Frequency Response Parameters of Expression (2)

Danyang Xu

Let us revisit Expression (2) in the manuscript:

$$\Delta f\left(\tau\right)= \begin{cases} \frac{c_1}{b}\left(1-e^{-\frac{b}{a}\tau}\right) & 0<\tau\leq t_W \\ \lambda_2 e^{-\frac{b}{a}\tau}+\frac{k_2}{b}\tau+\frac{c_2 b-ak_2}{b^2} & t_W<\tau\leq t_G \\ \lambda_3 e^{-\frac{b}{a}\tau}+\frac{k_3}{b}\tau+\frac{c_3 b-ak_3}{b^2} & t_G<\tau\leq t_W+T_W \\ \lambda_4 e^{-\frac{b}{a}\tau}+\frac{k_4}{b}\tau+\frac{c_4 b-ak_4}{b^2} & t_W+T_W<\tau\leq t_G+T_G \\ \lambda_5 e^{-\frac{b}{a}\tau}+\frac{c_5}{b} & t_G+T_G<\tau \end{cases} . \tag{S1}$$

The specific expressions for each parameter are as follows:

$$\begin{cases} \lambda_1=0 \\ \lambda_2=\lambda_1+e^{\frac{b}{a}t_1}\left(\frac{c_1}{b}\left(1-e^{-\frac{b}{a}t_1}\right)-\frac{k_2}{b}t_1-\frac{c_2 b-ak_2}{b^2}\right) \\ \lambda_3=\lambda_2+e^{\frac{b}{a}t_2}\left(\frac{k_2}{b}t_2+\frac{c_2 b-ak_2}{b^2}-\frac{k_3}{b}t_2-\frac{c_3 b-ak_3}{b^2}\right) \\ \lambda_4=\lambda_3+e^{\frac{b}{a}t_3}\left(\frac{k_3}{b}t_3+\frac{c_3 b-ak_3}{b^2}-\frac{k_4}{b}t_3-\frac{c_4 b-ak_4}{b^2}\right) \\ \lambda_5=\lambda_4+e^{\frac{b}{a}t_4}\left(\frac{k_4}{b}t_4+\frac{c_4 b-ak_4}{b^2}-\frac{c_5}{b}\right) \end{cases} , \tag{S2}$$

$$\begin{cases} a=\frac{2H^{sys}}{f_0} \\ b=D^{sys} \end{cases} , \tag{S3}$$

$$\begin{cases} k_1=k_5=0 \\ k_2=\frac{R^W}{T_W} \\ k_3=\frac{R^W}{T_W}+\frac{R^G}{T_G} \\ k_4=\frac{R^G}{T_G} \end{cases} , \tag{S4}$$

$$\begin{cases} c_1=-\Delta P^{imb} \\ c_2=-\frac{R^W}{T_W}t_W-\Delta P^{imb} \\ c_3=-\frac{R^W}{T_W}t_W-\frac{R^G}{T_G}t_G-\Delta P^{imb} \\ c_4=R^W-\frac{R^G}{T_G}t_G-\Delta P^{imb} \\ c_5=R^W+R^G-\Delta P^{imb} \end{cases} , \tag{S5}$$

where $t_1=t_W, t_2=t_G, t_3=t_W+T_W, t_4=t_G+T_G$.