Lecture 9

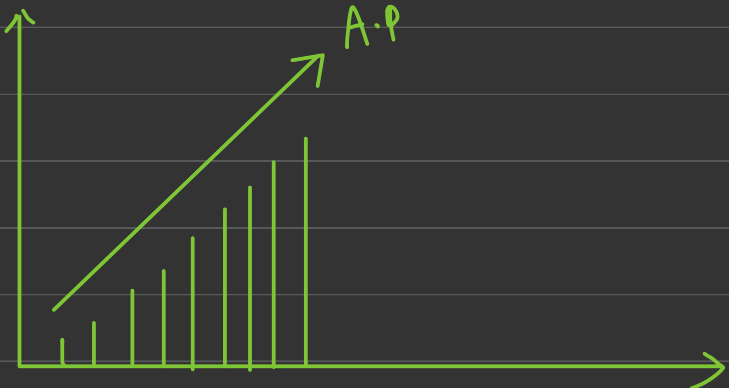
21/Jan/2023

- Arithmetic progression :
  - nth term of an AP = a + (n-1) d
  - Arithmetic Mean = Sum of all terms in the AP / Number of terms in the AP
  - Sum of 'n' terms of an AP = 0.5 n (first term + last term) = 0.5 n [2a + (n-1) d]

AP is a mathematical sequence in which the difference b/w two consecutive terms is always constant.

eg: 1, 4, 7, 10, 13, 16, ..

This is an AP with a common difference of 3



Notation:

First term = a

Common difference = d

n^{th} term = a_n

Sum of first n terms = S_n

eg: 3, 6, 9, 12, 15, - - ... .

(3 is a; the difference between consecutive terms is 3)

$a = 3$

$d = 3$

$a,\ a+d,\ a+d+d,\ a+d+d+d,\ a+d+d+d+d,\ \cdots$

$a,\ a+d,\ a+2d,\ a+3d,\ a+4d,\ \cdots,\ a+(n-1)d$

$2+(10-1)3$

Lets say $a=2,\ d=3,\ n=10$

What is AP?

$2,\ 2+3,\ 2+2(3),\ 2+3(3)+2+4(3)+\cdots,\ 2+9(3)$

$= 2, 5, 8, 11, 14, 17, 20, 23, 26, 29$

Sum of N terms:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Here, $n=5,\ a=2,\ d=3$

$$\therefore S_n = \frac{5}{2}\left[2(2)+(5-1)3\right]$$

$$= 2.5\left[4+4(3)\right]$$

$$= 2.5(16) = 8 \times 5$$

$$= 40$$

Sum of AP when Last term is given:

$$S = \frac{n}{2}(\text{1st term} + \text{last term})$$

eg: Last term = 14

$n = 5$

$a = 2$

$$\therefore S = \frac{5}{2}[2 + 14]$$

$$= \frac{5}{2} \times 16^{8}$$

$$= 40$$

eg: Find 1st term of AP whose 7th & 11th terms are 37 & 57 respectively

$11 - 7 = 4$

$57 - 37 = 20$

$\frac{20}{4} = 5$

$\therefore d = 5$

Soln: n^{th} term $= a + (n-1)d$

7^{th} term $= a + (7-1)d$ i.e. $a + 6d$ — (1)

11^{th} term $= a + (11-1)d$ i.e. $a + 10d$ — (2)

$\because 7^{th}$ term $= 37$ (given)

$\therefore a + 6d = 37$ — (3)

$a + 6d = 37$

$a = 37 - 6(5)$

$= 7$

$\because 11^{th}$ term $= 57$ (given)

$\therefore a + 10d = 57$ — (4)

(3) − (4)

$a + 6d = 37$

$-\quad a + 10d = 57$

$-4d = -20$

$d = \frac{20}{4}$

$\boxed{d = 5}$

$\therefore a+6d=37$

$\therefore a = 37-6d$

$= 37-6(5)$

$= 37-30$

$\therefore \boxed{a = 7}$ (1st term)

eg: How many natural nos b/w 200 & 500 which r multiples of 3?

Soln:

$201, 204, 207, 210, \ldots, 498$

$d=3 \quad a=201$

Total terms $= \dfrac{(\text{last term} - \text{first term})}{\text{diff}} + 1$

$= \dfrac{498-201}{3} + 1$

$= \dfrac{297}{3} + 1$

$= 99+1$

$= 100$

- **Geometric progression :**
  - nth term of a GP = a rn-1
  - Geometric Mean = nth root of product of n terms in the GP
  - Sum of 'n' terms of a GP (r < 1) = [a (1 – rn)] / [1 – r]
  - Sum of 'n' terms of a GP (r > 1) = [a (rn – 1)] / [r – 1]
  - Sum of infinite terms of a GP (r < 1) = (a) / (1 – r)

G.P. is a type of sequence whr each succeeding term is produced by multiplying each preceding term by a fixed no, which is called as common ratio.

Eg: 2, 4, 8, 16, 32, 64, ..., whr common ratio is 2.

If common ratio is r & 1st term is a.

GP is:

$a \cdot r^0, ar^1, ar^2, ar^3, ar^4, ar^5, \ldots$

$a=2, r=2$

$2 \cdot 2^0, 2 \cdot 2^1, 2 \cdot 2^2, 2 \cdot 2^3, 2 \cdot 2^4, 2 \cdot 2^5, \ldots$

$= 2 \cdot 1, 2 \cdot 2, 2 \cdot 4, 2 \cdot 8, 2 \cdot 16, 2 \cdot 32$

$= 2, 4, 8, 16, 32, 64$

Note: $\frac{3rd\ term}{2nd\ term} = \frac{ar^2}{ar^1} = r$

$$\frac{4\text{th term}}{3\text{rd term}} = \frac{ar^3}{ar^2} = r.$$

eg. Find sum of GP: 10, 30, 90, 270, 810 (use formula only)

Soln: $a = 10$

$r = \frac{30}{10} = 3$

$n = 5$

Sum of 'n' terms of a GP (r > 1) = [a (rn – 1)] / [r – 1]

Sum of GP $= a[(r^n - 1)/(r-1)]$

$= 10\left[(3^5 - 1)/(3-1)\right]$

$= 10\left[(243-1)/2\right]$

$= 10 \times \frac{242}{2}$

$= 10 \times 121$

$= 1210$

eg: The 8th term of a GP is 16 times the 4th term. What will be the first term when its 6th term is 64

Soln:

nth term of a GP = $a r^{n-1}$

n^{th} term $= a \cdot r^{n-1}$

8^{th} term $= a \cdot r^{7}$

4^{th} term $= a \cdot r^{3}$

$\because$ 8th term is 16 times 4th term

$$a \cdot r^{7} = 16 \cdot (a \cdot r^{3})$$

$$r^{4} = 16$$

$$\therefore r = \sqrt[4]{16}$$

$$\therefore \boxed{r = 2}$$

$\because$ 6^{th} term $= 64$

$$a \cdot r^{6-1} = 64$$

$$a r^{5} = 64$$

$$a \cdot 2^{5} = 64$$

$$\therefore a = \frac{64}{32}$$

$$\therefore \boxed{a = 2}$$ 1st term.

Completing the Number Series:

eg: 4, 18, x, 100, 180, 294, 448

Kartik's Solution:

Soln: $2^3 - 2^2 = 8 - 4 = 4$ $\quad 1 \times 2 \times 2 = 4$

$3^3 - 3^2 = 27 - 9 = 18$ $\quad 2 \times 3 \times 3 = 18$

$4^3 - 4^2 = 64 - 16 = 48$ $\quad 3 \times 4 \times 4 = 48$

$5^3 - 5^2 = 125 - 25 = 100$ $\quad 4 \times 5 \times 5 = 100$

$5 \times 6 \times 6 = 180$

$6 \times 7 \times 7 = 294$

eg: Find out the wrong no. in the series given:

1, 2, 6, 15, 31, 56, 91.

Soln: 91 is wrong

$1 + 1^2 = 2$

$2 + 2^2 = 6$

$6 + 3^2 = 15$

$15 + 4^2 = 31$

$31 + 5^2 = 56$

$56 + 6^2 = 92$

$2 - 1 = 1 = 1^2$

$6 - 2 = 4 = 2^2$

$15 - 6 = 9 = 3^2$

$31 - 15 = 16 = 4^2$

$56 - 31 = 25 = 5^2$

$91 - 56 = 35 \neq 6^2$

$\therefore$ 91 is wrong

H/W: Find wrong no

(a) 1, 2, 8, 33, 148, 760, 4626

(b) 7, 28, 63, 124, 215

Find the next no. in the given sequence.

11, 17, 39, 85, ?