

## Worksheet 1.5, Solution Sets of Linear Systems

*Recitations are meant to be active: students are encouraged to work with other students in recitation. As students are working through exercises, the TA should circulate around the room, helping students. Students may be asked to present their work using a document camera or write a solution on a whiteboard.*

### Worksheet Exercises

#### 1. Written Explanation Exercise

- (a) When a homogeneous system has a nontrivial solution, what properties does that system have? List at least two.
- 2. Indicate whether the statements are true or false.
  - (a) A non-trivial solution  $\vec{x}$  to  $A\vec{x} = \vec{0}$  has all non-zero entries.
  - (b) If  $A\vec{x} = \vec{b}$  and  $A\vec{y} = \vec{b}$ , then  $A(\vec{x} - \vec{y}) = \vec{0}$ .
  - (c) Any  $3 \times 2$  matrix  $A$  with two pivotal positions has a non-trivial solution to  $A\vec{x} = \vec{0}$ .

#### 3. Example Construction

- (a) Give an example of a non-zero  $2 \times 3$  matrix  $A$  such that  $\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$  is a solution of  $A\vec{x} = \vec{0}$ .
  - (b) Give an example of a non-trivial solution to  $A\vec{x} = \vec{0}$ , where  $A = \begin{pmatrix} 2 & 5 \\ 0 & 0 \\ 4 & 10 \end{pmatrix}$ .
4. Express the solution to  $A\vec{x} = \vec{0}$  in parametric vector form, where  $A = \begin{pmatrix} 1 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ .