A Literature Review of the N-Queens Problem and Best-First Search Algorithms

The N-Queens problem is a classical benchmark problem in artificial intelligence, combinatorial optimization, and constraint satisfaction. Due to its simple formulation yet exponentially growing search space, it has been extensively used to evaluate the effectiveness of heuristic search strategies. This project proposes solving the N-Queens problem using Best-First Search, specifically a Greedy Best-First Search strategy guided by conflict-based heuristics. This literature review examines academic work related to (1) the formal definition and computational properties of the N-Queens problem, (2) Best-First Search and its principal variants, and (3) heuristic functions suitable for guiding informed search in this domain. The reviewed literature directly informs the algorithmic design and pseudocode of the proposed solution.

The N-Queens Problem: Background and Computational Characteristics

The N-Queens problem, originally proposed in 1848, requires placing N queens on an $N \times N$ chessboard such that no two queens attack each other horizontally, vertically, or diagonally. While the problem admits solutions for all $N \geq 4$, the number of valid configurations grows rapidly as N increases, resulting in a large and complex search space.

Bell and Stevens (2009) provide a comprehensive survey of known results and research directions related to the N-Queens problem. Their work documents historical developments, symmetry properties, mathematical constructions, and algorithmic approaches. Importantly, the authors clarify that while finding a single solution is computationally feasible and can be achieved in polynomial time using constructive methods, the task of enumerating all possible solutions exhibits exponential growth, making the problem a challenging benchmark for search algorithms. As a result, N-Queens is frequently used to evaluate heuristic and informed search strategies rather than brute-force enumeration.

The value of N-Queens in artificial intelligence lies in its clear constraint structure and the necessity of balancing completeness and efficiency. Partial board configurations naturally form search states, allowing the problem to be modeled as a state-space search task. This representation directly supports tree-based algorithms such as Best-First Search, where nodes correspond to partial queen placements and edges represent valid queen insertions.

Dealy's project, *Common Search Strategies and Heuristics With Respect to the N-Queens Problem* (2004), experimentally compares several classical search strategies, including Depth-First Search, Branch and Bound, and Beam Search. Although the work primarily emphasizes DFS-based approaches, it demonstrates a crucial insight relevant to this project: the choice of heuristic has a greater impact on performance than the underlying

search strategy itself. Dealy shows that heuristics that minimize queen conflicts drastically reduce the explored search space. This conclusion directly motivates the use of a conflict-based heuristic within a Best-First Search framework, as reflected in the evaluation function used in the proposed pseudocode.

Best-First Search and Its Variants

Best-First Search (BestFS) is an informed search strategy that expands the most promising node according to an evaluation function. It should be clearly distinguished from Breadth-First Search, which is an uninformed strategy. In BestFS, nodes are typically stored in a priority queue ordered by a heuristic evaluation function $f(n)$.

Rios and Chaimowicz (2011) present a detailed survey and classification of A*-based Best-First heuristic search algorithms. Their work formalizes Best-First Search as a general framework and positions A* as a specific instantiation where the evaluation function is defined as:

$$f(n) = g(n) + h(n)$$

where $g(n)$ is the cost to reach node n and $h(n)$ is a heuristic estimate of the remaining cost to reach a goal state. The authors emphasize that A* is both complete and optimal when the heuristic is admissible. However, they also note that in domains where path optimality is irrelevant, the $g(n)$ component can be omitted, yielding Greedy Best-First Search, which evaluates nodes solely using $h(n)$.

This distinction is particularly important for the N-Queens problem. Unlike path-finding problems, the goal is not to minimize the number of steps or placements but simply to reach any valid solution. Consequently, Greedy Best-First Search aligns more naturally with the problem structure. In the proposed algorithm, the priority queue ordering in the pseudocode is defined entirely by a conflict-based heuristic, reflecting the Greedy Best-First Search paradigm discussed by Rios and Chaimowicz.

Setyobudhi (2022) provides an empirical comparison between A* and Greedy Best-First Search when solving the Fifteen Puzzle. The study demonstrates that Greedy Best-First Search often reaches a solution faster than A*, albeit without guaranteeing optimality. This trade-off is highly relevant to the N-Queens problem, where solution optimality is not a concern. The findings support the design decision in this project to prioritize search speed and heuristic guidance over path-cost considerations. Accordingly, the pseudocode does not maintain or update a cumulative path cost $g(n)$, focusing instead on heuristic ordering.

Heuristic Functions for the N-Queens Problem

The effectiveness of Best-First Search depends critically on the quality of the heuristic function $h(n)$. In the context of N-Queens, a heuristic must estimate how close a partial board configuration is to a valid solution.

Minton et al. (1993) introduced the Minimizing Conflicts heuristic, a landmark contribution to constraint satisfaction and scheduling problems. The heuristic evaluates a state based on the number of violated constraints and attempts to iteratively reduce conflicts. In the N-Queens problem, this corresponds to minimizing the number of pairs of queens that attack each other. The authors demonstrate that conflict-based heuristics can solve very large instances efficiently, even when traditional backtracking fails.

Although the original Min-Conflicts algorithm is formulated as a local search method, its core principle is directly applicable to informed search. In the proposed Best-First Search pseudocode, each node represents a partial queen placement, and the heuristic $h(n)$ is computed as the number of conflicts currently present or expected to arise from remaining placements. Nodes with fewer conflicts are expanded first, guiding the search toward promising regions of the state space.

Sosic and Gu (1990) present a polynomial-time algorithm for the N-Queens problem based on heuristic repair and complete board assignments. Their work demonstrates that domain-specific strategies can outperform general-purpose search algorithms. However, these approaches rely on problem-specific constructions that are not easily generalized to other constraint satisfaction problems. In contrast, Best-First Search combined with a conflict-based heuristic offers a general and adaptable framework, making it suitable for educational purposes and comparative evaluation across different CSPs.

Limitations of Best-First Search for the N-Queens Problem

Despite its effectiveness, Greedy Best-First Search exhibits several limitations that must be acknowledged. Unlike A*, Greedy Best-First Search does not guarantee completeness or optimality, as it relies exclusively on heuristic guidance without considering path cost. Consequently, if the heuristic is poorly designed or misleading, the algorithm may expand suboptimal regions of the search space or become trapped in locally promising but ultimately unproductive states. In the context of the N-Queens problem, this risk is mitigated by the use of a conflict-based heuristic, which has been empirically shown to correlate strongly with solution quality. However, as the board size increases, heuristic evaluation becomes more computationally expensive, and the priority queue may grow significantly, leading to increased memory consumption. Furthermore, compared to local search methods such as Min-Conflicts, Best-First Search may explore a larger number of intermediate states before reaching a solution. Despite these drawbacks, the algorithm remains valuable for analyzing heuristic-driven node expansion and for educational comparisons with other informed search strategies, justifying its selection for this project.

Implications for the Proposed Algorithm and Pseudocode Design

The reviewed literature collectively informs the structure of the proposed solution. The N-Queens problem is modeled as a state-space search where each state corresponds to a partial board configuration. A Greedy Best-First Search strategy is employed, using a

priority queue ordered by a conflict-minimization heuristic. The algorithm terminates upon reaching a conflict-free configuration of size *N*.

The absence of a path-cost component in the evaluation function aligns with findings from Setyobudhi (2022) and the theoretical framework described by Rios and Chaimowicz (2011). Furthermore, the heuristic function directly reflects the principles established by Minton et al. (1993), ensuring that the pseudocode expands states most likely to lead to a valid solution.

Despite the existence of polynomial-time and local-search solutions, this approach remains valuable for analyzing heuristic guidance, node expansion order, and search efficiency. The reviewed literature thus provides both the theoretical justification and empirical motivation for implementing a Greedy Best-First Search with a conflict-based heuristic for the N-Queens problem.

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