Lecture 11

24 Jan, 2023

Permutation & Combinations:

$$^nP_r = \frac{n!}{(n-r)!}$$

Permutation: It is an act of arranging objects/nos in order.

Combination: They are the way of selecting objects/nos from a group of objects or collections, in such a way that the order of the objects does not matter.

$$^nC_r = \frac{n!}{r! \cdot (n-r)!}$$

eg: My fruit salad is a combn. of apple, chickoo, banana. → Combination (order doesn't matter)

eg: The combn. to my locker access is 462. Now we do care abt the order. → Permutation (order matters)

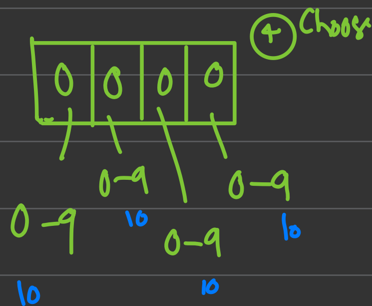
Permutation (2 Types)

(1) Repetition is allowed: Cycle Lock PIN: 2222

(2) No Repetition: The first three ppl in a running race.
You cant be be first & second.

When a thing has 'n' different types ....we hv n choices each time.

eg: In cycle lock,



10 X 10 X 10 X 10 = 10,000 permutations

Formula: n^r $= 10^4 = 10{,}000$.

where n = no. of things to choose from,
& we choose r of them.
repetition is allowed &
order matters.

② **Permutation w/o Repetition:** In this case, we hv to reduce the no. of available choices each time.

eg: What order could 16 pool balls be in?

Soln: $16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Evaluate: $\dfrac{n!}{(n-r)!}$

eg:

(i) $n=9,\ r=5$ $\dfrac{9!}{(9-5)!}$

$$= \frac{9!}{4!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!}$$

$$= 9 \times 8 \times 7 \times 6 \times 5$$

$$= 15120$$

eg: In how many ways a committe consisting of 5 men & 6 women can be formed from 8 men & 10 women?

Soln:

$$^{8}C_5 \times {}^{10}C_6$$

$$= \frac{8!}{5!\,(8-5)!} \times \frac{10!}{6!\,(10-6)!}$$

$$= \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \times 3!} \times \frac{10 \times 9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!} \times 4!}$$

$$= \frac{10 \times \overset{3}{\cancel{9}} \times \cancel{8} \times 7 \times 8 \times 7 \times \cancel{6}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 10 \times 3 \times 7 \times 8 \times 7$$

$$= 11760.$$

Distribution of Identical Objects :

While distributing identical object, it does not matter which object is given to which person, what matter that how many objects are given.

Given two integers N & R, The task is to calculate the no. of ways to distribute N identical objects into R distinct groups.

Let us suppose that x_1 objects are placed in first group,
x_2 objects are placed in second group,
& x_R objects are placed in R^{th} group.

$$x_1 + x_2 + x_3 + \dots + x_R = N$$

The solution of this eqn:

$$^{N+R-1}C_{R-1}$$

Division & Distribution of identical or distinct objects :

5 different balls

↓

Divide into 2 groups

G1 | G2
2 | 3

↳ ∴ # of ways of selecting 2 balls out of 5 is $^5C_2 = \dfrac{5!}{2!\,(5-2)!}$

eg: In how many ways 10 different objects can be divided into 3 groups of group sizes 2, 3 & 5 resp

Soln:

10

G1 G2 G3

2 3 5

$^{10}C_2$ $^{8}C_3$ $^{5}C_5$

$\therefore$ # ways $= {}^{10}C_2 \cdot {}^{8}C_3 \cdot {}^{5}C_5$.

eg: In how many ways can 5 rings of different type be worn in 4 fingers?

Soln: R1, R2, R3, R4, R5

$\Downarrow$

F1, F2, F3, F4

4 ways of wearing it

Each one of the other rings can be worn in 4 ways.

$\therefore$ reqd. no. of ways $= 4 \times 4 \times 4 \times 4 \times 4 = \underline{\underline{4^5}}$.

eg: The total no. of ways in which 5 balls of different colors can be distributed among 3 person so that each person gets atleast one ball.

Sol^n:

5 balls $\longrightarrow$ 3 ppl

$\therefore$ Total # of distributions possible $= 3 \times 3 \times 3 \times 3 \times 3$

$= 3^5$

Of these distributions, there are cases when one person does not get any ball.

$\hookrightarrow$ ${}^3C_1 \times$

I will get back on this.

Let me recheck the Q.

X