FM Synthesis and Digital Sound

Sound (Art & Technology)

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[Course info]

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I. Computer Music Foundations

> “ There are no theoretical limitations to the performance of the computer as a source of musical sounds, in contrast to the performance of ordinary instruments. ”

— Max Mathews

The Digital Computer as a Musical Instrument (1963)

Max Mathews with the GROOVE system, ca. 1972 | © Bell Laboratories | Courtesy of Max Mathews

Max Vernon Mathews (1926-2011)

American pioneer of computer music at Bell Laboratories:

- Developed **MUSIC I (1957)**, first widely-used program for computer sound generation
- Created **GROOVE (1970)**, first real-time interactive music synthesis system
- Programmed **"Daisy Bell" (1961)**, computer-synthesized singing (inspired HAL 9000)

→ *Max/MSP software named in his honor*

Audio Example 1:

Daisy Bell (1961) - IBM 7094

▶ Play excerpt

Audio: Daisy Bell (1961) | IBM 7094, Bell Labs | Max Mathews & Joan Miller | Source: Internet Archive | Educational fair use


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Daisy Bell (1961)

The first computer-synthesized song created with IBM 7094 computer at Bell Labs:

- Demonstrated music synthesis potential
- Milestone in computer music history

- Inspired HAL 9000 singing in *2001: A Space Odyssey* (1968)

→ *The MUSIC-N programming language lead to current software like Max/MSP, SuperCollider, and Reaktor*

Computer music

Computer as universal instrument without physical limitations

1. Composition
Algorithmic generation of musical structures and notation

2. Sound synthesis
Creating sounds from mathematical descriptions

3. Sound control
Real-time manipulation through performance interfaces

The computer as instrument

Discrete numbers represent sound pressure samples, enabling digital representation and processing.

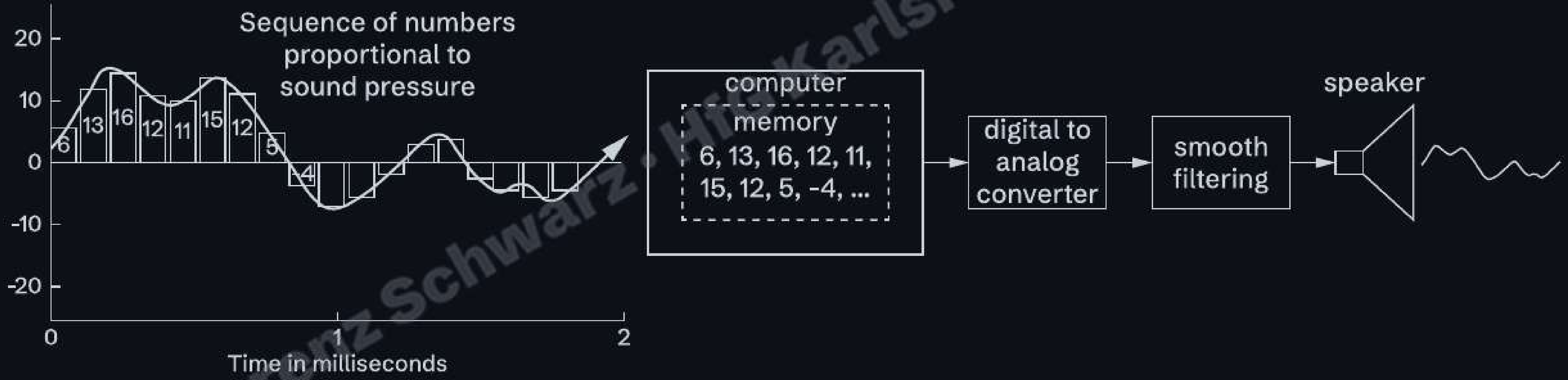


Diagram: Lorenz Schwarz, 2025 (after Mathews, 1963) | CC BY 4.0 | Original: Mathews, M.V. Science 142(3592), 1963

II. John Chowning and FM Synthesis

Photo: Histeria | Source: Sound on Sound

John Chowning (*1934)

American composer and computer music pioneer

Education and influences:

- Studied composition with Nadia Boulanger, Paris (1959–61)
- Doctoral degree, Stanford University (1966) with Leland Smith
- Worked with Max Mathews at Bell Telephone Laboratories

Chowning's research at Stanford AI Lab

Acquired MUSIC IV from Max Mathews, studied:

- Computer programming and hardware
- Psychoacoustics (spatial perception, localization)
- Acoustics (physics of sound and reverberation)
- Mathematics (Doppler shift for movement simulation)

→ *Fascination with composing for loudspeakers as spatial instruments*

Compositional approach

- Spatial compositions with sounds possessing "internal dynamism"
- Liberation of musical sound from physical constraints through computer synthesis

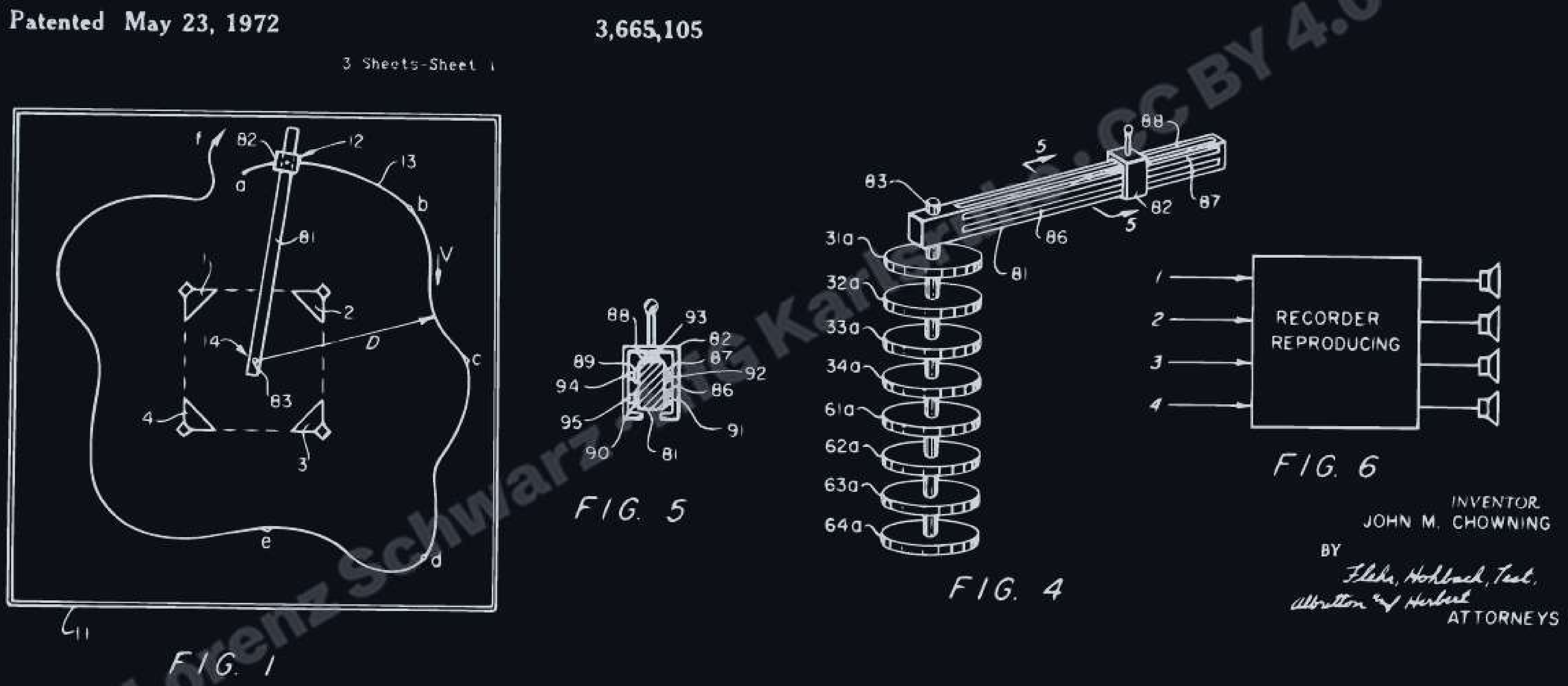
Constraint: Non-real-time processing required 10+ minutes to compute a few seconds of audio

Spatial music through quadraphonic synthesis

System: 4-channel DAC creating illusory acoustic environments

Psychoacoustic parameters controlled:

- **Movement:** Doppler shift simulates sound trajectory
- **Distance:** Direct-to-reverberant signal ratio
- **Azimuth/Position:** Energy distribution across speakers



US Patent 3,665,105 (1972) | Inventor: John M. Chowning | Public domain

Discovery of FM synthesis

Accident through experimentation (1967):

- Jean-Claude Risset mentioned additive synthesis from his projects with trumpet sounds
- Chowning experimented with simple two-oscillator system
- Discovered FM's ability to create complex timbres efficiently

First commercially sucessful digital synth

- Simple algorithm (two oscillators) generates rich, controllable spectra
- Analysis-by-synthesis approach to trumpet timbre
- Both harmonic and inharmonic spectra possible

→ *Patent licensing of the technology by Stanford University to Yamaha (~20 million dollars)*

Founding of CCRMA (1975)

Center for Computer Research in Music and Acoustics

- First university center dedicated to computer music
- Home to innovations in spatialization, FM, physical modeling
- Continues as leading research facility today

III. Audio Fundamentals

Important audio concepts for FM

- **Timbre and Spectrum**
  - Distribution of frequency components
- **Partials and harmonics**
  - Building blocks of complex sounds
- **Fourier transform**
  - Analyzing sounds into sine wave components

Timbre and spectrum

Timbre (perceptual):
The sonic quality that distinguishes instruments playing the same pitch

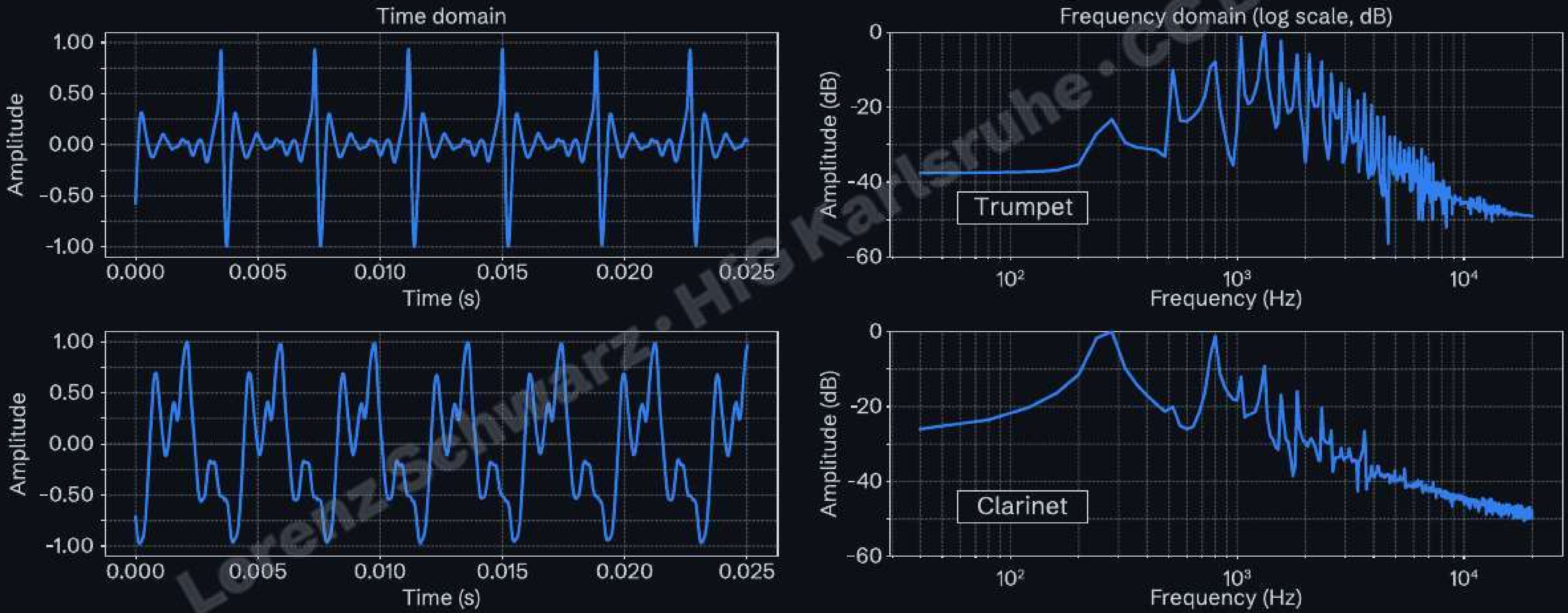
Spectrum (technical):
The distribution of frequency components and their amplitudes

Listen to the same pitch (C4 ≈ 261.6 Hz):

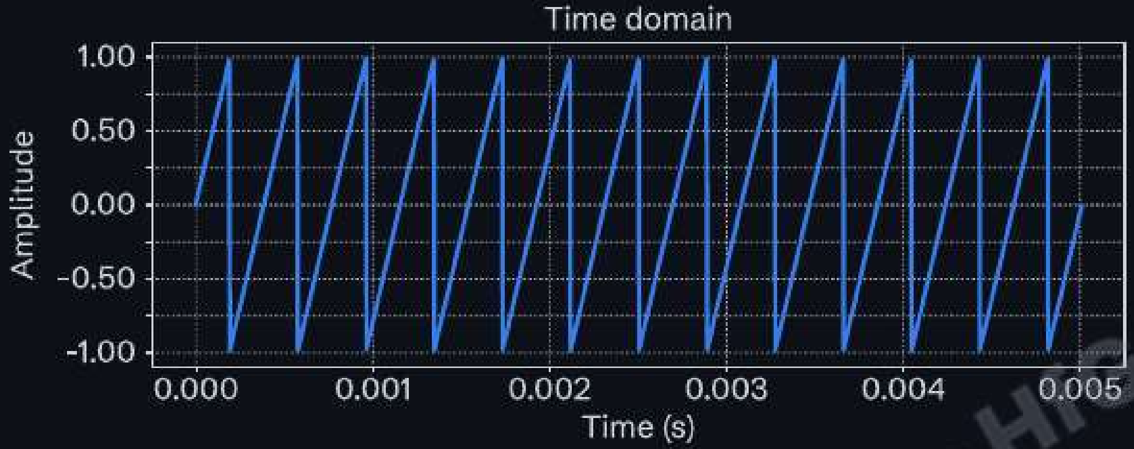
Trumpet: ▶ Play **Clarinet:** ▶ Play

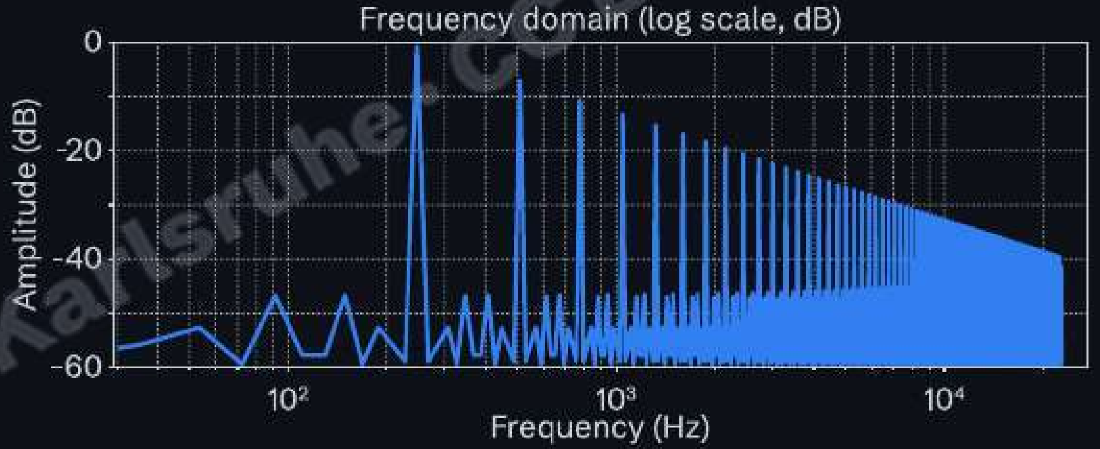
→ *Same pitch, different timbre*

Comparing clarinet and trumpet at C4



Understanding spectra with a sawtooth wave



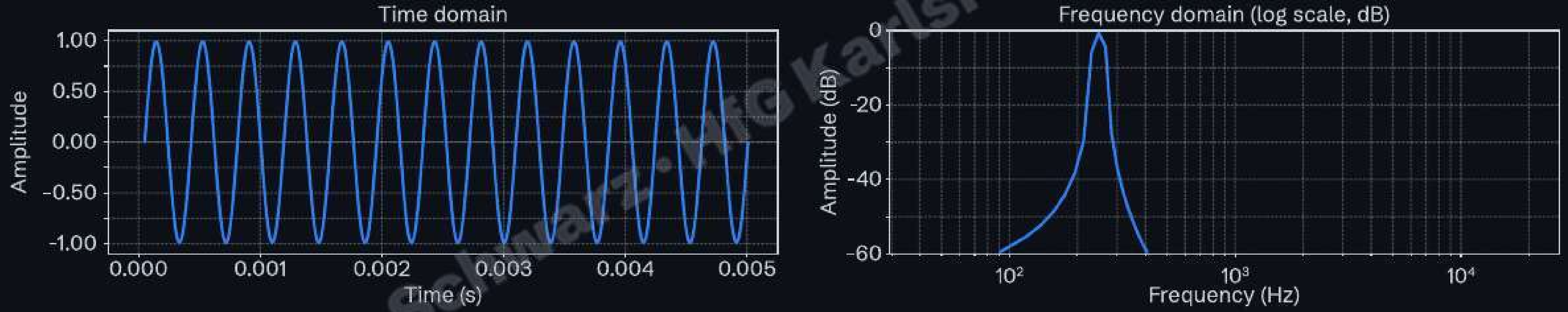


- Each peak in the spectrum represents one sine wave (partial or harmonics)
- Harmonic series 260, 520, 780, 1040... Hz

▶ Play sawtooth wave C4 ≈ 260 Hz

Pure tone (sine wave)

A sine wave is a single frequency component, the fundamental building block of all sounds

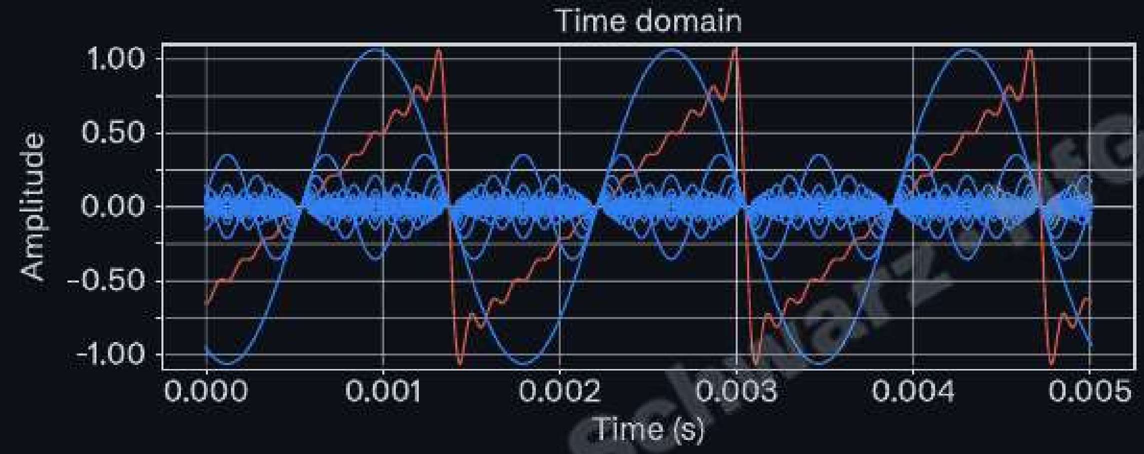


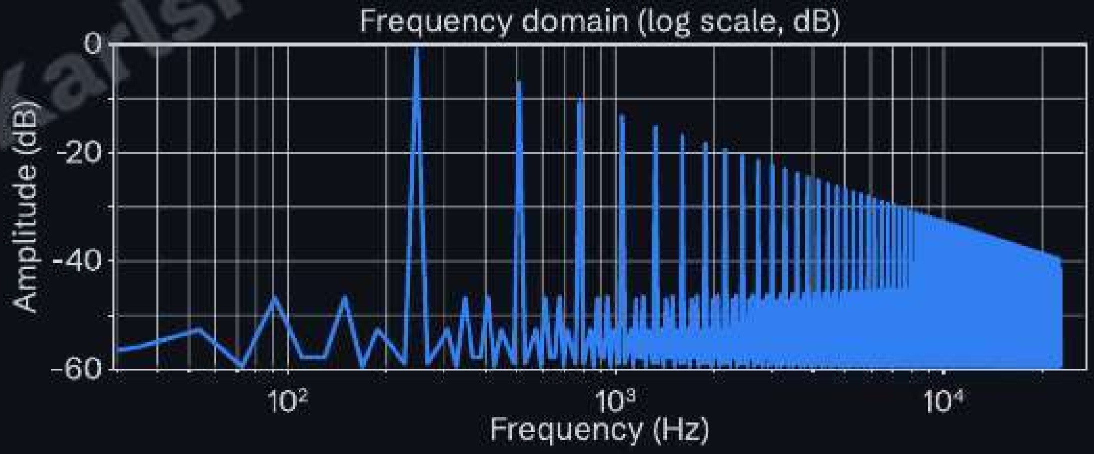
$$x(t) = A\sin(2\pi f_0 t + \varphi)$$

View sine wave on Desmos

Complex tones (example: sawtooth)

Musical instrument sounds and basic waveforms (except sine) contain many sine waves (click for graphing calculator)





$$x(t) = \frac{2A}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(2\pi n f_0 t)$$

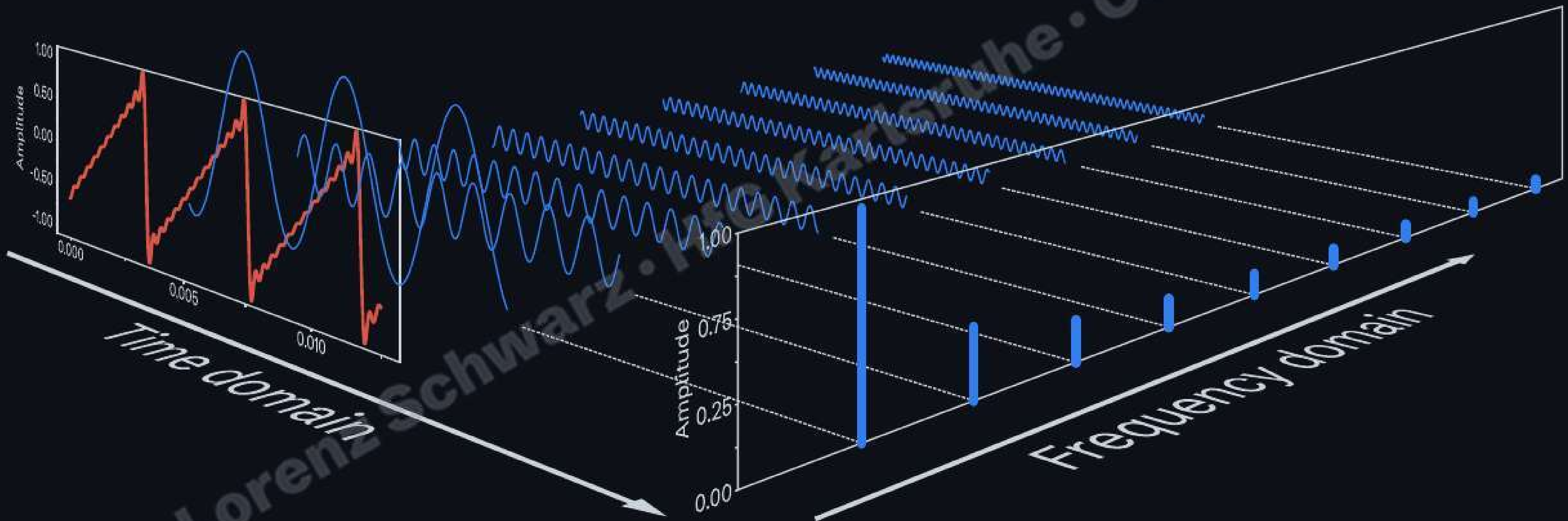
Fourier transform

Fourier transform converts time domain (waveform) to frequency domain (spectrum):

- Reveals the individual sine wave components of any sound
- Shows *which* frequencies are *how strong*

→ *Proves that complex sounds are sums of sine waves*

Fourier transform



Additive synthesis

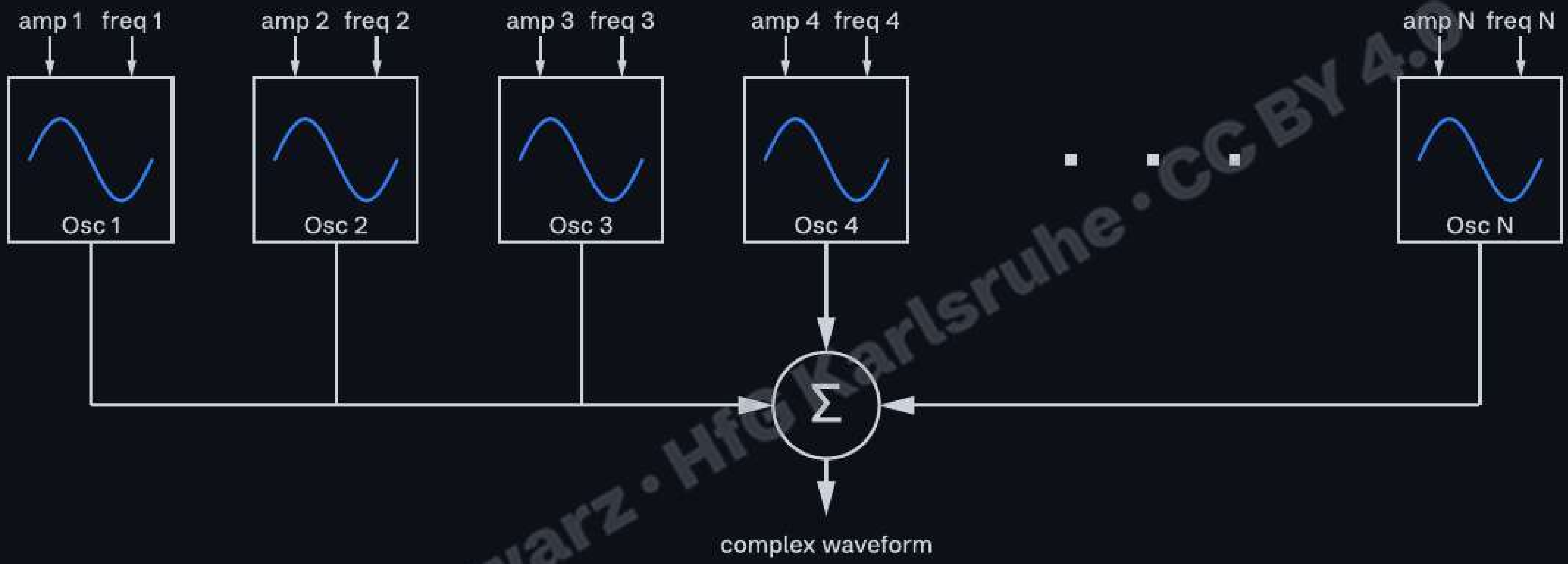
Concept: Build complex sounds by adding sine waves together

$$x(t) = \sum_{k=1}^{K} A_k(t) \sin(2\pi k f_0 t + \varphi_k)$$

Challenges:

- Requires many oscillators (one per partial)
- Controlling timbre changes over time is complex
- Computationally expensive

→ *Additive synthesis: conceptually simple, practically expensive*



Simplified schematic of additive synthesis

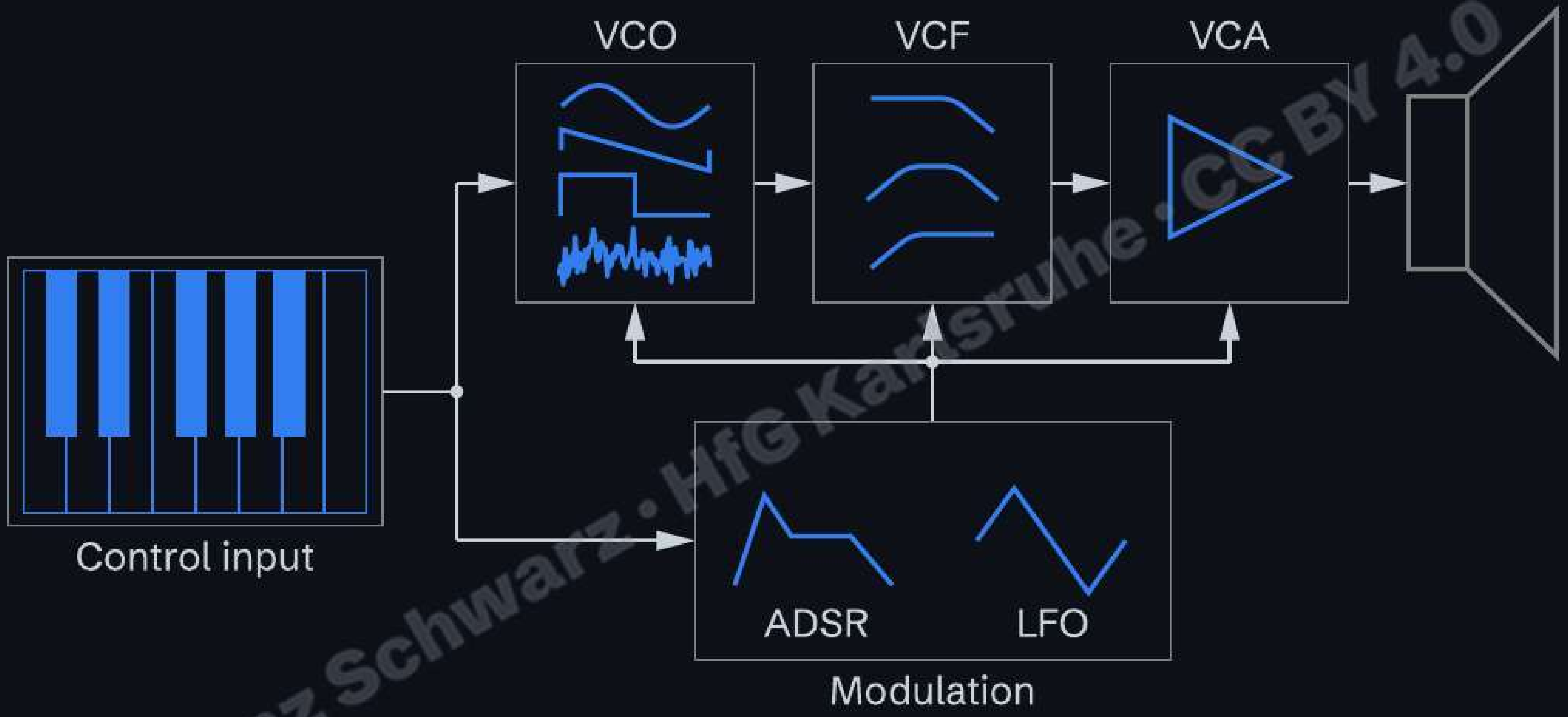
Subtractive synthesis

Alternative approach: Generate rich harmonic content, then filter out unwanted frequencies

Process:

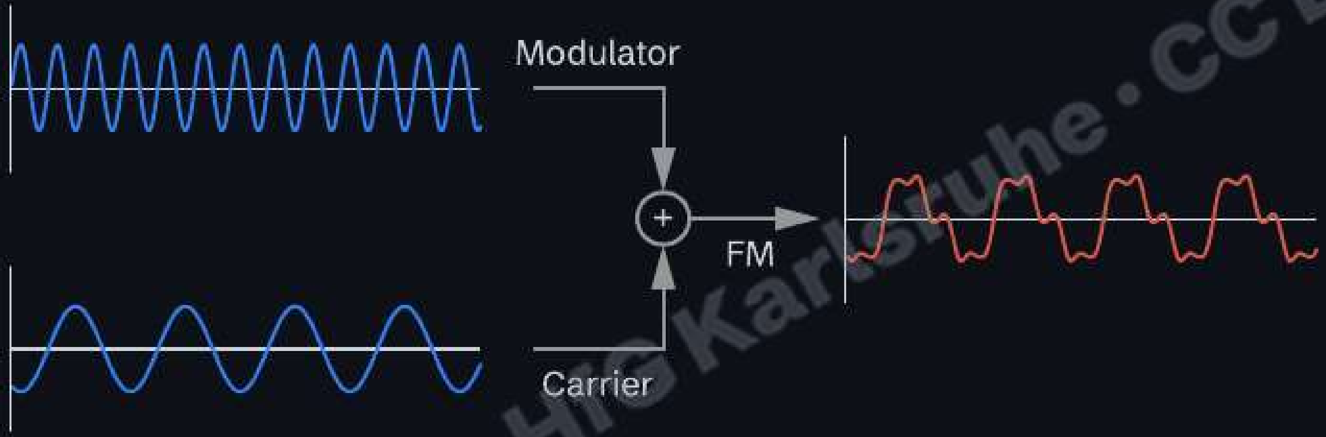
- Oscillator generates complex waveform (sawtooth, square)
- Filter removes unwanted frequency components
- Envelope controls amplitude over time

→ *More efficient than additive synthesis, but offers less spectral control*



Simplified schematic of voltage controlled subtractive synthesis.

Efficiency of FM synthesis



- Uses **two oscillators** (carrier + modulator)
- Generates rich, controllable spectra through frequency modulation
- Simple algorithm creates both harmonic and inharmonic sounds

→ *Achieves rich timbral control with minimal parameters*

IV. FM Theory

Origins of frequency modulation

Mainly developed by radio broadcasting engineer Edwin Armstrong (1890 - 1954) for transmitting high-fidelity sound over broadcast radio (since the late 1930)

- FM Radio - demodulation

Simple frequency modulation (FM)

In FM synthesis, the instantaneous frequency of a carrier oscillator (C) is varied according to the output of a modulator oscillator (M).

Carrier frequency (C) - Sets the perceived pitch

Modulator frequency (M) - Determines harmonic/inharmonic character

Modulation depth (D) - Controls spectral brightness/richness

Effects of FM synthesis

- *vibrato*-effect for modulator frequencies at sub-audio level (below 30 Hz)
- *rich spectra* with increasing modulator amplitude and frequency

▶ sine wave with vibrato → complex waveform with rich spectrum

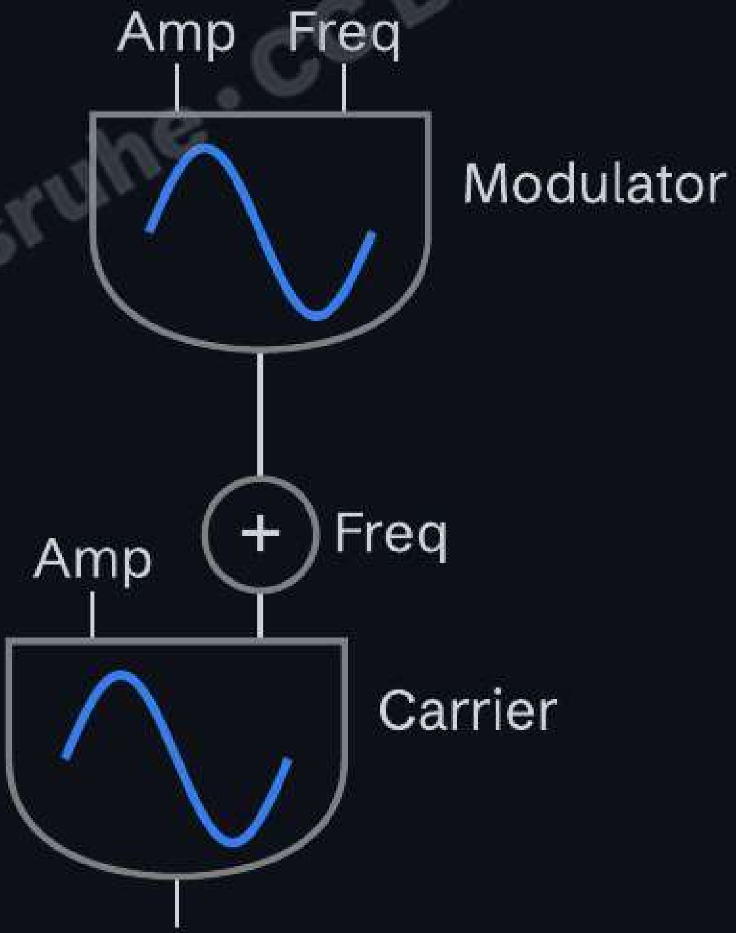
FM synthesis parameters

The resulting frequency components are determined by:

- Ratio between Carrier and Modulator
- Modulation depth (D)

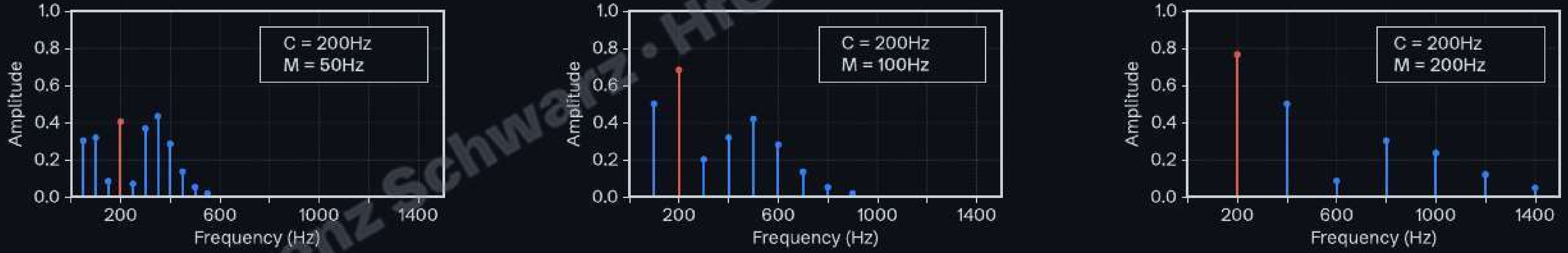
→ *These two parameters control the entire spectral output.*

click for graphing calculator



Detuning the oscillators

- If the carrier is detuned, the entire harmonic spectrum shifts up or down by that same amount.
- Detuning the modulator compresses or expands the separation between the sidebands



From left to right: increasing modulator frequency results in wider spacing

FM synthesis formula

Basic formula:

$$x(t) = A\sin(2\pi Ct + D\sin(2\pi Mt))$$

Parameters:

- $A(t)$ - Carrier amplitude
- f_c (or C) - Carrier frequency (perceived pitch)
- f_m (or M) - Modulator frequency (sideband spacing)
- D - Modulation depth (frequency deviation or amount of modulation)

Derived FM parameters

Harmonicity ratio:

- Determines harmonic (integer) or inharmonic (non-integer) spectrum

$$H = \frac{M}{C}$$

Modulation index:

- Controls the number of significant frequency components (sidebands)

$$I = \frac{D}{M}$$

Sidebands (spectral components)

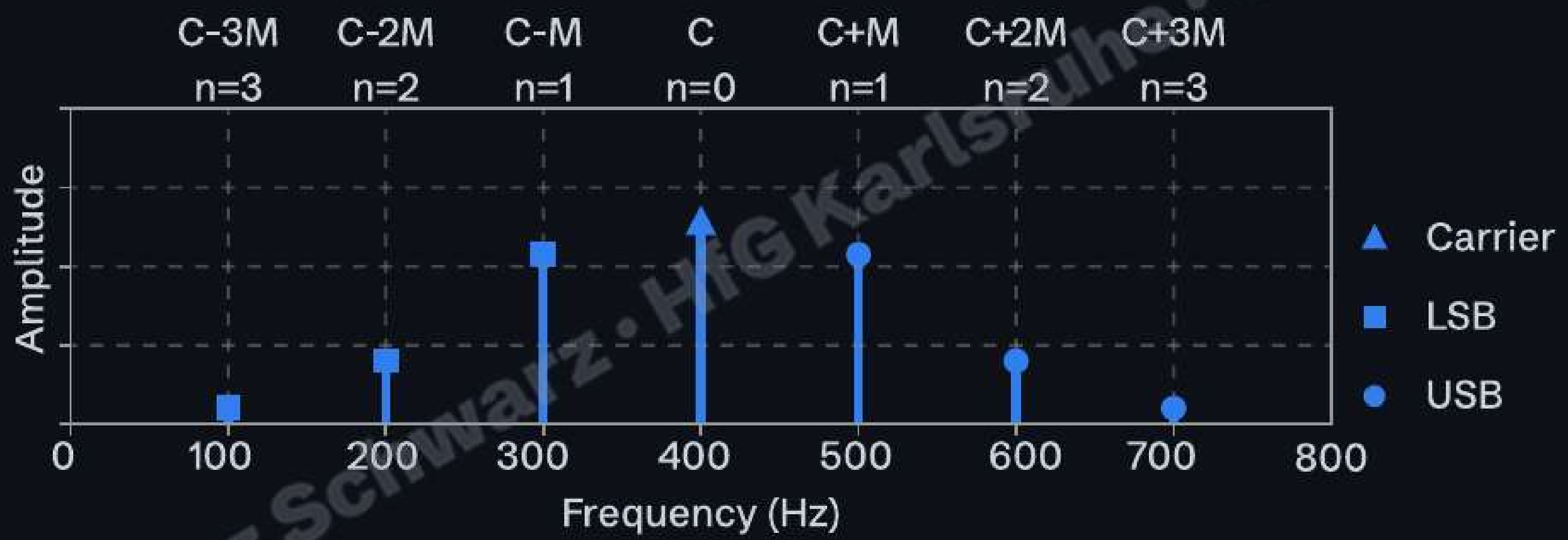
New frequency components appear in pairs symmetrically around the carrier frequency and define the timbre of the sound:

$$C \pm kM$$

- k is an integer that determines the order of the sidebands
- C carrier
- M modulator

Calculating the sidebands

$$C = 400\text{Hz},\ M = 100\text{Hz}$$



Each sideband pair has the same amplitude.

Reflected sidebands and interference

Lower sidebands extending below 0 Hz reflect at zero with a 180° phase shift, potentially interfering with positive-frequency components.

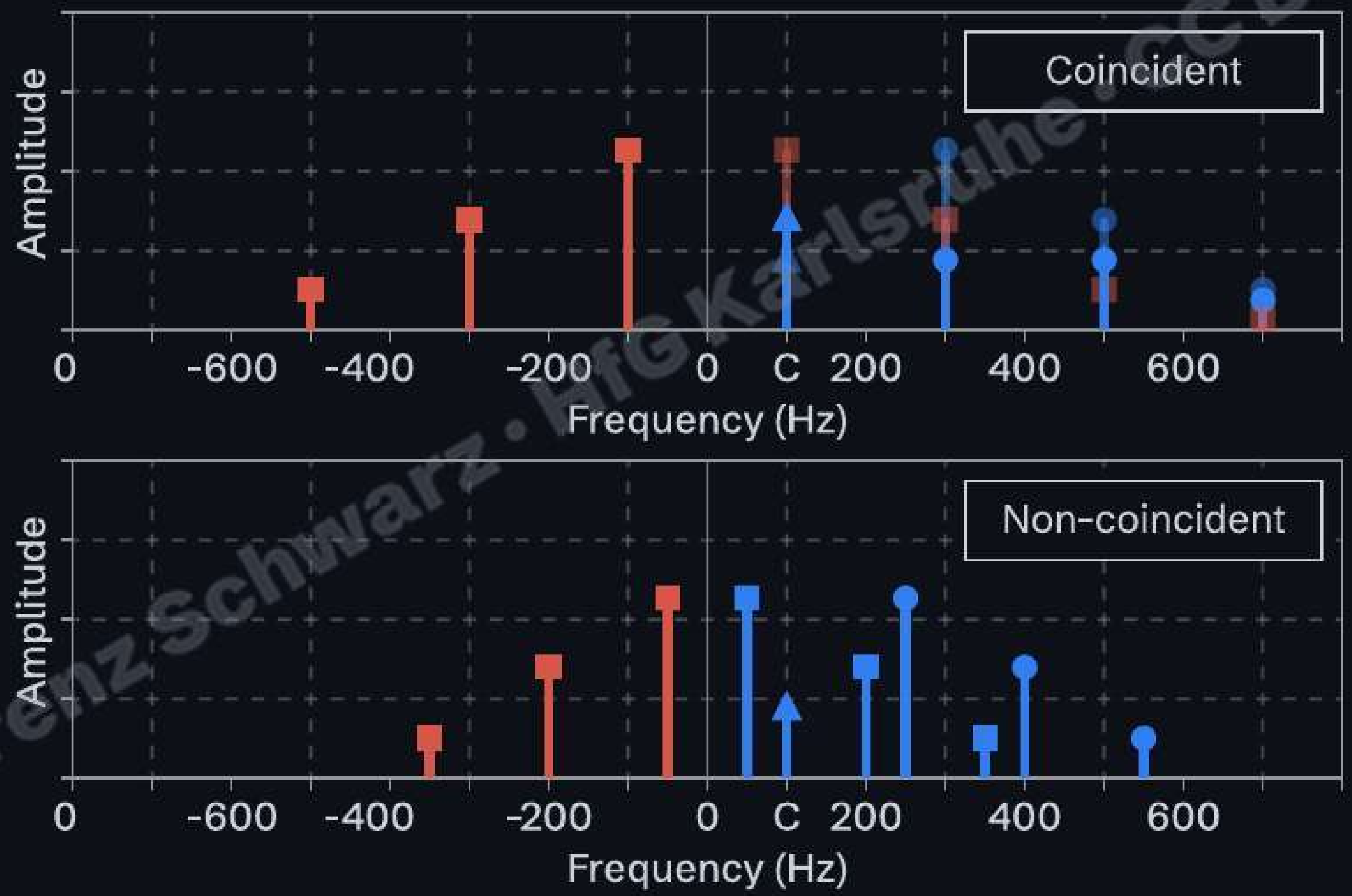
Coincident series (e.g., 1:1, 2:3 ratio):

- Reflected sidebands land on existing positive frequencies → regular harmonic spacing with interference

Non-coincident series (e.g., 3:5 ratio):

- Reflected sidebands fall between positive frequencies → irregular spacing, no interference

Coincident vs. non-coincident series



Control over a sound's "brightness"

Modulation index I

Number of significant (perceivable) frequency components increases with I:

$$I = \frac{D}{M}$$

→ *Increasing the modulation index creates more sidebands with greater amplitudes, redistributing energy across the spectrum and increasing spectral richness.*

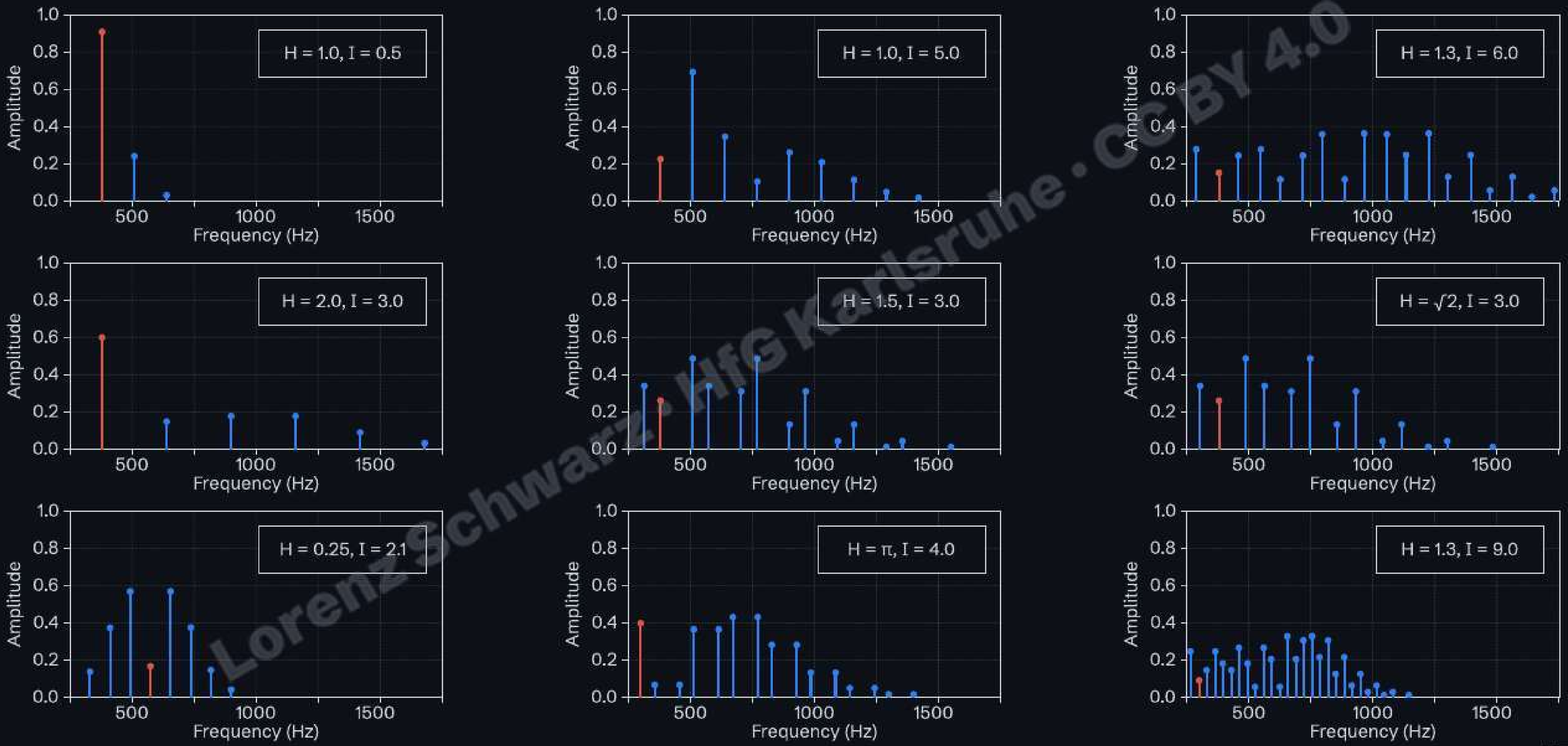
Harmonicity ratio and spectral character

The ratio between M and C determines the harmonicity of the resulting spectrum.

Harmonicity ratio: $H = \frac{M}{C}$ (modulator ÷ carrier)

→ *If H is rational ($H = p/q$), the spectrum contains only harmonic frequencies*

→ *If H is irrational, the spectrum is inharmonic*


H = 1.0, I = 0.5
H = 1.0, I = 5.0
H = 1.3, I = 6.0
H = 2.0, I = 3.0
H = 1.5, I = 3.0
H = √2, I = 3.0
H = 0.25, I = 2.1
H = π, I = 4.0
H = 1.3, I = 9.0
Amplitude
Frequency (Hz)

Listening examples: FM spectra

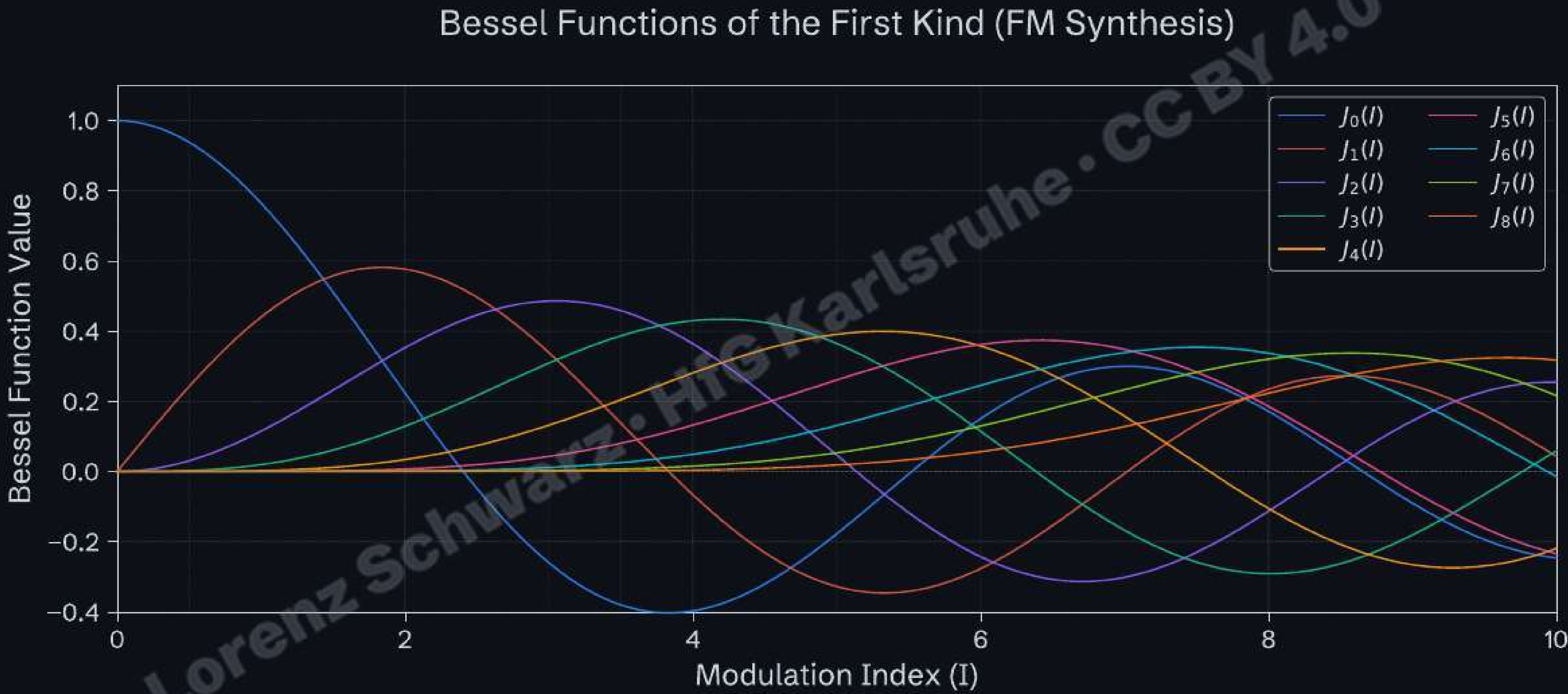
1. **Simple** (C=260, H=1.0, I=0.5) ▶
2. **Rich harmonic** (C=260, H=1.0, I=5.0) ▶
3. **Complex rational** (C=260, H=1.3, I=6.0) ▶
4. **Odd harmonics** (C=260, H=2.0, I=3.0) ▶
5. **2:3 ratio** (C=260, H=1.5, I=3.0) ▶
6. **Inharmonic √2** (C=260, H=1.414, I=3.0) ▶
7. **Sub-harmonic** (C=650, H=0.25, I=2.1) ▶
8. **Irrational π** (C=100, H=3.14159, I=4.0) ▶
9. **Extreme** (C=100, H=1.3, I=9.0) ▶

Calculating sideband amplitudes

Sideband amplitudes are determined by mathematical scaling factors known as Bessel functions of the first kind:

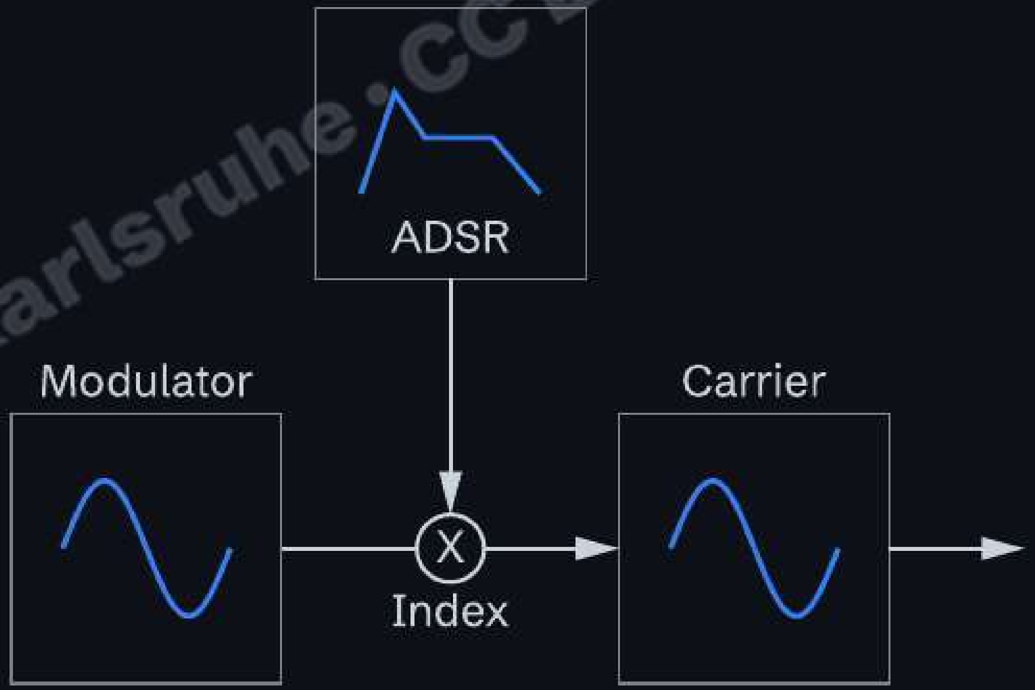
- The amplitude of the k-th sideband is calculated as $J_k(I)$, where k is the sideband order and I is the modulation index.
- Total average power of the signal remains constant, only the spectral distribution of that energy changes.

→ *Bessel functions act as a mathematical "lookup table"*


Bessel Functions of the First Kind (FM Synthesis)
Bessel Function Value
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0
2
4
6
8
10
Modulation Index (I)
$J_0(I)$
$J_1(I)$
$J_2(I)$
$J_3(I)$
$J_4(I)$
$J_5(I)$
$J_6(I)$
$J_7(I)$
$J_8(I)$

Dynamic timbres

Coupling an envelope to both the carrier amplitude and modulator level creates realistic, brass-like dynamic changes in both loudness and brightness



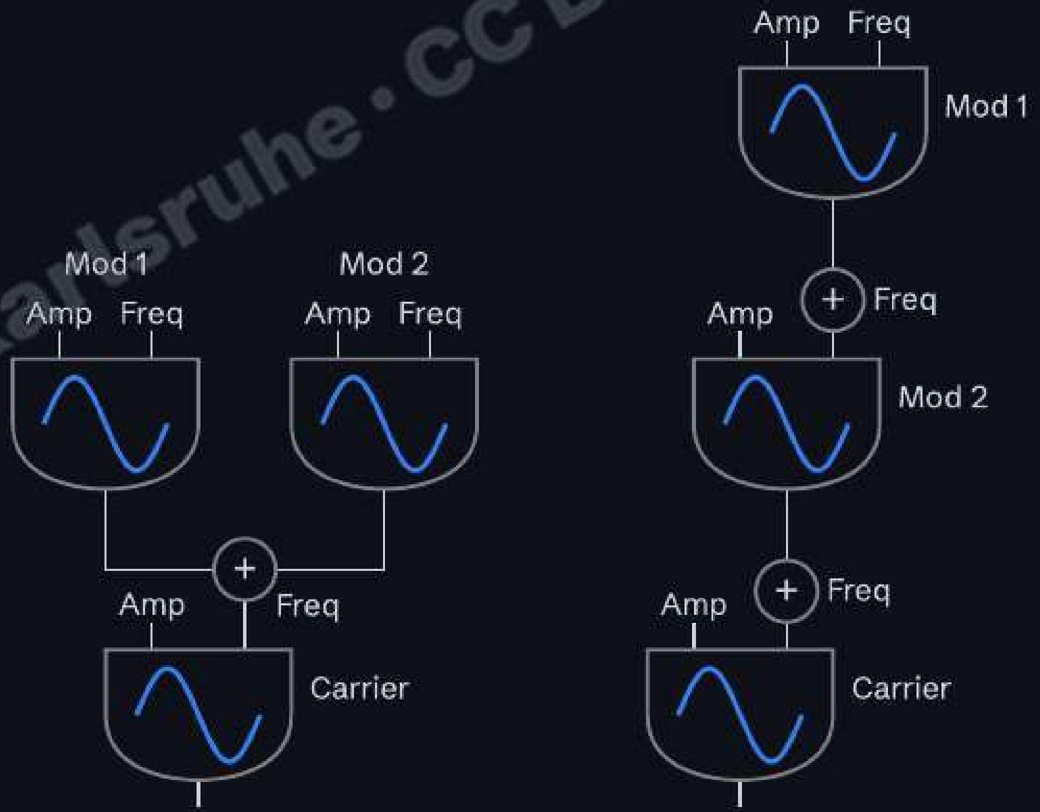
Multiple modulators

Parallel modulators (M1→C, M2→C):

- Sideband series add together
  $C \pm k \cdot M_1$ and $C \pm k \cdot M_2$

Cascaded modulators (M1→M2→C):

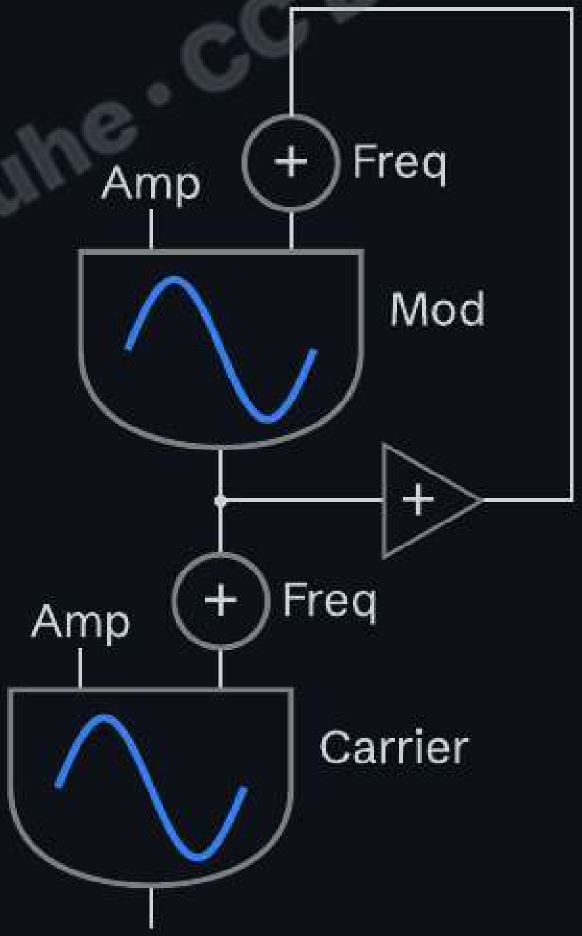
- Each sideband from M1→M2 acts as a sine wave modulator for the carrier
- Result: sidebands of sidebands (exponential spectral growth)



Feedback in FM

Feedback routes an operator's output back to its own input:

- Generates spectral complexity without additional oscillators
- Creates additional sideband frequencies beyond standard FM pairs



Phase modulation (PM)

PM is the derivative of FM. It varies the phase angle rather than frequency, but produces identical sidebands to FM.

Digital FM synthesis uses PM because:

- More stable with feedback loops
- Easier to implement digitally
- Same audible result as true FM

→ *All digital FM synthesizers (DX7, etc.) actually use PM*

V. FM Applications

Yamaha DX7 digital synthesizer

▶️ Audio example: DX7 electric piano preset

Yamaha DX7 (1983) | Photos: Leo-setä, iixorbiusii, Georgfotoart | Composite: Pittigrilli | Wikimedia Commons | CC BY 4.0

Yamaha DX7 (1983)

The Yamaha DX7 brought Chowning's academic research to consumer market and defined 1980s sound (pop, new wave, film scores)

- First affordable digital synthesizer using FM
- Over 2 million units sold (best-selling synth ever)
- 16-note polyphony
- 61-note keyboard with velocity sensitivity
- 32 algorithms (6 sine wave operators each)

→ *Stanford earned ~$20 million from Yamaha patent*

Examples of 4 algorithms (configurations of operators) to generate sounds through carrier/modulator relationships.

Other FM synthesizers

Current hardware and software FM synthesizers:

- Arturia DX7 V
- Elektron Digitone II
- Korg opsix SE
- Korg Volca FM
- Native Instruments FM8

FM synthesis in Max/MSP

Basic implementation:

- Two `cycle~` objects (oscillators)
- `*~` object for introducing modulation depth control
- `+~` object to add modulation to carrier frequency

→ *simpleFM~ abstraction*

VI. *Turenas*

Turenas (1972) (quadraphonic version)

Title: Anagram of "Natures"

- All sounds created through FM synthesis
- No recordings or traditional instruments
- Quadraphonic spatialization with Doppler shifts
- Movement through virtual acoustic space
- Demonstrates FM's ability to create complex, organic timbres

→ *Established FM as legitimate compositional tool*

VII. Artistic Research

FM Synthesis and artistic research

- Discovery emerged from compositional practice (sound spatialization)
- Required teaching himself advanced mathematics, programming, and signal processing
- Six years of systematic research to make it musically controllable
- Continuous artistic application used in his own compositions and widely adopted by other artists
- A major patent licensing success for Stanford with significant impact on the music industry

→ *Artistic inquiry and scientific understanding enabled genuine innovation*

Appendix

Determining the fundamental frequency

For rational harmonicity ratios $H = p/q$ (in lowest terms), the fundamental frequency can be calculated:

$$\text{Fundamental} = \frac{C}{q}$$

Example: $H = 1.9 = \frac{19}{10}$, $C = 260$ Hz

- $q = 10$
- Fundamental $= \frac{260}{10} = 26$ Hz

→ *All sideband frequencies are integer multiples of 26 Hz.*

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