

# Sample Quiz 1, Math 1554

**PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS**

First Name \_\_\_\_\_ Last Name \_\_\_\_\_

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Section Number (e.g. A4, QH3, etc.) \_\_\_\_\_ TA Name \_\_\_\_\_

## Student Instructions

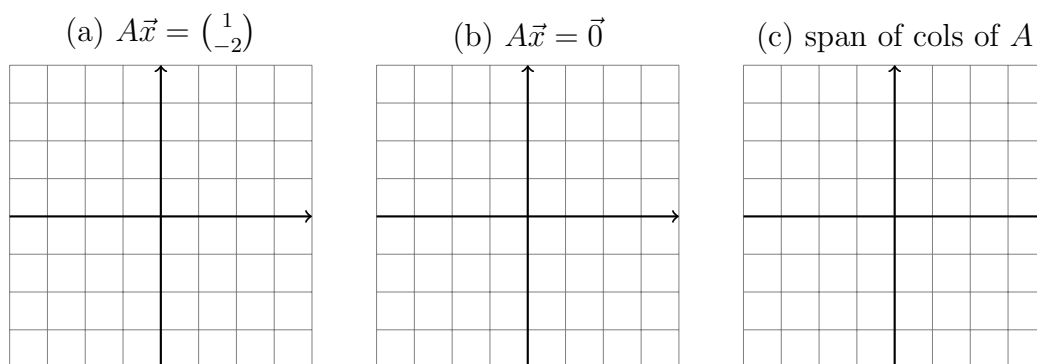
- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- Any work done on scratch paper will not be collected and will not be graded.

You do not need to justify your reasoning for questions in this quiz.

1. (4 points) Suppose  $A$  is an  $m \times n$  matrix,  $\vec{b} \in \mathbb{R}^m$ . Indicate **true** if the statement is true, otherwise, indicate **false**.

	true	false
a) If $A$ is a $3 \times 2$ matrix with 2 pivot columns, $A\vec{x} = \vec{0}$ has a unique solution.	<input type="radio"/>	<input type="radio"/>
b) If a linear system has more unknowns than equations, then the system always has at least one solution.	<input type="radio"/>	<input type="radio"/>
c) If the columns of $A$ span $\mathbb{R}^m$ , then there must be a pivot in every column of $A$ .	<input type="radio"/>	<input type="radio"/>
d) If $A$ is a $6 \times 7$ matrix and has two pivot columns, $A\vec{x} = \vec{b}$ has 4 free variables.	<input type="radio"/>	<input type="radio"/>

2. (3 points) Consider the matrix  $A = \begin{pmatrix} 1 & 2 \\ -2 & -4 \end{pmatrix}$ . In the grids below, draw (a) the solution set of  $A\vec{x} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ , (b) the solution set of  $A\vec{x} = \vec{0}$ , and (c) the span of the columns of  $A$ .



3. (3 points) If possible, write down an example of a matrix with the following properties. If it is not possible to do so, write *not possible*.

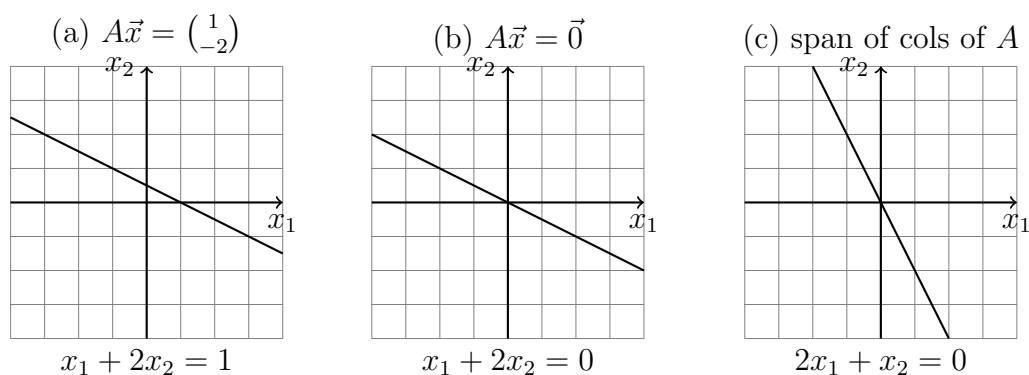
(a) A  $3 \times 4$  matrix  $A$ , in RREF, with just the second and third columns pivotal.

(b) An augmented matrix that can be used to compute the coefficients  $a_0, a_1, a_2$  of the polynomial  $y(x) = a_0x + a_1x^2 + a_2x^3$  that takes the values  $y(-1) = 4$ ,  $y(0) = 1$ ,  $y(2) = -1$ .

## Answers

No justification needed for any questions in this quiz.

1. (a) True
  - (b) False, for example the system with two equations  $x + y + z = 1$  and  $x + y + z = 2$  has no solutions.
  - (c) False, columns could be  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ .
  - (d) False. (number of cols) = (number of pivots) + (number of free variables).
2. Answers here assume grid spacing equal to one.



Justification not necessary for this quiz, but for part (c), the span is a line, and a vector in the span of the columns of  $A$  is the first column of  $A$ ,

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

The points  $(1, -2)$  and  $(0, 0)$  are in this line, two distinct points define a line, the line is  $2x_1 + x_2 = 0$ .

3. (a)  $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

- (b) Not possible (there is no polynomial of the form  $y(x) = a_0x + a_1x^2 + a_2x^3$  that satisfies  $y(0) = 1$ )