Lecture 6

15 Jan 2023

1. When the interest is compounded annually / yearly

$$A = P\left(1+\frac{R}{100}\right)^{n} \qquad CI = A - P$$

2. When Interest is compounded half yearly

$$A = P\left(1+\frac{R}{200}\right)^{2n} \qquad CI = A - P$$

where A = Amt P = Principle R = ROI

n = no. of yrs

eg: $P = 6000$ $R = 9\%$ $n = 2$ yrs

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$= 6000\left(1+\frac{9}{100}\right)^{2}$$

$$= 7128.6$$

$$CI = A - P$$

$$= 7128.6 - 6000$$

$$\boxed{CI = Rs.\ 1128.6}$$

eg: $P = 8000$ $R = 10\%$ per annum $n = 1$ yr

Compounded half yearly.

Soln: $A = P\left(1 + \frac{R}{200}\right)^{2n}$

$= 8000\left(1 + \frac{10}{200}\right)^{2\times 1}$

$= 8820$

$CI = A - P$

$= 8820 - 8000$

$\therefore$ **CI = Rs. 820**

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

Where: A = Final amt / Future value

P = Principle / Original amt

r = Annual Interest rate

n = no. of periods per yer

t = time per year

1. Investing an original $1000 (P) at 5% (r) compounded annually, how much would u hv after 7 years? (t)

Soln: n=1

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$5\% = \frac{5}{100} = 0.05$$

$$= 1000\left(1 + \frac{0.05}{1}\right)^{1\times 7}$$

$$= 1000 \times (1.05)^7$$

$$\therefore A = \$1407.1$$

2. Investing an original $2000 (P) at 12% (r) compounded monthly, how much would you hv after 5 yrs? (t)

Soln: n=12

$$A = P\left(1 + \frac{r}{n}\right)^{tn}$$

$$= 2000\left(1 + \frac{0.12}{12}\right)^{5\times 12}$$

$$= \$3633.39$$

3 Maria put Rs 20000 in a savings a/c paying 8% annual interest compounded monthly. At this rate, how much money will be in her a/c after 10 yrs?

(Rs 20000 = P; 8% = r; monthly: n = 12; 10 yrs = t)

Soln:

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

$$= 20000\left(1 + \frac{0.08}{12}\right)^{12 \times 10}$$

$$A = 44392.8$$

$$\therefore CI = A - P = 44392.8 - 20000 = \text{Rs. } 24392.8$$

4. James wants to have 2,000,000 (A) for retirement in 45 years (t). He invests in a mutual fund paying on average of 9.5% (r) each year compounded quarterly (n = 4). How much should he deposit into his mutual funds?

Soln:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\therefore P = \frac{A}{\left(1+\frac{r}{n}\right)^{t}}$$

H/W

$\therefore$ P =

$CI = A - P$

$\therefore CI = 2000000 -$ $=$.

5. Sarah wishes to turn her 10000 (P) investment into 1,00,000 (A) in 20 yrs. (t) How much interest does she need to receive (r = ?) compounded annually (n = 1) to reach her goal?

Soln:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$1,00,000 = 10,000\left(1+\frac{r}{1}\right)^{1\cdot 20}$$

$$1,00,000 = 10,000\,(1+r)^{20}$$

Divide by 10000 on both sides,

$$\frac{100000}{10000} = \frac{10000}{10000}(1+r)^{20}$$

$$10 = (1+r)^{20}$$

Raise both sides by $\frac{1}{20}$

$$10^{\frac{1}{20}} = \left((1+r)^{20}\right)^{\frac{1}{20}}$$

$$10^{\frac{1}{20}} = 1+r$$

$$\therefore 1+r = 10^{\frac{1}{20}}$$

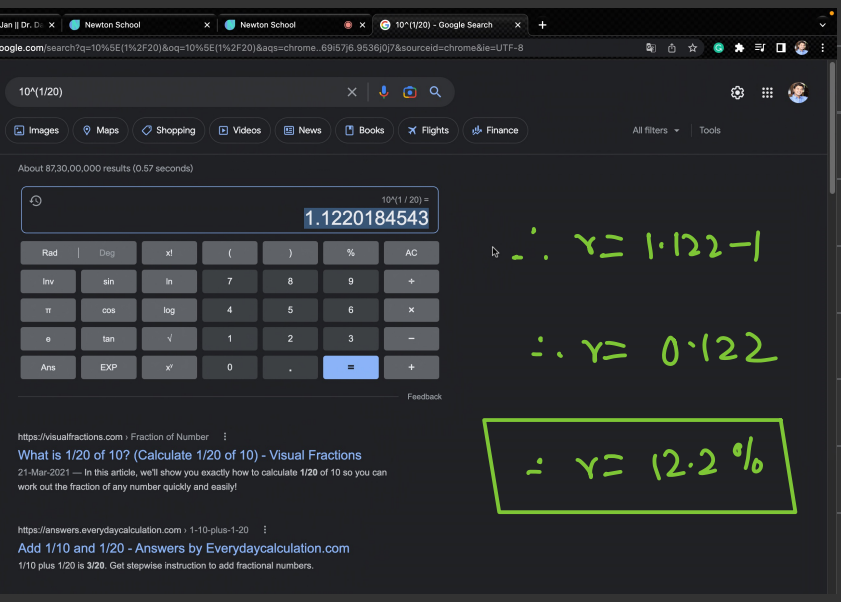
$$\therefore 1+r = 1.122$$

$$2^2 = 4$$

$$2 = \sqrt{4}$$

$$2^3 = 8$$

$$2 = \sqrt[3]{8}$$



$$\therefore r = 1.122 - 1$$

$$\therefore r = 0.122$$

$$\therefore r = 12.2\,\%$$

6. $A = 1{,}000{,}000$ $P = 50{,}000$ $r = 8.4\%$ $n = 2$ $t = ?$

Soln: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$1{,}000{,}000 = 50000\left(1 + \frac{0.084}{2}\right)^{2t}$$

$$\frac{1{,}000{,}000}{50000} = \frac{50000}{50000}\left(1 + \frac{0.084}{2}\right)^{2t}$$

$$20 = 1.042^{2t}$$

Taking log on both sides,

$$\log 20 = \log 1.042^{2t}$$

$$\log 20 = 2t \cdot \log 1.042$$

$$\therefore t = \frac{\log 20}{\log 1.042 \times 2}$$

$$= \frac{1.301}{0.017 \times 2}$$

$$\therefore \quad \boxed{t = 38.26 \text{ yrs}}$$

Time & Work

1. Work Done = Time taken x Rate of Work
2. Rate of work = $1 /$ Time taken

3. If a piece of work is done in x no. of days, then the work done in one day $= \frac{1}{x}$

4. Total Work Done = No. of Days x Efficiency

5. Efficiency $\propto \frac{1}{\text{Time}}$

6. $x : y$ is the ratio of no. of men which r reqd to complete a piece of work, then the ratio of time taken by them to complete the work will be $y : x$.

Q1. A builder appoints 3 construction workers Akash, Lohit, & Rakesh on one of his sites. They take 20, 30 & 60 days resp to do a piece of work. How many days will it take Akash to complete the entire work if he is assisted by Lohit & Rakesh every third day?

Soln:

Total work done by all 3 in one day $= \frac{1}{20} + \frac{1}{30} + \frac{1}{60}$

$$= \frac{3+2+1}{60}$$

$$= \frac{6}{60} = \frac{1}{10}$$

Work done by Akash in 2 day $= 2 \times \frac{1}{20} = \frac{1}{10}$

Work done in 3 days = 2 days of Akash + 1 day of all three

$$= \frac{1}{10} + \frac{1}{10}$$

$$= \frac{2}{10} = \frac{1}{5}.$$

∴ Time taken to complete the work $= 5 \times 3 = 15$ days.

eg: To complete a piece of work, Samir takes 6 days & Tanvir takes 8 days alone resp. Samir & Tanvir took Rs 2400 to do this work. When Aamir joined them, the work was done in 3 days. What amt was paid to Aamir?

Soln:

Total work done by Samir & Tanvir $= \frac{1}{6} + \frac{1}{8} = \frac{4+3}{24}$

$= \frac{7}{24}$

Work done by Aamir in 1 day $= \frac{1}{3} - \frac{7}{24} = \frac{1}{24}$

Amt distributed b/w each of them $= \frac{1}{6} : \frac{1}{8} : \frac{1}{24}$

$= \frac{24}{6} : \frac{24}{8} : \frac{24}{24}$

$= 4 : 3 : 1$

Aamir amt $= \cancel{2400}^{300} \times \frac{1}{\cancel{24}^{8}} \times \cancel{3}^{} = \text{Rs. } 300/-$

eg: Sonal & Preeti started working on a project & they can complete it in 30 days.
Sonal worked for 16 days & Preeti completed the remaining work in 44 days. How many days would Preeti have taken to complete the entire proj. all by herself?

Soln:

Let the work done by Sonal in 1 day $= x$

& —11— Preeti —11— $= y$

$\therefore x + y = \frac{1}{30}$ — (1)

$16x + 44y = 1$ — (2)

Mult. (1) X 16

$16x + 16y = \frac{16}{30}$ — (3)

$-16x + 44y = 1$

$-28y = \left(\frac{16}{30} - 1\right)$

$$\therefore 28y = -\left(\frac{16-30}{30}\right)$$

$$28y = -1\left(\frac{-14}{30}\right)$$

$$\therefore y = \frac{14 \times 2}{30 \times 28 \times 2}$$

$$\therefore \boxed{y = \frac{1}{60}}$$

$$x + y = \frac{1}{30}$$

$$\therefore x = \frac{1}{30} - y$$

$$= \frac{1}{30} - \frac{1}{60}$$

$$= \frac{2-1}{60} = \frac{1}{60}$$

$\therefore$ Preeti can complete the entire work in 60 days

X