Probability

Probability = $\dfrac{\text{No. of favorable outcomes}}{\text{Total no. of outcomes}}$

Tossing a coin

$$P(H) = \frac{1}{2} = 0.5$$

Throwing a dice

$$P(3) = \frac{1}{6}$$

$$P(\text{even no.}) = \frac{3}{6} = \frac{1}{2} = 0.5$$

$$\{1, 2, 3, 4, 5, 6\}$$

$$[0 - 1]$$

eg: Two coins r tossed simultaneously. What is the prob of getting Head on only one of the two coins?

Soln:

HH , HT, TH, TT
        —   —

$$\frac{2}{4} = \frac{1}{2} = 0.5$$

eg: 3 coins r tossed simultaneously 150 times & it is found that 3 tails appeared 24 times,
$$2 \quad —11 \quad —— 45 —11,$$
$$1 \quad —11 \quad —— 72 —11$$

If 3 coins r tossed simultaneously at random, find the prob of getting 3 tails, 1 tail & 0 tail.

Soln:

Total # of outcomes = 150

3 tails = 24	2 tails = 45	1 tail = 72
$P = \frac{24}{150} = \frac{4}{25}$	$P = \frac{45}{150} = \frac{9}{30} = \frac{3}{10}$	$P = \frac{72}{150} = \frac{24}{50} = \frac{12}{25}$

0 tail = 150 − (24 + 45 + 72)
= 150 − 141 = 9

$$P(0 \text{ tail}) = \frac{9}{150} = \frac{3}{50}.$$

eg: If 2 coins r tossed then what is the prob of getting 2 tails?

Soln: When we toss 2 coins:

1st coin	2nd coin
H	H
H	T
T	H
T	T

4 outcomes

$$P = \frac{1}{4}.$$

eg: Prob. of getting 3 heads & 3 tails in

   tossing a   coin 3 times is _____ .

Soln:
___
      3H & 3T → 3 times

Possible Outcomes :   H H H
                           H H T
                           H T H
                           H T T
                           T H H
                           T H T
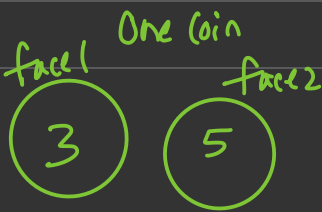   T T H   Total outcomes = 8
                           T T T
                                              favourable outcomes = 2

                           $\therefore P = \dfrac{2}{8} = \dfrac{1}{4}$.

              One Coin
     face1
            face2

      (3)   (5)

eg: A coin whose faces are marked 3 & 5 is
    tossed 4 times. The prob that sum of nos.
    thrown is 12 is _____ .

Soln:
    $P(sum=12) = P\left(3,3,3,3\right)$

    $= \dfrac{1}{2} * \dfrac{1}{2} * \dfrac{1}{2} * \dfrac{1}{2}$

    $= \dfrac{1}{16}$ //

eg: A coin whose faces are marked 3 &5 is tossed
    4 times The prob. that the sum of nos. is
    greater than 15 is _____ .

Soln:

1st	2nd	3rd	4th	Sum
5	5	5	5	20
5	5	5	3	18
5	5	3	3	16
5	3	3	3	14
3	3	3	3	9

$2^h = 2^4 = 16$

$$P(\text{sum} > 15) = P(5,5,5,5) + P(5,5,5,3) + P(5,5,3,3)$$

$$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16}$$

$$= \frac{11}{16} \ .$$

eg: Prob of rolling a sum of 6 with two dice = ?

Soln:

	Dice 1					
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Ruander.com

$$\frac{5}{36}$$

Face Cards

Knaves

Jack   Queen   King

---
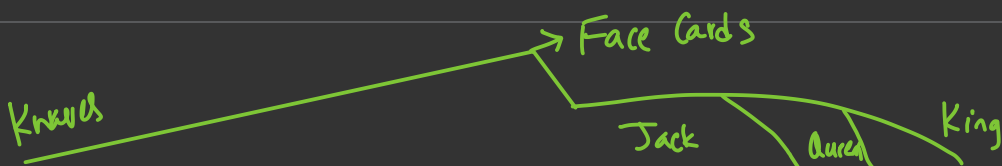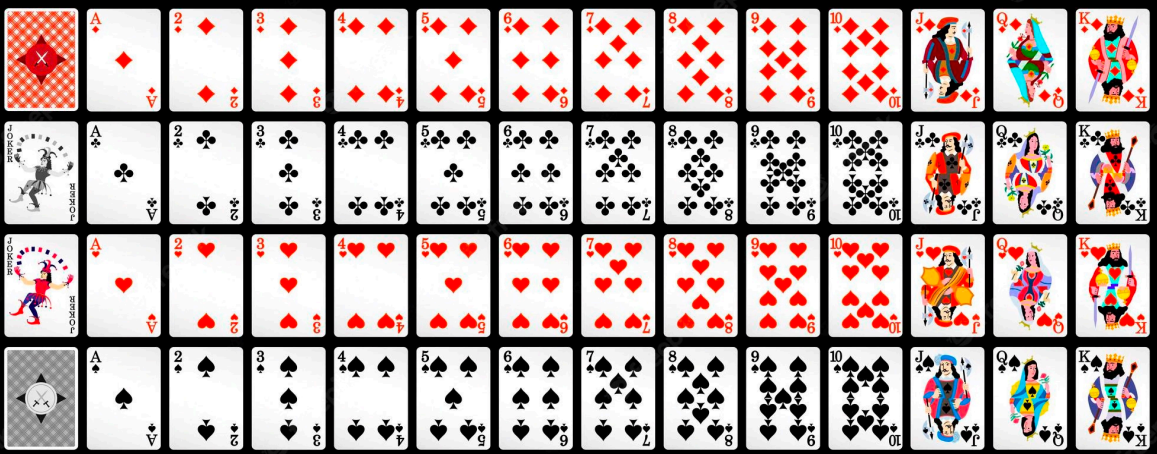
52 cards
---
4 suits of 13 cards

Diamond, Club,
Heart, Spade

eg. A card is drawn at random from a well-shuffled deck of
   playing cards. Find the prob. that card drawn is:
   (a) Spade or Ace   (c) neither jack nor a king
   (b) Black King   (d) either king or a queen.

Soln: Total Outcomes = 52

(a) # ♠ = 13 $\frac{13}{52}$

     #A = 4 $\frac{4}{52}$

     # $\boxed{A♠}$ = 1 $\frac{1}{52}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(\phi) + P(A) - P(A\phi)$$
$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}.$$

(b) Black King

Total black kings = 2

Total cards = 52

$$P = \frac{2}{52} = \frac{1}{26}.$$

(c) Neither jack nor king.

#J = 4

#K = 4

$$P(J \text{ or } K) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$$

$$P(N \ J \text{ or } K) = 1 - \frac{8}{52} = \frac{52-8}{52} = \frac{44}{52} = \frac{11}{26}$$

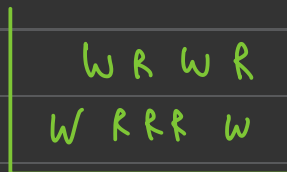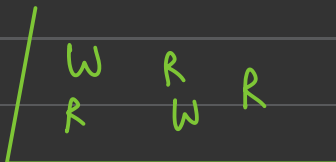(d) **Either king or queen:**

$$\#K = 4$$
$$\#Q = 4$$

$$P = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13} \; .$$

eg: Box1 contains 2 white & 3 red balls &
Box2 — " — 4 — 4 — 5 — " — .
One ball is drawn at random from one of the boxes &
is found to be red. Then, the prob that it was from Box2
is ?

Soln:

| W   R | | W R W R |
| R   W | R | W R R R W |

Formulas
- Conditional Probability:
  $$P(B|A) = \frac{P(A \cap B)}{P(A)}$$
- General multiplication rule:
  $$P(A \cap B) = P(A)\,P(B|A)$$
- Bayes' Rule
  $$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

(ignore this pb from

Java)

—— X ——