Worksheet 1.8, An Introduction to Linear Transforms

Worksheet Exercises

1. **Written Explanation Exercise**
   Suppose $T(x) = Ax$ for all x where A is a matrix and T is onto.

   (a) What can we say about pivotal rows of A?

   (b) What can we say about the existence of solutions to $Ax = b$?

2. Let A be an 3×4 matrix. What must c and d be if we define the linear transformation $T : \mathbb{R}^c \to \mathbb{R}^d$ by $T(\vec{x}) = A\vec{x}$?

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

   Construct a matrix A so that $T(\vec{x}) = A\vec{x}$ for all vectors $\vec{x}$.

4. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation such that

$$T\begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \end{bmatrix} = T\begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix} \neq \vec{0}.$$

   Identify a non-trivial solution $\vec{x}$ to $T\vec{x} = \vec{0}$.

5. Let T_A be the linear transformation with the matrix below. Match each choice of A on the left with the geometric description of the action of T_A on the right.

$$\begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix}$$
rotation by 90°

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$
A shear

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$
Projection onto y axis

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
dilation by $1/2$