《概率统计》(理工)参考答案及评分标准.

一、填空题(每题5分,共20分)

1. $\underline{1/8}$ 2. $\underline{1/2}$ 3. $\underline{2e^{-3}(1-e^{-3})\text{或}0.095}$ 4. $\underline{\frac{17}{2}e^{-3}\text{或}0.425}$

二、解答题

1. 解: 设事件A表示信源发出"0", $\bar{A}$ 表示发出"1"

15分 B表示接收端收到"1", 则由题意

$P(A)=0.55 \qquad P(\bar{A})=0.45$

由全概率公式, $P(B)=P(A)\cdot P(B|A)+P(\bar{A})\cdot P(B|\bar{A})$

$=0.55\cdot 0.05+0.45\cdot 0.85=0.41$

所求概率为, $P(A|B)=\dfrac{P(A)\cdot P(B|A)}{P(B)}=\dfrac{0.55\times 0.05}{0.41}=\dfrac{11}{164}$.

2. 解: 3分 (1) 由 $\int_{-\infty}^{\infty} f(x)dx=1$ 可得 $a=\frac{3}{4}$

6分 (2)
$$F_X(x)=\begin{cases}0, & x\le -2\\ \frac{1}{4}(x+2), & -2<x\le 0\\ \frac{1}{2}+\frac{3}{4}x-\frac{1}{4}x^3, & 0<x<1\\ 1, & x\ge 1\end{cases}$$

6分 (3) $R(Y)=[1,5]$

$\forall y\in R(Y)$, $F_Y(y)=P(Y\le y)=P(X^2\le y-1)=F_X(\sqrt{y-1})-F_X(-\sqrt{y-1})$

当 $1\le y\le 2$ 时, $F_Y(y)=\sqrt{y-1}-\frac{1}{4}(\sqrt{y-1})^3$

当 $2<y\le 5$ 时 $F_Y(y)=1-\frac{1}{4}(-\sqrt{y-1}+2)=\frac{1}{2}+\frac{1}{4}\sqrt{y-1}$

故 $Y=X^2+1$ 的密度函数为:
$$f_Y(y)=\begin{cases}\frac{1}{2}(y-1)^{-\frac{1}{2}}-\frac{3}{8}(y-1)^{\frac{1}{2}}, & 1\le y\le 2\\ \frac{1}{8}(y-1)^{-\frac{1}{2}}, & 2<y\le 5\\ 0, & \text{others}\end{cases}$$

3. 解：$\because \{Z=1\} = \{X=1, Y=1\} + \{X=0, Y=0\}$

(1) 12分

$\{Z=0\} = \{X=0, Y=1\} + \{X=1, Y=0\}$

$\therefore P(X=1, Z=1) = P(X=1, Y=1) = P(X=1)P(Y=1) = p^2$

$P(X=1, Z=0) = P(X=1, Y=0) = p\cdot(1-p)$

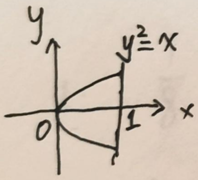
同理可得

$X \backslash Z$	0	1	$p_{i\cdot}$
0	$p(1-p)$	$(1-p)^2$	$1-p$
1	$p(1-p)$	p^2	p
$p_{\cdot j}$	$2p(1-p)$	$p^2+(1-p)^2$	1

(2) 4分 为使X与Z独立，必有 $p_{ij} = p_{i\cdot}\, p_{\cdot j}$，$i=1,2,\ j=1,2$.

即 $2p(1-p)\cdot(1-p) = p(1-p) \Rightarrow p=\frac{1}{2}$

4. 解：6分 (1) $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{-\sqrt{x}}^{\sqrt{x}} \frac{3}{4}\,dy = \frac{3}{2}\sqrt{x}$，$0\le x\le 1$



故 $f_X(x) = \begin{cases} \frac{3}{2}\sqrt{x}, & 0\le x\le 1 \\ 0, & \text{others} \end{cases}$

$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{y^2}^{1} \frac{3}{4}\,dx = \frac{3}{4}(1-y^2)$，$-1\le y\le 1$

故 $f_Y(y) = \begin{cases} \frac{3}{4}(1-y^2), & -1\le y\le 1 \\ 0, & \text{others} \end{cases}$

6分 (2) 对 $-1<y<1$，

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{\frac{3}{4}}{\frac{3}{4}(1-y^2)} = \frac{1}{1-y^2}\quad y^2\le x\le 1$$

6分 (3) 当 $Y=\frac{1}{4}$ 时 $f_{X|Y}(x|y) = \frac{16}{15}$，$\frac{1}{16}\le x\le 1$

$$P(X\le\tfrac{1}{2}\mid Y=\tfrac{1}{4}) = \int_{\frac{1}{16}}^{\frac{1}{2}} \frac{16}{15}\,dx = \frac{7}{15}$$

6分 (4) $E(\eta)=\int_{-1}^{1} y\, f_\eta(y)\,dy=\int_{-1}^{1} y\cdot\frac{3}{4}(1-y^2)\,dy=0$

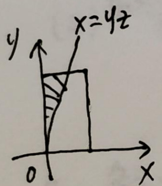
$E(\eta^2)=\int_{-1}^{1} y^2 f_\eta(y)\,dy=\int_{-1}^{1} y^2\cdot\frac{3}{4}(1-y^2)\,dy=\frac{1}{5}$

$D(\eta)=E(\eta^2)-(E\eta)^2=\frac{1}{5}$

5. 解: (1) $R(Z)=[0,+\infty)$

10分 (2) $\forall z\in R(Z)$, $F_Z(z)=P(\frac{\xi}{\eta}\le z)$

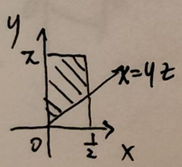
当 $0<z\le\frac{1}{2\pi}$ 时,



$F_Z(z)=\int_0^{\pi}\int_0^{yz}\sin y\,dx\,dy$

$=\int_0^{\pi}\sin y\cdot yz\,dy=z\cdot\int_0^{\pi} y\cdot\sin y\,dy=z\cdot\pi.$

当 $z>\frac{1}{2\pi}$ 时.



$F_Z(z)=\int_0^{\frac{1}{2}}\int_{\frac{1}{z}x}^{\pi}\sin y\,dy\,dx$

$=\frac{1}{2}+z\cdot\sin(\frac{1}{2z})$

(3) $f_Z(z)=\begin{cases}0, & z<0\\ \pi, & 0<z\le\frac{1}{2\pi}\\ \sin\frac{1}{2z}-\frac{1}{2z}\cos\frac{1}{2z}, & z>\frac{1}{2\pi}\end{cases}$