

VIBRATION AND SOUND

Lorenz Schwarz · HfG Karlsruhe · CC BY 4.0

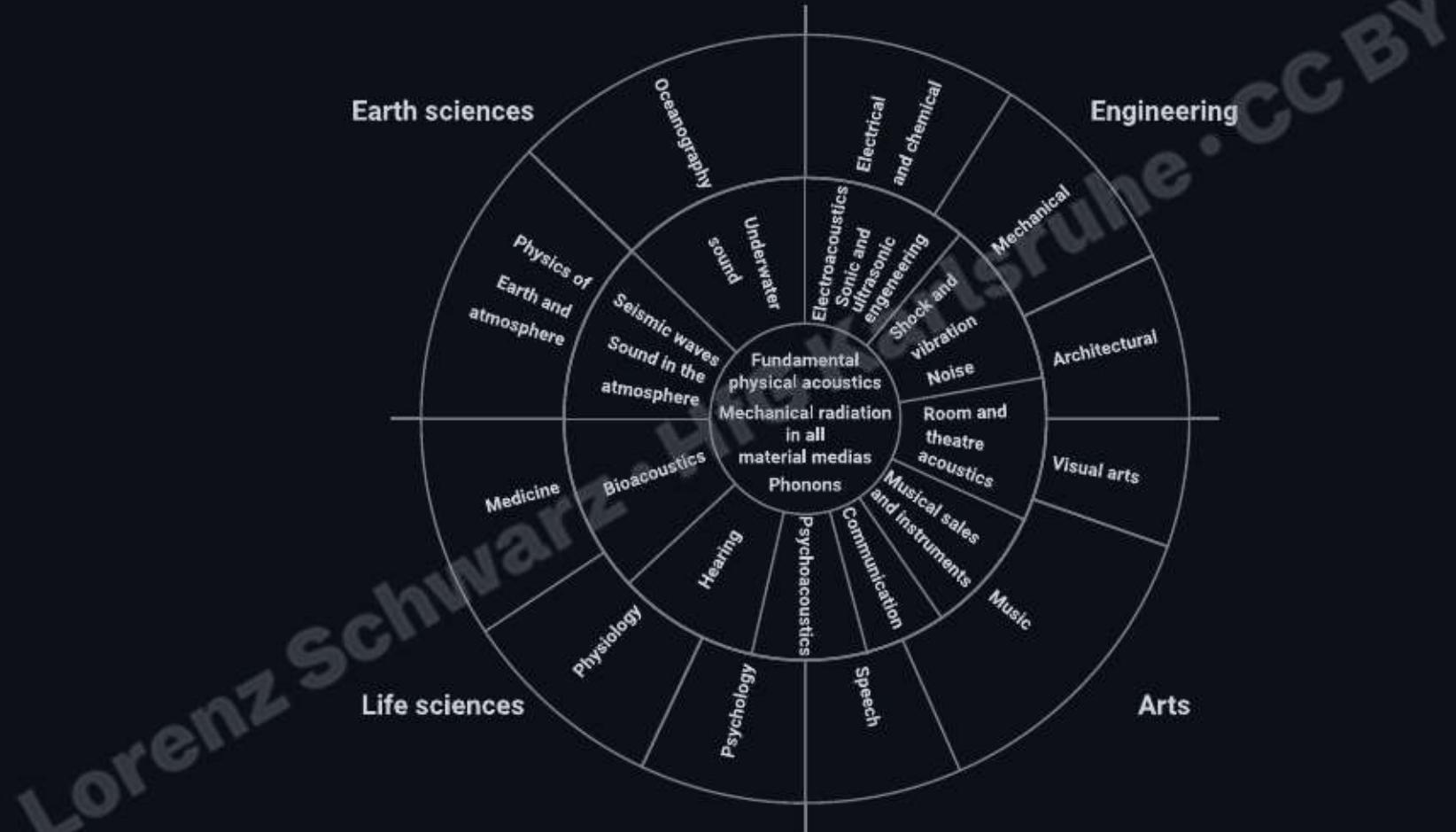
The physical and perceptual nature of sound

A complex relationship between:

1. Physical disturbance in a medium and transfer of energy
2. Psycho-physical perception and sensory experience of the physical stimuli

Lorenz Schwarz · HfG Karlsruhe · CC BY 4.0

Wheel of acoustics



(Beyer 300)

Sound (physics)

Etymology: Derived from Latin *sonare* (to sound)

Pressure or density variations in an elastic medium (e.g., air):

- particle displacement (e.g., air molecules)
- particle velocity

Elasticity and inertia

- **Elasticity:** The property of a material or medium that enables it to return to its original shape or equilibrium after being deformed, once the applied force is removed.
- **Inertia:** An object in motion remains in motion, and an object at rest stays at rest, unless acted upon by an external force (Newton's First Law of Motion).

Vibration and sound

Mechanical vibration is capable of producing sound, e.g.:

- strings (chordophones)
- membranes (membranophones)
- plates (struck idiophones)

Lorenz Schwarz · HfG Karlsruhe · CC BY 4.0

Oscillation

A process that returns to the same state after repeating periods:

- periodic vibration or cyclical process
- number of occurrences of a repeating event per second
- measured in hertz (Hz)

Lorenz Schwarz · HfG Karlsruhe · CC BY 4.0

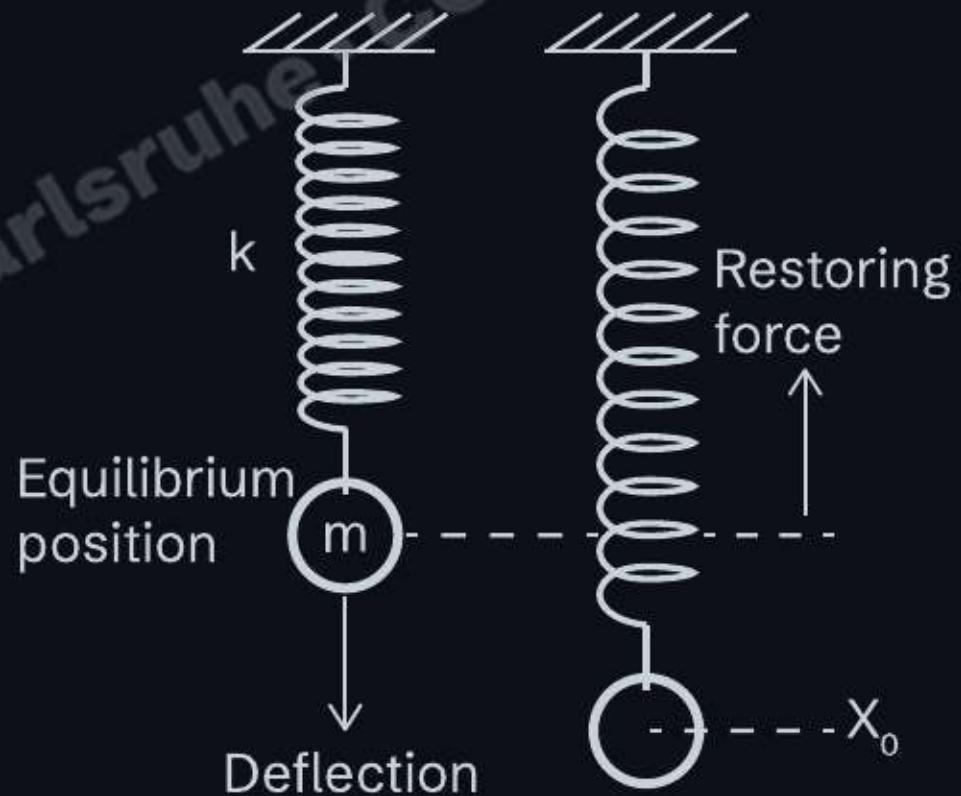
Case study: spring-mass system

Vertical back and forth movement of a mass on a spring:

- Newton's Second Law: $F = m \frac{dv}{dt} = ma$
- Hooke's Law (restoring force): $F = -kx$

constant:

- k spring constant
- m mass



Simple harmonic motion of a spring-mass system

- Newton's Second Law: $F = ma$
- Hooke's Law (restoring force): $F = -kx$

$$ma = -kx$$

variables:

- a acceleration
- x displacement from equilibrium

Acceleration

$a = \frac{dv}{dt}$ first derivative of the velocity with respect to time

or

$a = \frac{d^2x}{dt^2}$ second derivative of the position with respect to time

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

(second derivative of the function is the function)

Sine and cosine



The gradient of the tangent equals the derivative of the function at the point where the curve and tangent line meet.

[view in graphing calculator](#)

Solving the differential equation

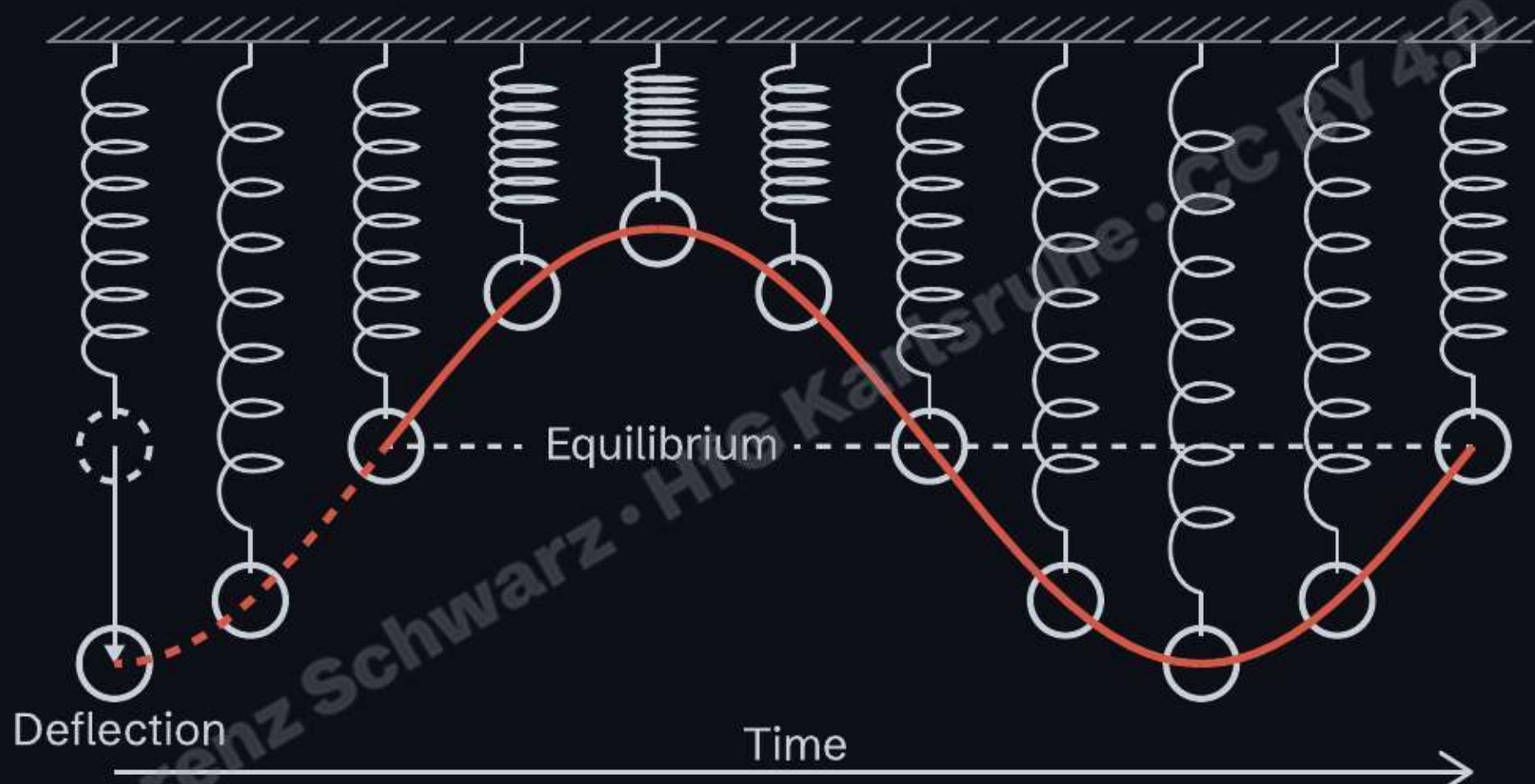
- first derivative of $\sin(\omega t)$ is $\omega \cos(\omega t)$
- and second derivative of $\sin(\omega t) \rightarrow -\omega^2 \sin(\omega t)$

$$-\omega^2 \sin(\omega t) = -\frac{k}{m} \sin(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$2\pi f = \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$



Simple harmonic motion of a mass-spring system

Oscillation of a mass-spring system

The mathematics confirms what we observe: a mass on a spring oscillates sinusoidally. The sine wave is the fundamental pattern underlying all sound.



► Sine wave 400 Hz

Sine wave

Medieval Latin *sinus*, from Latin, curve

Displacement plotted against time describes a curved and symmetrical rise and fall with no abrupt changes:

- simplest periodic function
- describing periodic phenomena (vibration)
- "pure tone", because it has no other constituent frequencies.

Sine wave function

The time dependence of a harmonic motion is described by a sine (or cosine) oscillation whose argument is a linear function of time:

$$x(t) = A \sin(2\pi ft + \varphi) = A \sin(\omega t + \varphi)$$

- A peak amplitude (nonnegative)
- $\omega = 2\pi f$ angular frequency (radians/seconds and f in Hertz)
- t time (seconds)
- φ initial phase (radians)

→ All complex oscillations can be related to the sine wave.

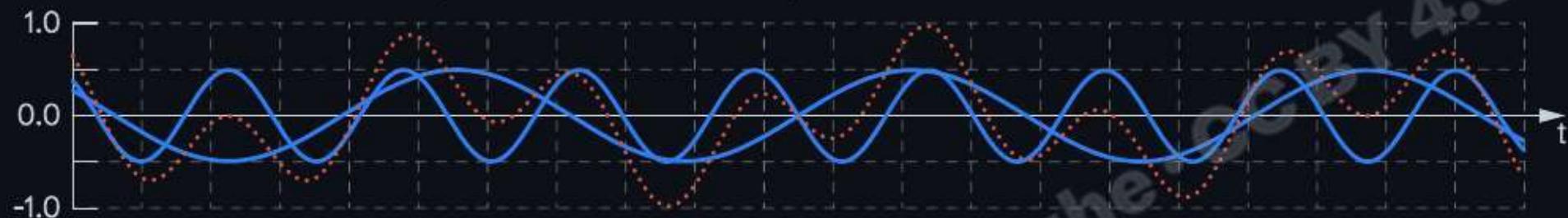
Superposition of sine waves

When a particle undergoes two or more simultaneous oscillatory movements in the same direction, the result is a combined oscillatory movement, determined by the sum of the individual oscillations.

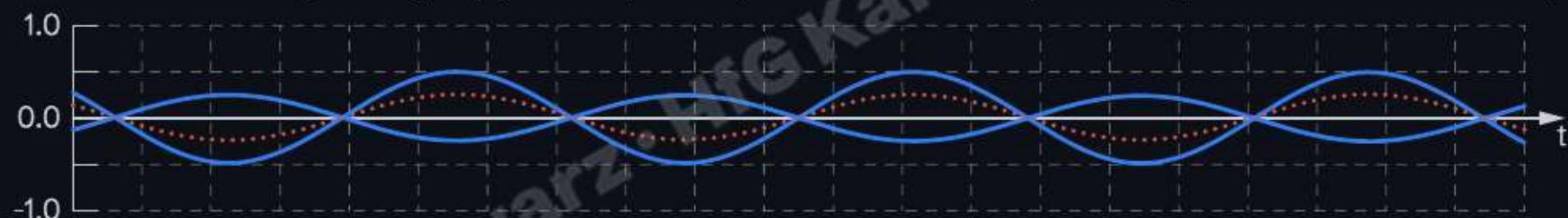
$$x(t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t + \varphi)$$

[view in graphing calculator](#)

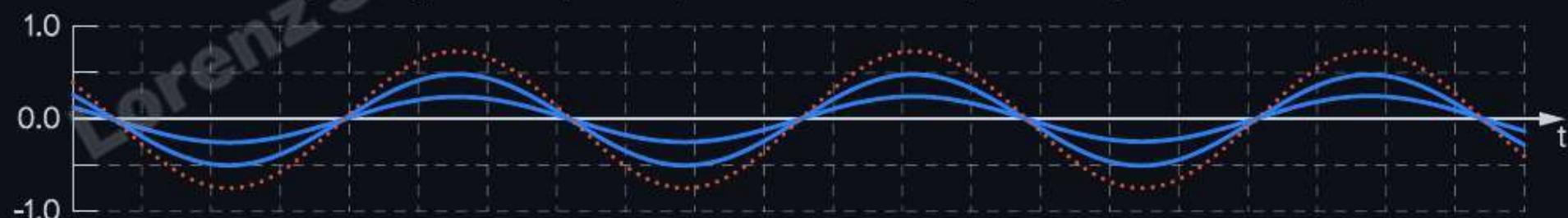
1. Two different frequencies, same amplitude



2. Same frequency, opposite phase, one at half amplitude (partial cancellation)



3. Same frequency, same phase, one at half amplitude (constructive)



Oscillation and pressure waves

The spring-mass system showed periodic oscillations.

Other mechanical systems like strings or speakers create periodic displacements.

In air, this displacement creates:

- Compression (molecules pushed together)
- Rarefaction (molecules spread apart)

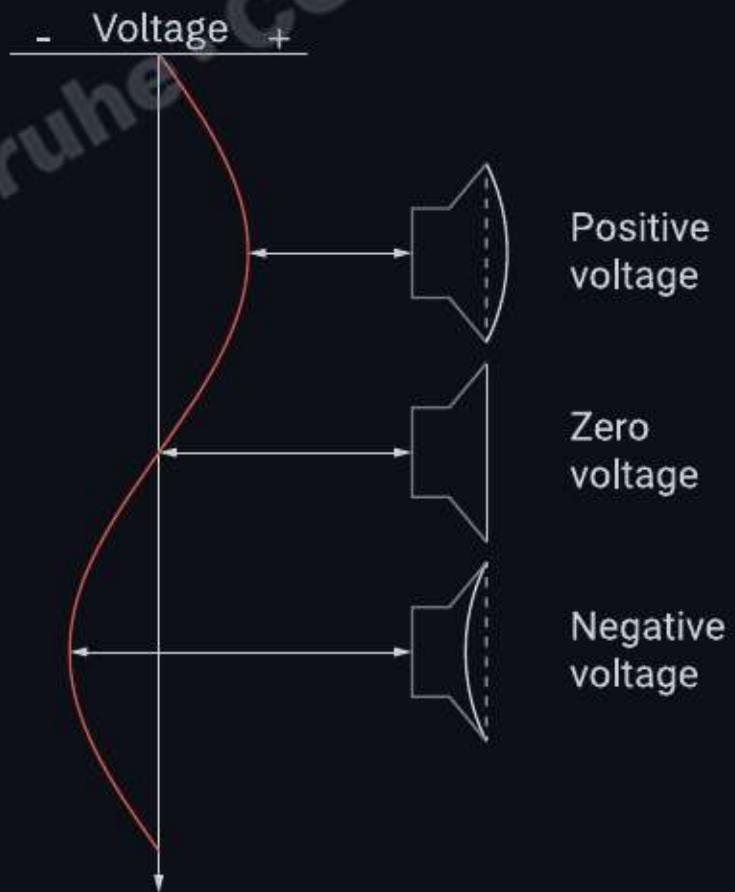
→ *These periodic displacements (pressure variations) propagate as sound waves.*

Back and forth movement of a speaker

The electrical audio signal causes the diaphragm of the speaker to move in an analogous manner:

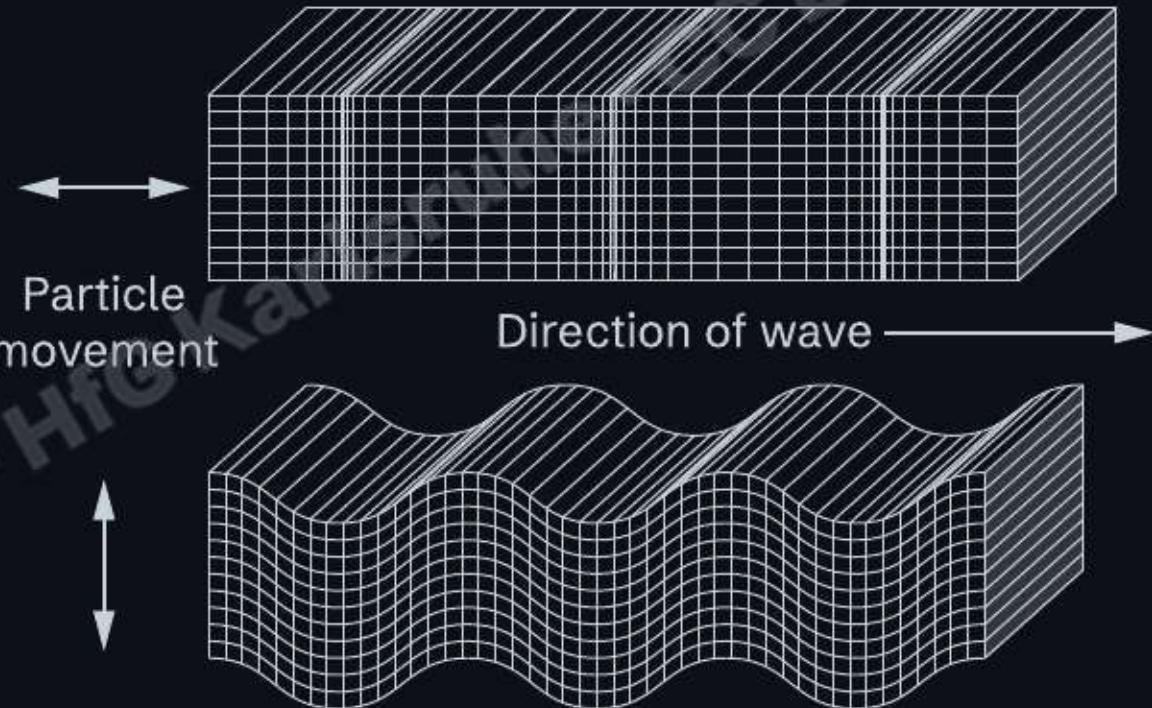
- When it moves forward, it compresses the air particles in front of it.
- When it moves backward, it creates a region of lower pressure.

→ *These alternating compressions and rarefactions propagate through the air as sound waves.*



Sound wave propagation

- Sound is transmitted as longitudinal waves (compression waves) through *gases* and *liquids*.
- It can be transmitted as both longitudinal and transverse waves through *solids*.



Longitudinal wave (top) and transverse wave (below)

Transverse and longitudinal waves

Transverse wave:

- particles move perpendicular to the direction of the wave

Longitudinal wave:

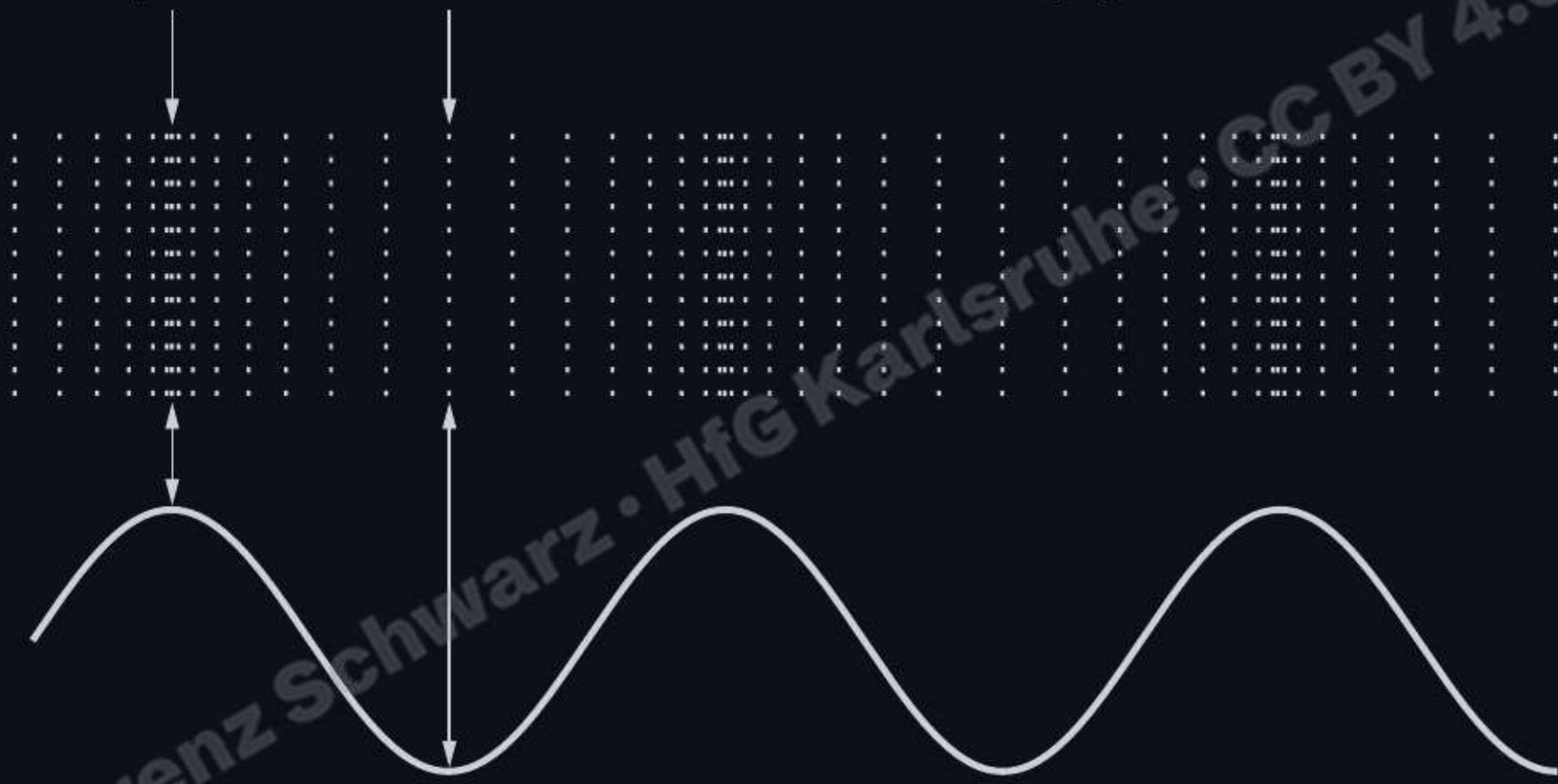
- particles move parallel to the direction of the wave

→ *Longitudinal waves are considered for airborne sound.*

[view in graphing calculator](#)

Compression Rarefaction

Propagation of sound →



Longitudinal waves are also called compression waves.

Quantifying sound in space

- **Field quantities** (at a point in space):
 - Sound pressure (Pa) — pressure deviation from ambient atmospheric pressure
 - Particle velocity (m/s) — velocity of particle oscillation around equilibrium
- **Energy quantities** (rate of energy transfer):
 - Intensity (W/m²) — energy flow per unit area
 - Power (W) — total energy radiated from source

→ *Impedance (Pa·s/m) links pressure and velocity as their ratio.*

Sound pressure p (sound field quantity)

Sound pressure is a property of the sound field at a specific point in space.

It represents variations in air pressure (local compressions and rarefactions) caused by sound waves, typically measured with a microphone, relative to the ambient (static) atmospheric pressure.

$$p_{total} = p_{stat} + p$$

- p = time-varying pressure
- p_{stat} = static pressure
- p in pascals (Pa) = N/m²

Sound pressure level (SPL)

Sound pressure level (L_p) expresses sound pressure on a logarithmic scale in decibels:

$$L_p = 20 \log_{10} \left(\frac{p}{p_0} \right) \text{ dB SPL}$$

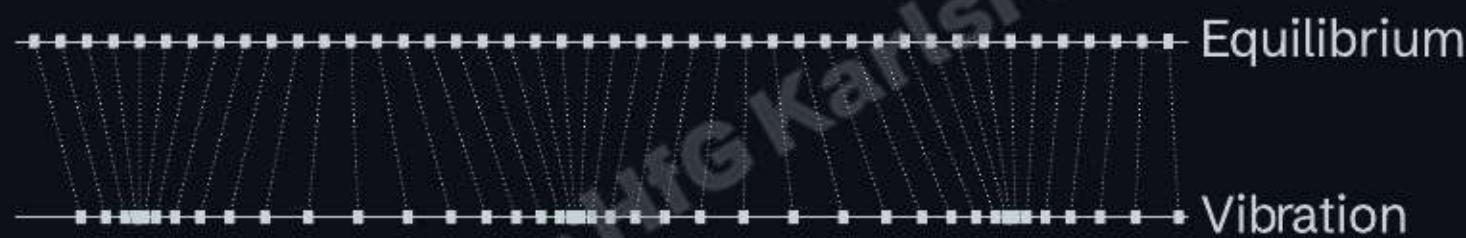
p — measured sound pressure (Pa)

p_0 — reference sound pressure

- Reference: $p_0 = 20 \mu\text{Pa} = 0 \text{ dB SPL}$ (threshold of human hearing at 1 kHz)
- Pain threshold: $p \approx 20\text{--}60 \text{ Pa} \approx 120\text{--}130 \text{ dB SPL}$

Particle velocity v

Particle velocity is the speed of the particles vibrating around their rest position (equilibrium).



→ *Particle velocity must not be confused with the speed of sound.*

Sound power

Sound is a form of energy:

- Property of the sound source, equal to the total power emitted by that source in all directions.

→ *Sound power is neither dependent on room nor distance*

Sound intensity I (sound energy quantity)

- Sound intensity is acoustical power per unit area (W/m^2).
- Sound intensity level (SIL) is its logarithmic representation (dB).

$$L_I = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

Reference sound intensity for the auditory threshold (at 1000Hz):

$$I_0 = 10^{-12} \text{W/m}^2$$

Impedance Z

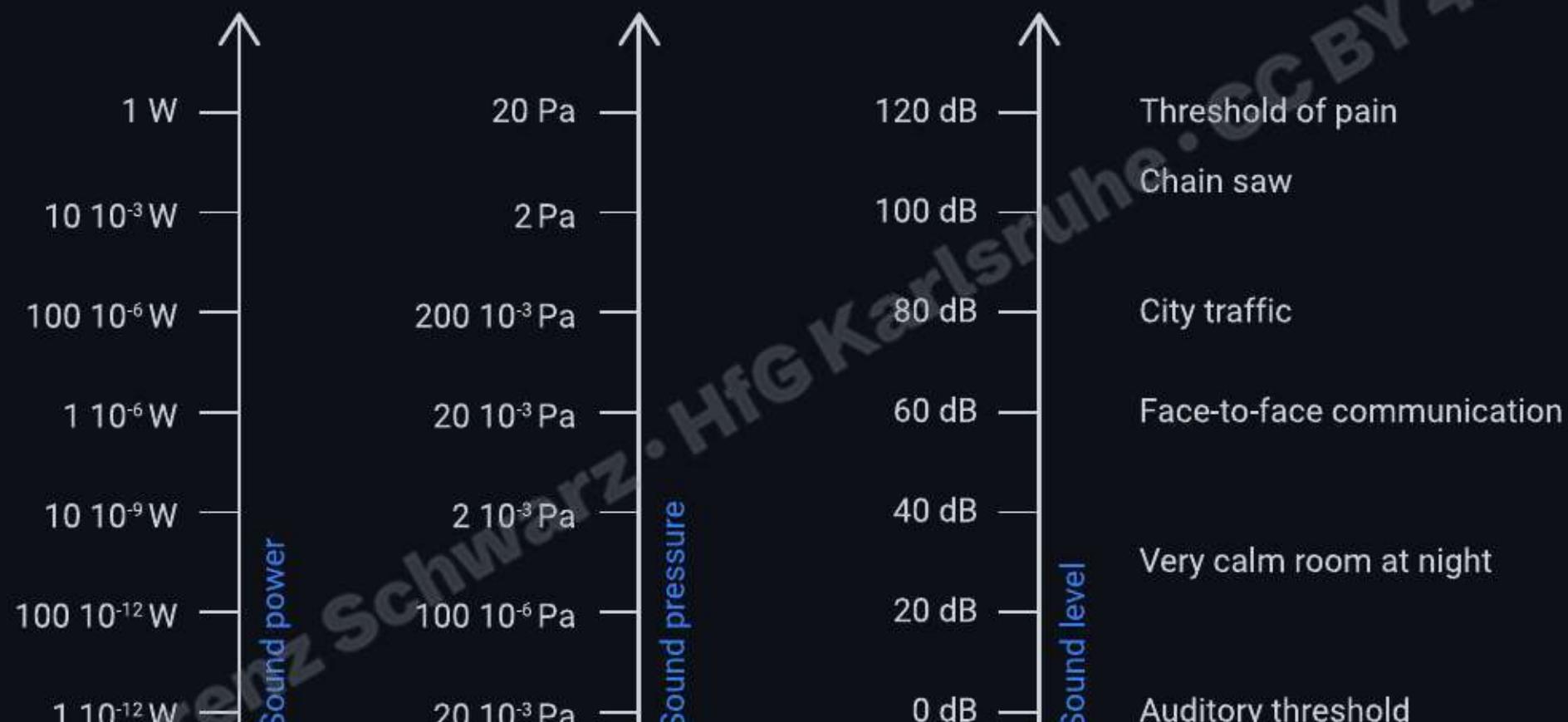
Acoustic Impedance (Z) is the ratio of sound pressure (p) to particle velocity (v) in a sound wave:

$$Z = \frac{p}{v}$$

Specific Acoustic Impedance (Z_0):

For a plane wave or in the far field, the specific acoustic impedance for air at standard temperature and pressure is approximated as:

$$Z_0 \approx 413 \text{ Pa}\cdot\text{s}/\text{m} \approx 413 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$



Sound intensity and sound pressure level

Sound pressure level L_p :

$$L_p = 20 \log_{10} \left(\frac{p}{p_0} \right)$$

Sound intensity level L_I :

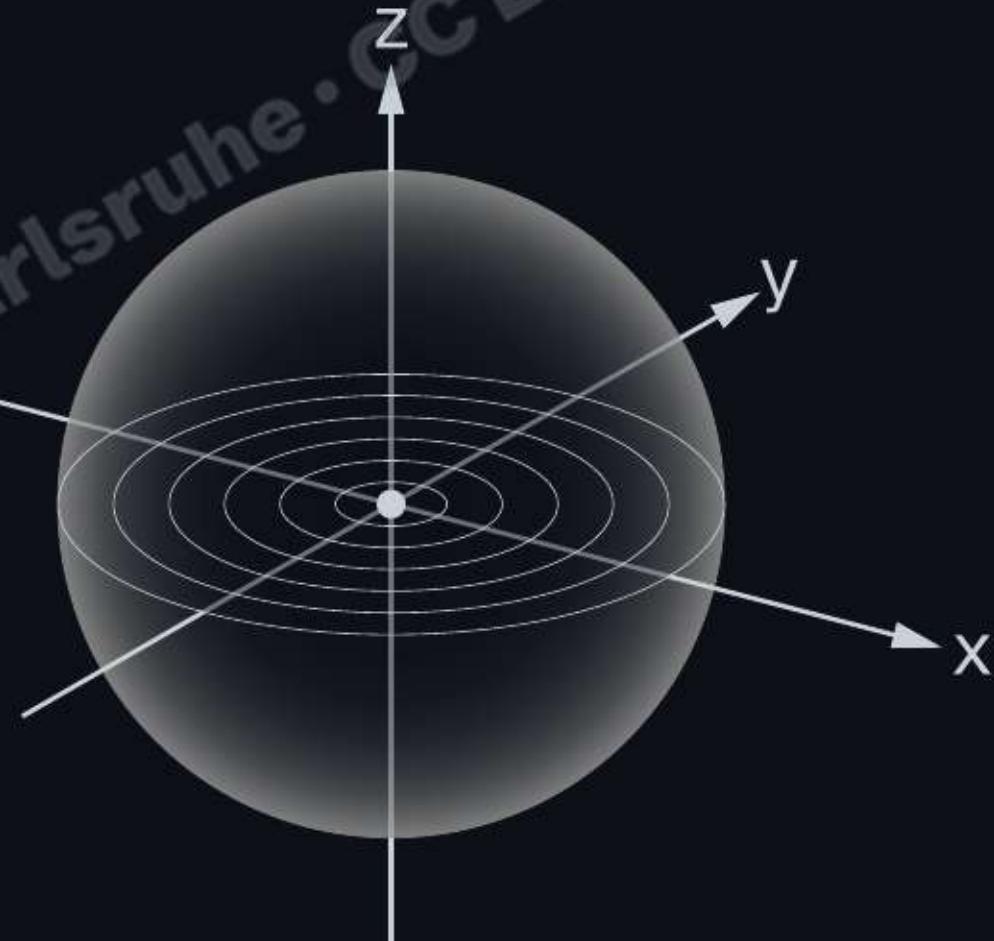
$$L_I = 10 \log_{10} \left(\frac{I}{I_0} \right)$$

→ Human hearing primarily responds to sound pressure.

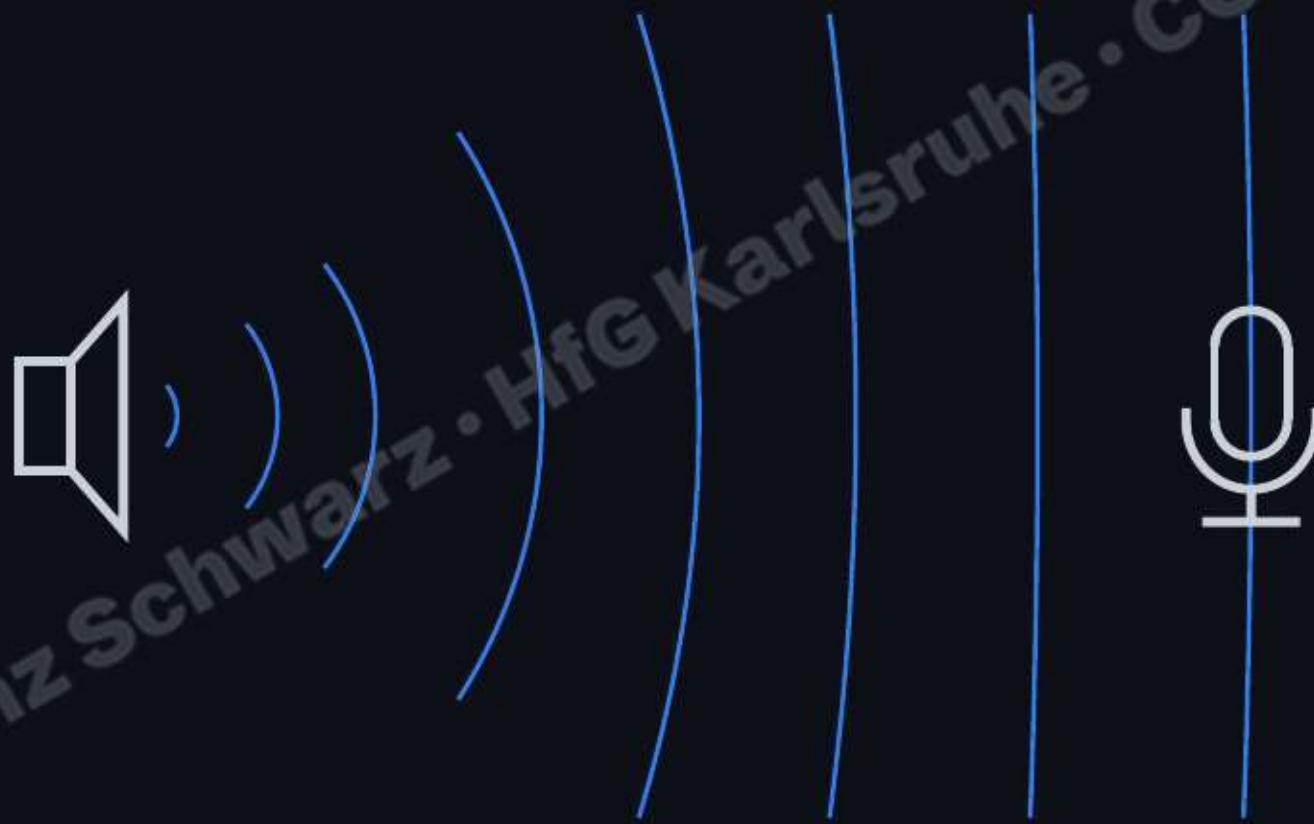
Spatial behavior

An idealized wave that radiates uniformly in all directions from a single point source in 3D space and attenuates with distance:

- The acoustic field variables depend only on the radial coordinate (r) and time (t).
- Surface area of spherical wavefront:
$$A = 4\pi r^2$$



Wave propagation: spherical (short distance), plane (long distance)



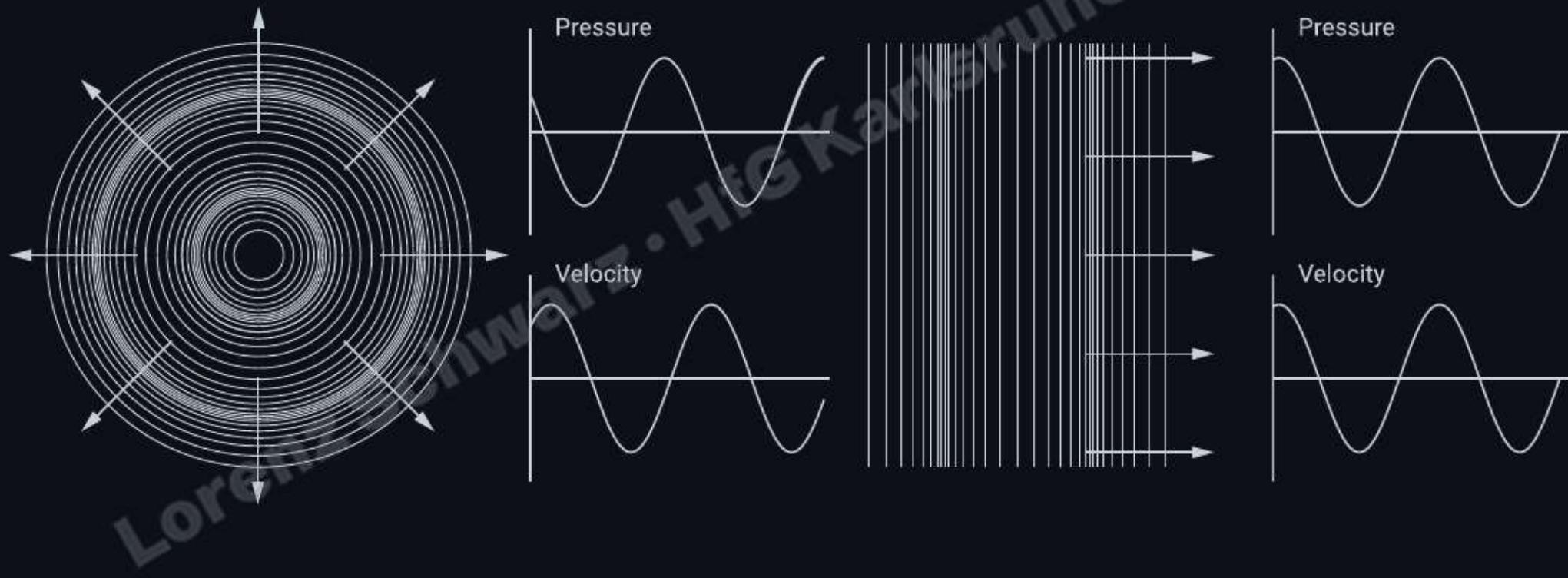
Acoustic fields and their properties

- **Near Field:** A property of the source; the region close to the sound source where the sound pressure and particle velocity are not proportional (non-linear behavior).
- **Far Field:** A property of the source; the region farther from the sound source where sound waves are proportional to the inverse of the distance (linear behavior).
- **Free Field:** A property of the environment; sound propagates without reflections or obstructions.
- **Diffuse Field:** A property of the environment; sound energy is uniformly distributed due to multiple reflections.

Near and far field

- Near field:
 - For point sources, the near field is often approximated as $r < \lambda$
 - Particle velocity shows strong deviations in the near field.
 - (Where λ = wavelength, distance over which wave repeats)
- Far field:
 - Ratio of sound pressure and particle velocity is constant (in phase).
 - The curvature of the wavefront becomes plane.
 - Sound pressure approximately follows inverse-distance behavior

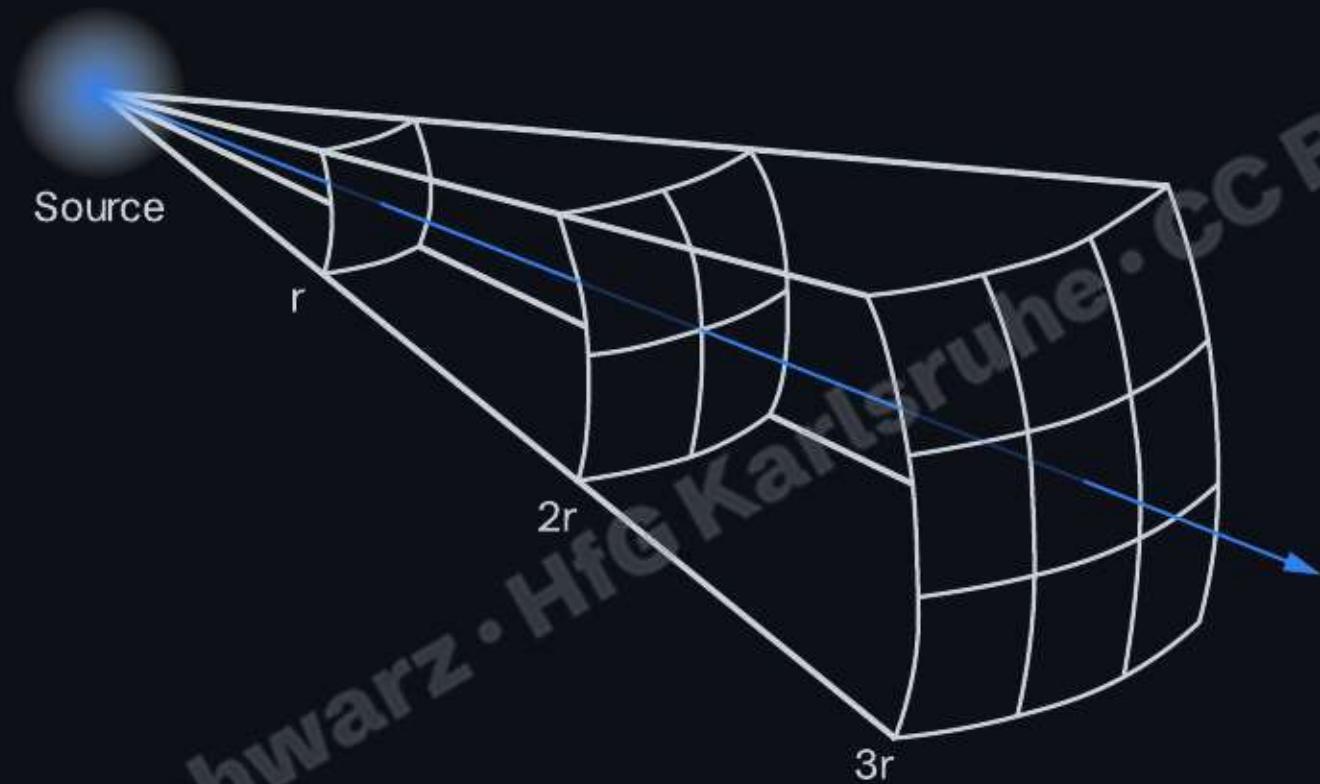
Spherical wave and plane wave



Free field

Region where sound propagates without any interference from reflective surfaces, obstacles, or boundaries, resulting in no reverberation or echo (only direct sound).

→ *Sound is attenuated according to the inverse-square law.*



Sound gets weaker as the distance from the sound source increases.
(Doubling the radius increases the surface area of a spherical wavefront by a factor of four.)

Sound propagation with distance

In free-field conditions, sound level decreases as sound energy spreads over a larger area with distance.

- **Sound intensity:** Doubling distance decreases the level by about 6 dB (intensity level)

$$I \propto \frac{1}{r^2}$$

- **Sound pressure:** Doubling distance decreases the level by about 6 dB SPL

$$p \propto \frac{1}{r}$$

→ Both intensity level and sound pressure level drop by about 6 dB per distance doubling

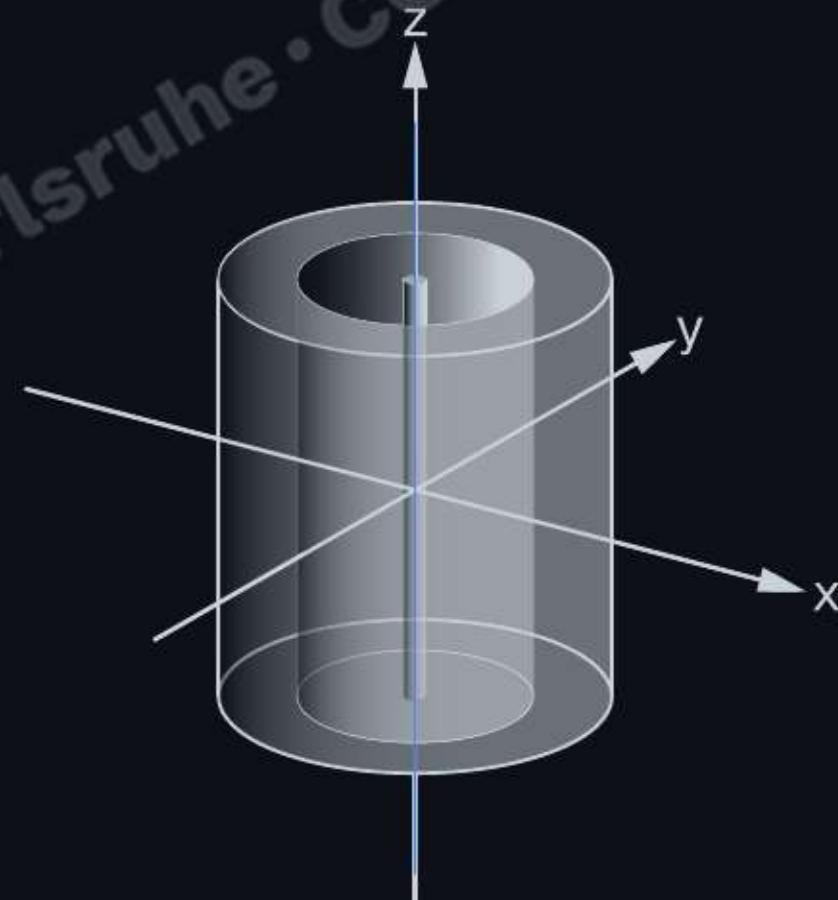
Line source

Cylindrical wavefront radiating from a one-dimensional line source (no vertical dispersion).

$$A_{cylinder} = 2\pi r h$$

Line source attenuates with the inverse of distance ($1/r$), which is a decrease of approximately -3 dB

Applications: Sound reinforcement situations (as much energy as possible for the audience, e.g., line arrays)



Copyright and Licensing

Original content: © 2025 Lorenz Schwarz

Licensed under [CC BY 4.0](https://creativecommons.org/licenses/by/4.0/). Attribution required for all reuse.

Includes: text, diagrams, illustrations, photos, videos, and audio.

Third-party materials: Copyright respective owners, educational use.

Contact: lschwarz@hfg-karlsruhe.de

[← Chapters](#)