1 a

$$\mathbb{E}\left(2^{\sum_{m=1}^{n} \xi_m}\right) = \mathbb{E}\left[\prod_{m=1}^{n} 2^{\xi_m}\right] = \prod_{m=1}^{n} \mathbb{E}\left[2^{\xi_m}\right] = \mathbb{E}\left[2^{\xi_1}\right]^n$$

$$= (p+1)^n$$

$$\mathbb{E}\left(2^{\xi_1}\right) = p \cdot 2^1 + (1-p)\,2^0 = 2p - p + 1 = p+1$$

2 b

$$\mathbb{P}\left(\tfrac{1}{n}\sum_{i=1}^{n} X_i - p > 0.02\right) \le e^{-2n(0.02)^2}$$

a) $e^{-2\bar{n}(0.02)^2} = 0.01$

$$-2\bar{n}(0.02)^2 = \log(0.01)$$

$$\bar{n} = \frac{\log(0.01)}{-2 \cdot 4 \cdot 10^{-4}} = \frac{-4.6}{-8 \cdot 10^{-4}}$$

$$\bar{n} = 5750$$

$\forall n \geq \bar{n}$ a desigualdade é certa $\Rightarrow \forall n \geq 5000$ não se cumpre

b) $e^{-2\bar{n}(0.02)^2} = 0.05$

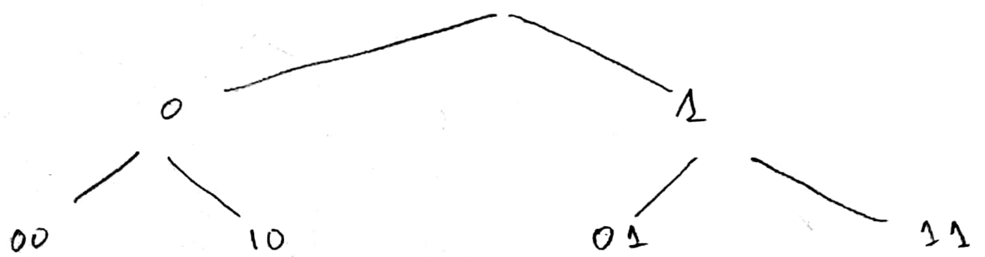
$$\bar{n} = \frac{\log(0.05)}{-8 \cdot 10^{-4}} = \frac{-3}{-8 \cdot 10^{-4}}$$

$$\bar{n} = 3750$$

$\forall n \geq \bar{n}$ a desigualdade é certa. $\Rightarrow \forall n \geq 4000$ também

3 d

árvore de altura $k=2$



$\hat{p}(0|00) = 1/3$ $\hat{p}(0|01) = 1/3$ $\hat{p}(0|10) = 2/3$ $\hat{p}(0|11) = 2/5$

$\hat{p}(1|00) = 2/3$ $\hat{p}(1|01) = 2/3$ $\hat{p}(1|10) = 1/3$ $\hat{p}(1|11) = 3/5$

$\hat{p}(0|0) = 3/7$ $\qquad$ $\hat{p}(0|1) = 3/8$

$\hat{p}(1|0) = 4/7$ $\qquad$ $\hat{p}(1|1) = 5/8$

- Testar se $w=0$ é contexto

$$\Delta(0) = \max_{a,b\in A} |\hat{p}(a|0) - \hat{p}(a|0b)|$$

$a=0,\ b=0$ $\quad \hookrightarrow |\hat{p}(0|0) - \hat{p}(0|00)| = |3/7 - 1/3| = 2/21$

$a=0,\ b=1$ $\quad |\hat{p}(0|0) - \hat{p}(0|01)| = |3/7 - 1/3| = 2/21$

$a=1,\ b=0$ $\quad |\hat{p}(1|0) - \hat{p}(1|00)| = |4/7 - 2/3| = 2/21$

$a=1,\ b=1$ $\quad |\hat{p}(1|0) - \hat{p}(1|01)| = |4/7 - 2/3| = 2/21$

$\Delta(0) = 2/21 = 0.095 > \delta \Rightarrow$ no podar (manter as sequencias $b0$)

- Testar se $w=1$ é contexto

$$\Delta(1) = \max_{a,b\in A} |\hat{p}(a|1) - \hat{p}(a|1b)|$$

$a=0,\ b=0$ $\quad \hookrightarrow |\hat{p}(0|1) - \hat{p}(0|10)| = |3/8 - 2/3| = 7/24$

$a=0,\ b=1$ $\quad |\hat{p}(0|1) - \hat{p}(0|11)| = |3/8 - 2/5| = 1/40$

$a=1,\ b=0$ $\quad |\hat{p}(1|1) - \hat{p}(1|10)| = |5/8 - 1/3| = 7/24$

$a=1,\ b=1$ $\quad |\hat{p}(1|1) - \hat{p}(1|11)| = |5/8 - 3/5| = 1/40$

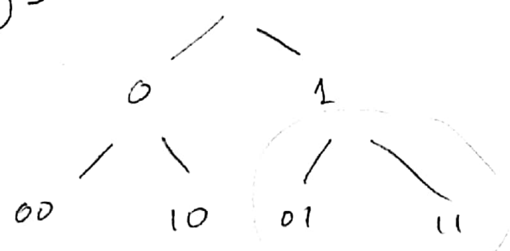
$\Delta(1) = 7/24 = 0.29 > \delta \Rightarrow$ no podar

A árvore resultante é $\tau = \{00, 10, 01, 11\}$

4 Opção correta (a)

5 opção correta (a)

6 a)



$\hat{p}(0|0) = 3/6 \qquad \hat{p}(0|1) = 2/7$

$\hat{p}(1|0) = 3/6 \qquad \hat{p}(1|1) = 5/7$

$\hat{p}(0|00) = 1/3 \qquad \hat{p}(0|01) = 1/2 \qquad \hat{p}(0|10) = 1/3 \qquad \hat{p}(0|11) = 1/4$

$\hat{p}(1|00) = 2/3 \qquad \hat{p}(1|10) = 1/2 \qquad \hat{p}(1|10) = 2/3 \qquad \hat{p}(1|11) = 3/4$

$$\Delta(0) = \max_{a,b \in A} |\hat{p}(a|0) - \hat{p}(a|0b)|$$

$\hookrightarrow |\hat{p}(0|0) - \hat{p}(0|00)| = |3/6 - 1/3| = 1/6$

$|\hat{p}(0|0) - \hat{p}(0|01)| = |3/6 - 1/2| = 0$

$\Rightarrow \Delta(0) = 1/6 = 0.17$

$$\Delta(1) = \max_{a,b \in A} |\hat{p}(a|1) - \hat{p}(a|1b)|$$

$\hookrightarrow |\hat{p}(0|1) - \hat{p}(0|10)| = |2/7 - 1/3| = 1/21$

$|\hat{p}(0|1) - \hat{p}(0|11)| = |2/7 - 1/4| = 1/28$

$\Rightarrow \Delta(1) = 1/21 = 0.047$

Para obter $\hat{\tau} = \{1, 00, 10\}$ precisamos $\Delta(0) > \delta$ e $\Delta(1) < \delta$

$\Rightarrow 0.047 < \delta < 0.17 \Rightarrow \delta = 0.05$

7) Opção correta é b. $\tau = \{0, 01, 11\}$

8) c

$X_{-1} = 0$

$X_0 = \mathbb{1}\{U_0 > p(0|c_\tau(X_{-1}))\} = \mathbb{1}\{U_0 > p(0|0)\} = 1$

$X_1 = \mathbb{1}\{U_1 > p(0|c_\tau(X_{-1}^{0}))\} = \mathbb{1}\{U_1 > p(0|10)\} = 1$

$X_2 = \mathbb{1}\{U_2 > p(0|c_\tau(X_{-1}^{1}))\} = \mathbb{1}\{U_2 > p(0|11)\} = 0$

$X_3 = \mathbb{1}\{U_3 > p(0|c_\tau(X_{-1}^{2}))\} = \mathbb{1}\{U_3 > p(0|0)\} = 1$

$X_4 = \mathbb{1}\{U_4 > p(0|c_\tau(X_{-1}^{3}))\} = \mathbb{1}\{U_4 > p(0|10)\} = 1$

$X_5 = \mathbb{1}\{U_5 > p(0|c_\tau(X_{-1}^{4}))\} = \mathbb{1}\{U_5 > p(0|11)\} = 0$

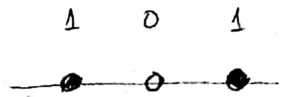
$\Rightarrow X_0 = 1,\ X_1 = 1,\ X_2 = 0,\ X_3 = 1,\ X_4 = 1,\ X_5 = 0 \Rightarrow$ c.

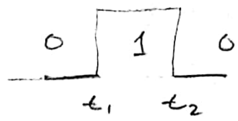
VC

Casos que não consigue classificar.

- $S(F,1) = 2^1 = 2$
  $S(F,2) < 2^2$ $\Rightarrow VC(F) = 1.$

- $S(F',1) = 2^1 = 2$
  $S(F',2) = 2^2 = 4$ $\Rightarrow VC(F) = 2$
  $S(F',3) < 2^3$

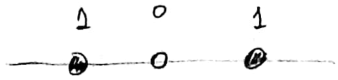


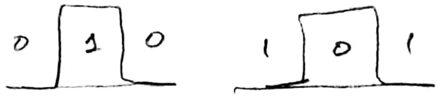


- $S(F,1) = 2^1 = 2$

  $S(F,2) = 2^2 = 4 \quad \Rightarrow VC(F) = 2$

  $S(F,3) < 2^3$



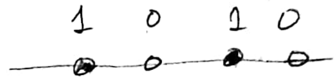


- $S(F',1) = 2^1 = 2$

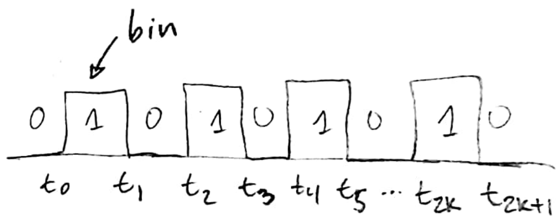
  $S(F',2) = 2^2 = 4$

  $S(F',3) = 2^3 = 8 \quad \Rightarrow VC(F') = 3$

  $S(F',4) < 2^4$



- Existem $(K+1)$ bins.!!!



Consegue classificar a seguinte configuração

$0\ 1\ 0\ 1 \ldots 0\ 1\ 0$ com $(K+1)$ 1's, $2(K+1)+1$ dados

mas nao consigue

$1\ 0\ 1\ 0 \ldots 1\ 0\ 1$ com $(K+2)$ 1's, $2(K+1)+1$ dados.

por tanto o maior $n = 2(K+1)$

$VC(\mathcal{F}_K) = 2(K+1) \quad [\, 2 * (\#\text{ de bins})\,]$.