Lecture 4 — 12 Jan, 2023

Percentages: of 100

One percent means $\frac{1}{100}$

50% means 50 per 100

100% means $\frac{100}{100}$ or 1

200% is $\frac{200}{100}$ or 2

eg: 25% of 80.

Soln: 25% of 80 = $\frac{25}{100}$ of 80

$= \frac{25}{100} \times 80$ (5 and 4)

$= 5 \times 4$

$= 20$

$\therefore$ 25% of 80 $= 20$.

eg: Restaurant : Menu Card 2022

Tea – 20 (Darshan will solve)

Burger – 45

Masala Dosa – 75

Idli Wada – 50

} U will solve it yourself later

In 2023, the owner wants to hike all the prices by 10%.. Create a new menu card for him.

Soln:

way1: Tea = 20 — (1)

10% of 20 $= \frac{10}{100} \times 20 = 2$ Rs. — (2)

New Tea Price = Old Price + 10% of Old Price

$= (1) + (2)$

$= 20 + 2$

$=$ Rs. 22

Way 2:

New price = old price + 10% of old price

Lets say old price = x

$\therefore$ New price = x + 10% of x

$$= x + \frac{10}{100} \text{ of } x$$

$$= x + \frac{10}{100} \times x$$

$$= x + 0.1x$$

$$= 1x + 0.1x$$

$$= (1 + 0.1)x$$

$\therefore$ **New Price = $1.1x$**

New menu:

Item	Old Price (x)	New Price ($1.1 \times x$)
Tea	20	$1.1 \times 20 = 22$
Burger	45	⋮
Masala Dosa	75	⋮
Idli Vada	50	⋮

Trick: $x\%$ of $y = y\%$ of x

Lets prove it.

$$\underbrace{10\% \text{ of } 20}_{\text{LHS}} = \underbrace{20\% \text{ of } 10}_{\text{RHS}}$$

LHS: 10% of $20 = \frac{10}{100} \times 20 = 2$

RHS: 20% of $10 = \frac{20}{100} \times 10 = 2$

$\therefore$ LHS = RHS

Hence, proved.

Percentage Difference: The difference between two values divided by the average of the two values. (shown as a percentage)

eg: Arun sold 15 tickets & Sunil sold 25.

Soln: ① Difference b/w 25 & 15 = 25 − 15 = 10

② Average $= \frac{25+15}{2} = \frac{40}{2} = 20$

③ $\left[\frac{10}{20} \times 100\right]\% = 50\%$

∴ The percentage diff b/w 25 & 15 is 50%.

Summarize the calculation: $\frac{(25-15)}{\left(\frac{25+15}{2}\right)} \times 100\% = 50\%$

When should we use it?

- Use it when both values mean the same kind of thing (ex: weights of 2 ppl)

Note:

1. But if there is an old value & a new value, we should use Percentage Change
2. If there is an approximate value & an exact value, we should use Percentage Error.

Can the difference be -ve?

eg: Arun works 6 hrs & Sunil works 9 hrs.

Soln:

$$\frac{(6-9)}{\left(\frac{6+9}{2}\right)} \times 100\% = \frac{-3}{7.5} \times 100\% = -40\%.$$

Note: Ignore minus sign

Percentage Change:

Intro:

1. Subtract old value from new value.
2. Divide result of Step1 by old value.
3. Convert it to %age.

Note: If the new value is > old value, it is %age increase, else it is decrease.

Mtd 2.

1. Divide new val by old val.
2. Convert it to %age.
3. Subtract 100% from that.

eg: A pair of shoes went from $5 to $6, what is the percentage change?

Soln: Mtd 1: 1. $6 - 5 = 1$

2. $\frac{1}{5} = 0.2$

3. $0.2 \times 100 = 20\%$ rise.

Mtd 2: 1. $\frac{6}{5} = 1.2$

2. $1.2 \times 100 = 120\%$.

3. $120 - 100 = 20\%$ rise.

Mtd 1: Formula: $\frac{\text{New\_val} - \text{Old\_val}}{\text{old val}} \times 100\%$.

H/w: Rahul Raj's salary increased from 65,000 Rs to 83,100 Rs. Calculate the percent change?

Percentage Error:

Steps: 1. Calculate error. (subtract one value from other), ignore any minus sign.

2. Divide the error by exact value.

3. Convert that to %age.

Formula: $\frac{|\text{Approx\_val} - \text{Exact\_val}|}{|\text{Exact\_Val}|} \times 100\%$

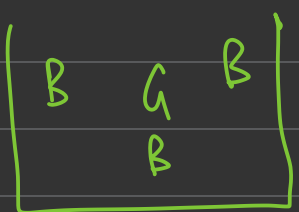
eg: I thought 100 people wud turn up for my class today, but only 60 came. Calc. how much %. age my prediction was wrong by?

Soln: $\frac{|100-60|}{|60|} \times 100\% = \frac{40}{60} \times 100\%$

$= 66.666\%$

I was in an error by 66.66%.

Ratio : It says how much of one thing there is compared to another thing.



$B : G = 3 : 1$

$= \frac{3}{1}$

$= 3$ to 1.

$\frac{3}{1}$ can be scaled up to any level.

$$\frac{3}{1} = \frac{6}{2} = \frac{30}{10} = \frac{75}{25}$$

Above are 'Part-to-Part' ratios

Let see, 'Part to Whole' ratios.

eg: There are 5 pups, 2 male & 3 females

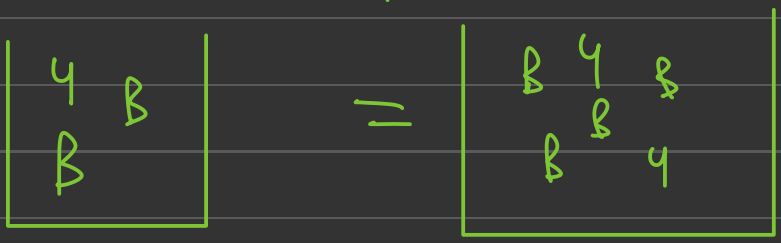
Soln: Part to Part: $\frac{M}{F} = \frac{2}{3} = 2:3$

& $\frac{F}{M} = \frac{3}{2} = 3:2$

Part to Whole: $\frac{M}{\text{All pups}} = \frac{2}{5} = 2:5$

$$\frac{F}{\text{All pups}} = \frac{3}{5} = 3:5$$

Proportions: It says that two ratios (or fractions) are equal.



$$\frac{4:B}{\frac{1}{3}} = \frac{4:B}{\frac{2}{6}}$$

∴ The ratios are same, they are in proportion.

eg: What is 25% of 160?

Soln: We want to find x.

$$\frac{x}{160} = \frac{25}{100}$$

$$x = \frac{25}{100} \times 160$$

$$= \frac{400}{10} = 40$$

eg: There are 12 bucket of stones in a mixer. How much cement & how much sand should add to mk it 1:2:6 mixture?

Soln:

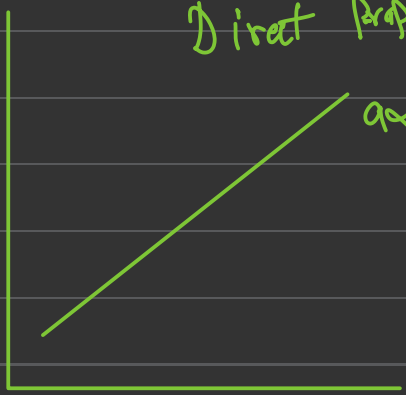
1 : 2 : 6

×2 ×2 ×2

2 : 4 : 12

Types:

Direct Prop.



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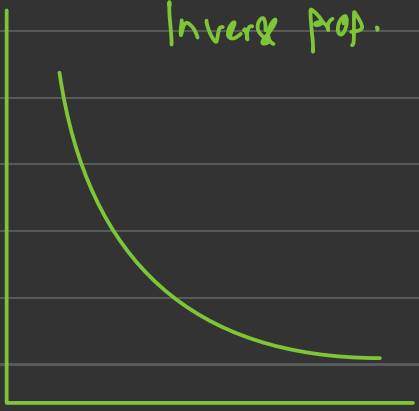
as a increases, b also increases at same rate

eg: You are paid Rs. 1000 an hour.

→ How much u earn is directly prop to how many

Inverse prop.



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when one value decreases at the same rate that other increases,

eg: Speed & travel time is inversely proportional.

$$\text{Speed} \propto \frac{1}{\text{Travel time}}$$

hrs a work.

earning of hrs wkd

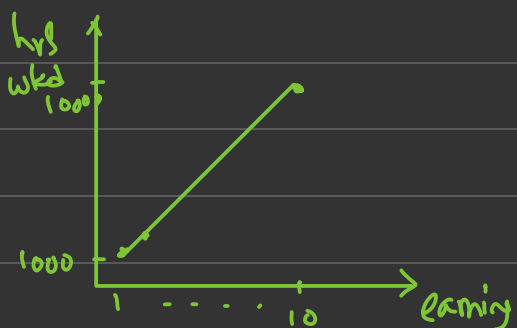
1 → 1000

2 → 2000

3 → 3000

⋮

10 → 10000



Mixture & Alligation:

Mixture — When two or more elements are mixed in a certain ratio

Alligation — The rule which enables us to find the ratio in which two or more elements are mixed together.

Formula: ALLIGATION RULE:

$$\frac{\text{Qty of Cheaper}}{\text{Qty of Dearer}} = \frac{\text{CP of Dearer} - \text{Mean Price}}{\text{Mean price} - \text{CP of Cheaper}}$$

CP of Cheaper Qty — CP of Dearer Qty

Mean Price

CP of Cheaper Qty – Mean price — Mean price – CP of Cheaper Qty

Q1. A grocer wishes to sell a mixture of two variety of pulses worth Rs. 16 per kg. In what ratio must he mix the pulses to reach this selling price, when the cost of one variety of pulses is Rs. 14 per kg & other is Rs. 24 per kg?

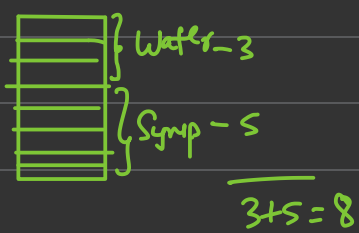
Soln:

$$\frac{\text{Qty of Dearer}}{\text{Qty of Cheaper}} = \frac{24-16}{16-14} = \frac{8}{2} = \frac{4}{1}$$

i.e. 4:1

eg: A vessel is filled with liquid, 3 parts of which are water & 5 parts syrup. How much of mixture must be drawn off & replaced with water so that the mixture may be half water & half syrup?

Soln:



Let initially vessel hv 8 litre
& x litre of this liquid is replaced with water
in new mixture then

(1) qty of water in new mixture $= 3 - \frac{3x}{8} + 8$

(2) Qty of Syrup in new mixture $= 5 - \frac{5x}{8}$

After replacement, Qty of water = Qty of Syrup

$$\therefore \quad 3 - \frac{3x}{8} + x = 5 - \frac{5x}{8}$$

$$\therefore \frac{5x}{8} + \frac{-3x}{8} + x = 2$$

$$\therefore \frac{5x - 3x + 8x}{8} = 2$$

$$\frac{10x}{8} = 2$$

$$\therefore x = \frac{2 \times 8}{10}$$

$$= \frac{16}{10}$$

$$= \frac{8}{5}$$