1. Network representation

Assume that an active distribution network (ADN) is represented by a graph $(\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ denotes the set of nodes and $\mathcal{L}$ denotes the set of branches, with $i \in \mathcal{N}$ and $ij \in \mathcal{L}$. Let node 0 be designated as the root node of the ADN. The set of all non-root nodes is then defined as $\mathcal{N}^{\dagger} = \mathcal{N} \setminus \{0\}$. It is assumed that each node is equipped with an equivalent distributed energy resource (DER), which may include conventional generators (CG), energy storage systems (ESS), and renewable energy sources (RES).

2. Network constraints

The network constraints of the ADN are described using the linear DistFlow model. (1)–(4) represent the active and reactive power injection formulations at the root node and non-root nodes, respectively. (5) describes the voltage drop across distribution lines. (6) enforces that DERs operate under a constant power factor mode. (7) specifies the reference voltage at the root node. (8) imposes limits on the voltage magnitudes at all nodes. (9) defines the active power output of DERs at each node, while (10) corresponds to their reactive power output.

$$P_{0,t} = P_t^E - \sum_{j \in \{0\}} P_{0j,t} - P_{0,t}^{\mathrm{L}} + P_{0,t}^{\mathrm{D}}, \forall t \in \mathcal{T} \tag{1}$$

$$P_{i,t} = -\sum_{j \in \{i\}} P_{ij,t} - P_{i,t}^{\mathrm{L}} + P_{i,t}^{\mathrm{D}}, \forall i \in \mathcal{N}^{\dagger}, t \in \mathcal{T} \tag{2}$$

$$Q_{0,t} = -\sum_{j \in \{0\}} Q_{0j,t} - Q_{0,t}^{\mathrm{L}} + Q_{0,t}^{\mathrm{D}}, \forall t \in \mathcal{T} \tag{3}$$

$$Q_{i,t} = -\sum_{j \in \{i\}} Q_{ij,t} - Q_{i,t}^{\mathrm{L}} + Q_{i,t}^{\mathrm{D}}, \forall i \in \mathcal{N}^{\dagger}, t \in \mathcal{T} \tag{4}$$

$$V_{i,t} - V_{j,t} = 2\left(r_{ij} P_{ij,t} + x_{ij} Q_{ij,t}\right), \forall ij \in \mathcal{L}, t \in \mathcal{T} \tag{5}$$

$$P_{i,t}^{\mathrm{D}} = \tan \varphi_i Q_{i,t}^{\mathrm{D}}, \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{6}$$

$$V_{0,t} = 1, \forall t \in \mathcal{T} \tag{7}$$

$$\underline{V} \le V_{i,t} \le \bar{V}, \forall i \in \mathcal{N}^{\dagger}, t \in \mathcal{T} \tag{8}$$

$$P_{i,t}^{\mathrm{D}} = \sum_{g \in \mathcal{G}(i)} P_{g,t}^{\mathrm{CG}} + \sum_{e \in \mathcal{E}(i)} \left(P_{e,t}^{\mathrm{dch}} - P_{e,t}^{\mathrm{ch}}\right) + \sum_{s \in \mathcal{S}(i)} P_{s,t}^{\mathrm{RES}}, \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{9}$$

$$Q_{i,t}^{\mathrm{D}} = \sum_{g \in \mathcal{G}(i)} Q_{g,t}^{\mathrm{CG}} + \sum_{s \in \mathcal{S}(i)} Q_{s,t}^{\mathrm{RES}}, \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{10}$$

3. DER constraints

3.1 CGs constraints

The constraints of CGs consist of power output limits and ramping constraints, which can be expressed as follows:

$$\underline{P}_{g}^{\mathrm{CG}} \leq P_{g,t}^{\mathrm{CG}} \leq \overline{P}_{g}^{\mathrm{CG}}, \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{11}$$

$$P_{g}^{\mathrm{RD}} \leq P_{g,t+1}^{\mathrm{CG}} - P_{g,t}^{\mathrm{CG}} \leq P_{g}^{\mathrm{RU}}, \forall g \in \mathcal{G}, t \in \mathcal{T} \setminus \{|\mathcal{T}|\} \tag{12}$$

$$\underline{Q}_{g}^{\mathrm{CG}} \leq Q_{g,t}^{\mathrm{CG}} \leq \overline{Q}_{g}^{\mathrm{CG}}, \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{13}$$

$$Q_{g}^{\mathrm{RD}} \leq Q_{g,t+1}^{\mathrm{CG}} - Q_{g,t}^{\mathrm{CG}} \leq Q_{g}^{\mathrm{RU}}, \forall g \in \mathcal{G}, t \in \mathcal{T} \setminus \{|\mathcal{T}|\} \tag{14}$$

3.2 ESSs constraints

The constraints of ESSs consist of power and energy limitations, which are formulated as follows:

$$\begin{cases} 0 \leq P_{e,t}^{\mathrm{dch}} \leq \overline{P}_{e}^{\mathrm{dch}} \\ 0 \leq P_{e,t}^{\mathrm{ch}} \leq \overline{P}_{e}^{\mathrm{ch}} \end{cases}, \forall e \in \mathcal{E}, t \in \mathcal{T} \tag{15}$$

$$E_{e,t}^{\min} \leq \sum_{\tau=1}^{t} \left(\frac{P_{e,t}^{\mathrm{dch}}}{\eta_{e}^{\mathrm{dch}}} - \eta_{e}^{\mathrm{ch}} P_{e,t}^{\mathrm{ch}} \right) \Delta tp \leq E_{e,t}^{\max}, \forall e \in \mathcal{E}, t \in \mathcal{T} \tag{16}$$

3.2 RESs constraints

The output power of RESs is subject to upper and lower bounds determined by forecast availability:

$$0 \leq P_{s,t}^{\mathrm{RES}} \leq P_{s,t}^{\mathrm{Fore}}, \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{17}$$

$$0 \leq Q_{s,t}^{\mathrm{RES}} \leq Q_{s,t}^{\mathrm{Fore}}, \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{18}$$

4. Nomenclature

Indices and sets

i / $\mathcal{N}$	Index/Set of nodes in the ADN, where $i = 0$ represents the root node.
t / $\mathcal{T}$	Index/Set of time periods.
g / $\mathcal{G}$	Index/Set of CGs in ADN.
e / $\mathcal{E}$	Index/Set of ESSs in ADN.
s / $\mathcal{S}$	Index/Set of RESs in ADN.
ij / $\mathcal{L}$	Index/Set of branches in ADN.
$\mathcal{G}(i)$ / $\mathcal{E}(i)$ / $\mathcal{S}(i)$	Set of CGs/ESSs/RESs connected to node i.

Variables

$P_{i,t}$ / $Q_{i,t}$	Active/Reactive power of node i in the ADN at time period t.
$P_{i,t}^{\mathrm{D}}$ / $Q_{i,t}^{\mathrm{D}}$	Active/Reactive of DER connected to node i in the ADN at time period t.
$P_{ij,t}$ / $Q_{ij,t}$	Active/Reactive power flow of branch ij in the ADN at time period t.
P_{t}^{E}	Power injection from transmission system at time period t.

$V_{i,t}$	Voltage of node *i* in the AND at time period *t*.
$P_{g,t}^{\mathrm{CG}}$ / $Q_{g,t}^{\mathrm{CG}}$	Active/Reactive power of CG *g* in the ADN at time period *t*.
$P_{e,t}^{\mathrm{dch}}$ / $P_{e,t}^{\mathrm{ch}}$	Discharging/Charing power of ESS *e* in the ADN at time period *t*.
$P_{s,t}^{\mathrm{RES}}$ / $Q_{s,t}^{\mathrm{RES}}$	Active/Reactive power of RES *s* in the ADN at time period *t*.

Parameters

r_{ij} / x_{ij}	Resistance and Reactance of branch *ij* in the ADN.
$P_{i,t}^{\mathrm{L}}$ / $Q_{i,t}^{\mathrm{L}}$	Active/Reactive load demand of node *i* in the ADN at time period *t*.
$\tan\varphi_i$	Power factor of DER connected to node *i* in the ADN.
$\bar{V}$ / $\underline{V}$	Square of upper and lower voltage limit of node *i* in the ADN.
$\bar{P}_g^{\mathrm{CG}}$ / $\underline{P}_g^{\mathrm{CG}}$	Upper and Lower active power output limit of CG *g* in the ADN.
$\bar{P}_g^{\mathrm{CG}}$ / $\underline{P}_g^{\mathrm{CG}}$	Upper and Lower active power output limit of CG *g* in the ADN.
P_g^{RU} / P_g^{RD}	Upward and Downward active power ramping limit of CG *g* in the ADN.
$\bar{Q}_g^{\mathrm{CG}}$ / $\underline{Q}_g^{\mathrm{CG}}$	Upper and Lower reactive power output limit of CG *g* in the ADN.
Q_g^{RU} / Q_g^{RD}	Upward and Downward reactive power ramping limit of CG *g* in the ADN.
$\bar{P}_e^{\mathrm{dch}}$ / $\bar{P}_e^{\mathrm{ch}}$	Discharging power/Charing power upper limit of ESS *e* in the ADN.
η_e^{dch} / η_e^{ch}	Discharging /Charing efficiencies of ESS *e* in the ADN.
$E_{e,t}^{\max}$ / $E_{e,t}^{\min}$	Upper and Lower energy limit of ESS *e* in the ADN.
Δtp	Time interval length
$P_{s,t}^{\mathrm{Fore}}$ / $Q_{s,t}^{\mathrm{Fore}}$	Active/Reactive forecast power of RES *s* in the ADN at time period *t*.