Worksheet 1.7, Linear Independence

Worksheet Exercises

Recitations are meant to be active: students are encouraged to work with other students in recitation. As students are working through exercises, the TA should circulate around the room, helping students. Students may be asked to present their work using a document camera or write a solution on a whiteboard.

1. **Written Explanation Exercise**

   (a) How are span and linear dependence related to each other?

   (b) Suppose T is a linear map.
   (a) If $v_1, \ldots, v_k$ are dependent, why are $T(v_1), \ldots, T(v_k)$ dependent?
   (b) If $v_1, \ldots, v_k$ are independent, need $T(v_1), \ldots, T(v_k)$ be independent?

2. In the problems below, $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are three linearly independent vectors in $\mathbb{R}^3$. Which of the collections of vectors below are linearly independent?

   a) $\{\vec{v}_1, \vec{v}_2, \vec{0}\}$

   b) $\{\vec{v}_1, \vec{v}_1 + \vec{v}_2, \vec{v}_2\}$

   c) $\{\vec{v}_1, \vec{v}_1 + \vec{v}_2\}$

3. For what values of h are the columns of A linearly dependent?

$$A = \begin{bmatrix} 2 & 4 & -2 \\ -2 & -6 & 2 \\ 4 & 7 & h \end{bmatrix}$$

4. A 5×3 matrix $A = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \end{bmatrix}$ has all non-zero columns, and $\vec{a}_3 = 5\vec{a}_1 + 7\vec{a}_2$. Identify a non-trivial solution to $A\vec{x} = \vec{0}$.

5. The columns of a 7×5 matrix are linearly independent. How many pivots does it have?