▶ Low pass sweep

▶ Low pass sweep with resonance

▶ Low pass self-oscillating

Frequency ratio units



- **Octave:** Doubling or halving of frequency
- **Decade:** Tenfold increase or decrease in frequency (factor-of-ten)

Relationship of filter key parameters

Center frequency f_0

$$f_0 = \sqrt{f_1 \cdot f_2}$$

Bandwidth B

$$B = f_2 - f_1$$

Quality factor Q

$$Q = \frac{f_0}{B}$$

Example: calculating bandwidth

Relation between center frequency (f_0), bandwidth (B), and Q-factor (Q)

Example: $f_0 = 800Hz$, $Q = 10$

$$B = \frac{f_0}{Q} = \frac{800}{10} = 80Hz$$

- high Q $\longleftrightarrow$ narrow bandwidth
- low Q $\longleftrightarrow$ wide bandwidth

Relation between Q-factor and bandwidth

large Q = narrow bandwidth $\longleftrightarrow$ small Q = broad bandwidth.

BW in octaves	Q
2.0	0.667
1.0	1.414
2/3	2.145
1/2	2.871
1/3	4.318
1/6	8.651
1/10	14.424
1/30	43.280

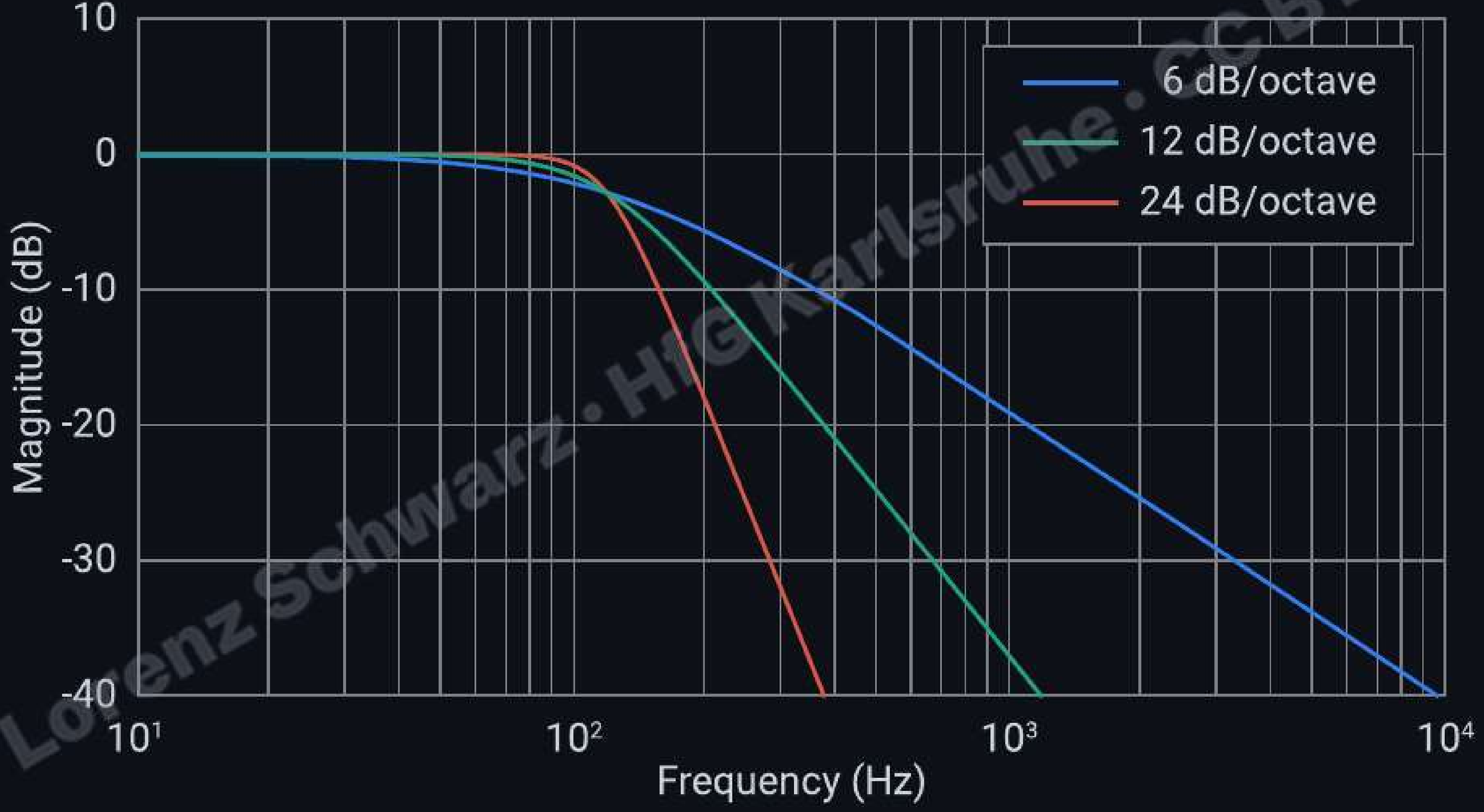
Roll-off (filter slope)

Roll-off describes the steepness of the transition from the passband to the stopband in a filter's transfer function graph.

- Indicates how rapidly the filter attenuates the signal beyond the cutoff frequency.
- Measured in decibels per octave (dB/octave) or decibels per decade (dB/decade).

▶ White noise corner frequency 500 Hz

Roll-off



6 dB/octave
12 dB/octave
24 dB/octave
Magnitude (dB)
10
0
-10
-20
-30
-40
10^1
10^2
10^3
10^4
Frequency (Hz)

Filter order

The filter order refers to the highest power of the variable in the polynomial of the filter's transfer function, which is the algebraic representation of the filter's behavior. The order determines the steepness of the filter's attenuation beyond the cutoff frequency.

For example 3rd order Butterworth polynomial:

$$B(s) = (s^2 + s + 1)(s + 1)$$

→ *Higher-order filters produce steeper slopes.*

Filter order and corresponding roll-off rates

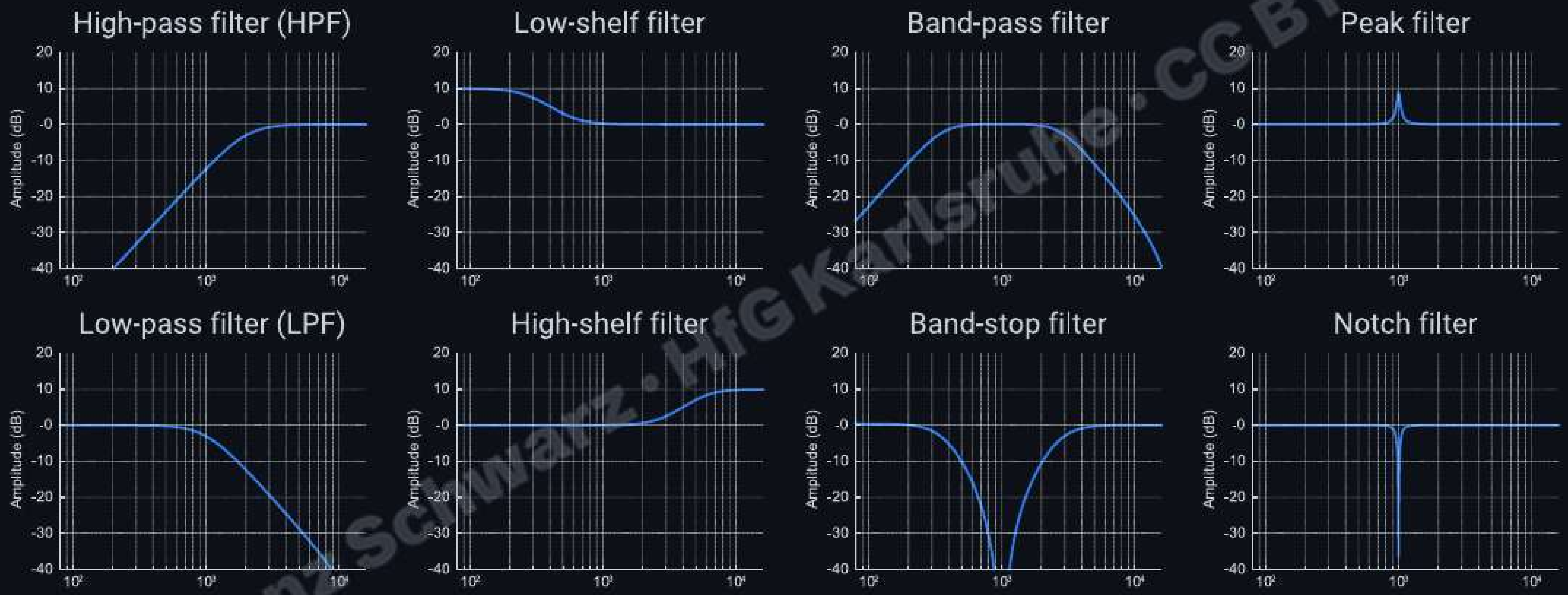
Each increase in filter order results in a roll-off rate increase of 6 dB per octave.

Filter order	dB/Octave	dB/decade
first order	6	20
second order	12	40
third order	18	60
fourth order	24	80
fifth order	30	100
sixth order	36	120
seventh order	42	140
eighth order	48	160

Filter types

- Low-pass filter
- High-pass filter
- Shelving filters
- Band-pass filter
- Band-stop filter
- Notch filter
- Peak filter

Overview of common filter types



Spectrum-limiting filters

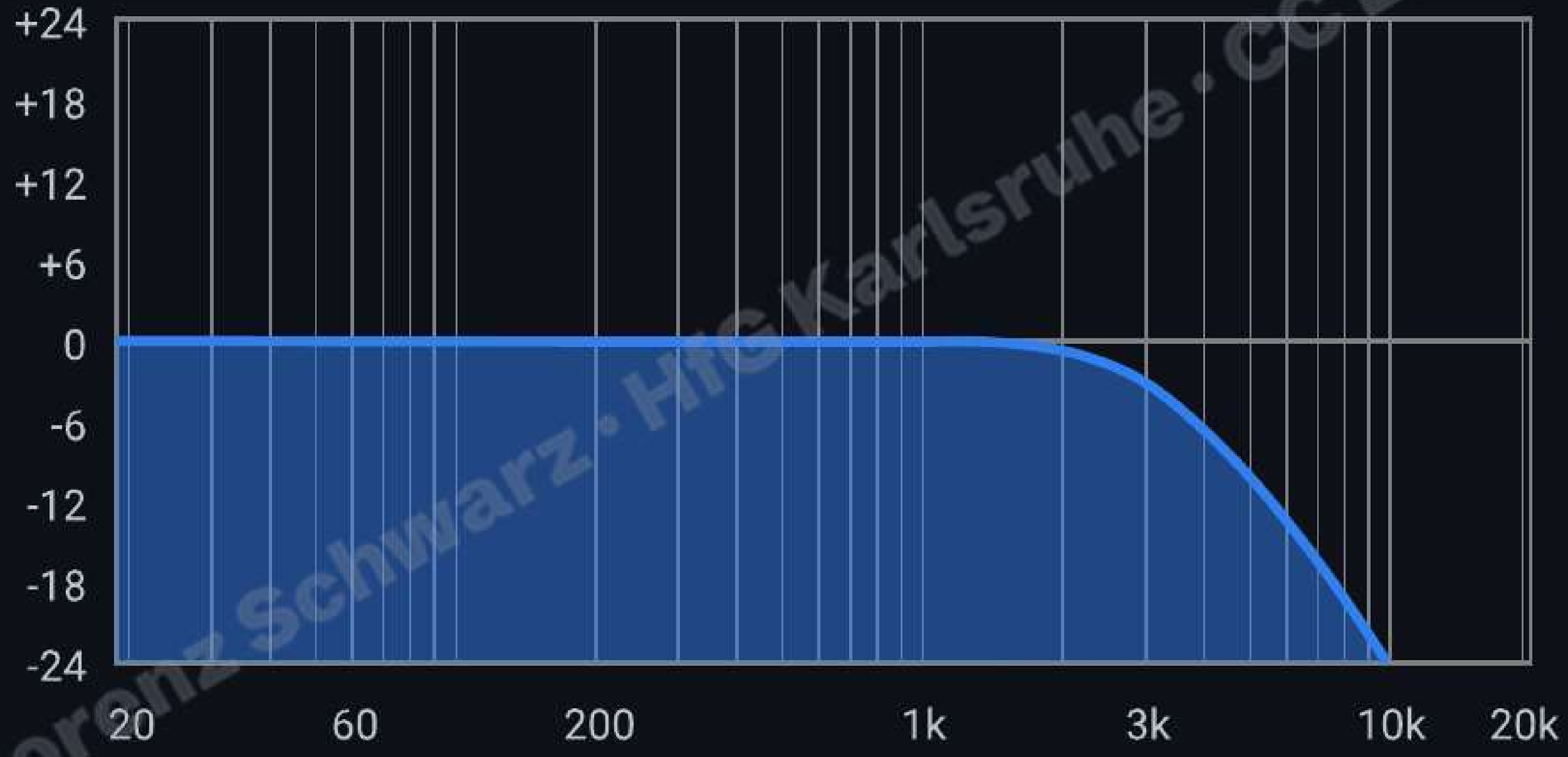
Low-pass and high-pass are the most fundamental filter types. They define a boundary frequency and progressively attenuate everything on one side of it.

Low-pass filter (LPF) - high cut

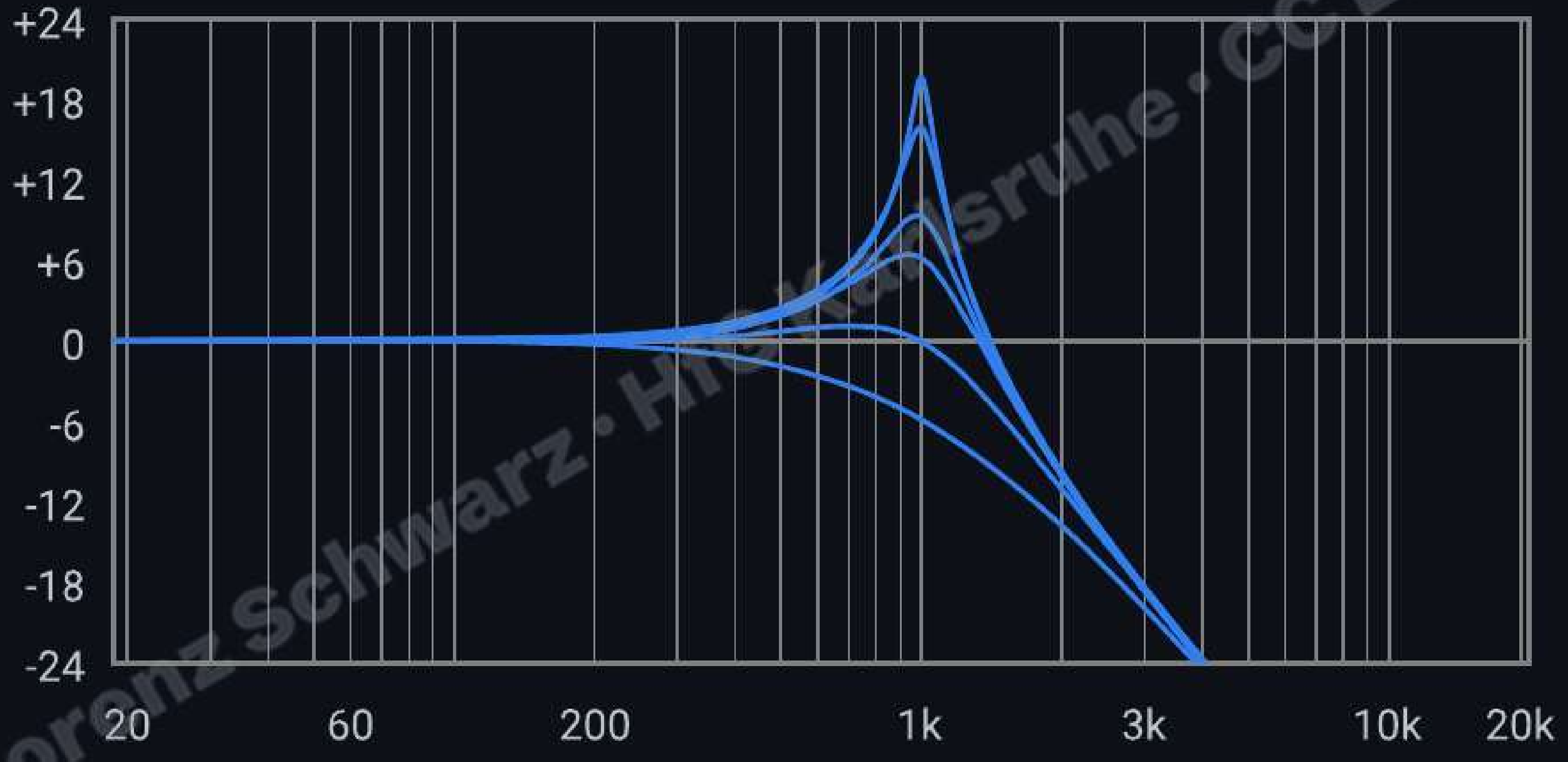
A low-pass filter is the opposite of a high-pass filter:

- It allows low-frequency components to pass while attenuating frequencies above the cutoff frequency.
- Commonly used to remove high-frequency noise.

Low-pass filter



Low-pass with different Q factors



Applications of low-pass filtering

Low-pass filters attenuate high-frequency content and are widely used in audio processing and sound design.

- Removing high-frequency hiss or noise from recordings
- Creating a muffled sound to simulate distance or obstruction
- Subtractive synthesis by shaping timbre from harmonically rich waveforms
- Anti-aliasing before digital sampling

High-pass filter (HPF) - low cut

A high-pass filter is the opposite of a low-pass filter:

- It allows high-frequency components to pass while attenuating frequencies below the cutoff frequency.
- Commonly used to remove low-frequency noise or rumble from audio signals.

High-pass filter

Applications of high-pass filtering

High-pass filters attenuate low-frequency content and are commonly used to improve clarity and technical efficiency in audio systems.

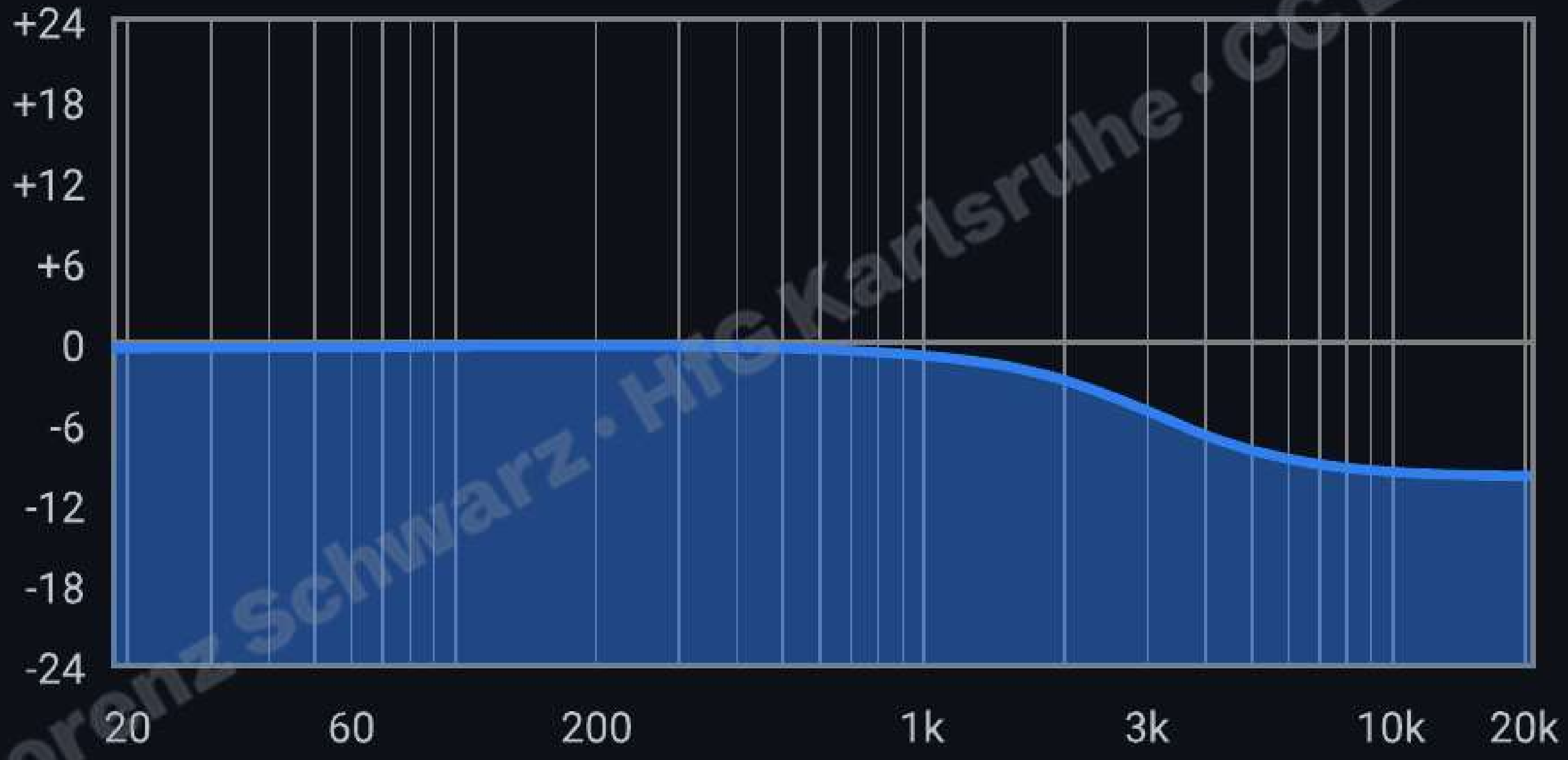
- Removing rumble or handling noise from recordings
- Filtering instruments that do not require bass content
- Preventing low frequencies from wasting amplifier headroom

Shelving filters

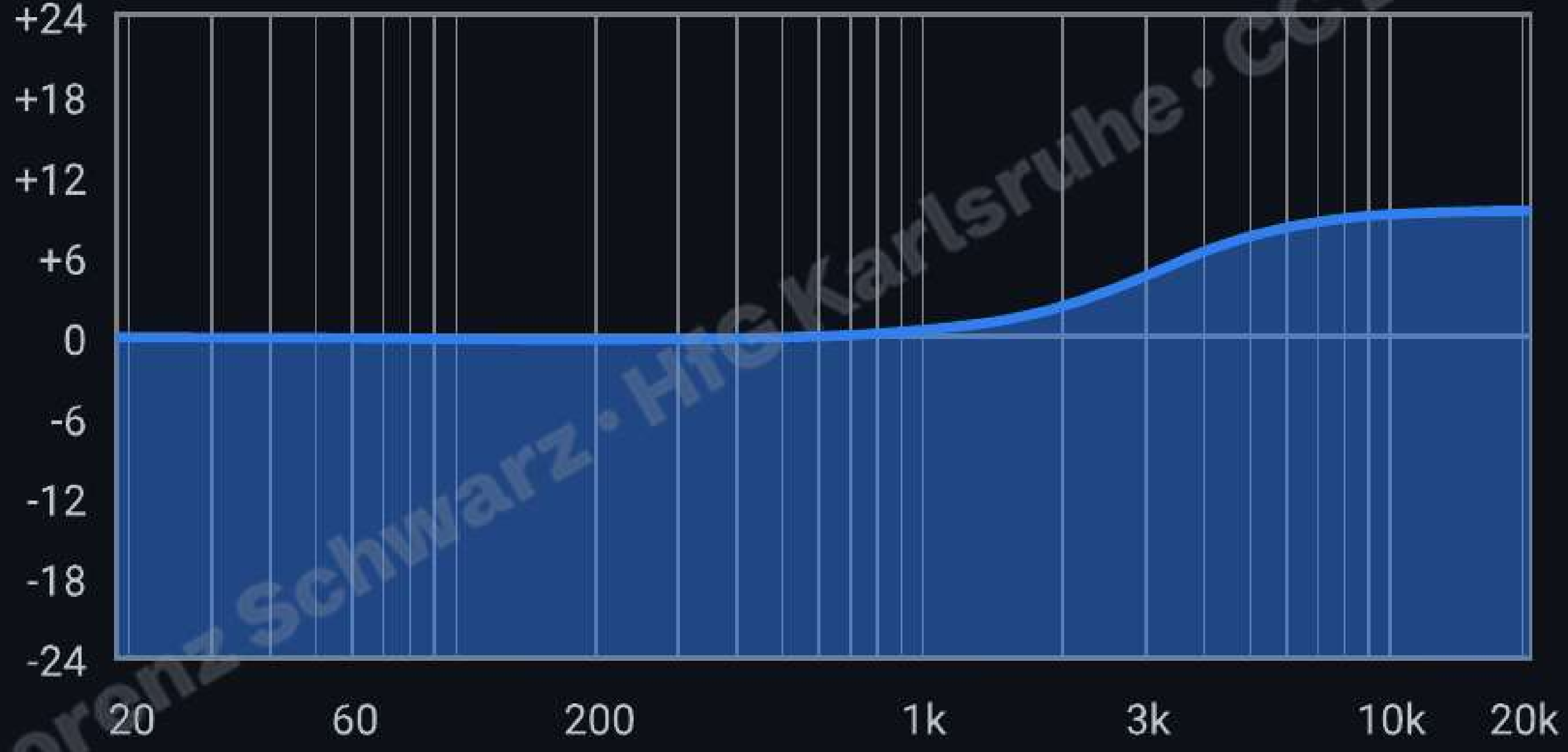
Unlike low-pass and high-pass filters, shelving filters boost or cut frequencies to a fixed level and then plateau rather than continuing to attenuate.

- Low-shelf filters affect frequencies below a cutoff frequency
- High-shelf filters affect frequencies above a cutoff frequency
- Commonly used for general tone control in audio systems and mixing desks

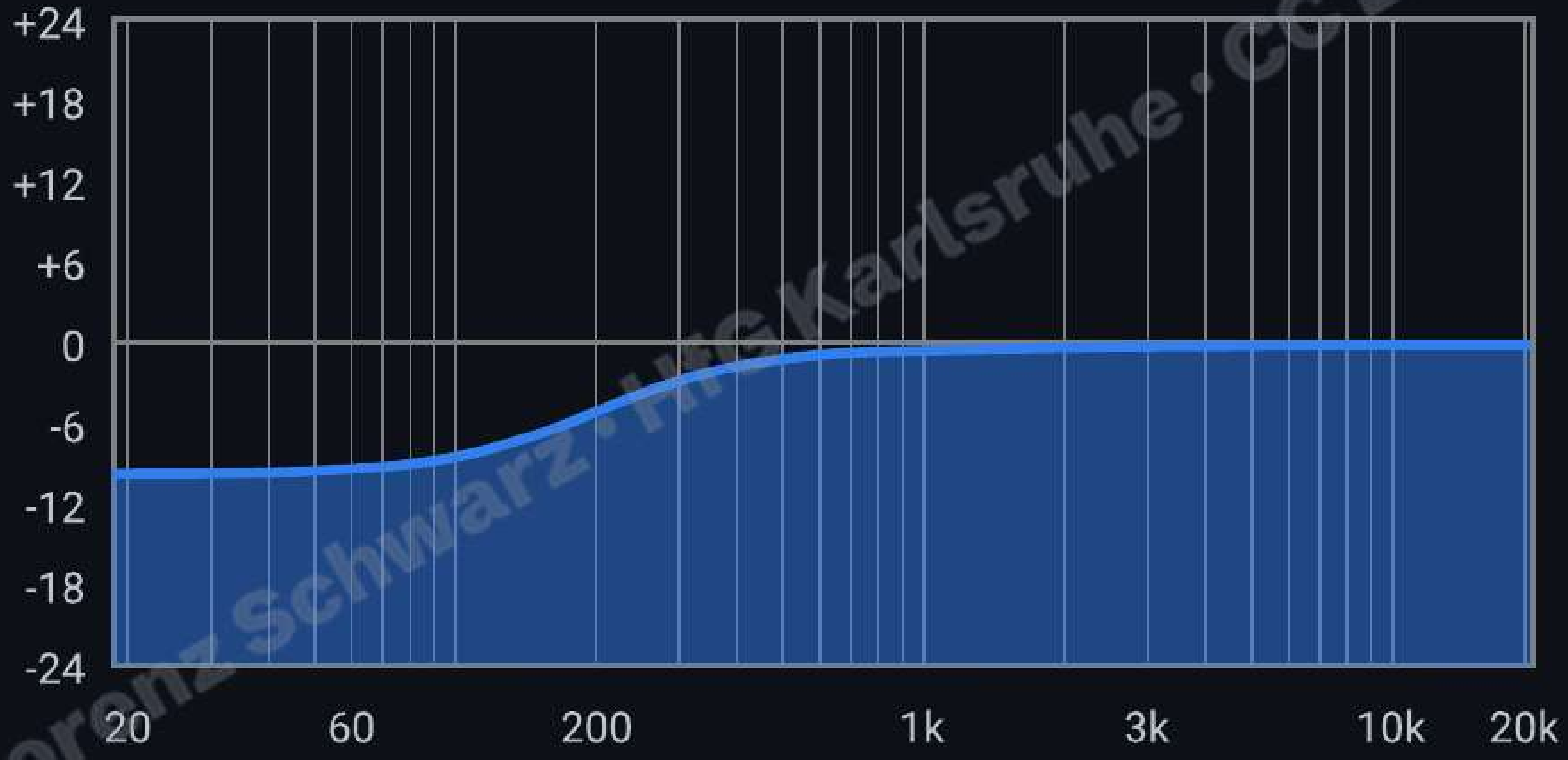
High shelf (cut)



High shelf (boost)



Low shelf (cut)



Low shelf (boost)

Creative use of shelving EQ

Shelving equalizers are commonly used for broad, musical tonal shaping rather than precise corrective filtering.

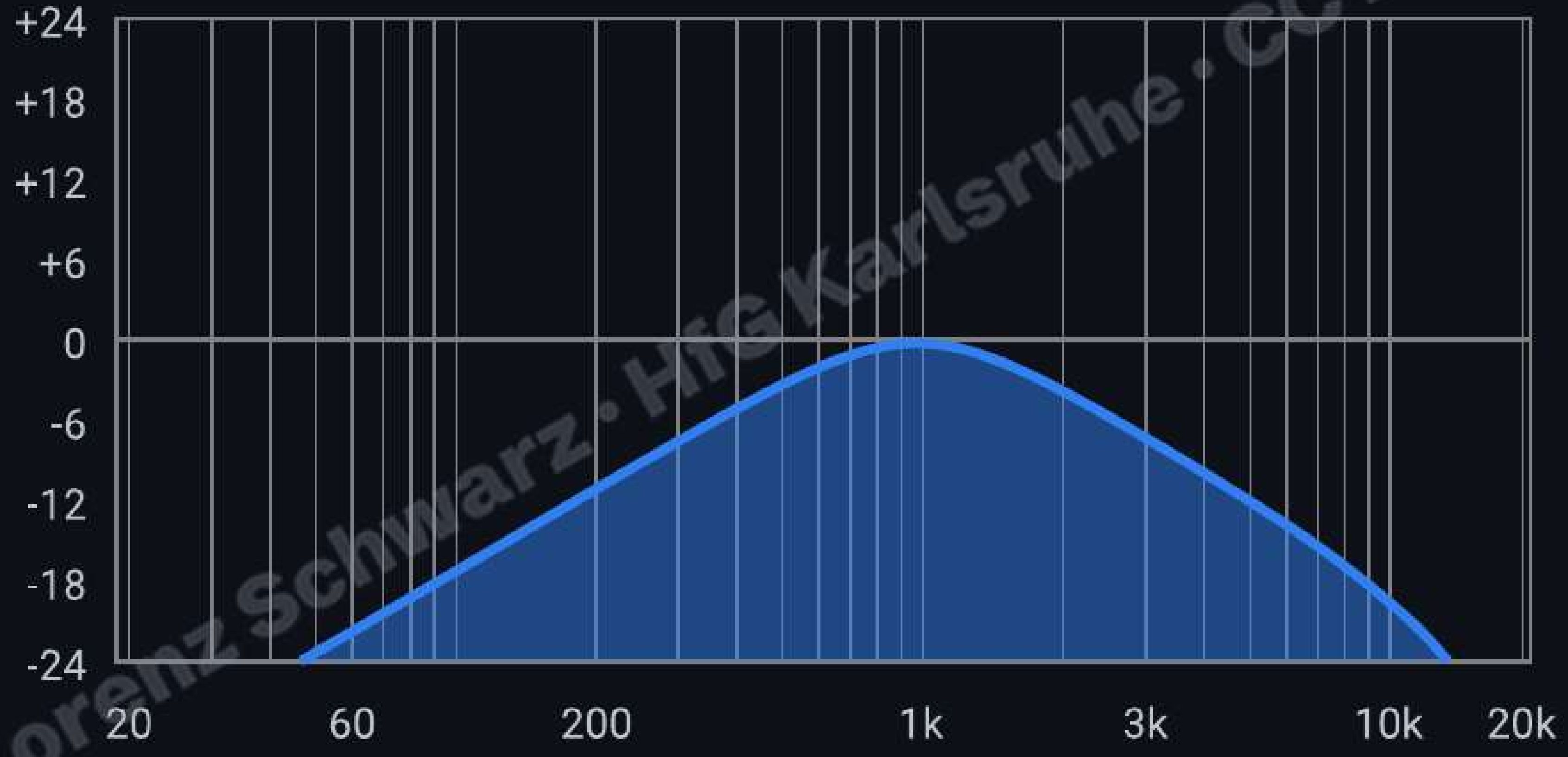
- **High-shelf boost:** adds “air” or presence (≈ 10–12 kHz and above)
- **High-shelf cut:** tames harshness; can act as a gentle de-essing alternative
- **Low-shelf boost:** adds warmth and weight (use carefully to avoid muddiness)
- **Low-shelf cut:** reduces boominess and proximity effect

Band-pass filters

Band-pass filters target a specific frequency range rather than everything above or below a single cutoff, making them useful for isolating or emphasizing selected spectral content.

- Defined by a lower and an upper cutoff frequency
- Frequencies outside this band are attenuated
- Commonly used to isolate or remove specific frequency components

Band pass filter



Applications of band-pass filtering

Band-pass filters isolate a defined frequency range.

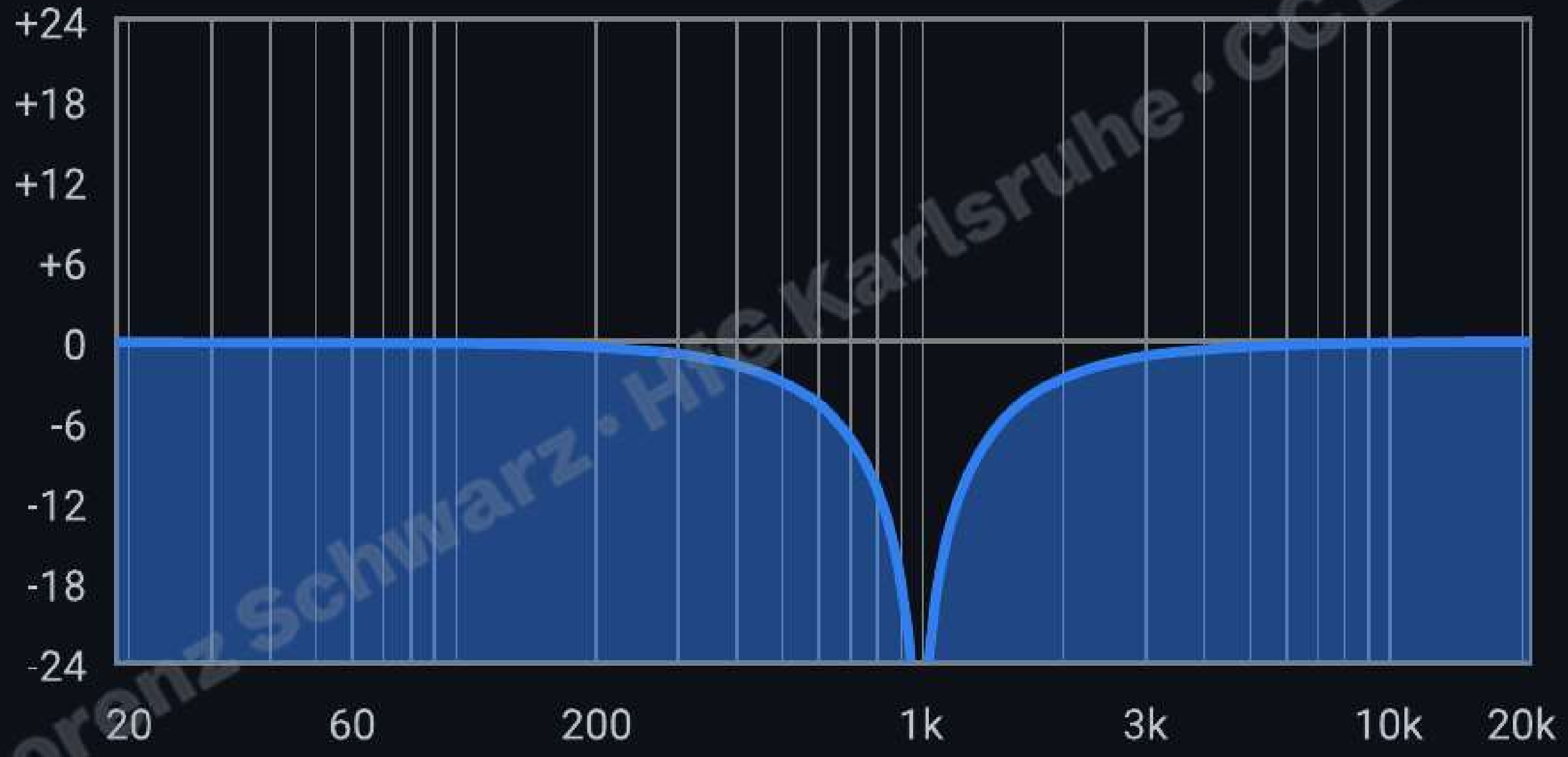
- Creating “telephone” or “radio” voice effects (narrow band-pass ≈ 300 Hz–3 kHz)
- Formant filtering in vocoders
- Wah-wah effects using a swept band-pass filter

Band-stop

A band-stop filter is the opposite of a band-pass filter:

- Frequencies within a specified range (the filter band) around the center frequency are suppressed, while frequencies outside this range are allowed to pass.

Band-stop filter

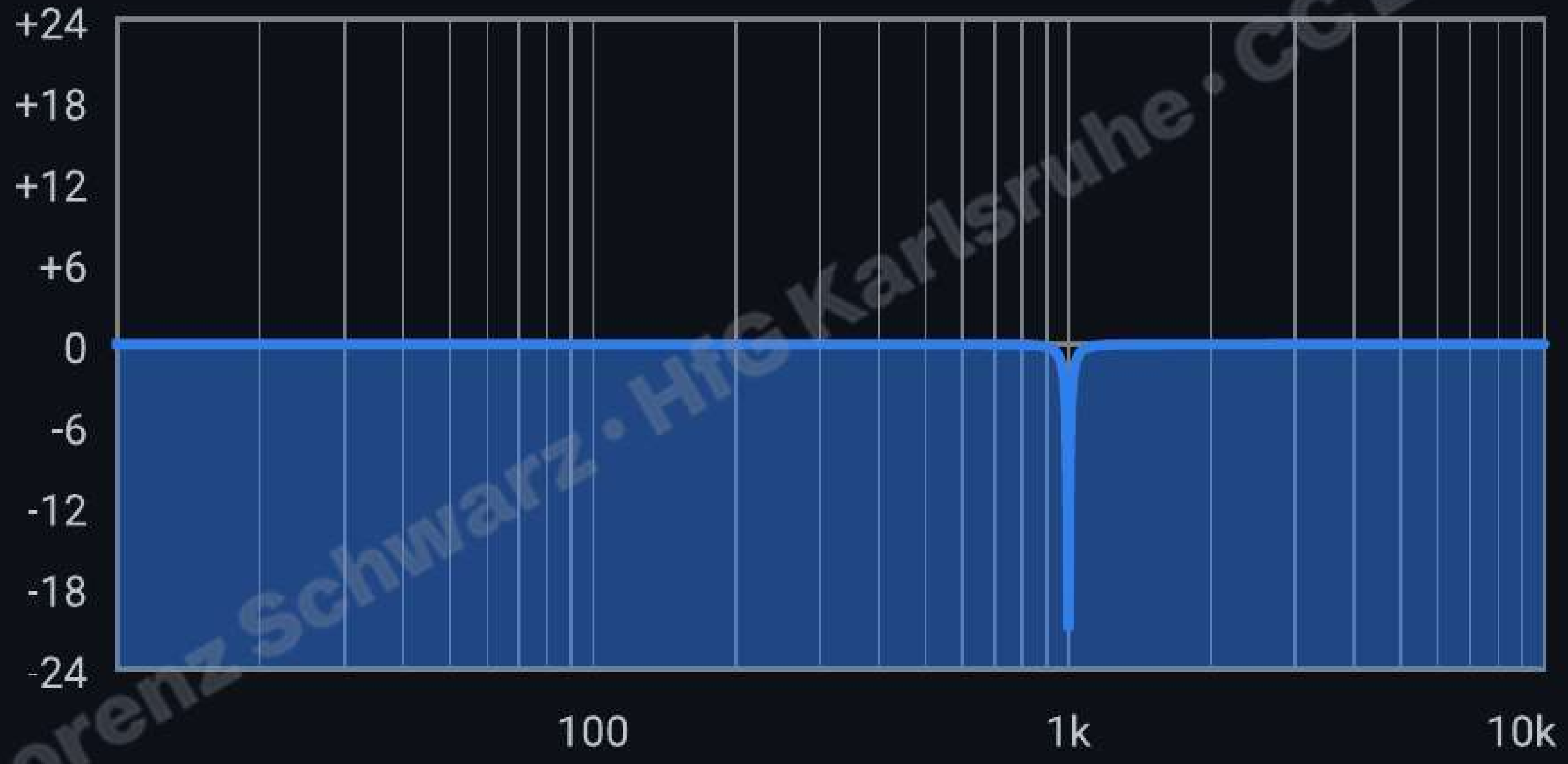


Notch filter

A notch filter is an extremely narrow banded type of band-stop filter, designed to attenuate a very specific frequency or a small range of frequencies while leaving other frequencies unaffected.

- Commonly used to eliminate unwanted sounds such as power line hum (e.g., 50 Hz or 60 Hz) or unwanted resonances (e.g., feedback).

Notch filter



Notch filters

Notch filters remove a very narrow frequency band while leaving the rest of the spectrum largely unaffected.

- Eliminating mains hum (50 Hz in Europe, 60 Hz in North America) and its harmonics
- Feedback suppression in live sound by targeting problematic resonant frequencies
- Removing room modes in live sound mixing or on location recordings

→ *Notch filters enable precise removal of unwanted frequencies without altering overall timbre.*

Peak filter

A peak filter boosts or attenuates frequencies within a specified range around a center frequency, forming a "bell-shaped" response.

- Commonly used in multi-band EQs to target specific frequency bands for precise adjustments without affecting surrounding frequencies.

Peak filter

Peak (bell) filters

Peak (bell) filters are the primary building blocks of parametric equalizers, allowing localized gain adjustments around a center frequency.

- Used for corrective (problem-focused) and tonal shaping
- Narrow bandwidth (high Q): precise attenuation of resonances
- Wide bandwidth (low Q): broad spectral shaping

Parametric EQ

Parametric equalizers are more versatile than graphic equalizers:

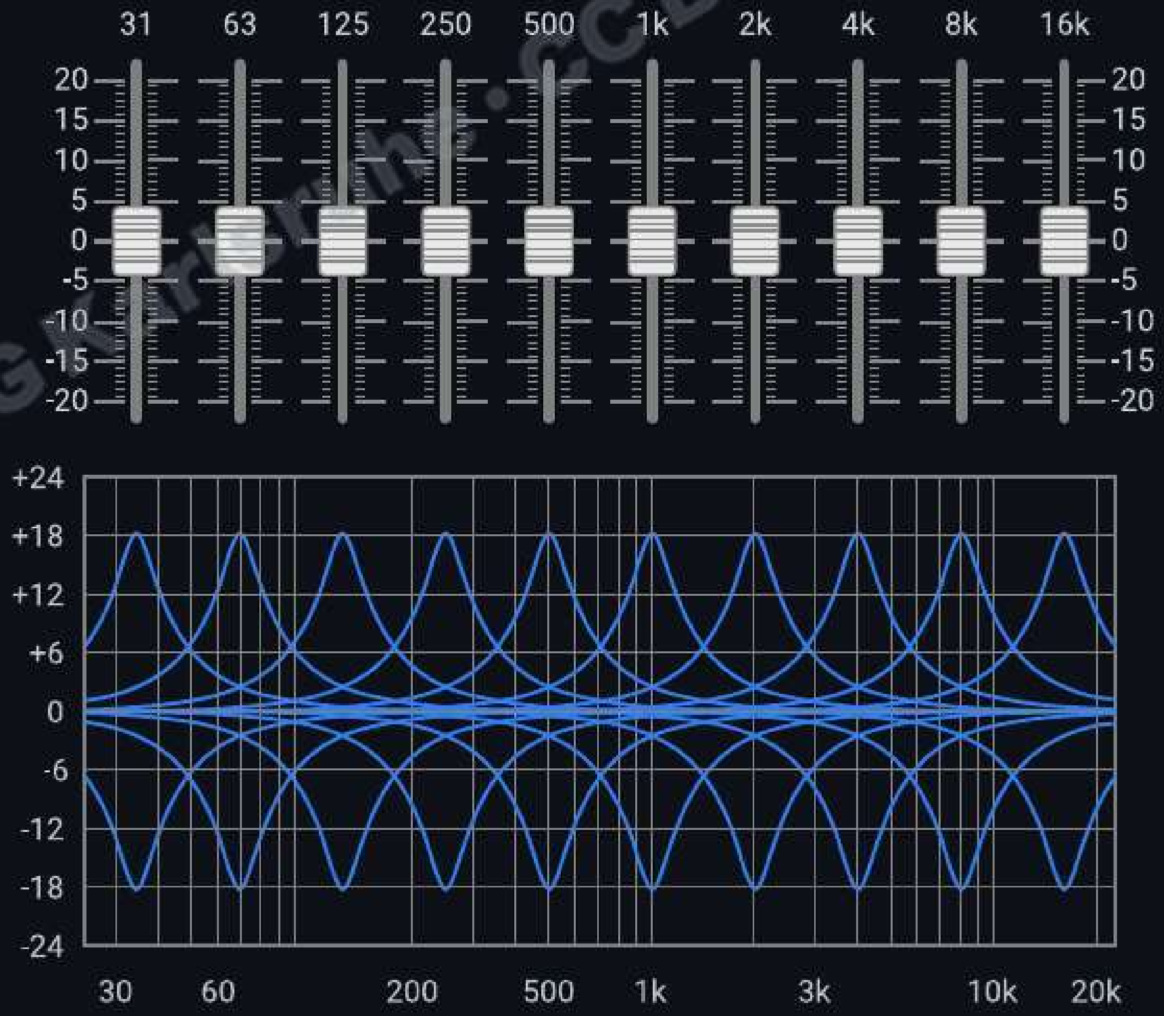
- Multiple bands can be adjusted independently.
- Variable Parameter such as center frequency, gain (boost or cut), bandwidth, and Q-factor of each band can be controlled.

4-band parametric EQ (low pass)

Graphic EQ

A graphic equalizer allows amplification or attenuation of predetermined frequency bands using adjustable faders.

- Position of the faders visually represent the frequency curve of the filter
- The width of each frequency band (Q) remains constant.



Parametric vs. graphic EQ

Aspect	Parametric EQ	Graphic EQ
Precision	Exact control of center frequency and bandwidth	Fixed frequency bands, less precise
Operation	More complex, multiple parameters per band	Intuitive, one control per band
Typical use	Studio mixing and mastering	Live sound and room correction

Common filter types

Filter Type	Design Characteristic	Typical Application
Butterworth	Maximally flat passband, no ripple	General-purpose filtering, natural response
Chebyshev	Steeper roll-off, ripple in passband or stopband	Narrow transition band required
Bessel	Linear phase response (linear group delay), gentle roll-off	Preserving transients, pulse shaping
Elliptic	Steepest roll-off, ripple in both bands	Maximum frequency selectivity needed

FIR and IIR filters (digital filters)

The impulse response (IR) describes a filter's output when excited by a single-sample impulse.

Finite Impulse Response (FIR)

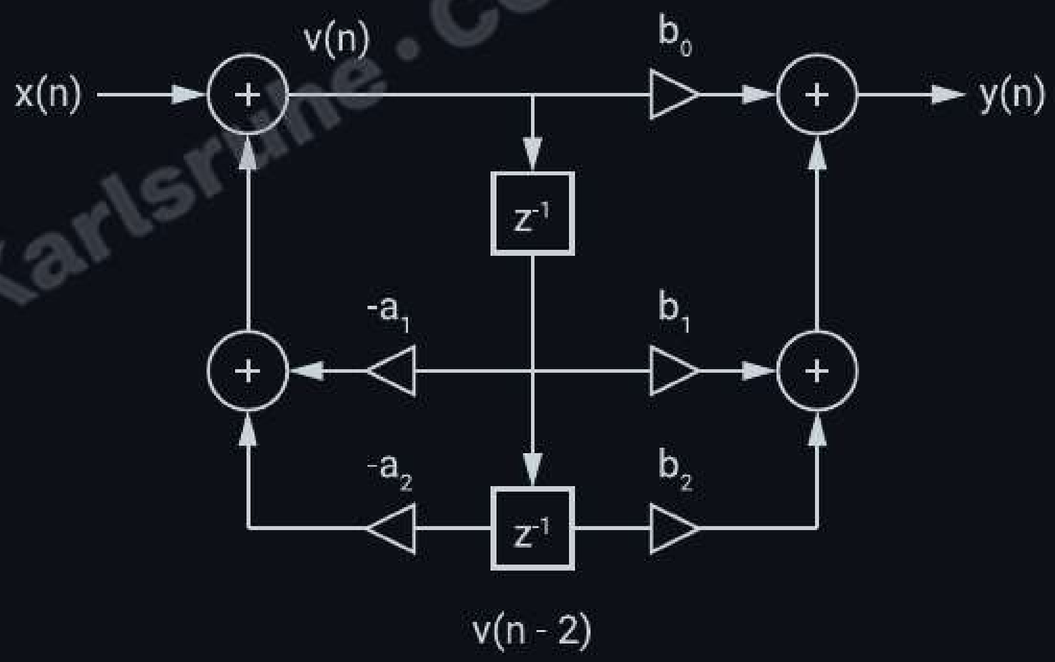
- IR decays to zero in finite time
- Linear phase response
- Higher computational cost and latency
- No direct analog counterpart
- Always stable
- Common in mastering

Infinite Impulse Response (IIR)

- IR is theoretically infinite
- Nonlinear phase response
- Computationally efficient, low latency
- Models traditional analog filters
- Stability depends on filter design
- Common in real-time processing

Biquad filter

Biquad filters are the building blocks of most digital EQs and filters. A single biquad can implement LP, HP, BP, notch, peak, or shelf filters by changing its coefficients. Higher-order filters are built by cascading multiple biquads.



Second-order DF-II structure

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