I. Logarithmic Scales in Acoustics

The human hearing range

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Human hearing range

The human ear **perceives** sound pressures from **20 µPa** (threshold of hearing) to **20 Pa** (pain threshold), a ratio of **1 : 1,000,000**.

$$\frac{20}{20 \times 10^{-6}} = 1 \times 10^{6}$$

The pain threshold is about a million times louder than the weakest audible sound.

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Scaling human hearing

The human hearing is too large for a linear scale, so the logarithmic decibel scale is used, matching human hearing.

$$(0.00002 \rightarrow 20Pa)$$

$$\Downarrow$$

$$(0 \rightarrow 120dBSPL)$$

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Examples in air at standard atmospheric pressure

Sound source	Distance	Pa	dB_{SPL}
Eruption of Krakatoa	165 km	—	172
Jet engine	1 m	632	150
Trumpet	0.5 m	63.2	130
Threshold of pain	At ear	20–200	120–140
Risk of instantaneous noise-induced hearing loss	At ear	20.0	120
Jet engine	30–100 m	6.32–200	110–140
Traffic on a busy roadway	10 m	0.20–0.63	80–90
Hearing damage (long-term exposure)	At ear	0.36	85
TV (home level)	1 m	6.32×10^{-3}–0.02	50–60
Normal conversation	1 m	2×10^{-3}–0.02	40–60
Light leaf rustling, calm breathing	Ambient	6.32×10^{-5}	10
Hearing threshold	—	20×10^{-6}	0

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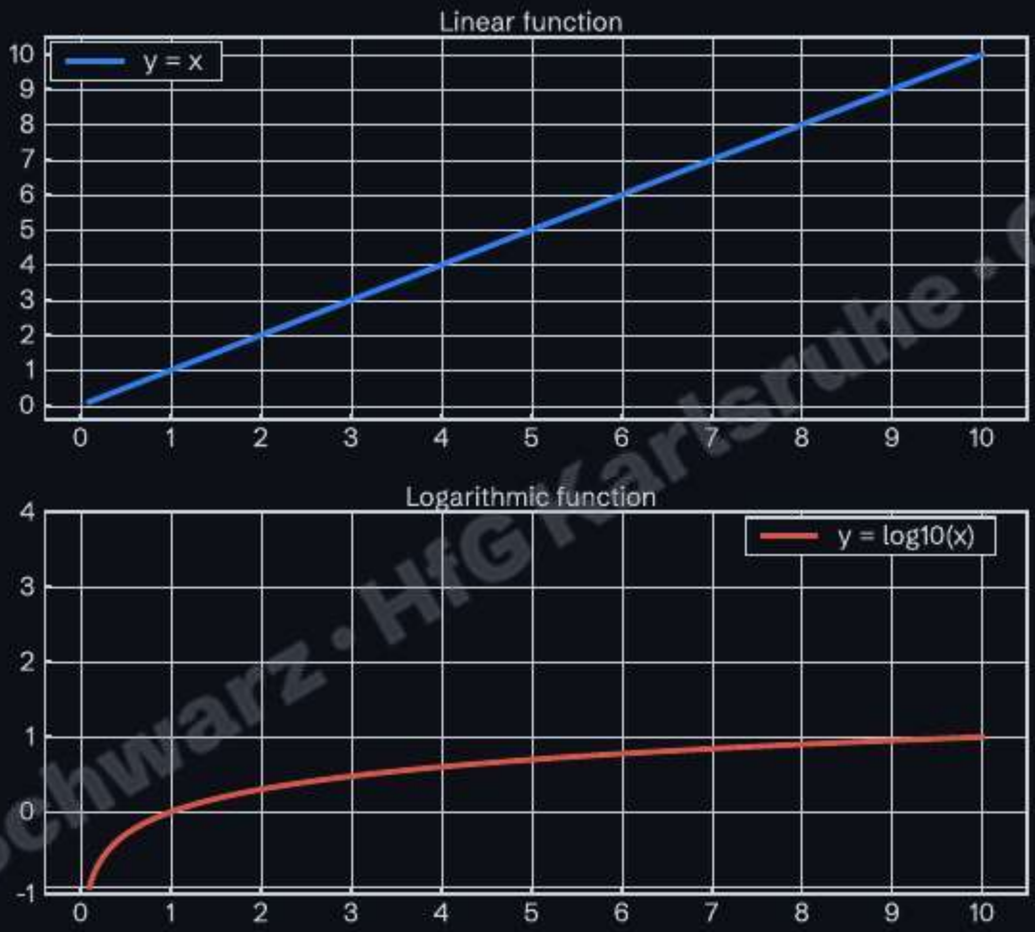

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Logarithmic compression makes the enormous range of human hearing manageable.

II. Logarithmic Principles

Decibels and ratios

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Decibel

A decibel is one-tenth of a Bel, a unit named after Alexander Graham Bell, expressing power ratios logarithmically.

- Expresses **relative change** (ratio between two physical quantities)
- A **logarithmic unit** (compresses large ranges into manageable numbers)

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Logarithm

The logarithm is the inverse function of exponentiation:

$$b^x = a \quad \Longrightarrow \quad x = \log_b(a)$$

Example:

$$2^5 = 32 \quad \Longrightarrow \quad 5 = \log_2(32)$$

Note: $\log_x(1) = 0 \;\Rightarrow\;$ ratio of 1 gives 0 dB

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Properties of logarithmic scales

Human perception of intensity follows an approximately logarithmic relationship.

Benefits:

- Matches nonlinear human perception
- Represent ratios (×2, ×10, etc.) consistently
- Display data spanning many orders of magnitude on one scale
- Reveal details at both low and high ends

→ Used for frequency and magnitude displays

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Linear and logarithmic scales

With a logarithmic scale, the values of the tick marks increase by the same factor over equal distances (e.g., a base value of 10 raised to the powers 0, 1, 2, 3, etc.)

With a linear scale, the values of the tick marks increase by the same amount over equal distances.

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Decibel relationships

Change (dB)	Power	Amplitude	Perception
+1 dB	~1.26×	~1.12×	Barely noticeable
+3 dB	~2×	~1.41×	Clearly noticeable
+6 dB	~4×	~2×	Noticeably louder
+10 dB	10×	~3.16×	About 2× as loud
-1 dB	~0.79×	~0.89×	Barely noticeable
-3 dB	~½	~0.71×	Clearly quieter
-6 dB	~¼	~½	Noticeably quieter
-10 dB	1/10	~0.32×	About ½ as loud

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Audio demonstration: Decibel changes

Pink noise demonstrating relative dB changes (each compared to reference):

▶ Play demonstration

Three A/B comparisons:

1. Reference → **+3 dB** (2× power, clearly noticeable)
2. Reference → **+6 dB** (4× power, 2× amplitude, noticeably louder)
3. Reference → **+10 dB** (10× power, about 2× as loud)

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Absolute and relative dB

- **With suffix** → absolute level (SPL, dBV, dBm, dBFS)
- **Without suffix** → relative ratio (input/output)

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III. Decibel Formulas

Power and magnitude ratios

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Decibel formulas

Power quantities:

$$X_{dB} = 10\log_{10}\left(\frac{X}{X_{\text{ref}}}\right)$$

Field quantities (magnitude):

$$X_{dB} = 20\log_{10}\left(\frac{X}{X_{\text{ref}}}\right)$$

- $10\log_{10}$ → Power, Intensity
- $20\log_{10}$ → Amplitude, Voltage, Current, Pressure

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Power and magnitude relationship

Power is proportional to the square of field quantities:

$$P = U \cdot I = R \cdot I^2 = \frac{U^2}{R}$$

When converting field quantities to decibels, this square relationship means:

$$20 \log_{10}\left(\frac{U}{U_{\text{ref}}}\right) = 10 \log_{10}\left(\frac{U^2}{U_{\text{ref}}^2}\right) = 10 \log_{10}\left(\frac{P}{P_{\text{ref}}}\right)$$

→ *The factor of 2 comes from the square relationship between power and field quantities.*

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Converting between linear values and decibels

Power ratio:

$$x\,\mathrm{dB} \;=\; 10\log_{10}(a) \quad\Longleftrightarrow\quad a \;=\; 10^{\frac{x}{10}}$$

Amplitude ratio:

$$x\,\mathrm{dB} \;=\; 20\log_{10}(a) \quad\Longleftrightarrow\quad a \;=\; 10^{\frac{x}{20}}$$

Example:

Two signals whose levels differ by 6 dB have an amplitude ratio of $10^{\frac{6}{20}} \approx 2$

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Example: doubling voltage and power in dB

- Voltage doubles (1 V → 2 V):

$$20 \log_{10}(2/1)\,\mathrm{dB} \approx 20 \cdot 0.3010\,\mathrm{dB} = +6.02\,\mathrm{dB}$$

- Power doubles (1 W → 2 W):

$$10 \log_{10}(2/1)\,\mathrm{dB} \approx 10 \cdot 0.3010\,\mathrm{dB} = +3.01\,\mathrm{dB}$$

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dB conversion table

Decibels (dB)	Magnitude (ratio, 20·log)	Power (ratio, 10·log)
-20	0.1	0.01
-12	0.25	0.06
-6	0.5	0.25
-3	0.7	0.5
0	1	1
+3	1.4	2
+6	2	4
+12	4	16
+20	10	100

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dB conversion graph

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IV. Reference Systems

Absolute measurements

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Decibel reference values

To express an **absolute value,** the suffix specifies the reference X_{ref}:

$$X_{dB} = k \cdot \log_{10}\left(\frac{X}{X_{\text{ref}}}\right), \quad k = \begin{cases} 10 & \text{power-like quantities} \\ 20 & \text{field-like quantities} \end{cases}$$

Unit	Quantity	Reference value X_{ref}
dB SPL	Sound pressure level	20 µPa
dBm	Power	1 mW
dBV	Voltage	1 V
dBu	Voltage	0.775 V
dBFS	Digital full scale	Maximum quantizing level

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Example: sound pressure (SPL)

$$L_p = 20 \log_{10}\left(\frac{p_{\text{rms}}}{p_{\text{ref}}}\right) \text{dB SPL}$$

p_{rms} is the root mean square of the measured sound pressure
p_{ref} is the standard reference sound pressure of 20 micropascals in air

$$p_{\text{ref}} = 20 \times 10^{-6}\,\text{Pa} = 0.00002\,\text{Pa} = 20\,\mu\text{Pa}$$

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Example: calculating dB SPL

Reference: $p_{\mathrm{ref}} = 20\,\mu\mathrm{Pa}$ (threshold of hearing)

$$L_p = 20\log_{10}\left(\frac{p_{\mathrm{rms}}}{p_{\mathrm{ref}}}\right)\mathrm{dB\ SPL}$$

Example with $p_{\mathrm{rms}} = 0.02\,\mathrm{Pa}$:

$$\Rightarrow \quad 20\log_{10}\left(\frac{0.02\,\mathrm{Pa}}{20\times 10^{-6}\,\mathrm{Pa}}\right)\mathrm{dB\ SPL} = 20\log_{10}(10^3)\,\mathrm{dB\ SPL} = 20\cdot 3\,\mathrm{dB\ SPL} = 60\,\mathrm{dB\ SPL}$$

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Example: dBFS (Full Scale)

Digital audio systems:

- Reference = maximum quantizing level (0 dBFS)
- Values typically negative (counted down from maximum)

16-bit system: Range = -32768 to +32767, so $X_{\text{ref}} = 32767$

Example calculation:

$$X_{dB} = 20\log_{10}\left(\frac{16384}{32767}\right) = 20\log_{10}(0.5) \approx -6\,\text{dBFS}$$

→ *Half maximum amplitude = -6 dBFS*

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V. Practical Applications

Using decibels in practice

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Example: adding gain stages

Stage →	1	2	3	4	Total
Linear Gain ×	2.0	3.0	1.5	0.5	4.5
Gain in dB +	+6 dB	+9.5 dB	+3.5 dB	-6 dB	+13 dB

→ *Since dB is logarithmic, multiplying ratios in linear terms becomes addition in dB.*

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Adding independent sound sources

Convert decibel values to linear form, perform the summation, then reconvert to decibels.

$$L_{\text{total}} = 10\log_{10}\left(10^{L_1/10} + 10^{L_2/10}\right)$$

Independent sources add **powers**, not pressures.

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Adding two sound sources 80 and 85 dB

$$10 \log_{10}\left(10^{80/10} + 10^{85/10}\right)$$

$$\Downarrow$$

$$10 \log_{10}\left(10^{8} + 10^{8.5}\right)$$

$$\Downarrow$$

$$10 \log_{10}(4.1623 \times 10^{8})$$

$$\Downarrow$$

$$86.2\,\mathrm{dB}$$

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VI. Reference

Conversion factors and relationships

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Decibel quick reference

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General relationships (relative changes):

- $+3\,\mathrm{dB}$ doubles power/intensity
- $+6\,\mathrm{dB}$ doubles amplitude/pressure
- $+10\,\mathrm{dB}$ doubles perceived loudness
- $0\,\mathrm{dB}$ = no change (ratio = 1)

Applied to sound pressure level (SPL):

- $+3\,\mathrm{dB}\ \mathrm{SPL}$ doubles intensity (since $I \propto p^2$)
- $+6\,\mathrm{dB}\ \mathrm{SPL}$ doubles pressure
- $+10\,\mathrm{dB}\ \mathrm{SPL}$ doubles perceived loudness

Common misconceptions

Adding dB SPL values directly

- Wrong: 70 dB + 70 dB = 140 dB
- Correct: Must convert to linear, add, convert back (≈ 73 dB)

Confusing 10 log vs 20 log

- **20 log** for: Voltage, Current, Pressure, Amplitude
- **10 log** for: Power, Intensity

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Example 1

Question: A sound measures 0.2 Pa. What is the SPL in dB?

Solution

$$L_p = 20\log_{10}\left(\frac{0.2}{20\times 10^{-6}}\right) = 20\log_{10}(10^4) = 20\times 4 = 80 \text{ dB SPL}$$

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Example 2

Question: Two independent sound sources each produce 70 dB SPL. What is the total SPL when both operate?

$$
\begin{aligned}
L_{\text{total}} &= 10\log_{10}(10^7 + 10^7) \\
&= 10\log_{10}(2 \times 10^7) \\
&= 10[\log_{10}(2) + \log_{10}(10^7)] \\
&= 10[0.301 + 7] \approx 73 \text{ dB SPL}
\end{aligned}
$$

→ *Independent sources add powers, not dB values.*

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