Lecture 3

10 Jan 2023

Greatest Common Factor (GCF)

The highest no. that divides exactly into two or more nos

eg: GCF of 12 & 16.

Soln:

Factors of	
12	1, 2, 3, 4, 6, 12
16	1, 2, 4, 8, 16

4 is Greatest Common Factor.

$6 = 2 \times 3$

↑ factor ↗

e.g: Common Factors of 15, 30, 105.

Factors of	
15	1, 3, 5, 15
30	1, 2, 3, 5, 6, 10, 15, 30
105	1, 3, 5, 7, 15, 21, 35, 105

15 is GCF

Real Life example:

Suppose u hv 15 apples & 40 bananas. You want to distribute them equally among orphans.
How many orphans can u help?

Soln: GCF/HCF/GCD of 15 & 40 i.e. 5.

LCM (Least common Multiple)

The smallest +ve no. that is a multiple of two or more nos.

eg: LCM of 3 & 5.

Soln:

Multiples of	
3	3, 6, 9, 12, 15, 18, ...
5	5, 10, 15, 20, 25, 30, ...

Find the first common value.

∴ LCM = 15

Real Life Example: Suppose u and ur gf's classes are at hand, & the bells of each other's classes get heard by you both. Your bell rings after some particular intervals say 40 min & ur gf's bell also rings after some different interval, say 60 min. Now, you both can meet each other only if the bells of both of your classes are rung simultaneously. When will u meet?

Soln: LCM of 60 & 40 → 120 min

Q. GCD of 1.08, 0.36, 0.9 ?

Soln: 1.08 , 0.36 , 0.9

$$\frac{108}{100}, \frac{36}{100}, \frac{90}{100}$$

Mtd 1:

2	108
2	54
3	27
3	9
3	3
	1

$2 \times 2 \times 3 \times 3 \times 3$

$= 2^2 \times 3^3$

2	36
2	18
3	9
3	3
	1

$36 = 2 \times 2 \times 3 \times 3$

$= 2^2 \times 3^2$

2	90
3	45
3	15
5	5
	1

$90 = 2 \times 3 \times 3 \times 5$

$= 2 \times 3^2 \times 5$

$108 = 2^2 \times 3^3$

$36 = 2^2 \times 3^2$

$90 = 2 \times 3^2 \times 5$

Common factors are: $2 \times 3^2 = 2 \times 9 = 18$ i.e $\frac{18}{100}$

∴ GCD of 1.08, 0.36 & 0.9 is 0.18

Q. LCM of 4, 6 & 9.

Soln: 3 Methods

Mtd1: By Division mtd.

2	4	6	9
2	2	3	9
3	1	3	9
3	1	1	3
	1	1	1

∴ LCM = $2 \times 2 \times 3 \times 3$

$= 36.$

Mtd2: By Prime Factorization.

Prime factors of	
4	$2 \times 2 = 2^2$
6	$2 \times 3 = 2^1 \times 3^1$
9	$3 \times 3 = 3^2$

LCM can be obtained by multiplying prime factors raised to their respective highest power i.e. $2^2 \times 3^2$

$= 4 \times 9$

$= 36.$

Mtd3: By Listing Multiples.

Multiples of	32,
4	4, 8, 12, 16, 20, 24, 28, (36), 40, ..
6	6, 12, 18, 24, 30, (36), 42, ...
9	9, 18, 27, (36), 45, ...

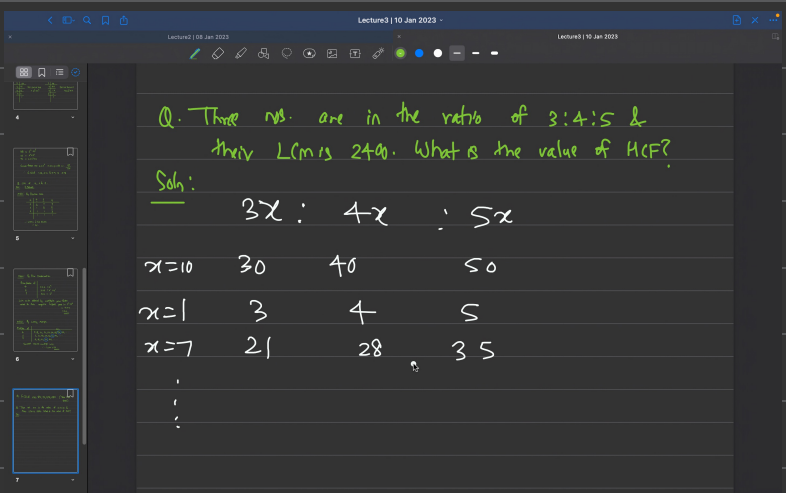
Smallest common multiple $= 36$

$\therefore$ LCM $= 36.$

Q. GCD of 600, 830, 90, 450, 280. (Take Home task)

Q. Three nos are in the ratio of 3:4:5 & their LCM is 2400. What is the value of HCF?

Soln:



Let us consider the number be $3x$, $4x$ & $5x$.

$\therefore$ LCM of 3 nos $= 60x$

$\because$ LCM is given as 2400

$\therefore \quad 60x = 2400$

$\therefore \boxed{x = 40}$

Then the number is:

$3x: \quad 3 \times 40 = 120$

$4x: \quad 4 \times 40 = 160$

$5x: \quad 5 \times 40 = 200$

$\because$ 40 is common.

$\therefore$ HCF of these 3 nos $= 40$.

eg: The HCF of two nos is 11 & LCM is 7700. If one of the nos. is 275, then what is the other number?

Soln: Product of two nos = Product of their HCF & LCM

Let the other number be x

$275 \times x = 11 \times 7700$

$\therefore \boxed{x = 308}$

eg: Let N be the greatest number that will divide 1305, 4665 & 6905 leaving the same remainder in each case. The sum of digits of N is ?

Soln:

Hint: 3 nos, lets say a, b, c

∴ Greatest no that will divide all 3 of them leaving the same rem. will be GCD of (b−a), (c−b), (c−a).

$= (4665 - 1305), (6905 - 4665), (6905 - 1305)$

$= 3360, 2240, 5600$

So the greatest no. N is the HCF of 3360, 2240, 5600 leaving the same remainder.

Factors of	
3360	$2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7 \times 5$
2240	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 5$
5600	$2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7$

$\therefore$ HCF $= 2\times2\times2\times2\times2\times7\times5 = 1120$

$\therefore N = 1120$

$\therefore$ Sum of digits of $N = 1+1+2+0 = 4$.

X
