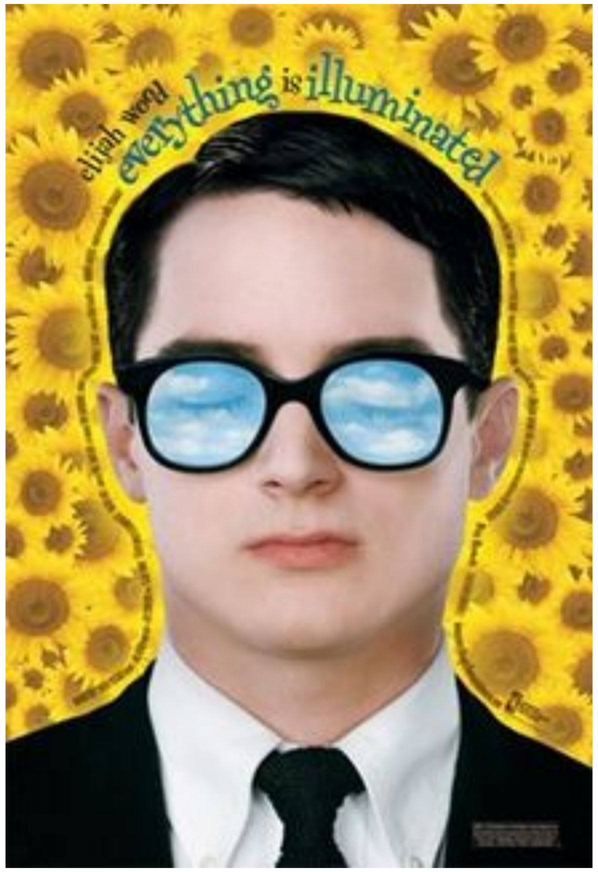
Language Models

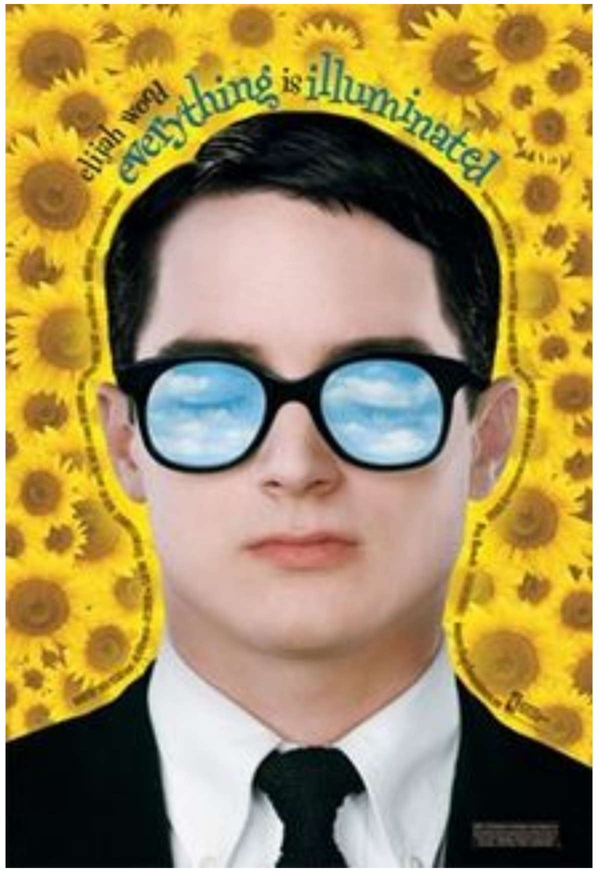
January 28, 2014

Still no MT??

- Today we will talk about models of p(sentence)
- The rest of this semester will deal with
  p(translated sentence | input sentence)
- Why do it this way?
  - Conditioning on more stuff makes modeling more complicated. That is: p(sentence) is easier than p(translated sentence | input sentence).
  - Language models are arguably the most important models in statistical MT



My legal name is Alexander Perchov.



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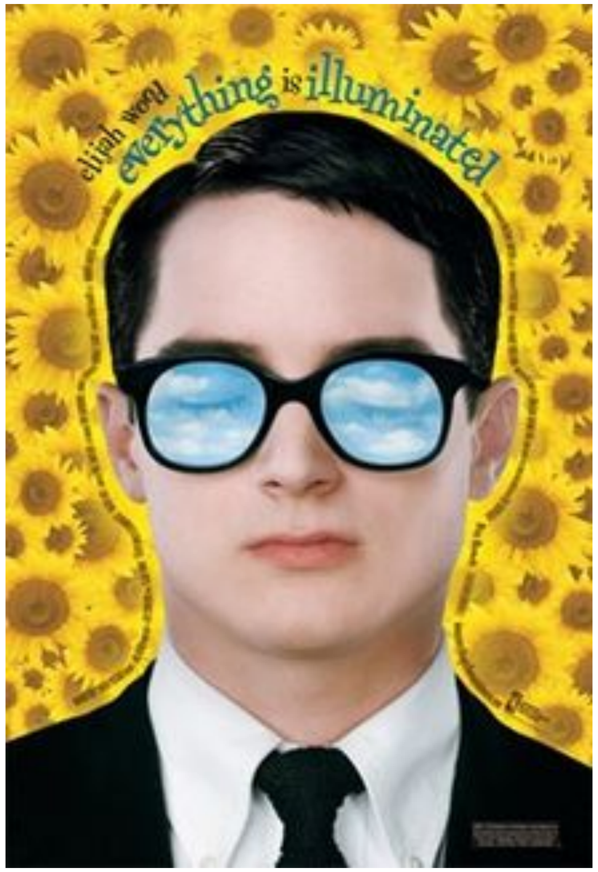

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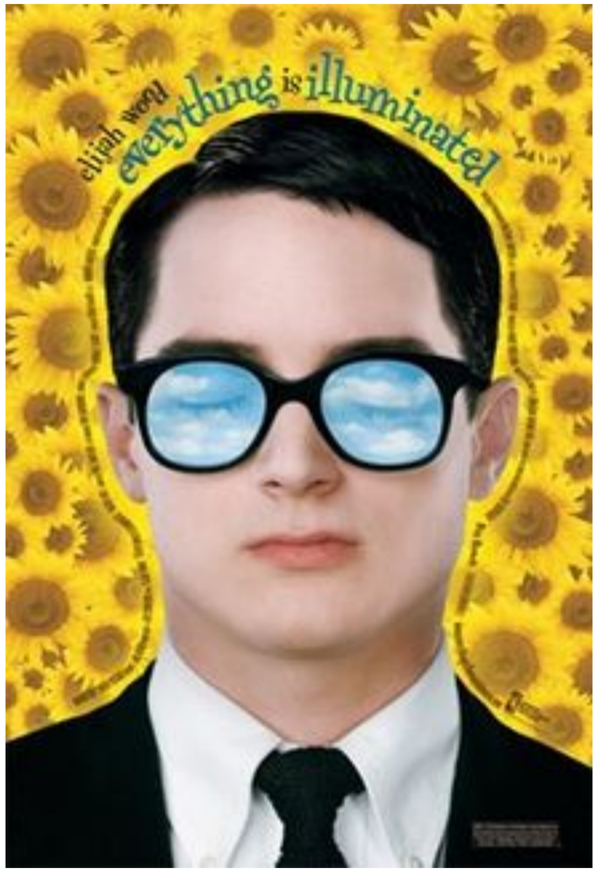

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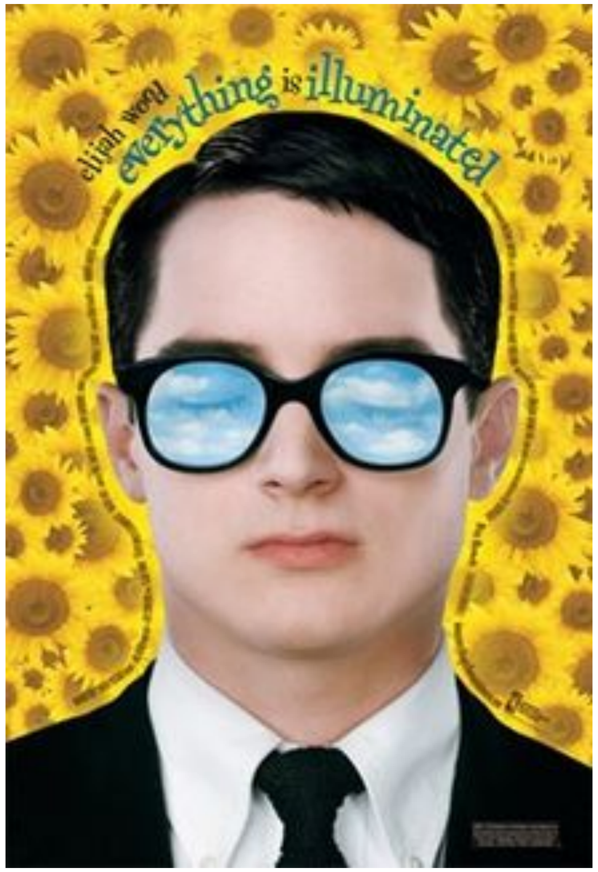

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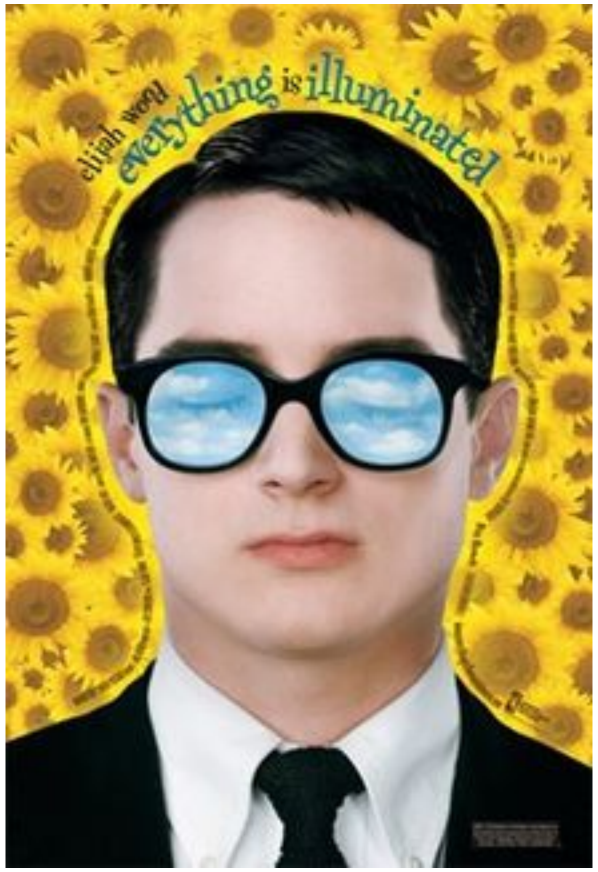

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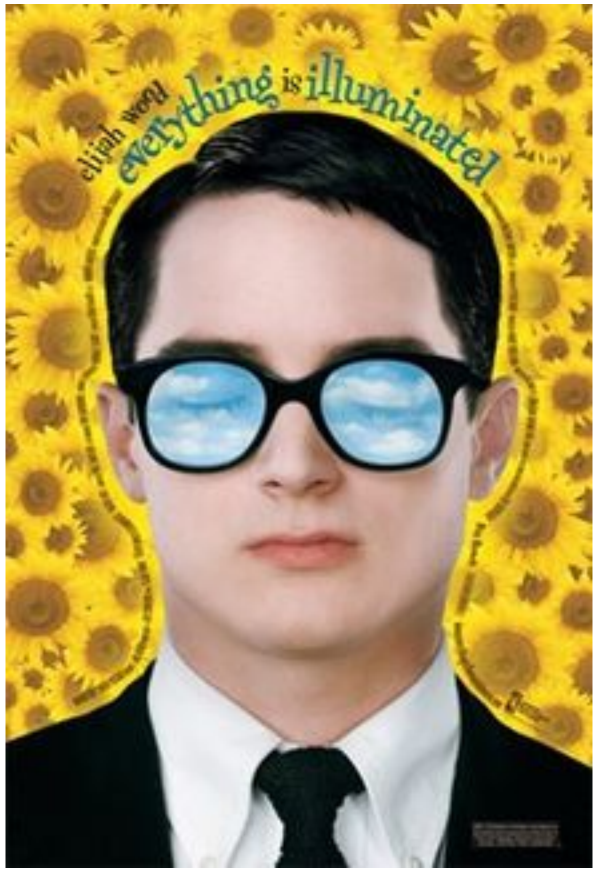

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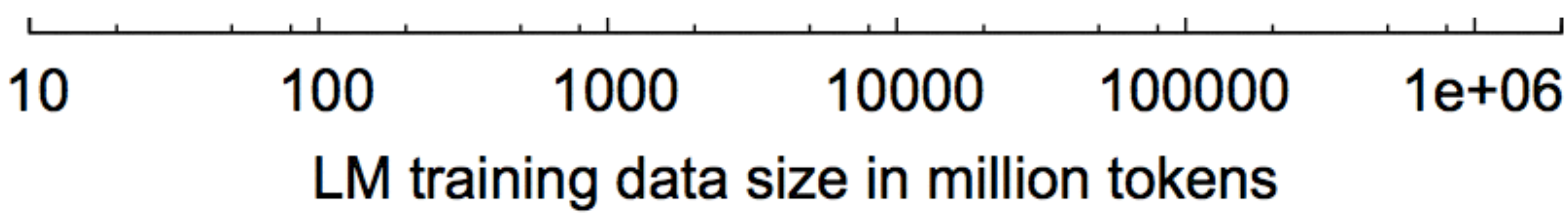
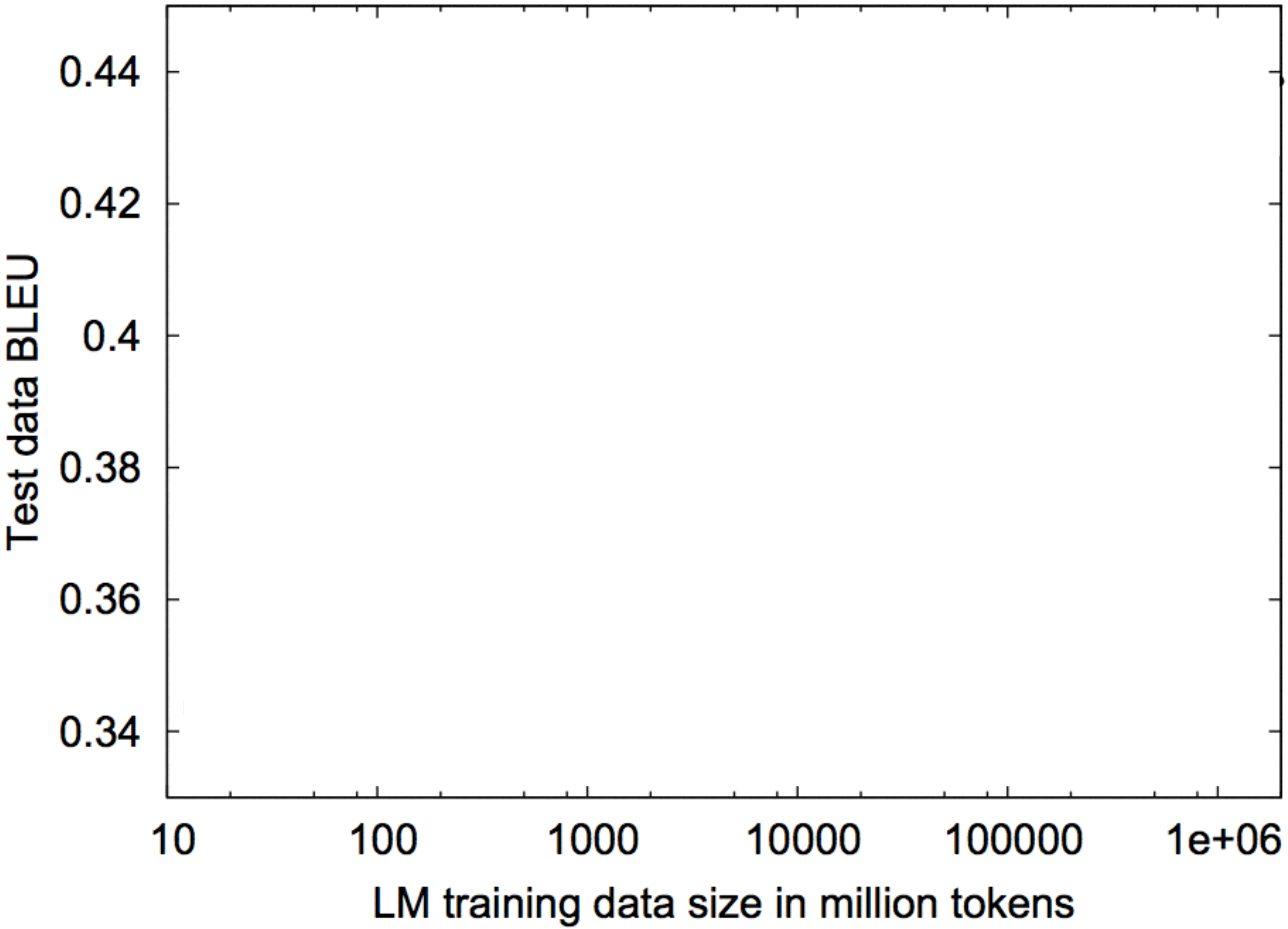
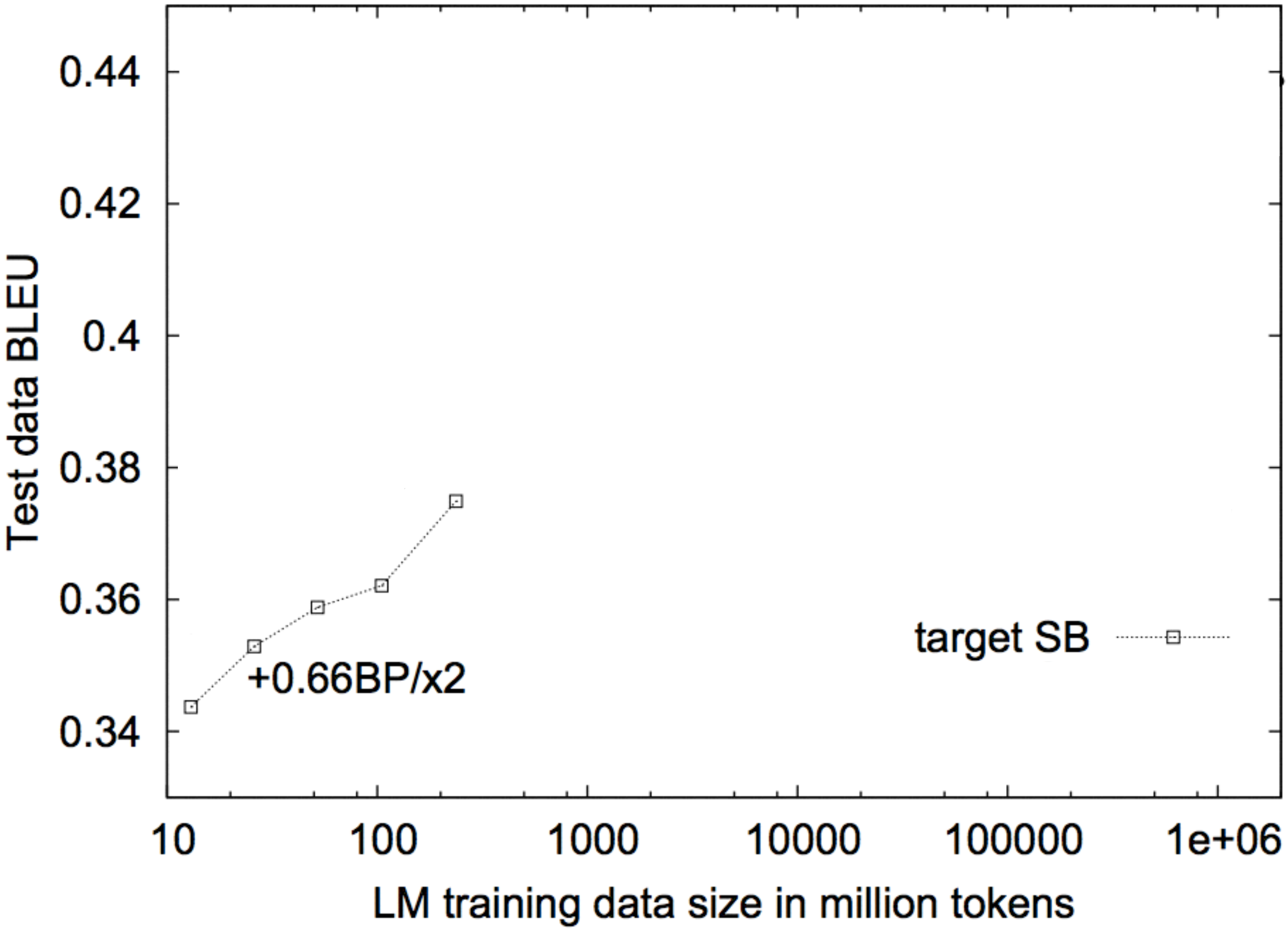
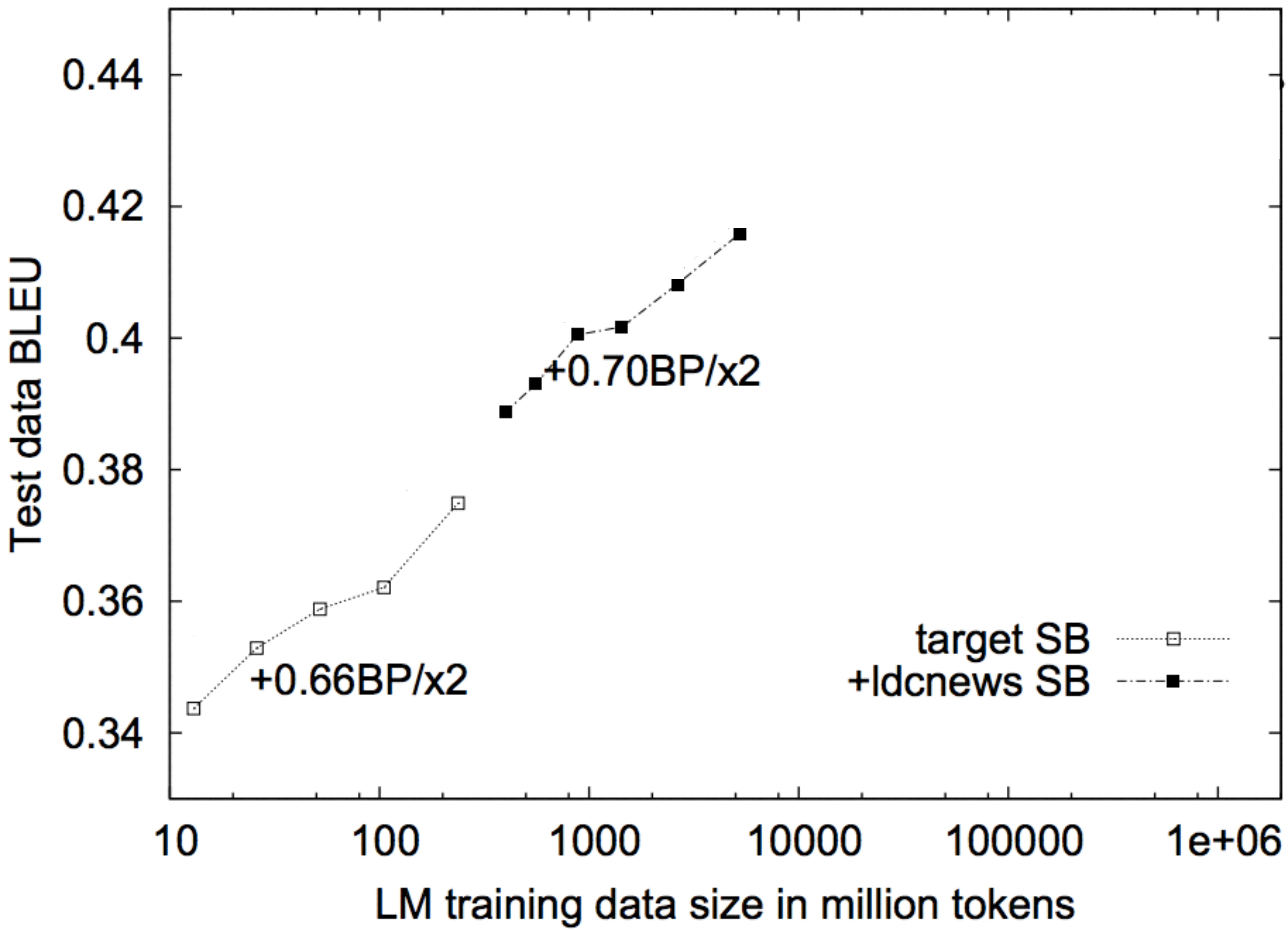

10
100
1000
10000
100000
1e+06
LM training data size in million tokens



Test data BLEU
0.44
0.42
0.4
0.38
0.36
0.34
10
100
1000
10000
100000
1e+06
LM training data size in million tokens



Test data BLEU
0.44
0.42
0.4
0.38
0.36
0.34
+0.66BP/x2
target SB
10
100
1000
10000
100000
1e+06
LM training data size in million tokens



0.44
0.42
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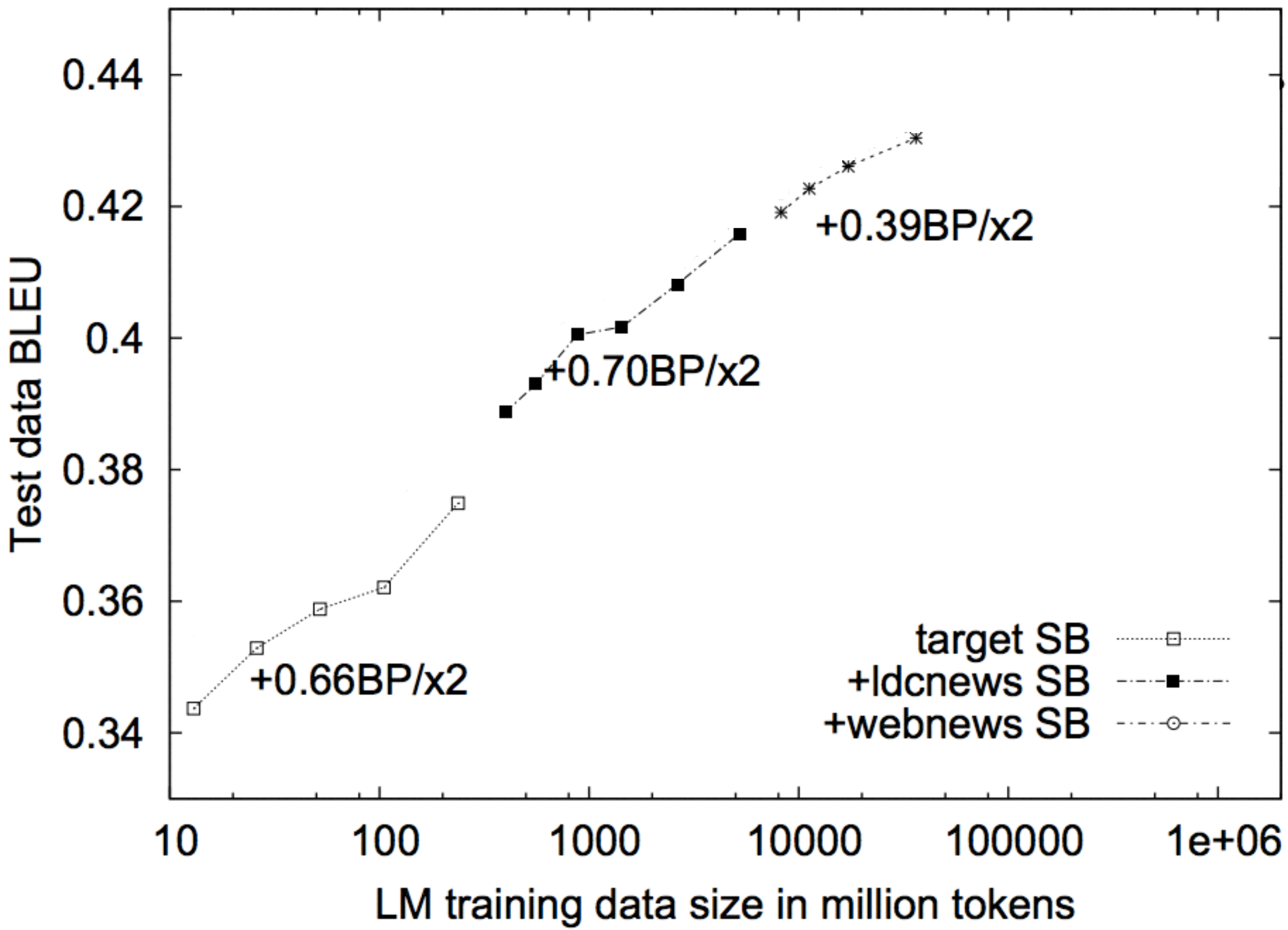
0.36
0.34
Test data BLEU
10
100
1000
10000
100000
1e+06
LM training data size in million tokens
+0.70BP/x2
+0.66BP/x2
target SB
+ldcnews SB



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0.36
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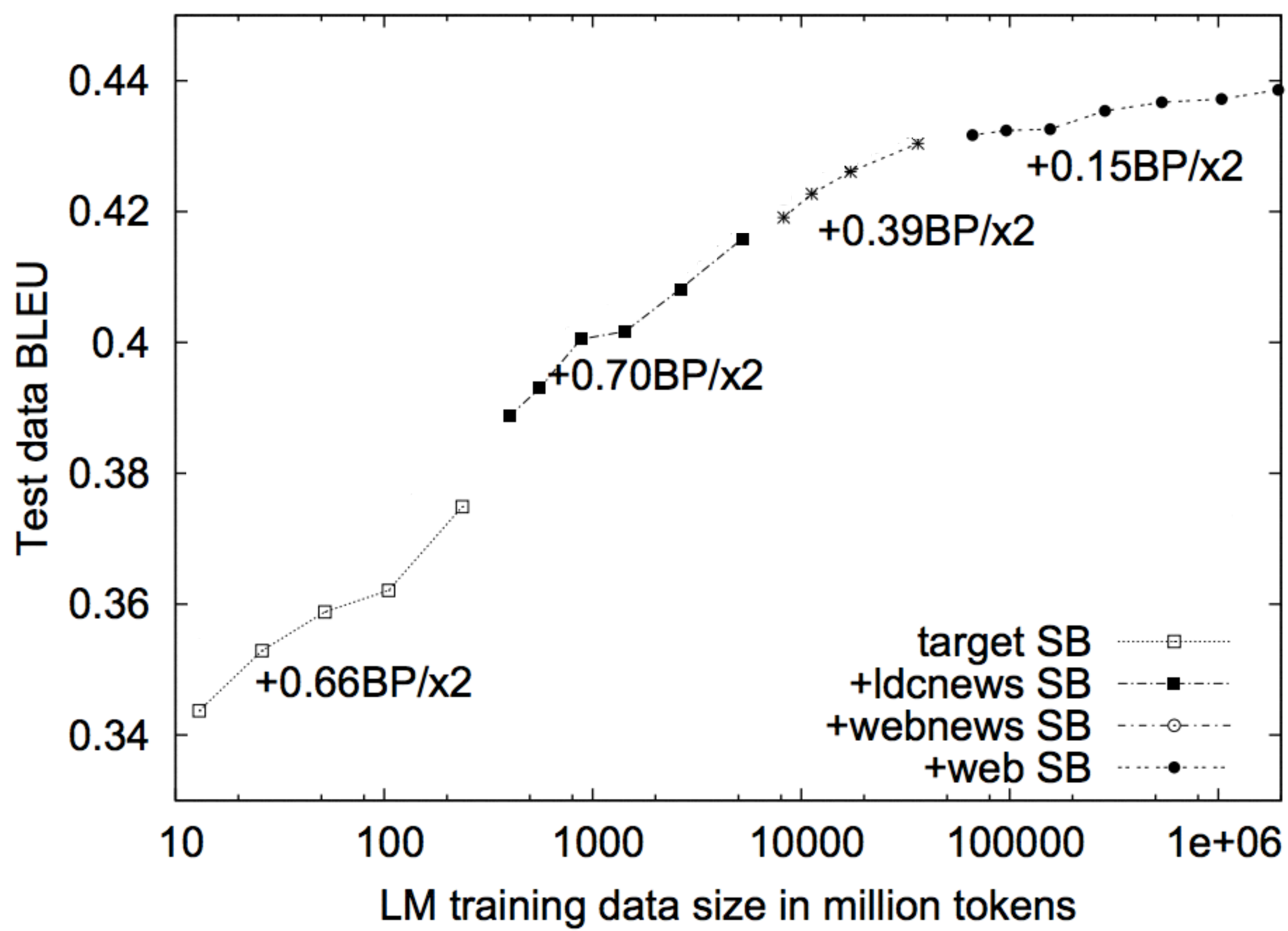
10
100
1000
10000
100000
1e+06
LM training data size in million tokens
+0.39BP/x2
+0.70BP/x2
+0.66BP/x2
target SB
+ldcnews SB
+webnews SB



Test data BLEU
0.44
0.42
0.4
0.38
0.36
0.34
+0.15BP/x2
+0.39BP/x2
+0.70BP/x2
+0.66BP/x2
target SB
+ldcnews SB
+webnews SB
+web SB
10
100
1000
10000
100000
1e+06
LM training data size in million tokens

Language Models Matter

- Language models play the role of ...
  - a judge of *grammaticality*
  - a judge of *semantic plausibility*
  - an enforcer of *stylistic consistency*
  - a repository of knowledge (?)

What is the probability of a sentence?

- Requirements
  - Assign a probability to *every sentence* (i.e., string of words)

What is the probability of a sentence?

- Requirements
  - Assign a probability to *every sentence* (i.e., string of words)

$$\sum_{\mathbf{e} \in \Sigma^*} p_{\text{LM}}(\mathbf{e}) = 1$$

$$p_{\text{LM}}(\mathbf{e}) \geq 0 \quad \forall \mathbf{e} \in \Sigma^*$$

Why do we want to estimate the probability of a sentence?

- Goal: Assign a higher probability to *good sentences in English*

p_{LM}*(the house is small)* > p_{LM}*(small the is house)*

translations of German *Haus: home, house …*

p_{LM}*(I am going home)* > p_{LM}*(I am going house)*

n-gram LMs

$$p_{\mathrm{LM}}(\mathbf{e})$$

Vector-valued random variable

n-gram LMs

$$
\begin{aligned}
p_{\mathrm{LM}}(\mathbf{e}) = & p(e_1, e_2, e_3, \ldots, e_\ell) \\
\approx & p(e_1) \times \\
& p(e_2 \mid e_1) \times \\
& p(e_3 \mid \cancel{e_1,} e_2) \times \\
& p(e_4 \mid \cancel{e_1, e_2,} e_3) \times \\
& \cdots \times \\
& p(e_\ell \mid \cancel{e_1, e_2, \ldots, e_{\ell-2},} e_{\ell-1})
\end{aligned}
$$

Chain rule

The **chain rule** is derived from a repeated application of the definition of conditional probability:

$$p(a, b, c, d)$$

Conditional Independence

$$
\begin{aligned}
p(a, b, c) &= p(a \mid b, c)p(b, c) \\
&= p(a \mid b, c)p(b \mid c)p(c)
\end{aligned}
$$

"If I know B, then C doesn't tell me about A"

$$p(a \mid b, c) = p(a \mid b)$$

$$
\begin{aligned}
p(a, b, c) &= p(a \mid b, c)p(b, c) \\
&= p(a \mid b, \cancel{c})p(b \mid c)p(c) \\
&= p(a \mid b)p(b \mid c)p(c)
\end{aligned}
$$

Is the Markov assumption valid for Language?

- the old man are/is
- the pictures are/is
- The old man in the pictures is my dad.

n-gram LMs

$$
\begin{aligned}
p_{\mathrm{LM}}(\mathbf{e}) =& p(e_1, e_2, e_3, \ldots, e_\ell) \\
\approx& p(e_1) \times \\
& p(e_2 \mid e_1) \times \\
& p(e_3 \mid \cancel{e_1,}\, e_2) \times \\
& p(e_4 \mid \cancel{e_1, e_2,}\, e_3) \times \\
& \cdots \times \\
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\end{aligned}
$$

$$
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& \cdots \times \\
& p(e_\ell \mid \cancel{e_1, e_2, \ldots,}\, e_{\ell-2}, e_{\ell-1})
\end{aligned}
$$

Which do you think is better? Why?

n-gram LMs

$$
\begin{aligned}
p_{\mathrm{LM}}(\mathbf{e}) = & p(e_1, e_2, e_3, \ldots, e_\ell) \\
\approx & p(e_1) \times \\
& p(e_2 \mid e_1) \times \\
& p(e_3 \mid \cancel{e_1,}\ e_2) \times \\
& p(e_4 \mid \cancel{e_1, e_2,}\ e_3) \times \\
& \cdots \times \\
& p(e_\ell \mid \cancel{e_1, e_2, \ldots, e_{\ell-2},}\ e_{\ell-1})
\end{aligned}
$$

$$
= p(e_1 \mid \mathrm{START}) \times \prod_{i=2}^{\ell} p(e_i \mid e_{i-1}) \times p(\mathrm{STOP} \mid e_\ell)
$$

START my friends call me Alex STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{call} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{call}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

START my friends dub me Alex STOP

$$p(\texttt{my} \mid \texttt{START}) \times p(\texttt{friends} \mid \texttt{my}) \times p(\texttt{dub} \mid \texttt{friends}) \times p(\texttt{me} \mid \texttt{dub}) \times p(\texttt{Alex} \mid \texttt{me}) \times p(\texttt{STOP} \mid \texttt{Alex})$$

These sentences have many terms in common.

Categorical Distributions

A **categorical distribution** characterizes a random event that can take on exactly one of *K* possible outcomes.

(*nb*. we often call these "multinomial distributions")

$$p(x) = \begin{cases} p_1 & \text{if } x = 1 \\ p_2 & \text{if } x = 2 \\ \ldots & \\ p_K & \text{if } x = K \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_i p_i = 1$$

$$p_i \geq 0 \quad \forall i$$

$$p(\cdot)$$

Outcome	p
the	0.3
and	0.1
said	0.04
says	0.004
of	0.12
why	0.008
Why	0.0007
restaurant	0.00009
destitute	0.00000064

Probability tables like this are the workhorses of language (and translation) modeling.

$p(\cdot \mid \text{some context})$

Outcome	p
the	0.6
and	0.04
said	0.009
says	0.00001
of	0.1
why	0.1
Why	0.00008
restaurant	0.0000008
destitute	0.00000064

$p(\cdot \mid \text{other context})$

Outcome	p
the	0.01
and	0.01
said	0.003
says	0.009
of	0.002
why	0.003
Why	0.0006
restaurant	0.2
destitute	0.1

~~$p(\cdot \mid \text{some context})$~~ $p(\cdot \mid \texttt{in})$

Outcome	p
the	0.6
and	0.04
said	0.009
says	0.00001
of	0.1
why	0.1
Why	0.00008
restaurant	0.0000008
destitute	0.00000064

~~$p(\cdot \mid \text{other context})$~~ $p(\cdot \mid \texttt{the})$

Outcome	p
the	0.01
and	0.01
said	0.003
says	0.009
of	0.002
why	0.003
Why	0.0006
restaurant	0.2
destitute	0.1

LM Evaluation

- Extrinsic evaluation: build a new language model, use it for some task (MT, ASR, etc.)
- Intrinsic: measure how good we are at modeling language

We will use **perplexity** to evaluate models

Given: $\mathbf{w}, p_{\mathrm{LM}}$

$$\mathrm{PPL} = 2^{\frac{1}{|\mathbf{w}|} \log_2 p_{\mathrm{LM}}(\mathbf{w})}$$

$$0 \leq \mathrm{PPL} \leq \infty$$

Perplexity

- Generally fairly good correlations with machine translation quality for *n*-gram models
- Perplexity is a generalization of the notion of branching factor
  - How many choices do I have at each position?
- State-of-the-art English LMs have PPL of ~100 word choices per position
- A uniform LM has a perplexity of $|\Sigma|$
- Humans do much better
- ... and bad models can do even *worse* than uniform!

Whence parameters?

Estimation.

$$p(x \mid y) = \frac{p(x, y)}{p(y)}$$

$$\hat{p}_{\text{MLE}}(x) = \frac{\text{count}(x)}{N}$$

$$\hat{p}_{\text{MLE}}(x, y) = \frac{\text{count}(x, y)}{N}$$

$$\hat{p}_{\text{MLE}}(x \mid y) = \frac{\text{count}(x, y)}{N} \times \frac{N}{\text{count}(y)}$$

$$\boxed{= \frac{\text{count}(x, y)}{\text{count}(y)}}$$

$$\hat{p}_{\text{MLE}}(\texttt{call} \mid \texttt{friends}) = \frac{\text{count}(\texttt{friends call})}{\text{count}(\texttt{friends})}$$

MLE & Perplexity

- What is the **lowest (best) perplexity possible** for your model class?
- Compute the MLE!
- Well, that's easy...

START	my	friends	call	me	Alex	STOP
	$p(\texttt{my} \mid \texttt{START})\times$	$p(\texttt{friends} \mid \texttt{my})\times$	$p(\texttt{call} \mid \texttt{friends})\times$	$p(\texttt{me} \mid \texttt{call})\times$	$p(\texttt{Alex} \mid \texttt{me})\times$	$p(\texttt{STOP} \mid \texttt{Alex})$
MLE	-3.65172	-2.07101	-3.32231	-0.271271	-4.961	-1.96773

START	my	friends	dub	me	Alex	STOP
	$p(\texttt{my} \mid \texttt{START})\times$	$p(\texttt{friends} \mid \texttt{my})\times$	$p(\texttt{dub} \mid \texttt{friends})\times$	$p(\texttt{me} \mid \texttt{dub})\times$	$p(\texttt{Alex} \mid \texttt{me})\times$	$p(\texttt{STOP} \mid \texttt{Alex})$
MLE	-3.65172	-2.07101	$-\infty$	-2.54562	-4.961	-1.96773

MLE assigns probability zero to unseen events

Zeros

- Two kinds of zero probs:
  - **Sampling zeros**: zeros in the MLE due to impoverished observations
  - **Structural zeros**: zeros that should be there. *Do these really exist?*
- Just because you haven't seen something, doesn't mean it doesn't exist.
- In practice, we don't like probability zero, even if there is an argument that it is a structural zero.

***the a** 's are nearing the end of their lease in oakland*

Smoothing

Smoothing an refers to a family of *estimation techniques* that seek to model important general patterns in data while avoiding modeling noise or sampling artifacts. In particular, for language modeling, we seek

$$p(\mathbf{e}) > 0 \quad \forall \mathbf{e} \in \Sigma^*$$

We will assume that Σ is known and finite.

Add-1 Smoothing

$$p(x \mid y) = \frac{p(x, y)}{p(y)}$$

$$\hat{p}_{\mathrm{MLE}}(x) = \frac{\operatorname{count}(x)}{N}$$

$$\hat{p}_{\mathrm{MLE}}(x, y) = \frac{\operatorname{count}(x, y)}{N}$$

$$\hat{p}_{\mathrm{MLE}}(x \mid y) = \frac{\operatorname{count}(x, y)}{N} \times \frac{N}{\operatorname{count}(y)}$$

$$= \frac{\operatorname{count}(x, y) + 1}{\operatorname{count}(y) + V}$$

What's wrong with this?

$$p(x \mid y) = \frac{p(x, y)}{p(y)}$$

$$\hat{p}_{\mathrm{MLE}}(x) = \frac{\mathrm{count}(x)}{N}$$

$$\hat{p}_{\mathrm{MLE}}(x, y) = \frac{\mathrm{count}(x, y)}{N}$$

$$\hat{p}_{\mathrm{MLE}}(x \mid y) = \frac{\mathrm{count}(x, y)}{N} \times \frac{N}{\mathrm{count}(y)}$$

$$= \frac{\mathrm{count}(x, y) + 1}{\mathrm{count}(y) + \Sigma}$$

Add-α Smoothing

- Add α<<1
- An optimal α can be analytically derived so that it gives an appropriate weight to unseen n-grams
- Simplest possible smoother
- But it doesn't work well for language models

Interpolation

- "Mixture of MLEs"

$$\begin{aligned}\hat{p}(\texttt{dub} \mid \texttt{my friends}) = &\lambda_3 \hat{p}_{\mathrm{MLE}}(\texttt{dub} \mid \texttt{my friends}) \\ &+ \lambda_2 \hat{p}_{\mathrm{MLE}}(\texttt{dub} \mid \texttt{friends}) \\ &+ \lambda_1 \hat{p}_{\mathrm{MLE}}(\texttt{dub}) \\ &+ \lambda_0 \frac{1}{|\Sigma|}\end{aligned}$$

Where do the lambdas come from?

Discounting

Discounting adjusts the frequencies of observed events downward to reserve probability for the things that have not been observed.

Note $f(w_3 \mid w_1, w_2) > 0$ only when $\mathrm{count}(w_1, w_2, w_3) > 0$

We introduce a **discounted frequency**:

$$0 \leq f^*(w_3 \mid w_1, w_2) \leq f(w_3 \mid w_1, w_2)$$

The total discount is the zero-frequency probability:

$$\lambda(w_1, w_2) = 1 - \sum_{w'} f^*(w' \mid w_1, w_2)$$

Back-off

Recursive formulation of probability:

$$\hat{p}_{\mathrm{BO}}(w_3 \mid w_1, w_2) = \begin{cases} f^*(w_3 \mid w_1, w_2) & \text{if } f^*(w_3 \mid w_1, w_2) > 0 \\ \alpha_{w_1,w_2} \times \underbrace{\lambda(w_1, w_2)}_{\text{“Back-off weight”}} \times \hat{p}_{\mathrm{BO}}(w_3 \mid \cancel{w_1}, w_2) & \text{otherwise} \end{cases}$$

Question: how do we discount?

Witten-Bell Discounting

Let’s assume that the probability of a zero off can be estimated as follows:

a b c a b c a b x a b c c a b a b x c

$\lambda(\texttt{a}, \texttt{b}) \propto$ | + | + | =3

$$t(\texttt{a}, \texttt{b}) = |\{x : \text{count}(\texttt{a}, \texttt{b}, x) > 0\}|$$

$$\lambda(\texttt{a}, \texttt{b}) = \frac{t(\texttt{a}, \texttt{b})}{\text{count}(\texttt{a}, \texttt{b}) + t(\texttt{a}, \texttt{b})}$$

$$f^*(\texttt{c} \mid \texttt{a}, \texttt{b}) = \frac{\text{count}(\texttt{a}, \texttt{b}, \texttt{c})}{\text{count}(\texttt{a}, \texttt{b}) + t(\texttt{a}, \texttt{b})}$$

Example: spite v. constant

- *spite* and *constant* both appear in the Europarl corpus 993 times
- *spite* only has 9 words that follow it (979 times it was followed by *of* because it is used in the phrase *in spite of*)
- *constant* has 415 words that follow it: *and* (42 times), *concern* (27 times), *pressure* (26 times), plus long tail including 268 that only appear once
- How likely is it that we'll see a previously unseen bigram that starts with *spite* v. *constant*?

Example: spite v. constant

$$t(\textit{spite}) = 9$$
$$\lambda(\textit{spite}) = t(\textit{spite}) \,/\, (\mathrm{count}(\textit{spite}) + t(\textit{spite}))$$
$$\lambda(\textit{spite}) = 9/(9+993) = .00898$$

$$t(\textit{constant}) = 415$$
$$\lambda(\textit{constant}) = t(\textit{constant}) \,/\, (\mathrm{count}(\textit{constant}) + t(\textit{constant})$$
$$\lambda(\textit{constant}) = 415/(415+993) = .29474$$

Previously unseen bigrams starting with *spite* are multiplied by a smaller value and are therefore less likely

Kneser-Ney Discounting

- State-of-the-art in language modeling for 15 years
- Two major intuitions
  - Some contexts have lots of new words
  - Some words appear in lots of contexts
- Procedure
  - Only register a lower-order count the *first time* it is seen in a backoff context
  - Example: bigram model
    - "San Francisco" is a common bigram
    - But, we only count the unigram "Francisco" the first time we see the bigram "San Francisco" - **we change its unigram probability**

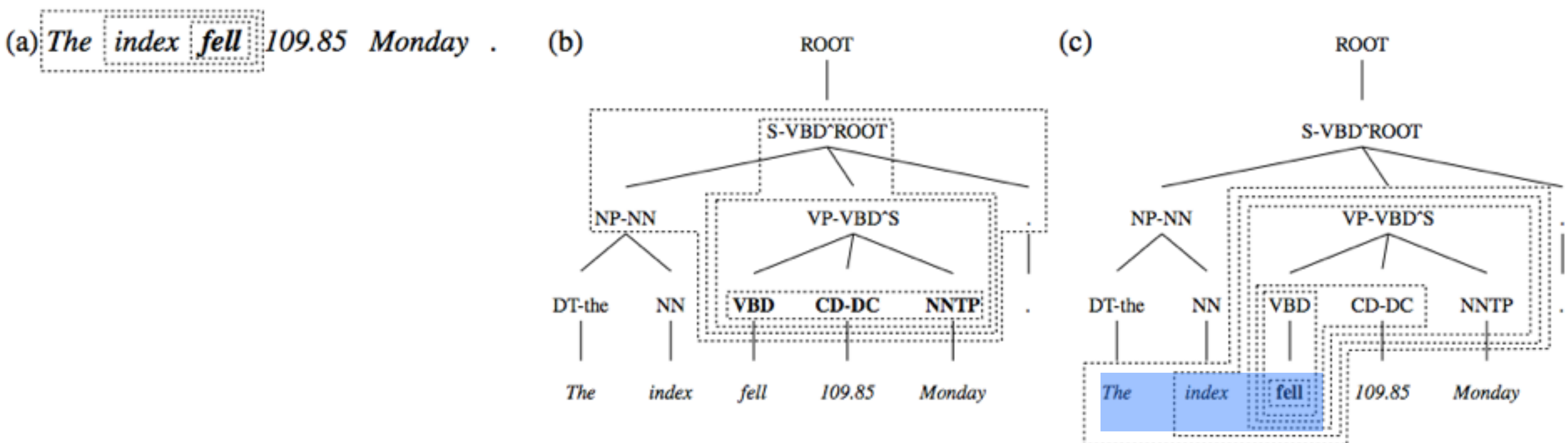
Other Formulations

- *N*-gram class-based language models

$$p(\mathbf{w}) = \prod_{i=1}^{\ell} p(c_i \mid c_{i-n+1}, \ldots, c_{i-1}) \times p(w_i \mid c_i)$$

Pauls & Klein (2012)

$$p(\boldsymbol{\tau}, \mathbf{w}) = p(\boldsymbol{\tau}) \times p(\mathbf{w} \mid \boldsymbol{\tau})$$



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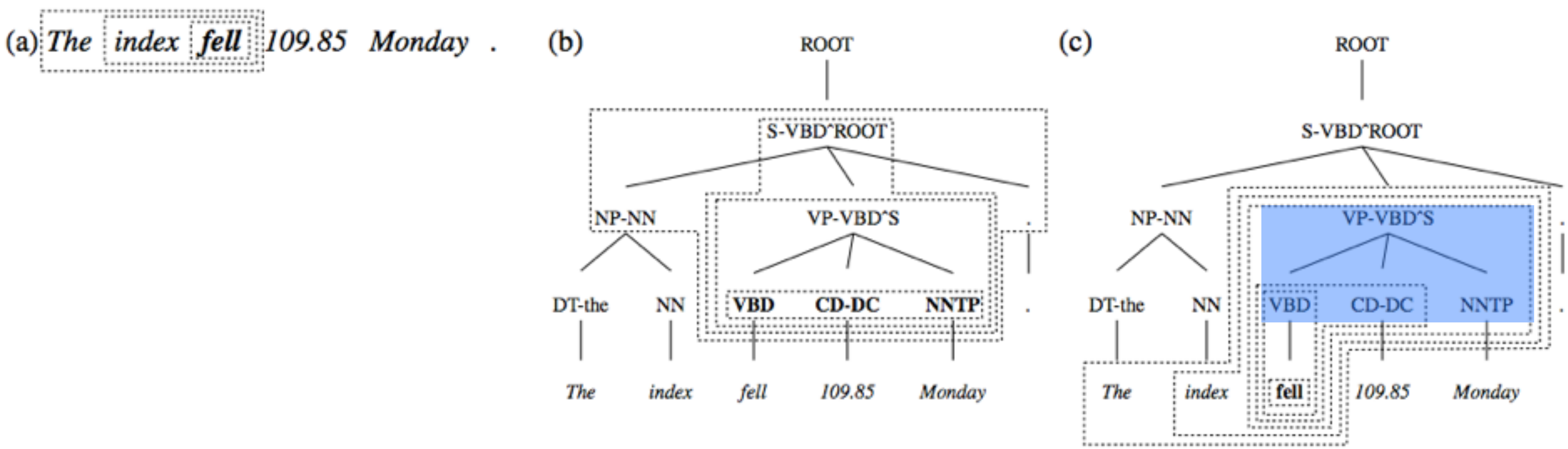


Google (2007)

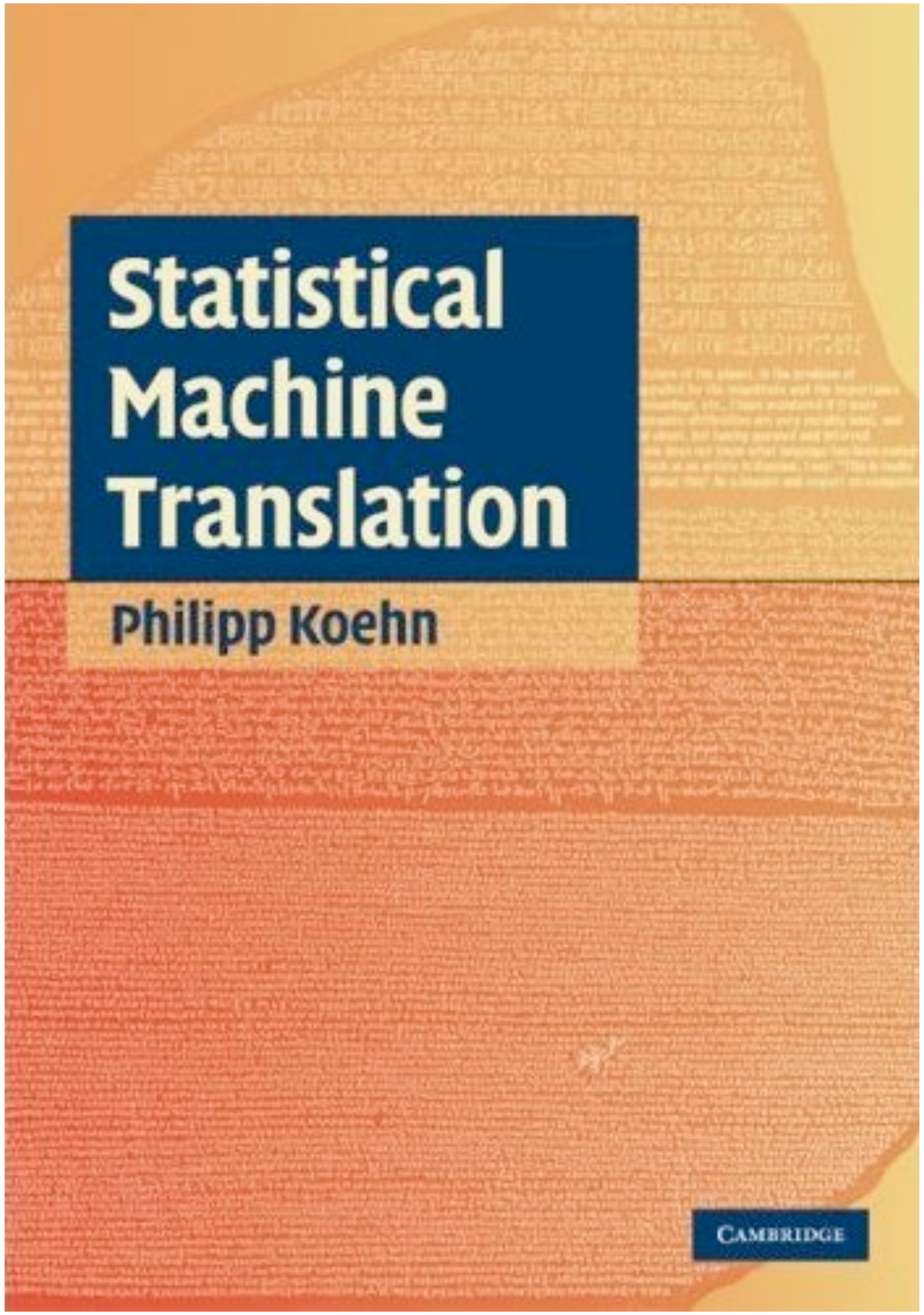
- "Stupid backoff"
- A simpler method than Kneser Ney smoothing, that is easier to estimate using MapReduce
- Approaches the same level of performance on tasks like MT when using large amounts of data

Summary

- *n*-gram language models are the standard method for estimating the probability of sentences for MT and for ASR
- Although the Markov assumption does not hold for language, it allows us to easily estimate probabilities by counting sequences of words in data
- Since data is finite, sparse counts are still a problem even when dealing with n-grams instead of sentences
- Smoothing is the most common solution.

Questions?

- Please read Chapter 7 from the textbook for more information on language models



Announcements

- Please sign up for a "language in 10 minutes" presentation slot
- If you are having problems with Assignment 1, go to Jonny's office hours tomorrow at 2pm