第三周

2.1 Consider the following length-7 sequences defined for $-3 \le n \le 3$:

$$x[n]=\{3\ -2\ 0\ 1\ 4\ 5\ 2\},$$

$$y[n]=\{0\ 7\ 1\ -3\ 4\ 9\ -2\},$$

$$w[n]=\{-5\ 4\ 3\ 6\ -5\ 0\ 1\}.$$

Generate the following sequences: (a) $u[n]=x[n]+y[n]$, (b) $v[n]=x[n]$□$w[n]$, (c) $s[n]=y[n]-w[n]$ and (d) $r[n]=4.5y[n]$.

2.3 Determine the even and odd parts of the sequences $x[n]$, $y[n]$, and w$[n]$ of Problem 2.1.

2.6 Determine the periodic conjugate symmetric and periodic conjugate antisymmetric parts of the following sequences:

(b) $\{h[n]\}=\{-2+j5\ \ 4-j3\ \ 5+j6\ \ 3+j\ \ -7+j2\}$

$$\uparrow$$

2.18 Express the sequence $x[n]=1, -\infty \prec n \prec \infty$, in terms of the unit step sequence $\mu[n]$.

2.21 Determine the fundamental period of the following periodic sequences:

(a) $\tilde{x}_1[n]=e^{-j0.4\pi n}$,

(d) $\tilde{x}_4[n]=3\sin(1.3\pi n)-4\cos(0.3\pi n+0.45\pi)$.

2.23 A continuous-time sinusoidal signal $x_a(t)=\cos\Omega_o t$ is sampled at $t=nT$, $-\infty \prec n \prec \infty$, generating the discrete-time sequence $x[n]=x_a(nT)=\cos(\Omega_o nT)$. For what values of T is $x[n]$ a periodic sequence? What is the fundamental period of $x[n]$ if $\Omega_o=18$ and $T=\pi/6$ seconds?

第四周

2.26 For each of the following discrete-time systems, where $y[n]$ and $x[n]$ are, respectively, the output and the input sequences, determine whether or not the system is (1) linear, (2)causal, (3) stable, and (4) shift-invariant:

(a) $y[n]=n^2 x[n]$,

(e) $y[n]=\alpha x[-n]$, α is a nonzero constant.

(f) $y[n]=x[n-5]$.

2.32 A periodic sequence $\tilde{x}[n]$ with a period N is applied as an input to an LTI discrete-time system characterized by an impulse response $h[n]$ generating an output $y[n]$. Is $y[n]$ a periodic sequence? If it is, what is its period?

2.45 Consider a causal discrete-time system characterized by a first-order linear, constant-coefficient difference equation given by $y[n] = ay[n-1] + bx[n],\ n \geq 0$, Where $y[n]$ and $x[n]$ are, respectively, the output and input sequences. Compute the expression for the output sample $y[n]$ in terms of the initial condition $y[-1]$ and the input samples.

(a)Is the system time-invariant if $y[-1]=1$? Is the system linear if $y[-1]=1$?

(b)Repeat part(a) if $y[-1]=0$.

(c)Generalize the results of parts (a) and (b) to the case of an Nth-order causal discrete-time system given by Eq.(2.93) $y[n]+0.7y[n-1]-0.45y[n-2]-0.6y[n-3]=0.8x[n]-0.44x[n-1]+0.36x[n-2]+0.02x[n-3]$

2.57 Determine the expression for the impulse response of each of the LTI systems shown in Figure P2.2.

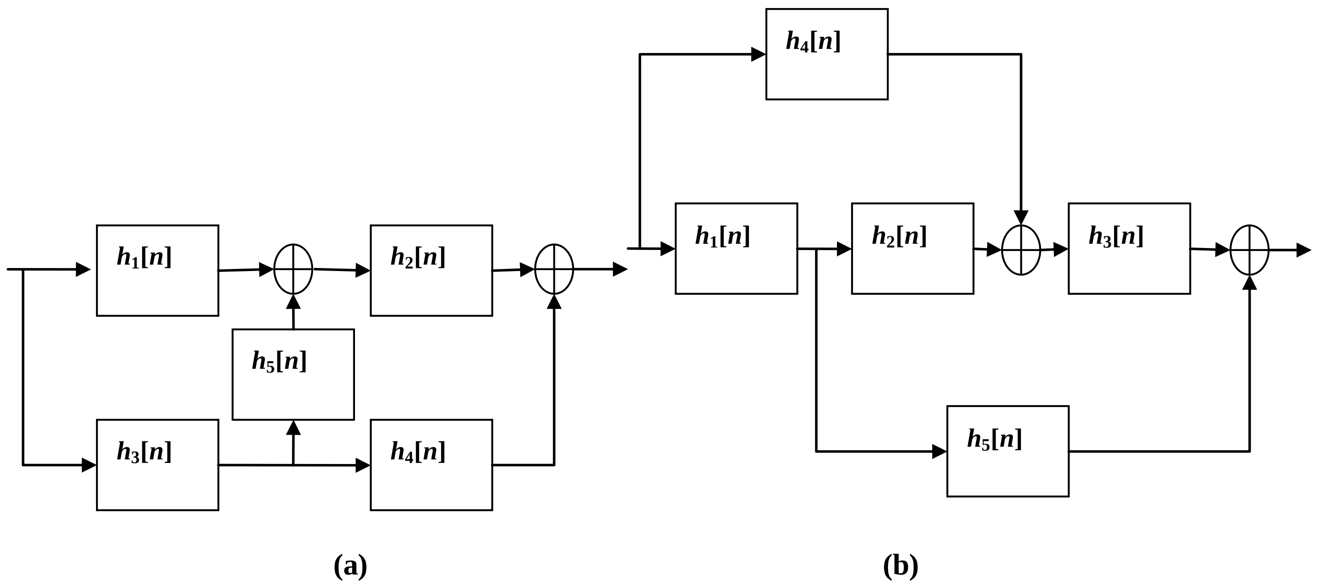

Figure P2.2

第五周

3.4 Show that the DTFT of $x[n]=1, -\infty \prec n \prec \infty$, is given by $X(e^{j\omega})=\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$.

3.14 Evaluate the inverse DTFT of each of the following DTFTs:

(a) $X_a(e^{j\omega})=\sum_{k=-\infty}^{\infty}\delta(\omega+2\pi k)$,

(b) $X_b(e^{j\omega})=\dfrac{1-e^{j\omega(N+1)}}{1-e^{j\omega}}$,

(c) $X_c(e^{j\omega})=1+2\sum_{l=0}^{N}\cos\omega l$.

3.15 Determine the inverse DTFT of each of the following DTFTs:

(a) $H_1(e^{j\omega})=1+2\cos\omega+3\cos 2\omega$,

(c) $H_3(e^{j\omega})=j(3+4\cos\omega+2\cos 2\omega)\sin\omega$.

3.22 Let $X(e^{j\omega})$ denote the DTFT of a real sequence $x[n]$. Determine the DTFT $Y(e^{j\omega})$ of the sequence $y[n]=x[n] \otimes x[-n]$ in terms of $X(e^{j\omega})$ and show that it is a real-valued function of ω.

3.25 Let $x[n]$ be a length-9 sequence given by

$$\{x[n]\}=\{3 \quad 0 \quad 1 \quad \underset{\uparrow}{-2} \quad -3 \quad 4 \quad 1 \quad 0 \quad -1\}$$

with a DTFT $X(e^{j\omega})$. Evaluate the following function of $X(e^{j\omega})$ without computing the transform itself.

(a) $X(e^{j0})$,

(d) $\int_{-\pi}^{\pi} \left|X(e^{j\pi})\right|^2 d\omega$,

(c) $\int_{-\pi}^{\pi} X(e^{j\omega})$.

3.27 Let $G_1(e^{j\omega})$ denote the discrete-time Fourier transform of the sequence $g_1[n]$ shown in Figure P3.3(a). Express the DIFTs of the remaining sequences in Figure P3.3(b),(c) in terms of $G_1(e^{j\omega})$. Do not evaluate $G_1(e^{j\omega})$.

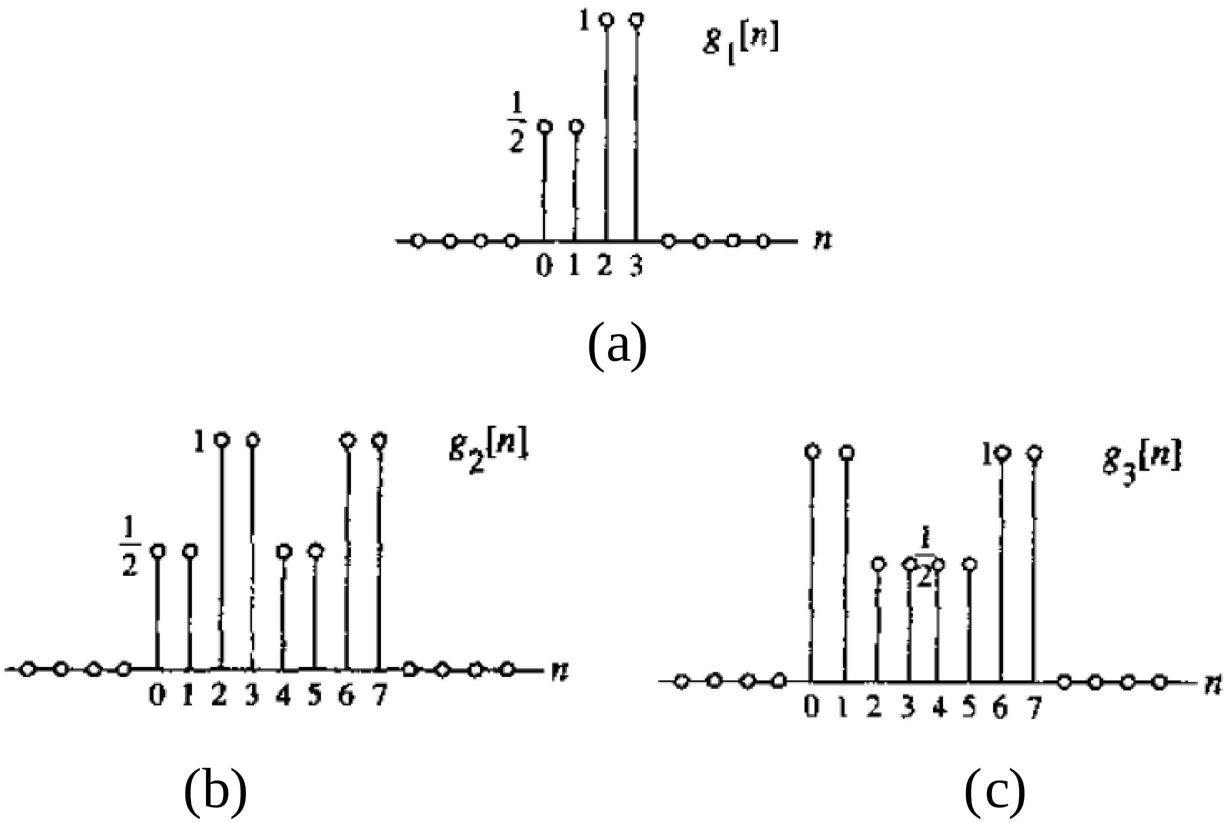

Figure P3.3

第六周

3.33 Determine the periodic sequence $\tilde{y}[n]$ obtained by a periodic convolution of the following two periodic sequences of period 5 each:

$$\tilde{x}[n]=\begin{cases} 2, & \text{for } n=0.2, \\ -1, & \text{for } n=1, \\ 3, & \text{for } n=3, \\ -2, & \text{for } n=4, \end{cases} \qquad \tilde{h}[n]=\begin{cases} 1, & \text{for } n \\ 2, & \text{for } n \\ -3, & \text{for } n = \\ 0, & \text{for } n \end{cases}$$

3.39 Prove the following general properties of the DFT listed in Table 3.5:

(a) linearity, (b) circular time-shifting,

(e) N-point circular convolution.

PS： The Third Edition : Table5.3 DFT Theorems.

Table 3.5: General properties of the DFT.

Type of Property	Length-N Sequence	N-point DFT
	$g[n]$ $h[n]$	$G[k]$ $H[k]$
Linearity	$\alpha g[n] + \beta h[n]$	$\alpha G[k] + \beta H[k]$
Circular time-shifting	$g[\langle n - n_o \rangle_N]$	$W_N^{kn_o} G[k]$
Circular frequency-shifting	$W_N^{-k_o n} g[n]$	$G[\langle k - k_o \rangle_N]$
Duality	$G[n]$	$N\,g[\langle -k \rangle_N]$
N-point circular convolution	$\sum_{m=0}^{N-1} g[m] h[\langle n - m \rangle_N]$	$G[k]H[k]$
Modulation	$g[n]h[n]$	$\frac{1}{N} \sum_{m=0}^{N-1} G[m] H[\langle k - m \rangle_N]$
Parseval's relation	$\sum_{n=0}^{N-1} \lvert x[n] \rvert^2 = \frac{1}{N} \sum_{k=0}^{N-1} \lvert X[k] \rvert^2$	

第六周

3.48 Let $x[n], 0 \le n \le N\text{-}1$, be a length-$N$ sequence with an N-point DFT $X[k], 0 \le k \le N\text{-}1$. Determine the N-point inverse DFTs of the following length-N DFTs in terms of $x[n]$:

(a) $W[k]=\alpha X[\langle k\text{-}m_1\rangle_N]+\beta X[\langle k\text{-}m_2\rangle_N]$, where m_1 and m_2 are positive integers less than N,

(b) $G[k]=\begin{cases} X[k], & \text{for } k \ even, \\ 0, & \text{for } k \ odd, \end{cases}$

(c) $Y[k]=X[k]$ Ⓝ $X[k]$.

3.51 Let $G[k]$ and $H[k]$ denote the 7-point DFTs of two length-7 sequences $g[n]$ and h[n], respectively.

(a) If $G[k]=\{1+j2 \quad -2+j3 \quad -1-j2 \quad 0 \quad 8+j4 \quad -3+j \quad 2+j5\}$ and $h[n]=g[(n-3)_7]$, determine $H[k]$ without computing the DFT.

(b) If $g[n]=\{-3.1 \ \ 2.4 \ \ 4.5 \ \ -6 \ \ 1 \ \ -3 \ \ 7\}$ and $H[k]=G[(k-4)_7]$, determine $h[n]$ without computing the DFT.

第六周

3.54 Consider the length-12 sequence, define for $0 \le k \le 11$, $\{x[n]\} = \{3\ \ -1\ \ 2\ \ 4\ \ -3\ \ -2\ \ 0\ \ 1\ \ -4\ \ 6\ \ 2\ \ 5\}$,

With a 12-point DFT give by $X[k],\ 0 \le k \le 11$. Evaluate the following functions of $X[k]$ without computing the DFT:

(a) $X[0]$, (c) $\sum_0^{11} X[k]$.

第七周

3.64 Let $g[n]$ and $h[n]$ be two finite-length sequences as given below:

$$\{g[n]\} = \{\underset{\uparrow}{-3}\ 2\ 4\},\ \{h[n]\} = \{\underset{\uparrow}{2}\ -4\ 0\ 1\}.$$

(a) Determine $y_l[n] = g[n] \circledast h[n]$.

(b) Extend $g[n]$ to a length -4 sequence $g_e[n]$ by zero-padding and compute $y_c[n] = g_e[n] \, ④ \, h[n]$.

3.84 Consider the z-transform

$$G(z) = \frac{(z+0.4)(z-0.91)(z^2+0.3z+0.4)}{(z^2-0.6z+0.6)(z^2+3z+5)}.$$

There are three possible nonverlapping regions of convergence (ROCs) of this z-transform. Disscuss the type of inverse z-transform (left-sided, right sided, or two sided sequences) associated with each of the three ROCs. It is not necessary to compute the exact inverse transform.

第七周

3.85 Consider the following sequences:

$$(1)\ x_1[n]=(0.4)^n\mu[n],\qquad (2)\ x_2[n]=(-0.6)^n\mu[n]$$

(a) Determine the ROCs of the z-transform of each of the above sequences.

(b) form the ROCs determined in part (a) , determine the ROCs of the following sequences:

$$(1)\ y_1[n]=x_1[n]+x_2[n],\qquad (2)\ y_2[n]=x_1[n]+x_3[n]$$

3.86 Derive the z-transforms and the ROCs give in Table 3.8 (Third Edition:Table 6.1) of the following sequences:

$$(a)\delta[n],(b)\alpha^n\mu[n],(c)(r^n\cos\omega_0 n)\mu[n],and\,(d)(r^n\sin\omega_0 n)\mu[n].$$

3.98 Evaluate the inverse z-transforms of the following z-transforms:

$$(a)Y_1(z)=\frac{z(z-1)}{(z+1)(z+\frac{1}{3})},|\,z\,|>1$$

第八周

4.8 An FIR LTI discrete-time system is described by the difference equation $y[n]=a_1x[n+k]+a_2x[n+k-1]+a_3x[n+k-2]+a_2x[n+k-3]+a_1x[n+k-4]$,Where $y[n]$ and $x[n]$ denote, respectively, the output and the input sequences. Determine the expression for its frequency response $H(e^{j\omega})$. For what values of the constant k will the system have a frequency response $H(e^{j\omega})$ that is a real function of ω?

4.9 Consider the cascade of two causal LTI systems: $h_1[n]=\alpha\delta[n]+\delta[n-1], and\ h_2[n]=\beta^n\mu[n]$, $|\beta|<1$. Determine the frequency response $H(e^{j\omega})$ of the overall system. For what values of α and β will $|H(e^{j\omega})|=1$?

第九周

4.16 An FIR filter of length 3 is defined by a symmetric impulse response, i.e., $h[0]=h[2]$. Let the input to this filter be a sum of two cosine sequences of angular frequencies 0.2rad/samples and 0.5 rad/samples, respectively. Determine the impulse response coefficients so that the filter passes only the high-frequency component of the imput.

4.17 (a) Design a length-5 FIR bandpass filter with an antisymmetric impulse response $h[n]$, i.e., $h[n]=-h[4-n], 0\le n\le 4$, satisfying the following magnitude response values: $|H(e^{j\pi/4})|=0.5 and |H(e^{j\pi/2})|=1$.
(b) Determine the exact expression for the frequency response of the filter designd, and plot its magnitude and phase response.

第十、十一周

4.6 A noncausal LTI FIR discrete-time system is characterized by an impulse response $h[n]=a_1\delta[n-2]+a_2\delta[n-1]+a_3\delta[n]+a_4\delta[n+1]+a_5\delta[n+2]$. For what values of the impulse response samples will its frequency response $H(e^{j\omega})$ have a zero phase?

4.7 A causal LTI FIR discrete-time system is characterized by an impulse response $h[n]=a_1\delta[n]+a_2\delta[n-1]+a_3\delta[n-2]+a_4\delta[n-3]+a_5\delta[n-4]+a_6\delta[n-5]+a_7$. For what values of the impulse response samples will its frequency response $H(e^{j\omega})$ have a linear phase?

4.21 The frequency response $H(e^{j\omega})$ of a length-4 FIR filter with a real and antisymmetric impulse response has the following specific values: $H(e^{j\pi})=8$, and $H(e^{j\pi/2})=-2+j2$. Determine $H(z)$.

第十二周

7.1 Determine the peak ripple values $\delta_p and \delta_s$ for each of the following sets of peak passband ripple α_p and minimum stopband attenuation α_s :

$$(a)\alpha_p = 0.15dB, \alpha_s = 41dB, \ \ (b)\alpha_p = 0.23dB, \alpha_s = 73dB.$$

7.2 Determine the peak passband ripple α_p and minimum stopband attenuation α_s in dB for each of the following sets of peak ripple values $\delta_p and \delta_s$:

$$(a)\alpha_p = 0.01, \alpha_s = 0.01, \ \ (b)\alpha_p = 0.035, \alpha_s = 023.$$

第十三周

7.4 Let $H(z)$ be the transfer function of a lowpass digital filter with passband edge at ω_p, stopband edge at ω_s, passband ripple of δ_p, and stopband ripple of δ_s, as indicated in Figure 7.1(Third Edition: Figure 9.1(a)). Sketch the magnitude response of the highpass transfer function $H_{LP}(-z)\, for -\pi \le \omega \le \pi$ and determine its passband and stopband edges in terms of ω_p and ω_s.

7.14 The following causal IIR digital transfer functions were designed using the bilinear transformation method with T=2. Determine their respective parent causal analog transfer functions.

$$(a) G_a(z) = \frac{5z^2 + 4z - 1}{8z^2 + 4z}, \quad (b) G_b = \frac{8(z^3 + 3z^2 + 3z + 1)}{(3z+1)(7z^2 + 6z + 3)}$$

第十四周

M7.2 Design a digital Butterworth lowpass filter operating at a sampling rate of 80kHz with a 0.5-dB cutoff frequency at 4kHz and a minimum stopband attenuation of 45dB at 20 kHz using the bilinear transformation method. Determine the order of the analog filter prototype using the formula give in Eq.(5.36)(Third Edition: Eq.4.35) and then design the analog prototype filter using the M-file buttap of Matlab. Transform the analog filter transfer function to the desired digital transfer function using M-file bilinear. Plot the gain and phase response using Matlab. Show all steps used in the design.

M7.3 Modify Program 7_3 to design a digital Butterworth lowpass filter using the bilinear transformation method. The input data required by the modified program should be the desired passband and stopband edges, and maximum passband deviation and the minimum stopband attenuation in dB. Using the modified program, design the digital Butterworth lowpass filter of Exercise M7.2.

第十四周

M7.13 Polt the magnitude response of a linear-phase FIR highpass filter by truncating the impulse response $h_{HP}[n]$ of the ideal highpass filter of Eq.(7.61 to length N=2M+1 for two different values of M and show that the truncated filter exhibits oscillatory bebavior on both sides of the cutoff frequency.

Ep.(7.61) $$h_{HP}[n]=\begin{cases} 1-\dfrac{\omega_c}{\pi}, & for\ n=0 \\ -\dfrac{\sin(\omega_c n)}{\pi n}, & for\ |n|>0 \end{cases}$$

M7.14 Polt the magnitude response of a linear-phase FIR highpass filter by truncating the impulse response $h_{BP}[n]$ of the ideal highpass filter of Eq.(7.62 to length N=2M+1 for two different values of M and show that the truncated filter exhibits oscillatory bebavior on both sides of the cutoff frequency.

Ep.(7.62) $$h_{BP}[n]=\frac{\sin(\omega_{c2}n)}{\pi n}-\frac{\sin(\omega_{c1}n)}{\pi n},\ |n|\geq 0$$