# Lecture 1

07 Jan, 2023

- Non-tech backgraud
- Zero pgming knowledge (thats ok)
- Programming is Mathematics & Logic

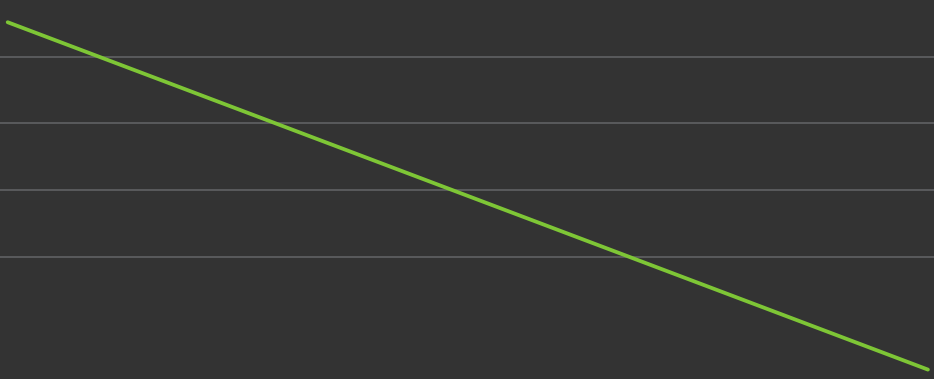
Mathematics: Universal language

Language:

Digits: 0, 1, 2, 3, ... 9

Symbols: +, −, ×, /, ...

Special Symbols: π, =, <, ≥, ...



# Evolution of Nos.

Simplest idea of a number?
→ Something to count with.

$1, 2, 3, 4, \ldots$

## Zero

Placeholder: $40 - 40 = 0$

Whole Nos: $\{0, 1, 2, 3, \ldots\}$

Natural Nos: $\{1, 2, 3, 4, \ldots\}$

Integers: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Fractions: $\{\frac{1}{2}, \frac{4}{3}, \ldots\}$

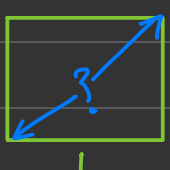
$\frac{p}{q}$

**Rational No**: any no. that can be written as fraction is called a Rational No.

eg: $\frac{3}{2}$

$\left\{ \frac{p}{q}, q \neq 0 \right\}$

Rational nos include: all integers
all fractions.




$\sqrt{1^2+1^2} = \sqrt{2}$

$= 1.414213562373095 0 \ldots$

Sq. root of $2 \neq \frac{p}{q}$ → Irrational

eg: $\sqrt{2}$, $\frac{22}{7}$

**Real Nos:** include – Rational nos
– Irrational nos

**Complex No:** When we put a real & imaginary no together, we get a complex no.

"a+bi"

eg: 3+2i

2–7.103i

**Odd No & Even No.**

| | |
|---|---|
| 1, 3, 5, 7, 9, 11, .... | 0, 2, 4, 6, 8, 10, . |

How to conclude mathematically if a no. is even or odd?

if no%2=0 → Even
if no%2=1 → Odd
} Programming

% gives remainder in Programming.

$$2\overline{)2}\ 1 \quad \text{– Quotient}$$
$$-2$$
$$0 \quad \text{– Remainder}$$

even

$$2\overline{)1382}\ 691$$
$$-2$$
$$18$$
$$-18$$
$$02$$
$$-2$$
$$0$$

even

$$2\overline{)1921}\ 96$$
$$-18$$
$$12$$
$$-12$$
$$01$$

odd

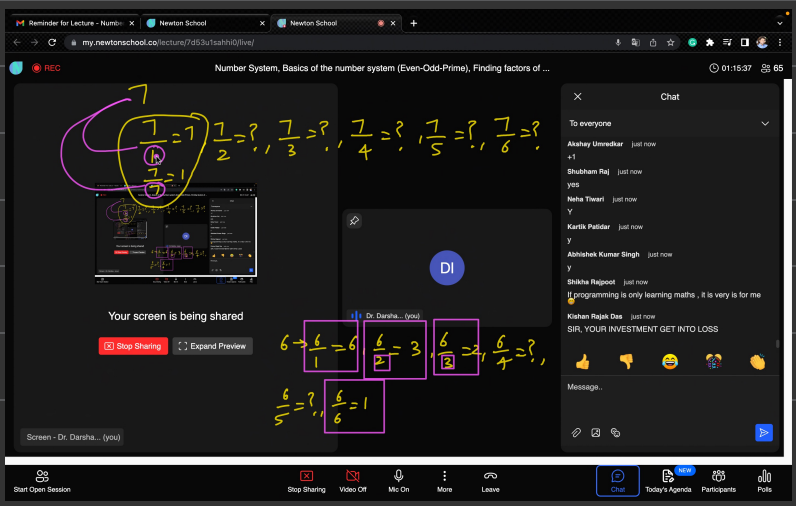
**Prime No.** : A whole no. above 1 that cannot be made by multiplying other whole nos.

eg: 5, 2, 3, 7, 11, ...

No. that is divisible by 1 & itself.

**Composite No:** Any no. which is not prime is Composite.

**Note:** 1 is neither a Prime no nor a Composite no.



# Fractions:

1. Proper fraction : eg: $\frac{3}{8}$, $\frac{1}{4}$, $\frac{4}{5}$

It has Numerator < Denominator

2. Improper : eg: $\frac{9}{5}$, $\frac{4}{3}$, $\frac{7}{7}$

Num $\geq$ Den.

3. Mixed eg: $2\frac{1}{3}$

It is whole no. & proper fraction together

$2\frac{1}{3}$ i.e. $\frac{(3\times2)+1}{3} = \frac{7}{3}$

**Coprime**: When two nos hv no common factors other than 1.

i.e. There is no whole no. that u could divide them both by exactly (w/o any remainder)

eg: 21 & 22 are coprime.

Explain: Factors of 21: 1, 3, 7, 21

Factors of 22: 1, 2, 11, 22

∴ only 1 is common, 21 & 22 r coprime.

Coprimes are also called 'Relatively Prime' or 'Mutually Prime'

**Twin Primes**: A pair of prime nos. that differ by 2 (successive odd nos that r both prime nos.)

eg: (3, 5), (5, 7), (11, 13), ...

**Greatest Common Factor**: The highest no. that divides exactly into 2 or more nos.

eg: Find GCF of 12 & 16

Soln.

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 16: 1, 2, 4, 8, 16

4 is the greatest.

∴ GCF = 4

**Least Common Multiple**: The smallest positive no. that is a multiple of two or more nos.

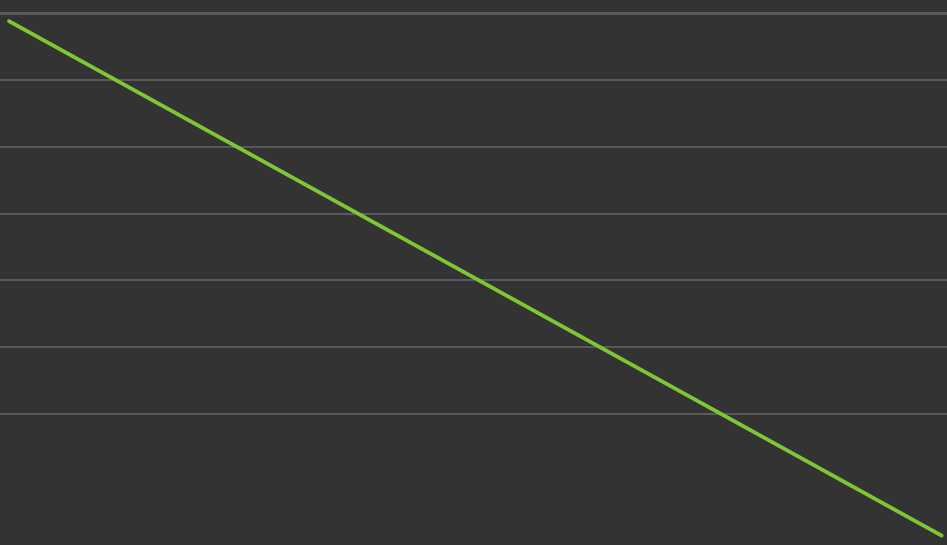
eg: LCM of 3 & 5.

Multiples of 3: 3, 6, 9, 12, 15, 18, ....

Multiples of 5: 5, 10, 15, 20, 25, ..

15 is the first common (same) value.

∴ LCM = 15



# Number System (NS):

1 **Decimal NS:** Nos. to base 10.
Formed using digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

eg: 10, −5, 70, 62, ....

eg: 512 i.e. $(512)_{10}$

$10^2$ $10^1$ $10^0$

$= 5 \times 10^2 + 1 \times 10^1 + 2 \times 10^0$

$= 5 \times 100 + 1 \times 10 + 2 \times 1$

$= 500 + 10 + 2$

$= 512$

eg: 32.12 $= (32.12)_{10}$

$10^1$ $10^0$ $10^{-1}$ $10^{-2}$

10 1 0.1 0.01

$= 3 \times 10 + 2 \times 1 + 1 \times 0.1 + 2 \times 0.01$

$= 30 + 2 + 0.1 + 0.02$

$= 32 + 0.10 + 0.02$

$= 32 + 0.12$

$= 32.12$

## 2. Binary: $\{0, 1\}$

To base 2.

eg: 101, 0101, 1110, 10110, etc.

eg: $(1\ 0\ 1\ 1\ 0)_2$

$2^4\ \ 2^3\ \ 2^2\ \ 2^1\ \ 2^0$

$16\ \ 8\ \ 4\ \ 2\ \ 1$

$= 1\times16 + 0\times8 + 1\times4 + 1\times2 + 0\times1$

$= 16 + 0 + 4 + 2 + 0$

$= (22)_{10}$

| Decimal | 16 | Binary 8 | 4 | 2 | 1 |
|---|---|---|---|---|---|
| 1 | | 0 | 0 | 0 | 1 |
| 2 | | 0 | 0 | 1 | 0 |
| 3 | | 0 | 0 | 1 | 1 |
| 4 | | 0 | 1 | 0 | 0 |
| 5 | | ... 0 | 1 | 0 | 1 |
| 6 | | 0 | 1 | 1 | 0 |
| 7 | | 0 | 1 | 1 | 1 |
| 8 | | 1 | 0 | 0 | 0 |
| 9 | | 1 | 0 | 0 | 1 |
| 10 | | 1 | 0 | 1 | 0 |
| 17 | 1 | 0 | 0 | 0 | 1 |

eg: Convert $(117)_{10}$ to its equivalent Binary form.

| | 117 | |
|---|---|---|
| | Quotient | Remainder |
| 2 | 58 | 1 |
| 2 | 29 | 0 |
| 2 | 14 | 1 |
| 2 | 7 | 0 |
| 2 | 3 | 1 |
| 2 | 1 | 1 |
| | ⋯⋯⋯⋯→ | 1 |

Least Significant Bit (LSB) ↑

Most Significant bit (MSB)

$$(117)_{10} = (1110101)_2$$

eg: $(5)_{10}$ to $(?)_2$

| | 5 | |
|---|---|---|
| | Quotient | Remainder |
| 2 | 2 | 1 LSB |
| 2 | 1 | 0 ↑ |
| | ⋯⋯⋯⋯→ | 1 MSB |

$(5)_{10} = (101)_2$

3. Octal: Base 8

$\{0, 1, 2, 3, 4, 5, 6, 7\}$

eg: Convert $(246)_b$ to equivalent Octal.

| | 246 | | |
|---|---|---|---|
| | Quotient | Remainder | |
| 8 | 30 | 6 | LSD |
| 8 | 3 | 6 | ↑ |
| | ! - - - - - - - - - → | 3 | MSD |

$(246)_{10} = (366)_8$

4 Hexadecimal: 16 digits

$\{0, 1, 2, \cdots, 9, A, B, C, D, E, F\}$

eg: $(2338)_{10}$ to $(?)_{16}$

| 16 | 2338 | | |
|---|---|---|---|
| | Quotient | Remainder | |
| 16 | 146 | 2 | LSD |
| 16 | 9 | 2 | ↑ |
| | ! - - - - - - - → | 9 | MSD |

$(2338)_{10} = (922)_{16}$

eg. $(0.122)_{10}$ to $(?)_{16}$

| Decimal fraction | | Base | | Recorded Decimal | Bit (Hex) | |
|---|---|---|---|---|---|---|
| 0.122 | × | 16 | $= 1.952$ | 1 | 1 | → MSD |
| 0.952 | × | 16 | $= 15.232$ | 15 | F | |
| 0.232 | × | 16 | $= 3.712$ | 3 | 3 | |
| 0.712 | × | 16 | $= 11.392$ | 11 | B | |
| 0.392 | × | 16 | $= 6.272$ | 6 | 6 | |
| 0.272 | × | 16 | $= 4.352$ | 4 | 4 | → (LSD) |

$(0.122)_{10} = (0.1F3B64)_{16}$