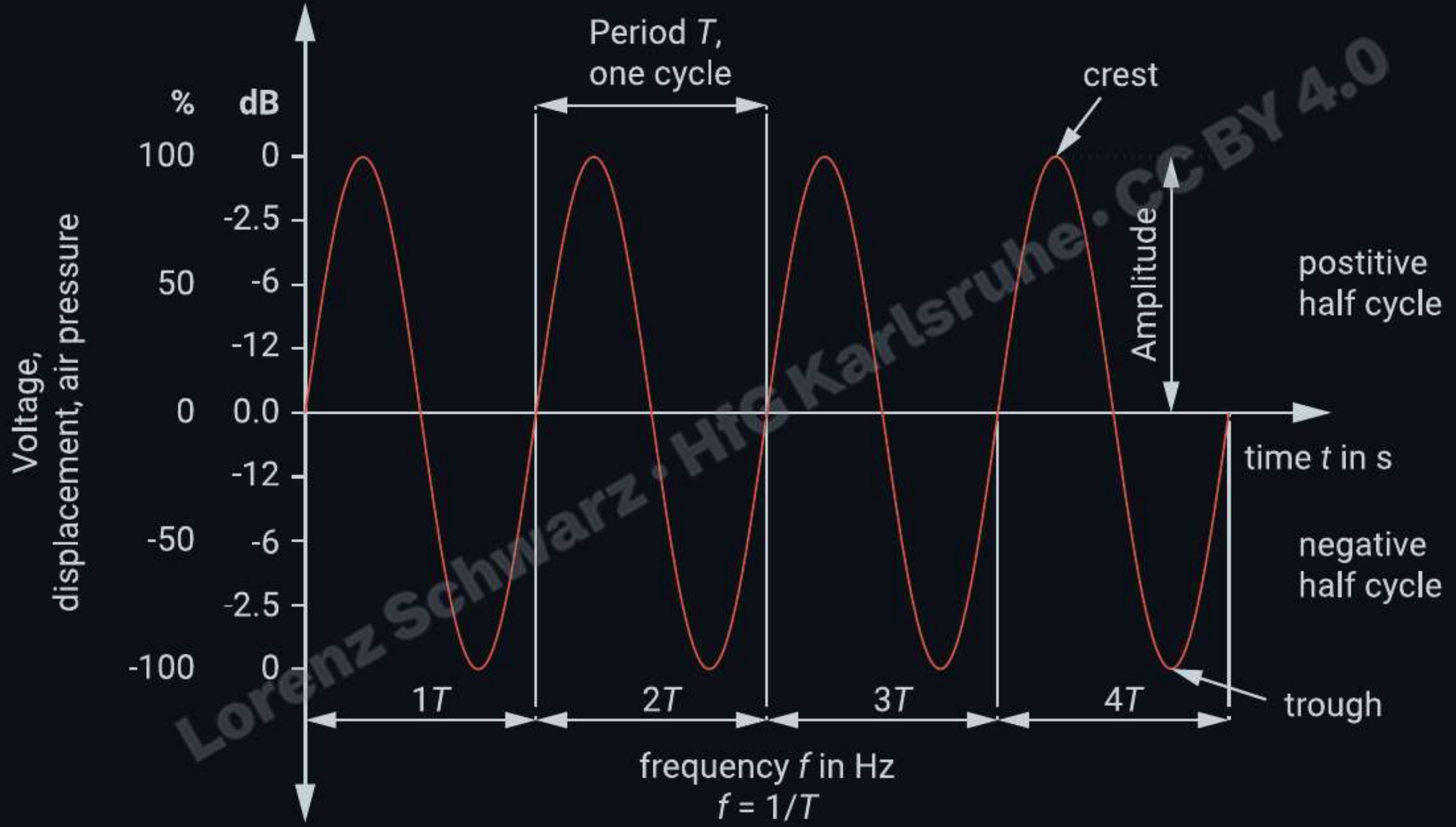
WAVE PROPERTIES

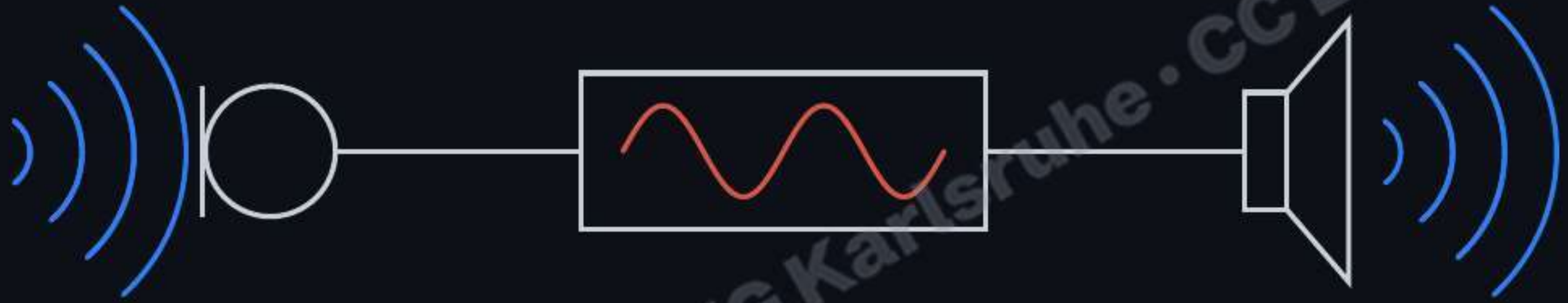


Period T,
one cycle
crest
%
dB
100
0
-2.5
50
-6
-12
0
0.0
-12
-50
-6
-2.5
-100
0
Voltage,
displacement, air pressure
Amplitude
postitive
half cycle
time t in s
negative
half cycle
1T
2T
3T
4T
trough
frequency f in Hz
$f = 1/T$

Audio signals (electrical representation)

A transducer (e.g., microphone) converts the varying air pressure of a sound wave into a continuous electrical signal, which is proportional to the sound pressure variations.

→ *Electrical sound signals in this form are analogous to the sound pressure levels.*



Acoustic Sound - Electrical Signal - Acoustic Sound

Electronic sound and audio signals

Electronic audio signals are variations in electrical energy (changes in voltage or current), that correspond to sound pressure variations.

→ *Audio signals can be converted into audible sound through a speaker or other transducers.*

Wave properties

The shape of the graph of a periodic function can be described using the following terms:

1. Amplitude $\hat{u}$ - The maximum instantaneous value of the wave.
2. Period T - The time interval after which the wave repeats.
3. Frequency f - The reciprocal of the period (oscillations per second).
4. Phase angle φ - Describes the offset or difference between two sine waves.
5. Wavelength λ - The distance over which the wave repeats (spatial period).

Related:
Zero crossing - Point where the wave crosses zero.

Amplitude

Amplitude describes the maximum variation of a periodic signal (such as air pressure, displacement, or voltage) within a single period:

- Maximum instantaneous value of the variable.
- Maximum distance between the resting position (equilibrium) and the point of maximum displacement.
- Perceived as the loudness of sound in the context of audio signals.

→ *Amplitude has no influence on frequency, wavelength, phase, period of time, and speed of sound.*

Amplitude and energy transfer in waves

Amplitude relates to the wave's energy:

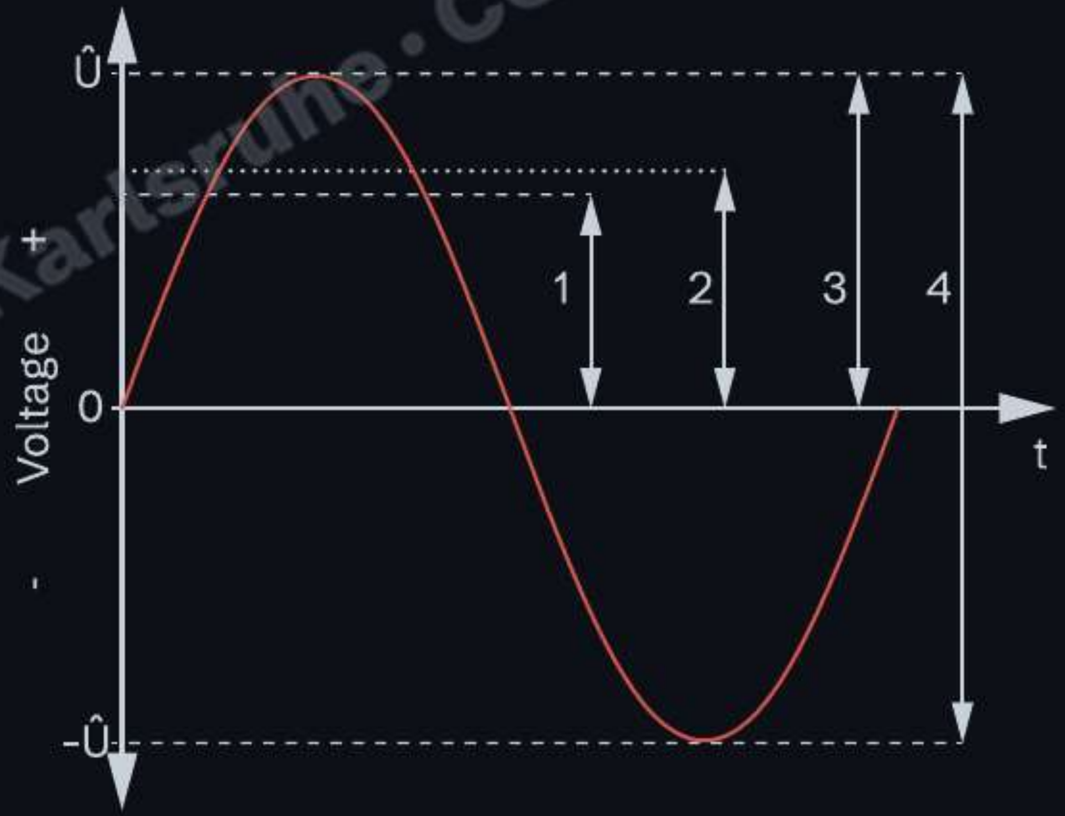
- Higher amplitude corresponds to greater energy transfer in the wave

This energy can be converted into work, heat, or other forms depending on the medium and interaction.

Amplitude

For a sinusoidal waveform:

1. Average rectified (ARV) $\frac{2\hat{u}}{\pi}$
2. Root mean square amplitude (RMS): $\frac{\hat{u}}{\sqrt{2}}$ (equivalent value of constant direct current)
3. Peak amplitude or semi-amplitude: $\hat{u}$ (maximum distance between resting position (equilibrium) and maximum displacement)
4. Peak to peak amplitude: $2\hat{u}$ (between maximum and minimum)



Root mean square (RMS)

The average value of a sine wave over a full cycle is zero (positive and negative halves cancel). RMS solves this by squaring values first, making them all positive:

RMS represents the *effective value*: the equivalent DC level that delivers the same power.

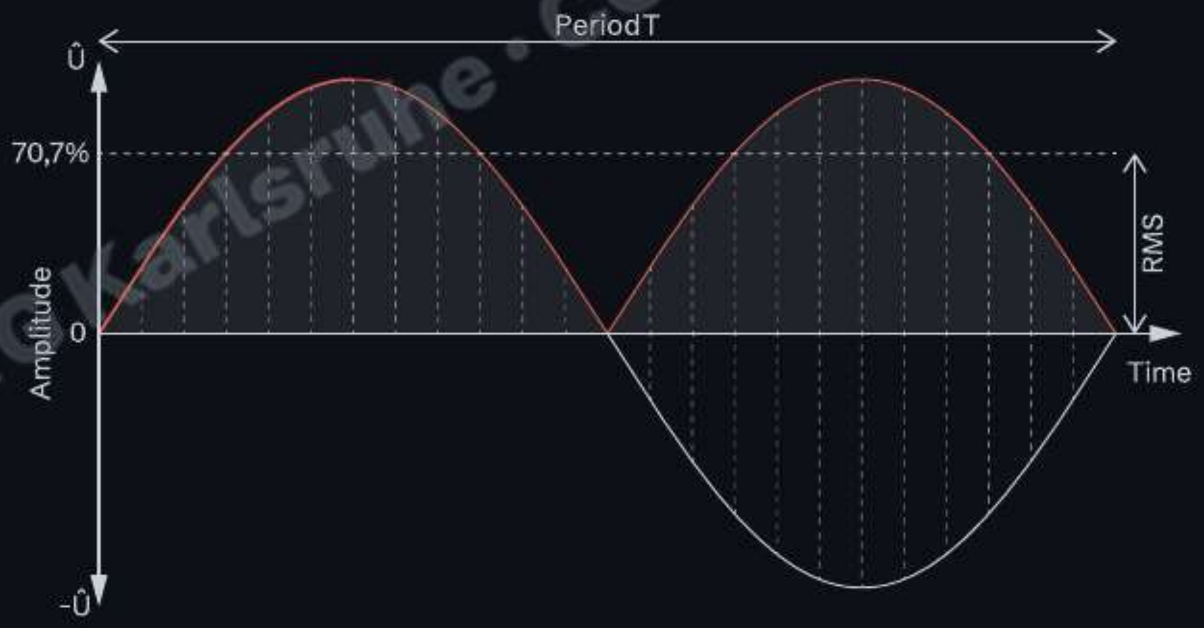
Waveform	RMS
Sine	$0.707 \times A_{\text{peak}}$
Square	A_{peak}

→ *Electrical and acoustic power are proportional to the square of the RMS value.*

Calculating RMS

Square root of the mean of the squares:

- Square all values of the signal: $a^2(t)$ (makes all values positive)
- Compute the area under the squared curve: $\int_0^T a^2(t)\,dt$
- Divide by the period T: $\frac{1}{T}\int_0^T a^2(t)\,dt$
- Take the square root: $\sqrt{}$



$$A_{\text{RMS}} = \sqrt{\frac{1}{T}\int_0^T a^2(t)\,dt}$$

RMS and power

Electrical and acoustic power are proportional to the square of RMS values:

- Amplifier power ratings use RMS voltage and current
- Sound intensity is proportional to RMS sound pressure squared

→ *RMS enables meaningful comparison of signal strength and power delivery across different waveforms.*

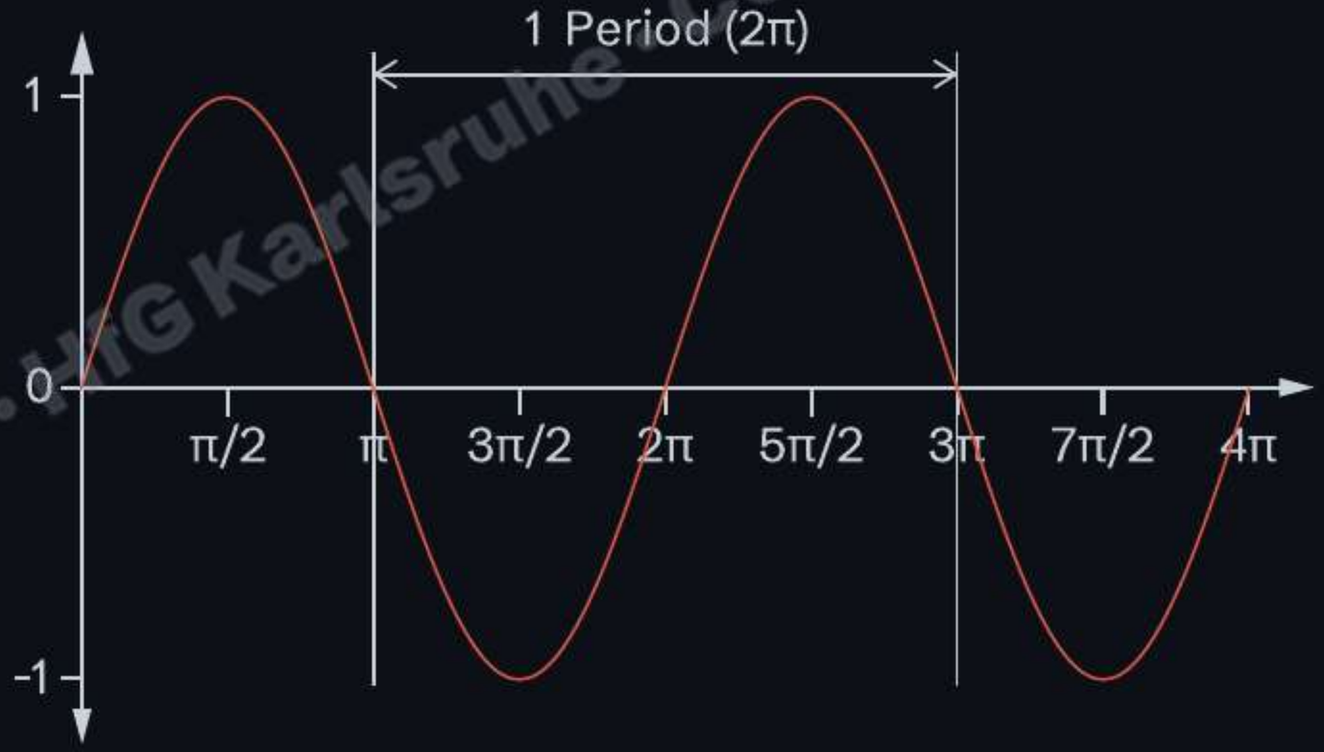
Period T

Time required for a wave to complete one wave cycle (2π)

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Each multiple of a period is also a period, but we usually refer to the smallest positive one as the period.

view in graphing calculator



Period

Example: $f = 440\,\mathrm{Hz}$

$$T = \frac{1}{f} = \frac{1}{440}\,\mathrm{s} = 2.27\,\mathrm{ms}$$

Frequency

Variations or alterations between 0,05 ms (20 kHz) and 50 ms (20 Hz) are perceived as sound.

- Ultrasound: higher than 20 000 Hz.
- Infrasound: lower than 20 Hz.

→ *Hearing range for humans is 20 Hz to 20 000 Hz.*

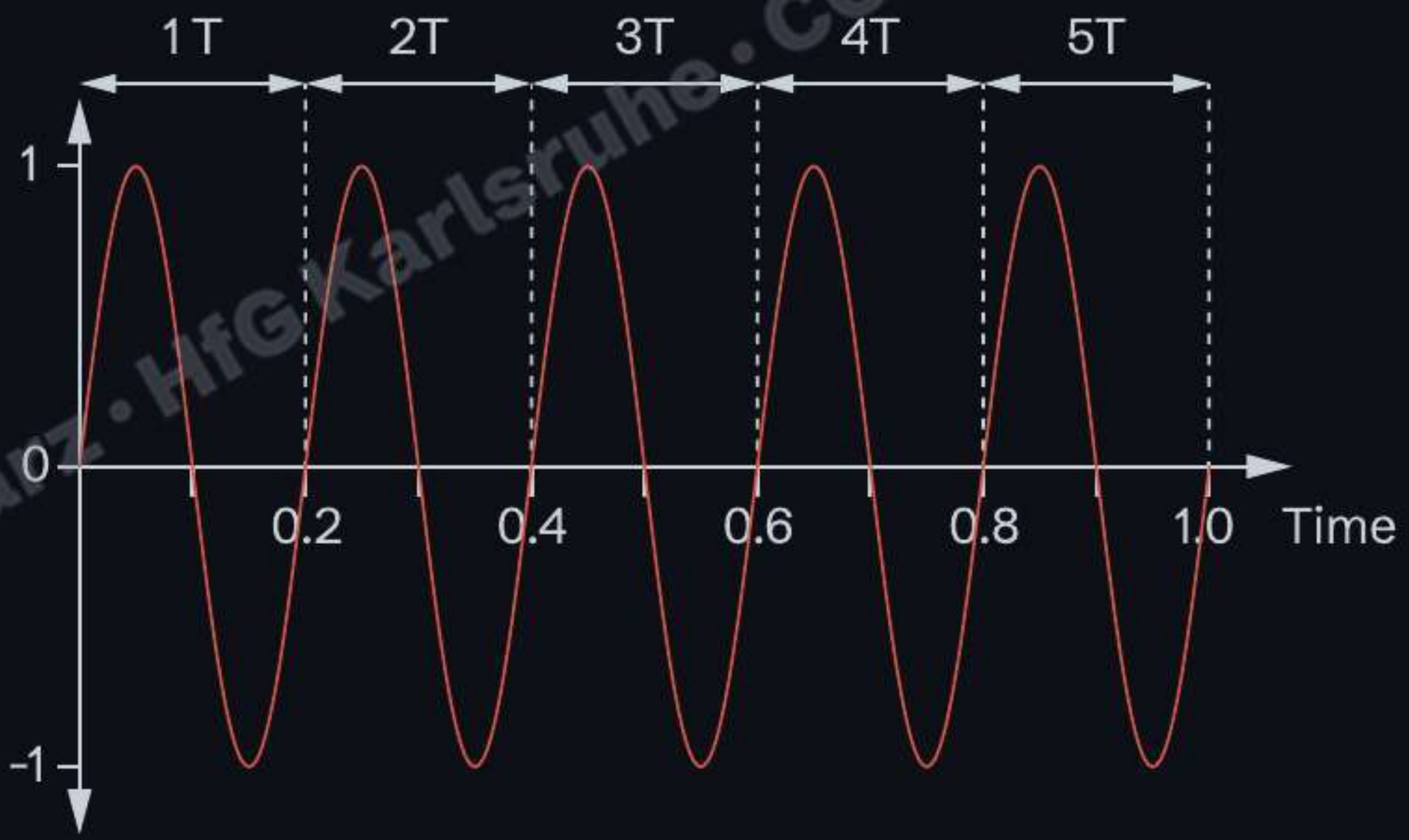
Frequency f

Number of wave cycles per second, expressed in Hertz [Hz]

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

f: frequency
T: period
ω: angular frequency

$$\omega = 2\pi f$$



Angular frequency (ω)

Whereas Hertz [Hz] counts cycles per second, radians per second [rad/s] measure the angle swept per second by a rotating pointer.

- **Hz**: cycles per second (cps)
- **rad/s**: angle swept per second (rotating motion)

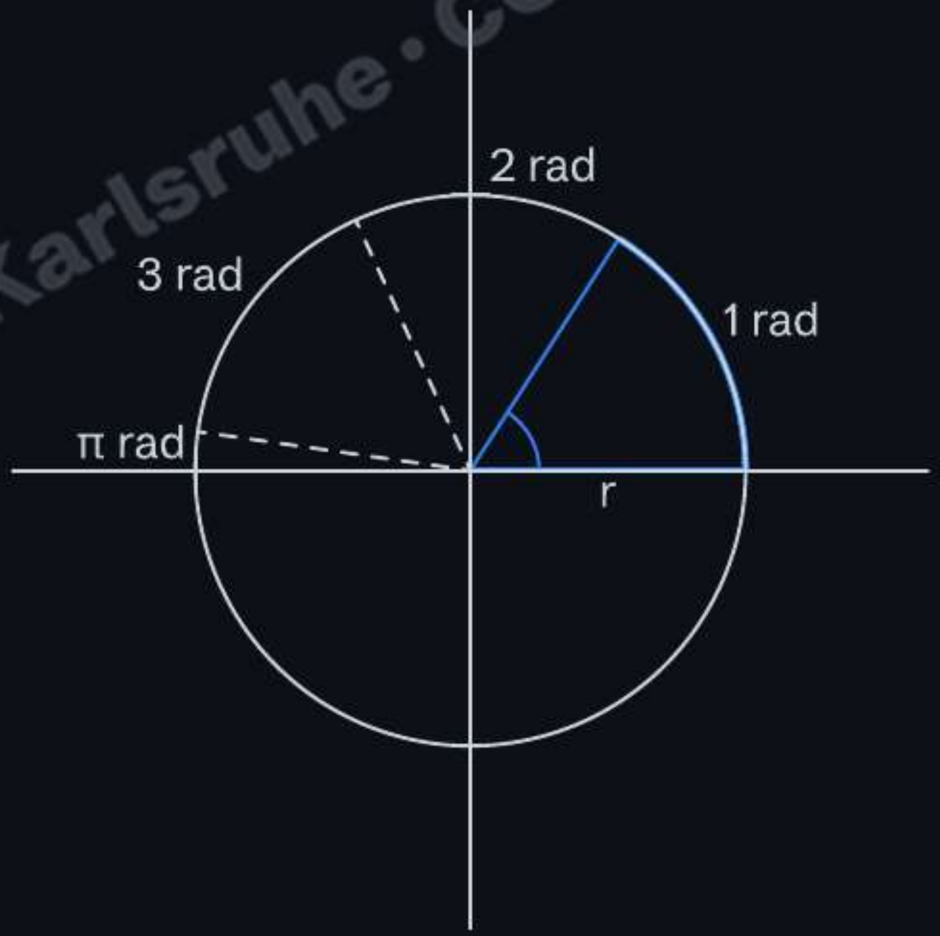
Hertz and radian can be expressed as reciprocal seconds:

$$[Hz] = [\omega] = s^{-1}$$

Radian

Radian is the angle subtended at the center of a circle by an arc equal in length to the radius.

- 1 full circle = 2π radians
- Radians are dimensionless (ratio of lengths)
- Used in angular measures: ω in rad/s, φ in rad



view in graphing calculator

Example: Relating rad/s to Hz

One radian per second:

$$f_{(Hz)} = \frac{\omega_{(rad/s)}}{2\pi} = 0.1591549433\,Hz$$

Frequency and period

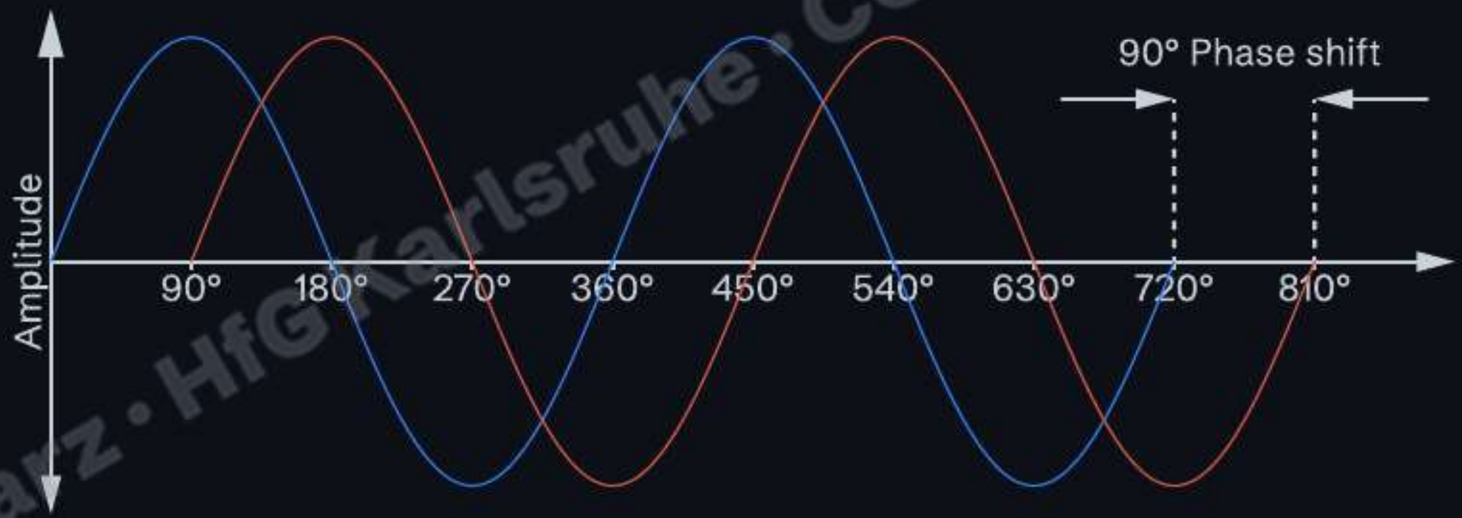
The period is the reciprocal of the frequency and vice versa.

$$T = \frac{1}{f} \qquad f = \frac{1}{T}$$

Phase

Position of a sine wave in time.

- defined for two sine waves
- not for music signals or noise



Cosine and sine have a mutual phase difference of 90°

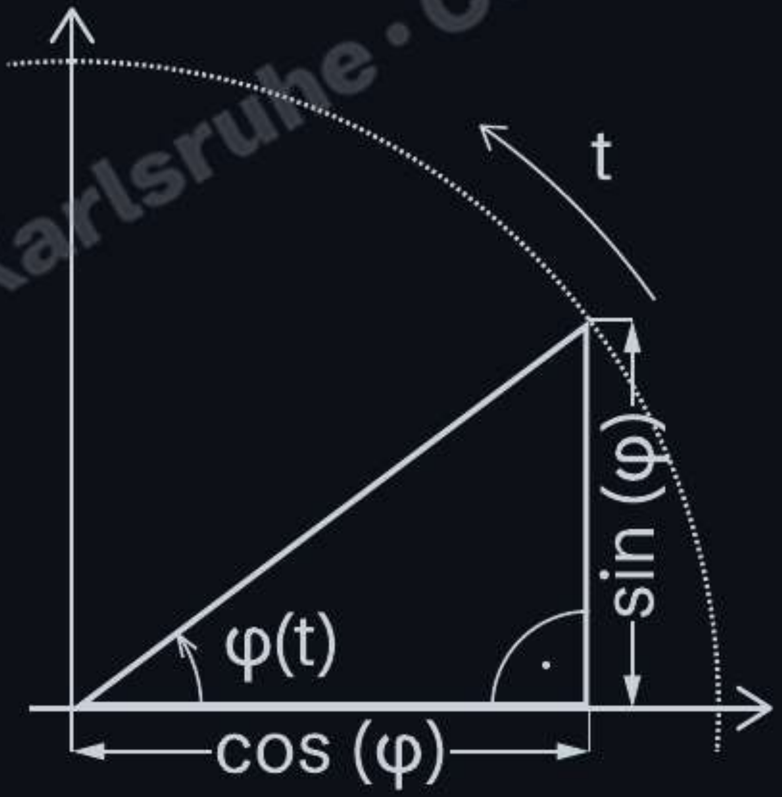
$$\varphi(t) = 90^{\circ} = \frac{\pi}{2}$$

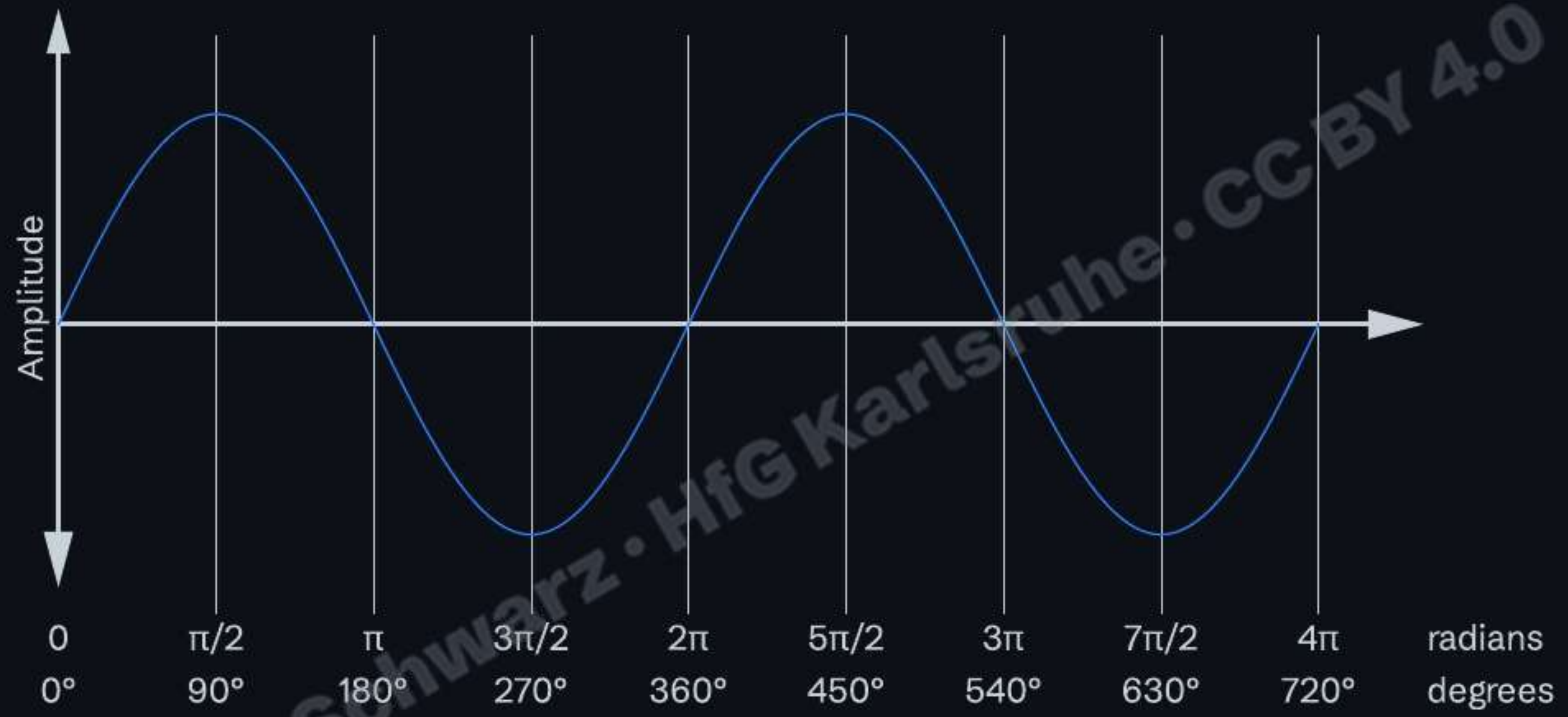
Phase angle φ

The phase indicates the angular position in the cycle of a periodic process as a function of time.

$$\varphi(t) = 2\pi f t = \omega t$$

view in graphing calculator





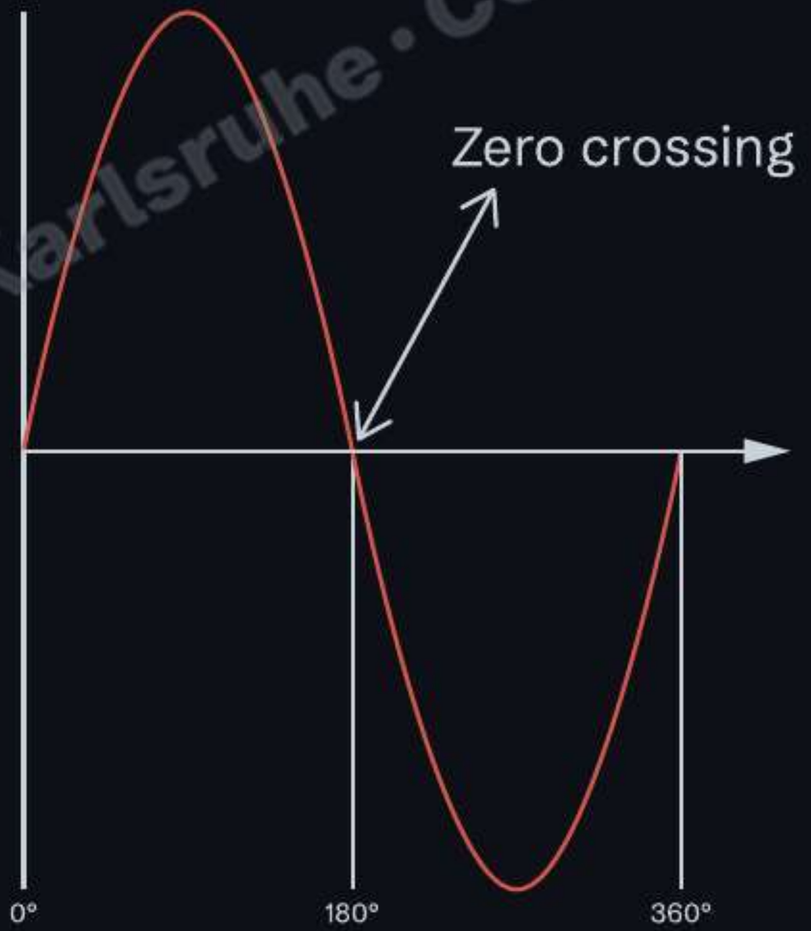
One complete cycle is 2π in radians or 360 in degrees:
0° starting point (zero position), 90° highest point, 270° lowest point

Zero crossing

Zero crossing is the point where the signal's amplitude is zero and it changes sign:

- Occurs twice per cycle in simple waveforms (e.g., sine, sawtooth, triangle, square)

→ *In speech processing, the zero-crossing rate helps distinguish between voiced and unvoiced speech sounds.*



The wave equation

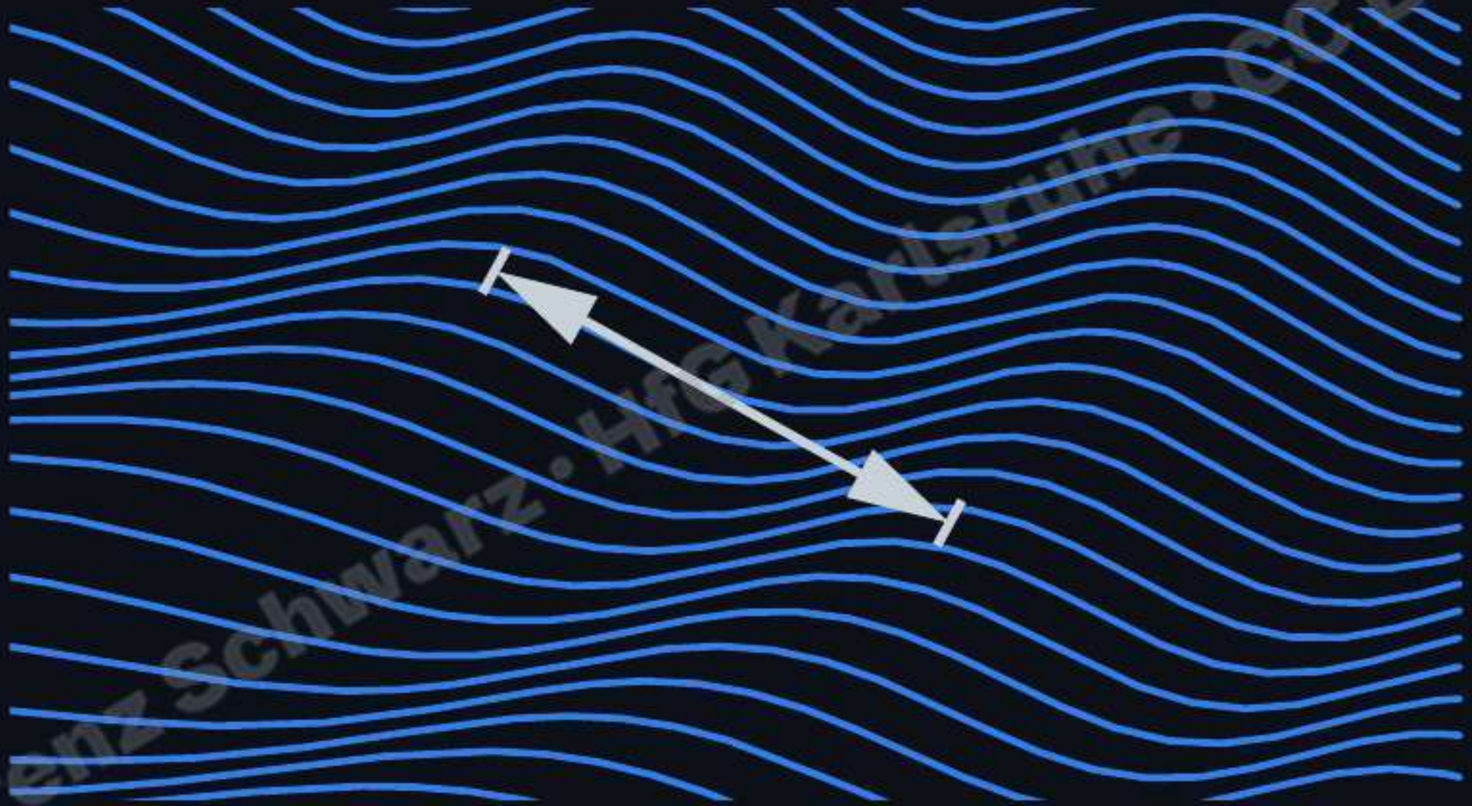
All wave properties are interconnected through a single relationship:

$$c = f\lambda = \frac{\lambda}{T}$$

- c is the propagation speed of the wave (e.g. 343 m/s in air)
- Amplitude determines energy, independent of the other properties
- Frequency and period are reciprocals: $f = \frac{1}{T}$
- Wavelength depends on both frequency and medium: $\lambda = \frac{c}{f}$
- Phase describes position within a cycle

→ *Changing one property (except amplitude) necessarily affects others.*

Wavelength λ



Wavelength

Spatial period:

- distance over which the wave's shape repeats (related to frequency)
- parallel to the direction of propagation

$$\lambda = \frac{c}{f}$$

λ Wavelength
f Frequency
c Speed of sound

Calculating wavelength from frequency

Example:

What is the wavelength of a 440 Hz tone in air, where sound speed is 343 m/s?

$$\lambda = \frac{c}{f}[m]$$

$$\lambda = \frac{343}{440} = 0.78m$$

Wavelength λ

Wavelength and frequency

Frequency and wavelength are inversely proportional to each other:

low frequency $\longleftrightarrow$ long wavelength
high frequency $\longleftrightarrow$ short wavelength

Wavelengths of various sound frequencies

Frequency (Hz)	Wavelength in Air (m)
31.5	11
63	5.5
125	2.7
250	1.4
500	0.7
1k	0.344
2k	0.172
4k	0.086
8k	0.043
16k	0.021

Speed of sound

The speed of sound is the distance a sound wave travels per unit of time through a medium.

Speed of sound:

$$c \approx 343\,\frac{\mathrm{m}}{\mathrm{s}} \text{ at 20°C in air}$$

$$c \approx 1481\,\frac{\mathrm{m}}{\mathrm{s}} \text{ in fresh water}$$

Factors affecting the speed of sound

- **Elasticity:** The more elastic (less compressible) the medium, the faster sound travels.
- **Density:** Higher density generally slows sound in gases but may increase speed in solids or liquids if accompanied by high elasticity.
- **Temperature:** In gases, higher temperatures increase the speed of sound by reducing the medium's density and increasing molecular energy.

Speed of sound in gases

$$c = \sqrt{\kappa \frac{p}{\rho}} = \sqrt{\kappa \frac{RT}{M}} = \sqrt{\kappa \frac{k_B T}{m}}$$

κ heat capacity ratio
ρ density
p pressure
R molar gas constant
$M = m \cdot N_A$ molar mass
T thermodynamic temperature in kelvin
k_B Boltzmann constant
m molecular mass

Example: Time-distance relationship of sound

- **34 cm/ms**
  - sound travels about 34 cm per millisecond
- **3 ms/m**
  - Sound takes roughly 3 ms to travel 1 meter

(Assuming speed of sound = 343 m/s)

Applying the speed of sound formula

Example:

Determining the distance of a lightning bolt (and thunderstorm cell):

- Every **3 seconds** of delay ≈ **1 kilometre** distance from the lightning bolt.

Question:

How long does sound need to travel **2 m**?

$$t = \frac{d}{c} = \frac{2\,\mathrm{m}}{343\,\mathrm{m/s}} \approx 5.8\,\mathrm{ms}$$

Speed of sound in different media (at 20°)

Media	Meters/Second
Air	344
Helium	981
Water, fresh	1480
Seawater	1500
Ice (-4°C)	3250
Acrylic Glass	2670
Beech wood	3300
Concrete	5850-5920
Mild Steel	5050
Aluminium	6250-6350

Wave propagation

Sound speed in an elastic medium depends on temperature.

- Lower temperature → lower speed
- 1°C change ≈ 60 cm/s change in sound speed

Practical use: delay lines in PA systems

When multiple speaker systems cover a large area, sound from distant speakers must be delayed to match the time it takes for sound from closer speakers to arrive.

Speed of sound and related terms

Generally, sound travels faster in denser and less compressible media.

- **Subsonic:** Motion or speed less than the speed of sound in a given medium.
- **Infrasonic:** Sound waves with frequencies lower than ~20 Hz (below the range of human hearing).

Doppler effect

The change in frequency or pitch of sound waves perceived by an observer due to the relative motion between the sound source and the observer.

- The pitch is higher than the stationary pitch as the source approaches.
- The pitch decreases as the source passes the observer.
- The pitch becomes lower than the stationary pitch as the source moves away.

Used in rotary (Leslie) speakers and film sound design plug-ins.

▶ Doppler effect applied to a moving sound source (plugin simulation)

Doppler effect formula

$$f' = \frac{c \pm v_r}{c \mp v_s} \cdot f_0$$

- f' = observed frequency
- f_0 = emitted (source) frequency
- c = speed of sound in the medium
- v_r = velocity of the receiver
- v_s = velocity of the source

For a stationary receiver ($v_r = 0$):

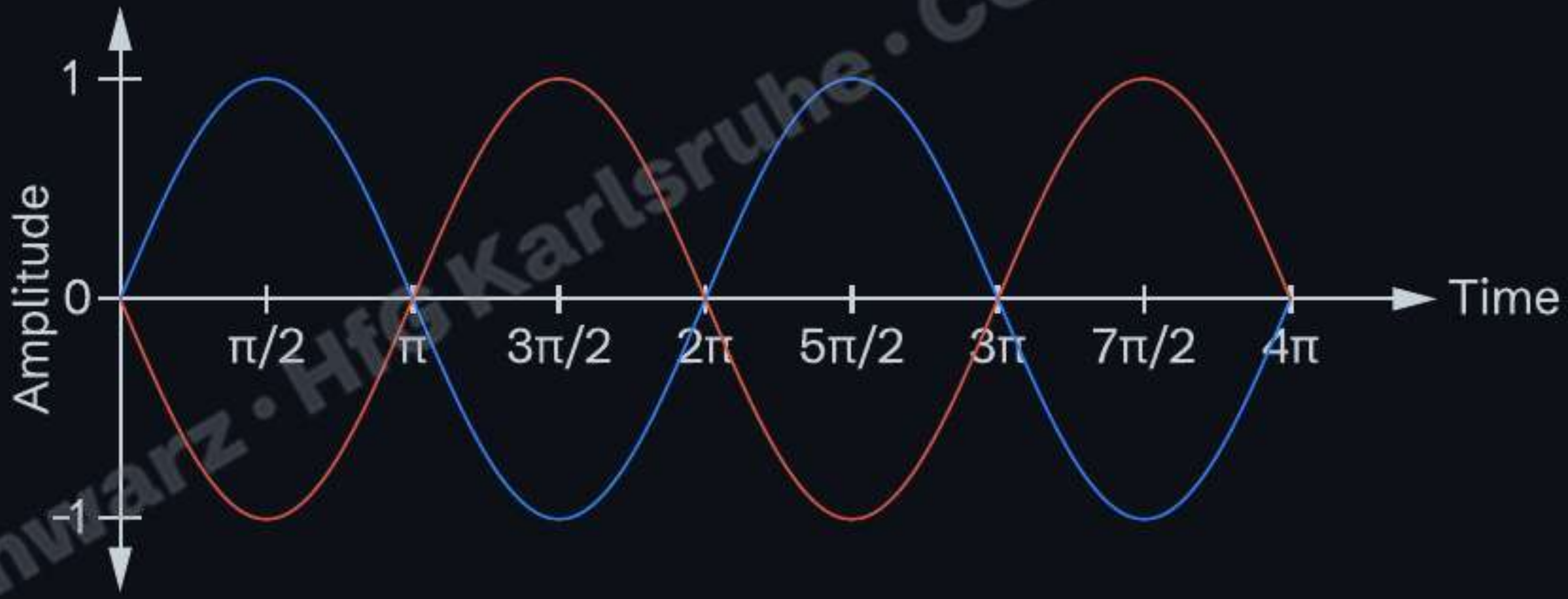
$$f' = \frac{c}{c \mp v_s} \cdot f_0$$

Doppler effect and sound barrier

Stopped Subsonic Speed of sound Supersonic

Polarity inversion

- opposite amplitude
- inverted signal
- no time shift



The red graph shows an inverted version of the blue graph (same shape, opposite sign)

Polarity inversion

Applications:

- Differential signalling for transmitting analog audio.
- "Phase" button on mixing desks to avoid phase cancellation

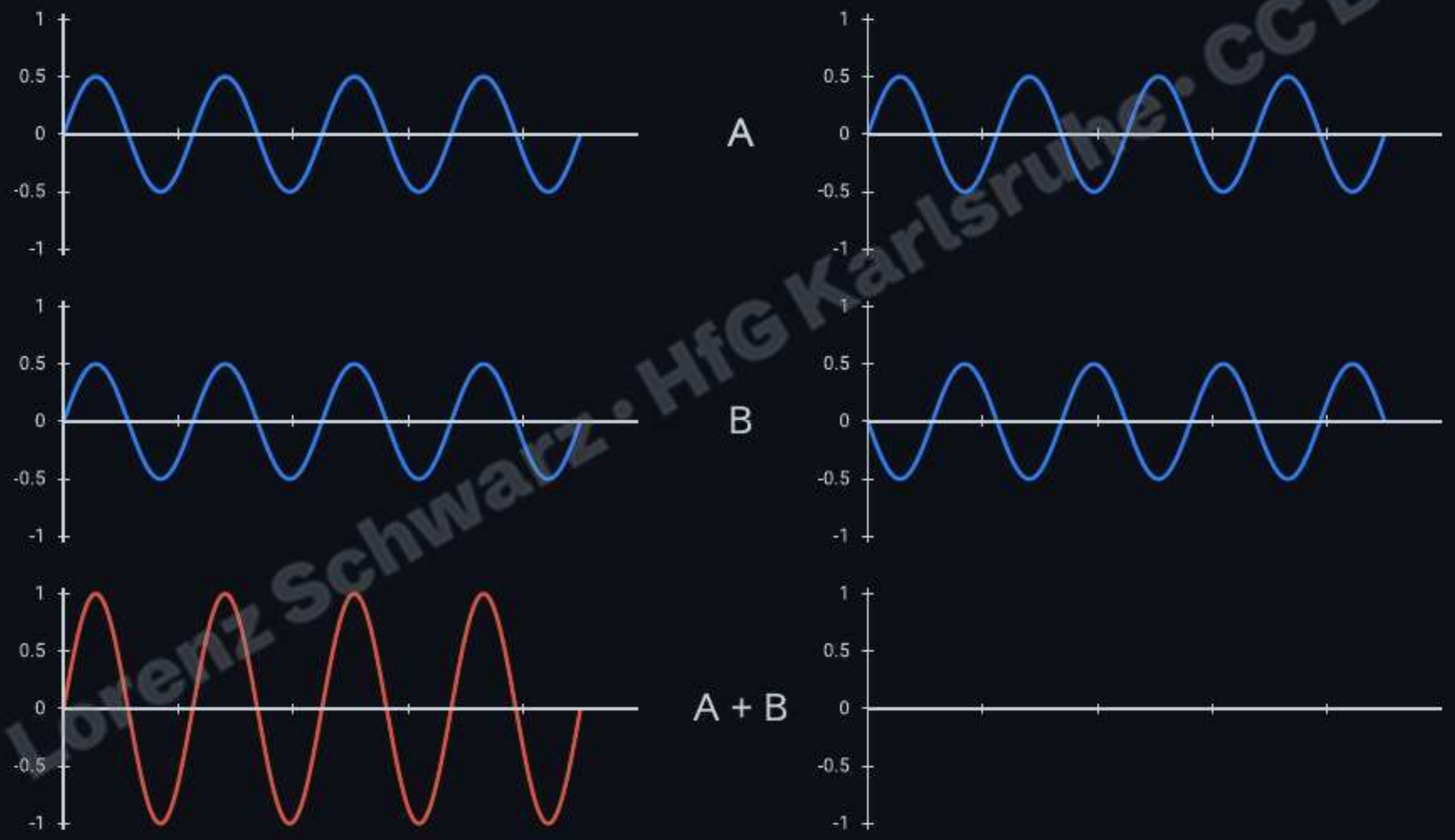
Superposition and interference

Interference, a consequence of superposition, describes the interaction between sound waves. The resultant amplitude is the sum of the individual amplitudes:

- amplification (constructive, even multiple of π)
- attenuation (destructive)
- cancellation (destructive, odd multiple of π)

view in graphing calculator

Constructive and destructive interference



Applications:

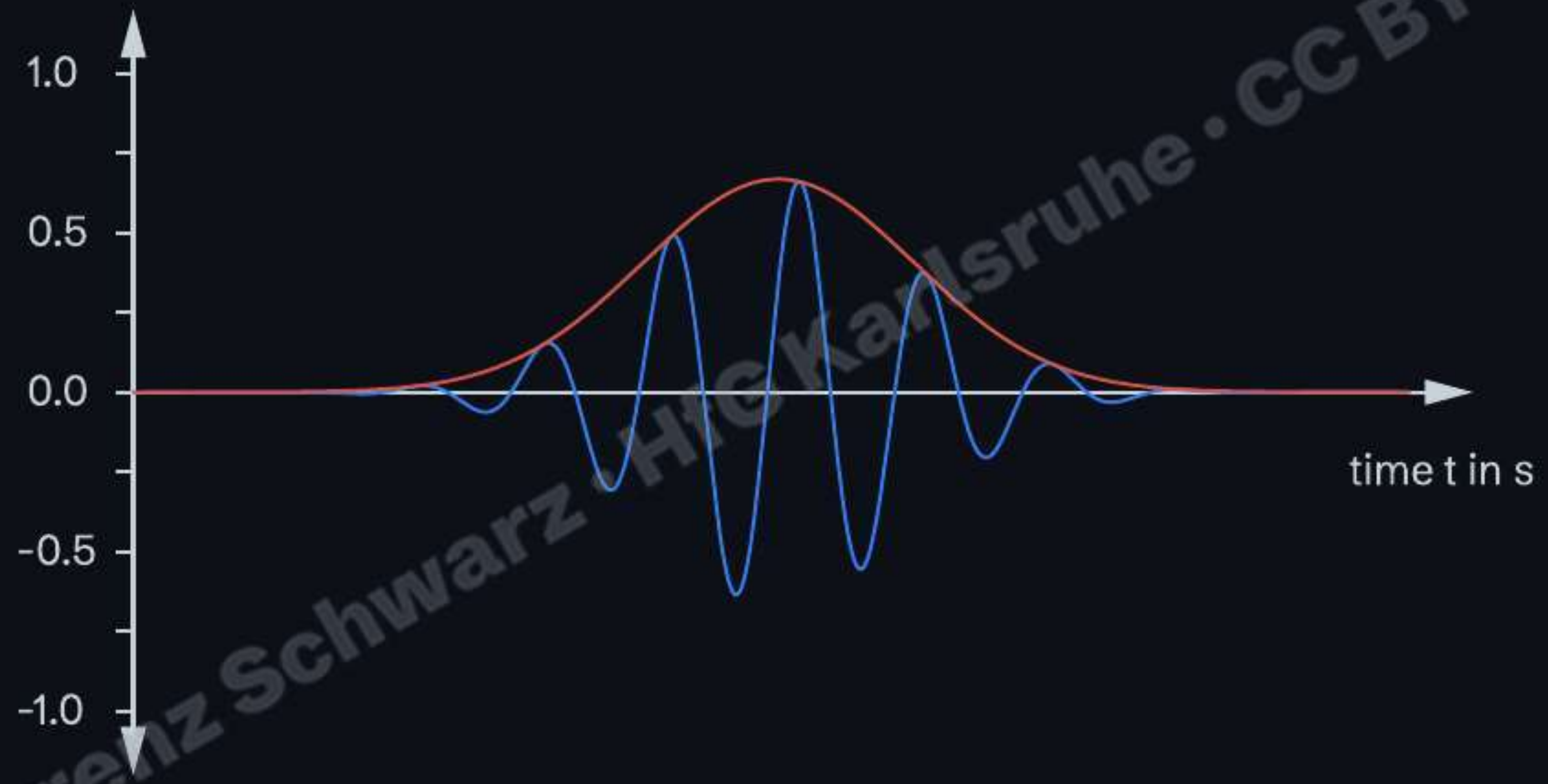
- Chorus (multiple copies of the same signal, slightly delayed and out of tune)
- Phaser (copied signal runs through an all-pass filter and is then mixed with its original)
- Active noise control

▶ Phasing effect applied to white noise

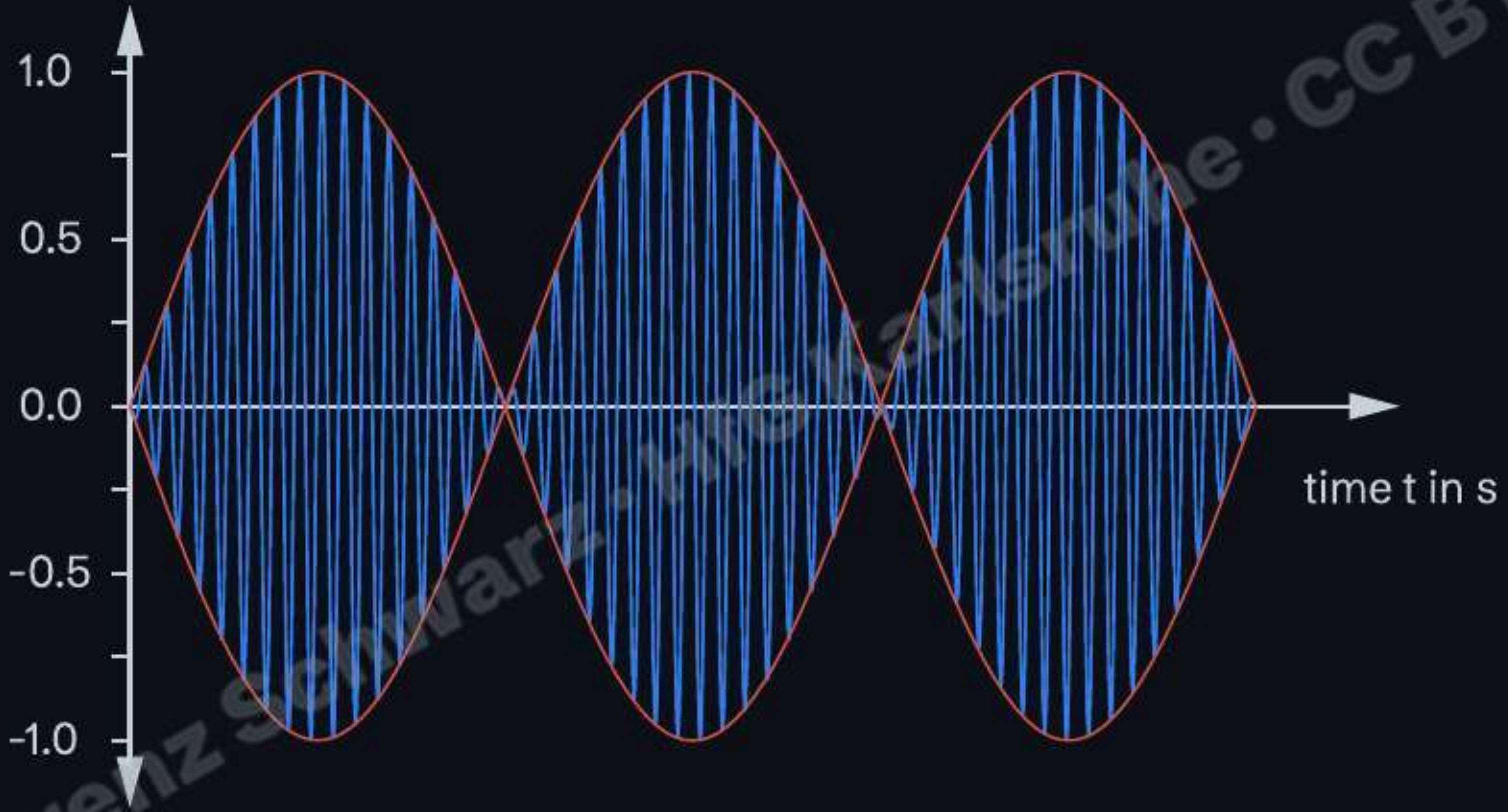
Wave properties and sound design

The following slides on envelope and amplitude modulation connect amplitude concepts to time-varying behavior.

Envelope over time



Upper and lower envelope



Listening examples: Amplitude modulation

Varying the amplitude of a 400 Hz sound with a lower-frequency modulation signal:

1. Slow rates (< 4 Hz) → Pulsation. ▶
2. Moderate rates (4 - 30 Hz) → Tremolo. ▶
3. Faster rates (30 - 70 Hz) → Roughness. ▶
4. Very fast rates (> 70 Hz) → Spectral coloration. ▶

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