

Worksheet 1.9, Linear Transforms

Worksheet Exercises

1. Indicate whether the statements are true or false.
 - (a) If A is a 3×2 matrix then the map $x \mapsto Ax$ cannot be one-to-one.
 - (b) If A is a 2×3 matrix then the map $x \mapsto Ax$ cannot be onto.
 - (c) The transformation made by performing one linear transformation and then another is not necessarily a linear transform.
 - (d) $T_A : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is one-to-one if and only if $A\vec{x} = \vec{0}$ only has the trivial solution.
2. Construct the standard matrix of the linear transformation T .

(a) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^4$, where $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 6 \\ 1 \\ 8 \end{bmatrix}$

(b) T is a vertical shear given by $T(\vec{e}_2) = 2\vec{e}_2$ and $T(\vec{e}_1) = \vec{e}_1 - 2\vec{e}_2$.

(c) A matrix $A \in \mathbb{R}^{2 \times 2}$ such that $T(\vec{x}) = A\vec{x}$. T is a linear transformation that first reflects vectors across the line $x_1 = x_2$, then rotates them counterclockwise by π radians about the origin, then reflects them across the line $x_2 = 0$.

(d) T rotates points around the origin by the angle $-\pi/4$ radians.

(e) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 5x_2 + x_3 \\ x_2 - 6x_3 \end{bmatrix}$.