2 Wireless Channel Characteristics

2.1 Path Loss

1, Free Space: Friis free space equation: $P_r(d)=\dfrac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$, L is the system loss factor;

2, Log-Distance Path Loss Model: $\overline{PL}(d)=\overline{PL}(d_0)+10n\lg(d/d_0)(\mathrm{dB})$

2.2 Doppler Shift

Doppler frequency shift $f_d=\Delta\phi/(2\pi\Delta t)=v\cos\theta/\lambda$, θ is the angle between speed v and the length to the signal source. λ is the wave length.

2.3 Impulse Response Model of a Multipath Channel

2.3.1 Fading Channel

The fading is typically composed of two multiplicative components as $\rho(t)=\rho_{s-slow}(t)\rho_{r-fast}(t)$.

1, Rayleigh Fading: $p(y)=\dfrac{y}{\sigma^2}e^{-\frac{y^2}{2\sigma^2}}(y\ge 0)$, σ^2 is the time-average power of y ,the $y_{mean}=1.2533\sigma$.

2, Ricean Fading: when there is a line-of-sight propagation path, Ricean fading occurs as $p(y)=\dfrac{y}{\sigma^2}e^{-\frac{y^2+S^2}{2\sigma^2}}I_0\left(\dfrac{ys}{\sigma^2}\right)(y\ge 0, S\ge 0)$, $I_0(z)=\sum_{n=0}^{\infty}\dfrac{z^{2n}}{2^{2n}n!n!}$, $K=S^2/2\sigma^2$,

when $K\to\infty$, Ricean fading becomes Rayleigh fading.

2.3.2 Coherence Bandwidth and Coherence Time

Coherence bandwidth B_c is the range of frequencies over which two frequency components have a strong potential for amplitude correlation. Two sinusoids with frequency separation greater than B_c are affected quite differently by the channel. In general, $B_c\approx 1/T_m$, where T_m is the multipath delay spread. $B>B_c$, the channel is frequency selective fading channel; $B<B_c$, the channel is flat fading channel.

Coherence time $T_c\approx 1/f_m$, where f_m is the maximum Doppler shift. When $T_s\ll T_c$, T_s is the base band signal pulse duration, the channel is a slow fading channel; when $T_s>T_c$, the channel is a fast fading channel.

3 Digital Modulation Techniques

3.1 Channel Capacity

$\eta_B=R/B(bps/Hz)$, Shannon's channel capacity theorem: $\eta_{B\max}=C/B=\log_2(1+S/N)$

3.2 Nyquist Criterion for ISI Cancellation

Nyquest bandwidth $W=R_b/2=1/2T_b$, one signal waveform that produces zero ISI is given by $p(t)=\sin c(2Wt)$, $p(f)=1/2W, -W<f<W$.

3.3 Raised Cosine Spectrum

$$P(f)=\begin{cases}\dfrac{1}{2W} & 0\le|f|<f_1\\ \dfrac{1}{4W}\left[1-\sin\left(\dfrac{\pi|f|-W}{2W-2f_1}\right)\right] & f_1\le|f|<2W-f_1\\ 0 & |f|\ge 2W-f_1\end{cases}$$

, where $\alpha=1-f_1/W$

3.4 Gaussian Pulse-Shaping Filter

3.5 Matched Filter

$h_{opt}(t)=kg(T-t)$, $\eta_{\max}=2E/N_0$

3.6 Bit Error Rate Due to Noise

Average probability of symbol error P_e in the receiver is $P_e=P_0P_{e0}+P_1P_{e1}=\dfrac{1}{2}erfc\left(\sqrt{E_b/N_0}\right)$.

3.7 Geometric Interpretation of Signals (Norm, Base, Inner-Product and so on…)

3.8 Coherent Detection of Signals in Noise (Maximum Likelihood Decoder)

$\dfrac{P_e}{\log_2 M}\le BER\le P_e$

3.9 Correlation Receiver…

4 Spread-Spectrum System

4.2 PN Sequence

4.3 Walsh Code

Each row of a Walsh matrix is a Walsh code that is exactly orthogonal to other Walsh codes in the matrix. A Walsh matrix of order 2^{N+1} is determined by the Walsh matrix of order 2^N as $\mathbf{H}_{2^{N+1}}=\begin{bmatrix}\mathbf{H}_{2^N} & \mathbf{H}_{2^N}\\ \mathbf{H}_{2^N} & \overline{\mathbf{H}}_{2^N}\end{bmatrix}$, $\mathbf{H}_1=0$.

4.4 Orthogonal Variable Spreading Factor (OVSF) Code

4.5 Spread Spectrum Communication System

4.6 DSSS with Coherent BPSK Modulation

4.7 Performance of the DSSS System

$(SNR)_{Output}=2N(SNR)_{Intput}$, note that 2 accounts for an additional gain that is obtained through the use of coherent detection (which presumes exact knowledge of signal phase by the receiver). N here accounts for the gain in SNR obtained by the use of spread spectrum and referred to as the processing gain.

4.8 CDMA System

We may support multiple users over a common communication channel in a spread spectrum environment by using a technique known as code-division multiple access (CDMA), which relies on the use of different spreading codes for the individual users. If every user transmits the same level of signal power, the received powers from users closer to the base station will be higher than that of the users farther away from the base station. Furthermore, a user close to the base station may contribute too much multiple access interference (MAI) to make a high BER. This prevents other users' signals from being received successfully. This is known as ***Near-Far Effect***. To reduce the near-far effect, the technique of power control is employed in CDMA wireless system. With power control, the signal power transmitted by a mobile user is automatically adjusted and only the minimum power needed to achieve the acceptable signal quality is transmitted.

4.9 Power Control

Open-Loop Power Control: $P_t(dBm)=L(dB)+I(dBm)+C$, P_t is the mobile transmit power, L is the path loss measured by the mobile unit, C is a constant, and I is the interference power. Usually a mobile unit expects a received signal power level. By making comparison between this power level and the actually received power, the path loss can be estimated. Notice that Open-Loop Power Control only achieves good performance within the coherent bandwidth of a fading channel.

Closed-Loop Power Control: this technique provides a more accurate power adjustment so that the system can track the fast fading channel. $P_t(k+1)=P_t(k)-\Delta PC(\gamma-\gamma_0)$ where $P_t(k)$ is the transmit power at the k^{th} iteration, ΔP is the step size, C is a constant, γ is the measured SNR and γ_0 is the threshold value for SNR.

4.10 Multiuser Detection

Multiuser detection intends to suppress the MAI in uplink and achieve an improved SIR.

5 Adaptive Filtering, Equalization and RAKE Reception

5.1 Linear Optimum Filtering

5.2 Principle of Orthogonality

The necessary and sufficient condition for the cost function $J=E\left[e(n)e^*(n)\right]=E\left[|e(n)|^2\right]$ to attain its minimum value is that the corresponding value of the estimation error $e_0(n)$ is orthogonal to each input sample that enters into the estimation of the desired response at the time n. this statement constitutes the principle of orthogonality; it represents one of the most elegant theorems in the subject of line optimum filtering.

5.3 Wiener-Hopf Equations

5.4 Channel Equalization

Zero-Forcing Equalizer: $\sum_{k=-N}^{N}w_k c_{n-k}=\begin{cases}1 & n=0\\ 0 & n=\pm1,\pm2,\dots,\pm N\end{cases}$

5.5 Adaptive Equalizer

5.7 RAKE Receiver

In a RAKE receiver, the coherent combining is implemented to increase the SINR.

6 Smart Antenna

6.1 Array Pattern

An array pattern is defined as $P(\theta)=\left|\mathbf{V}_s(\theta)^H\mathbf{w}\right|$

6.2 Adaptive Beamforming

6.2.1 Minimum Variance Beamforming

$\min_{\mathbf{w}}\left(\mathbf{w}^H\mathbf{R}\mathbf{w}\right)$ $\underrightarrow{Subject\ to}$ $\mathbf{Cw}=\mathbf{c}$, we have the optimum weight formula $\mathbf{w}_{opt}=\dfrac{\mathbf{R}^{-1}\mathbf{C}}{\mathbf{C}\mathbf{R}^{-1}\mathbf{C}^H}$

6.2.2 Frost Beamformer

Computing a matrix and its inverse to obtain the beamforming weight vector is computationally intense and it is an undesirable operation in reality. Frost beamformer gives a solution that does not require such computation. The weight vector is iteratively updated through a simple recursive algorithm as

$\mathbf{w}_{k+1}=\mathbf{w}_c+\mathbf{P}\left[\mathbf{w}_k-\mu\mathbf{S}_k^*\mathbf{S}_k^T\mathbf{w}_k\right]=\mathbf{w}_c+\mathbf{P}\left[\mathbf{w}_k-\mu y_k\mathbf{S}_k^*\right]$, where $\mathbf{w}_c=\mathbf{C}^H\left(\mathbf{C}\mathbf{C}^H\right)^{-1}\mathbf{c}$ and $\mathbf{P}=\mathbf{I}-\mathbf{C}^H\left(\mathbf{C}\mathbf{C}^H\right)^{-1}\mathbf{C}$.

6.2.3 Generalized Sidelobe Canceller (GSC)

$\mathbf{w}_a=\left(\mathbf{BRB}^H\right)^{-1}\mathbf{BRw}_c$, $\mathbf{y}_{B,k}=\mathbf{By}_k$, $Z_{c,k}=\mathbf{w}_c^T\mathbf{y}_k$, $Z_{a,k}=\mathbf{w}_a^T\mathbf{y}_{B,k}$, $Z_k=Z_{c,k}-Z_{a,k}$, $\mathbf{w}_{a,k+1}=\mathbf{w}_{a,k}-\mu Z_k\mathbf{y}_{B,k}^*$

6.2.4 MMSE Beamformer

$\min_{\mathbf{w}}\left(E\left[\left|\mathbf{w}^T\mathbf{r}_t - d_t\right|^2\right]\right)$, $\mathbf{w}_{opt} = \mathbf{R}^{-1}\mathbf{P}$, $\mathbf{w}_{k+1} = \mathbf{w}_k - \mu\mathbf{r}_k^*\left[z_k - d_k\right]$

6.3 Direction of Arrival (DOA) Estimation

6.3.1 Minimum Variance Method

$P(\theta) = \left[\mathbf{V}^T(\theta)\mathbf{R}^{-1}\mathbf{V}^*(\theta)\right]^{-1}$, as θ changes over a range of DOAs, a peak response of the spatial power spectrum $P(\theta)$ indicates the DOA of an incident signal. $P(\theta)$ is also called steered response.

6.3.2 The Conventional Method

Consider a steered response defined as $P(\theta) = \mathbf{V}^T(\theta)\mathbf{R}\mathbf{V}^*(\theta)$, which is beamformer output power with steering vector as its weight vector. The maximum response corresponds to the DOA of the incident signal.

6.3.3 Multiple Signal Classification (MUSIC) Algorithm

$P(\theta) = \dfrac{1}{\mathbf{V}_s^T(\theta)\mathbf{V}_N\mathbf{V}_N^H\mathbf{V}_s^*(\theta)}$, where $\mathbf{V}_S(\theta)$ is the test steering vector and $\mathbf{V}_N$ is the base matrix of noise subspace.

7 Space-Time Diversity

7.1 One Transmitter and Two Receivers (Space Diversity)

Space-time diversity is a very effective technique that exploits the principle of providing the receiver with multiple faded replicas of the same information-bearing signal from uncorrelated space-time channels. Received Signal: $\mathbf{r}_0 = h_0\mathbf{s}_0 + \mathbf{n}_0$, $\mathbf{r}_1 = h_1\mathbf{s}_0 + \mathbf{n}_1$ .Output: $\hat{\mathbf{s}}_0 = h_0^*\mathbf{r}_0 + h_1^*\mathbf{r}_1 = \left(\alpha_0^2 + \alpha_1^2\right)\mathbf{s}_0 + h_0^*\mathbf{n}_0 + h_1^*\mathbf{n}_1$, Symbol decision of the maximum likelihood detection applied in 1T2R is $d^2(\hat{\mathbf{s}}_0, \mathbf{s}_i) \le d^2(\hat{\mathbf{s}}_0, \mathbf{s}_k)$ $(\forall i \ne k)$

7.2 Two Transmitters and One Receiver (Time Diversity)

Transmission Scheme	Antenna 0	Antenna 1
t	$\mathbf{s}_0$	$\mathbf{s}_1$
$t+T$	$-\mathbf{s}_1^*$	$\mathbf{s}_0^*$

$\mathbf{r}_0 = \mathbf{r}(t) = h_0\mathbf{s}_0 + h_1\mathbf{s}_1 + \mathbf{n}_0$, $\mathbf{r}_1 = \mathbf{r}(t+T) = -h_0\mathbf{s}_1^* + h_1\mathbf{s}_0^* + \mathbf{n}_1$. The combined signals after $\mathbf{r}_0$ and $\mathbf{r}_1$ received are: $\hat{\mathbf{s}}_0 = h_0^*\mathbf{r}_0 + h_1\mathbf{r}_1^* = \left(\alpha_0^2 + \alpha_1^2\right)\mathbf{s}_0 + h_0^*\mathbf{n}_0 + h_1\mathbf{n}_1^*$,

$\hat{\mathbf{s}}_1 = h_1^*\mathbf{r}_0 - h_0\mathbf{r}_1^* = \left(\alpha_0^2 + \alpha_1^2\right)\mathbf{s}_1 - h_0\mathbf{n}_1^* + h_1^*\mathbf{n}_0$

7.3 Two Transmitters and Two Receivers

Definition of Channels	Receiving Antenna 0	Receiving Antenna 1
Transmitting Antenna 0	h_0	h_2
Transmitting Antenna 1	h_1	h_3

Notation for the Received Signals	Receiving Antenna 0	Receiving Antenna 1
t	$\mathbf{r}_0$	$\mathbf{r}_2$
$t+T$	$\mathbf{r}_1$	$\mathbf{r}_3$

$\mathbf{r}_0 = h_0\mathbf{s}_0 + h_1\mathbf{s}_1 + \mathbf{n}_0$, $\mathbf{r}_1 = -h_0\mathbf{s}_1^* + h_1\mathbf{s}_0^* + \mathbf{n}_1$, $\mathbf{r}_2 = h_2\mathbf{s}_0 + h_3\mathbf{s}_1 + \mathbf{n}_2$, $\mathbf{r}_3 = -h_2\mathbf{s}_1^* + h_3\mathbf{s}_0^* + \mathbf{n}_3$.

The combiner gives two outputs that are sent to the maximum likelihood detector:

$$\hat{\mathbf{s}}_0 = h_0^*\mathbf{r}_0 + h_1\mathbf{r}_1^* + h_2^*\mathbf{r}_2 + h_3\mathbf{r}_3^* = \left(\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2\right)\mathbf{s}_0 + h_0^*\mathbf{n}_0 + h_1\mathbf{n}_1^* + h_2^*\mathbf{n}_2 + h_3\mathbf{n}_3^*$$

$$\hat{\mathbf{s}}_1 = h_1^*\mathbf{r}_0 - h_0\mathbf{r}_1^* + h_3^*\mathbf{r}_2 - h_2\mathbf{r}_3^* = \left(\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2\right)\mathbf{s}_1 - h_0\mathbf{n}_1^* + h_1^*\mathbf{n}_0 - h_2\mathbf{n}_3^* + h_3^*\mathbf{n}_2$$

Space-time diversity can be obtained by transmitting data symbols at different time and different locations. Both transmitter and receiver can employ multiple antennas to increase the order of diversity and achieve the improved performance.