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Modelling Poverty Alleviation Strategies Using Resilience Thinking

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Declaration of Authorship

I, Namitha Tresa Joppan, declare that this thesis, entitled 'Modelling Poverty Alleviation Strategies Using Resilience Thinking' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at the University of Amsterdam.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
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Date: 19 August 2021

"From now on I will tell you of new things, of hidden things unknown to you."

Isaiah 48:6

UNIVERSITY OF AMSTERDAM

Abstract

Faculty of Science
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Modelling Poverty Alleviation Strategies Using Resilience Thinking

by Namitha Tresa JOPPAN

Twenty six years after the UN declaration of poverty as a multi-dimensional problem in 1995, 1 out of 10 goes to bed on an empty stomach, with many without a roof over their head. Policies implemented to eradicate poverty and help people overpower the poverty trap they are in, often fail due to their lack of consideration of the different dimensions of poverty. Hence financial assistance or a warm meal provided at a time only provide temporary solutions let alone protection against exogenous shocks. For a permanent solution, a thorough understanding of the factors aiding poverty in that situation should be identified so as to develop strategies that are resistant to exogenous shocks. These factors are unique to each circumstance and are part of the Community Capitals Framework(CCF). The CCF acts as a guide for developing an interconnected network of interaction between the different dimensions of poverty in a community/-household. The multi-dimensional poverty trap model in this thesis project takes into account the financial aspects, social interaction, and the innate ability of a person to fend for themselves as the three dimensions of the model. Under the present circumstance of COVID'19 and worldwide lockdowns pushing the poor into extreme poverty and poverty trap, the choice of social interaction and innate ability as the dimensions of the model is justified. Policies based on resilience thinking concepts are applied on this model of artificial society, to understand their effectiveness in helping individuals overcome the poverty trap they are in, in the presence of exogenous shocks. This further revealed the most important factor on which the investments should be made to help the poor overcome the poverty trap.

Acknowledgements

This thesis project is the culmination of hard work, faith, and prayer. I have been praying since December 2019, to be able to do a Computational Social Science thesis project and once I studied the course of Agent-Based Modelling in the 3rd block of 2019-20 academic year I was captivated by the idea and wanted to base the project on Agent-Based Modelling. There are many people whose guidance and altruism that I owe to. I thank my constant and the only person who ever knew my fears and confusions, Jesus. I also thank Mother Mary and my heavenly patron St. Antony, for they knew how much I have troubled them persistently for favors and guidance. These people worked together to answer my prayers to guide me to the best supervisor I can ask for, Dr.Debraj Roy. Often, I wondered 'why is he making me do that now?', and later I realized the reason being, 'time management, multi-tasking efficiently, getting the knack and having a flow'. So, without his careful supervision and encouragement, I wouldn't be able to complete this project. I gratefully remember my family, my Appa and Amma, even though they wouldn't understand what I am telling, they used to enquire, 'did you get the plot your Professor asked you to?, did you get the bulge (thick tail) you wanted in the graph? etc', which was more than enough. I thank my friends in India, especially Merrin, and my friends here namely Fiona, Jasper, Cillian, and Marcus, who many times let me borrow their powerful laptops for running some codes and helping me with Python issues. I gratefully remember my study advisor Cecilia Sigvardsdotter for her support and tips especially about writing the thesis. I thank Peter Heemskerk for organizing thesis coaching sessions and giving me elaborate feedback on my thesis report draft and presentation. I remember Nienke, our Programme Coordinator for her timely replies to my queries and constant reassurances. I also thank Deborah who was a sweet stranger I met on my morning jog and who became a great friend and family in the Netherlands and who has helped me in times of need when no one else would have. I also gratefully remember Fr. Peter Klos and Fr. Rijo for frequently asking 'how's your thesis going?' and constantly encouraging me to move forward. There are many other people I must have forgotten to mention here, thank you, everyone.

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Abbreviations

CSL	**C**omputational **S**ceince **L**ab
UvA	**U**niversitiet **v**an **A**msterdam
VU	**V**rije **U**niversitiet
UN	**U**nited **N**ations
COVID'19	**CO**rona **VI**rus **D**iease of 20**19**
i.e.	**i**d **e**st (that is)
UBI	**U**niversal **B**asic **I**ncome
PFD	**P**ermanent **F**und **D**ividend
CCF	**C**ommunity **C**apitals **F**ramework
MPI	**M**ulti-Dimensional **P**overty **I**ndex
OPHI	**O**xford **P**overty and **H**uman **D**evelopment **I**nitiative
UK	**U**nited **K**ingdom
POFED	**PO**litics, **F**ertility and **E**conomic **D**evelopment
AIC	**A**kaike **I**nformation **C**riterion
BIC	**B**ayesian **I**nformation **C**riterion
OFAT	**O**ne **F**actor **A**t a **T**ime
GDP	**G**ross **D**omestic **P**roduct
PSN	**P**roductive **S**afety **N**et
CN	**C**argo **N**et

Chapter 1

Introduction

The United Nations World Summit for Social Development held in Copenhagen in 1995 defined poverty as *"lack of income and access to resources, lack of access to basic social services such as education, and alienation from civil, social and cultural life"* [4]. The 186 then member countries of the UN, drafted policies and pledged to eradicate poverty. 20 years later in the year 2015, with more than 736 million people living below the international poverty line, the UN set an ultimatum to end global poverty by the year 2030 [5]. Although, there has been a significant reduction in the number of global poor ever since the collective initiative of the UN and year 2019 showed the lowest ever number in the world's poor [6], many thought that we were going in the right direction. However, nobody prepared us for what the world had in store for us since December 2019 - a global pandemic. The slowing of the global poverty reduction rate as a result of COVID'19 will make it strenuous and almost impossible to attain the global goal of reducing extreme poverty to 3 percent in the year 2030 [7]. In addition, more than 100 million people will be pushed into extreme poverty as a consequence of the COVID'19 pandemic [7]. What went wrong? Was the world not prepared, and, didn't the policymakers take into account exogenous shocks like a pandemic that could reverse the positive results?

1.1 Objective

This research aims to move beyond the conventional one-dimensional models of poverty, where poverty is the state of merely lacking money or income. As the 1995 United Nations World Summit for Social Development rightly [4] defined, poverty has many aspects or dimensions. However, before explaining the multiple dimensions of poverty it's important to define 'poverty trap'. There are many definitions for 'poverty trap' in literature. Nevertheless, it is what it means literally- i.e. trapped in poverty. Azariadis and Stachurski give a more formal definition as *"any self-reinforcing mechanism that causes poverty to persist"* [8]. This definition points to the fact that poverty is a multi-dimensional problem. Hence, to understand and study the causal mechanisms resulting in poverty and eventually poverty trap in individuals, households, or communities, the various factors aiding poverty must be identified. These factors are unique to each circumstance under study. This project focuses on the impact, human capital and social capital have on an individual's financial stability. The social capital and human capital are part of the Community Capital Framework(CCF) which will be addressed in section 2.3.

Addressing all the different dimensions of poverty [2] is beyond the scope of this research study owing to the time constraints that this study is subjected to. Besides, as explained above, the causal mechanism of poverty in each scenario is different. For example, in a community sustained by agriculture, the study of natural capital, consisting of soil quality, climate, geographic isolation, etc are most important amongst other factors [2][1]. Similarly, in a community of herders, the health and well-being of their livestock upon which their livelihood depends matters the most [1].

In the context of COVID'19 pandemic, the social contact that people maintained with others, the health of an individual and their accessibility to hospitals are most significant. Hence, the three major dimensions of this poverty trap model will be human, social and financial capital. 'Human Capital' can be formally defined as the innate ability of an individual to contribute to the process of earning income and it incorporates the health, physical stature, cognitive development and education with which an individual enter economy. 'Social Capital' is the network of relationships among individuals living in a society, enabling it to function effectively [9] whereas 'Financial Capital' is the amount of wealth possessed by an individual or household.

This project aims to answer the following questions:

1. What are the mechanisms characterizing and consolidating the poverty trap in an urban setting?
2. What are the effects human and social capital has on this process?

The poverty alleviation strategies based on the principles of resilience thinking will be applied to the multi-dimensional poverty trap model developed. This in turn will help in developing effective poverty alleviation strategies that are best suited to different situations and resistant to exogenous shocks, for instance, pandemics like COVID'19. Hence, the objective of this project is achieved by:

1. Developing a multi-dimensional agent-based poverty trap model that simulate the financial, social, and human capital aspects of an individual.
2. Applying resilience thinking principles on this model to develop poverty alleviation strategies and find their effectiveness in the presence of an exogenous shock.

This multi-dimensional poverty trap model can act as a virtual laboratory to test the developmental policies before enacting them.

1.2 Research Approach

A step-by-step approach is used in the making of the multi-dimensional poverty trap model of this project. Boltzmann Wealth Model which will be explained in section 2.4.2.1 formed the base model. Apart from financial capital, the other dimensions added to this model are social capital and human capital. These capitals are part of the Community Capitals Framework(CCF) and are discussed in detail in section 2.3. At each stage, the developed model is studied in detail to analyze the effectiveness of the model and the corresponding results to define the next step. The ideas and works done by many researchers have guided and inspired in the process of making this multi-dimensional poverty trap model and they are all explained in detail in the corresponding sections. Due to the lack of availability of data, the model was built and tested on an artificial society of a given number of agents who interact, move, and engage in financial transactions subjected to certain rules and conditions which will be explained in the corresponding sections.

1.3 Report Outline

The report is divided into seven major sections. The objective of this research project and the approach used are as explained in the first section. This is followed by the literature review, where the main ideas used in this project are explained in detail. The third section presents the model development in a step-by-step manner where each sub-section is either a method used or a model developed. This is followed by the sensitivity analysis of all the major parameters of the final multi-dimensional poverty trap model. In the sixth section two major poverty alleviation strategies based on one of the principles of resilience thinking namely regime shift - pushing over the barrier, are applied to the final model, and their effectiveness is studied. The final section answers the research questions and summarizes the project.

Chapter 2

Literature Review

This literature review aims to introduce the key concepts upon which this project is founded. While elaborating these concepts it is also explained in detail why certain approaches are relevant and beneficial.

2.1 Poverty: A Multidimensional Approach

Conventionally poverty is defined as the condition in which people live below a specified minimum income level [10]. Hagenaars and de Vos [11] defined poverty by categorizing it as below:

1. *Poverty is having less than an objectively defined, absolute minimum.*
2. *Poverty is having less than others in the society.*
3. *Poverty is feeling you do not have enough to get along.*

Although the last 2 categories are subjective, all the 3 categories defined poverty in terms of absence of money [11]

When a person remains poor no matter what mechanisms are used to help them change their condition, then that person is said to be trapped in inescapable poverty. This state is called the poverty trap. The poverty trap is formally defined by Azariadis and Stachurski, as *"any self-reinforcing mechanism that causes poverty to persist"* [8]. The poverty trap can be seen as a 'S- curve' as shown in figure 2.1. The poor people are trapped in the left-most region below the diagonal line in the 'Poverty Zone'. As evident from figure 2.1 their future income at time '$t+1$' is lower than the current income at time 't'. Once the agent crosses the threshold and reaches the 'No Poverty' zone, then they

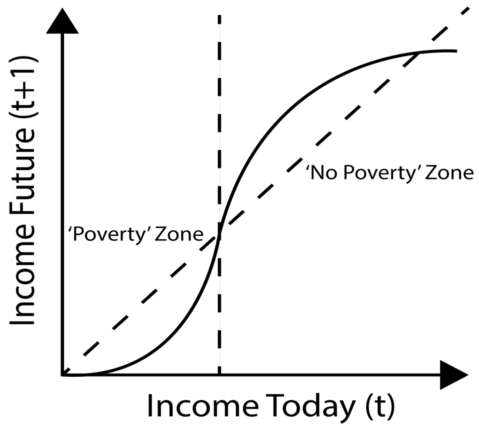


FIGURE 2.1: Savings Trap: S Curve [1]

are no longer in poverty. They have higher incomes both in the present and in the future. The agents who are in the 'No Poverty Zone' but not yet in the stable equilibrium tends to be impacted the most by an exogenous shock. This would then push them back to the 'Poverty Zone' depending on how far they're from the 'No Poverty Zone' equilibrium stage.

The process of moving from 'Poverty Zone' to 'No Poverty Zone' and maintaining that status quo is quite challenging. This is mainly because poverty is regarded only as a single-dimensional problem - lack of money. Policies and poverty alleviation strategies are hence made in a way to increase the income of the 'poor' people.

The definition of poverty wouldn't accurately categorize people as poor and non-poor. There are many indices that measure the level of poverty in an economy. A few of them are:

1. **Headcount Ratio:** This is the most basic method of measuring poverty and it measures the proportion of people living below an income threshold [12]. The disadvantage of headcount ratio is that it does not convey the measure of the poor getting poorer.

2. **Poverty Gap Index:** This gives the intensity of poverty and is defined as the average poverty gap in the population as a proportion of the poverty line [13]. Unlike headcount ratio, poverty gap index measures how far the people are from the poverty line, thereby measuring the depth of poverty.

3. **Sen Index:** "Sen Index is defined by the combination of three distinct measures of poverty and are, poverty rate, poverty gap ratio and the inequality of incomes among the poor as measured by the Gini index"[14]. Gini index also known as Gini ratio or Gini coefficient, is a measure of the income inequality or wealth inequality

within a nation or any other group of people. Sen index uses the Gini coefficient as a measure of the poverty gaps for the poor.i.e. it measures the poverty inequality amongst the poor.
Sen index = poverty rate X average poverty gap X (1 + Gini coefficient of poverty gaps for the poor)[15]
While Sen index successfully incorporates the effects of the number of poor, the depth of poverty, and the distribution of poverty in a group, it has a disadvantage attributed to it's dependence on the distribution of poverty. i.e. the Gini coefficient. Hence, Sen index has all the disadvantages associated with the Gini coefficient [16].

4. **Sen-Shorrocks-Thon (SST) Poverty Index:** SST index measures the poverty intensity.
SST index = poverty rate X average poverty gap X (1 + Gini coefficient)
SST is essentially same as the Sen index except that it measures the poverty distribution for the whole population [15].

5. **Watts Poverty Index:** Watt index is the average difference between the logarithm of the poverty line and the logarithm of incomes [16]. It is a good indicator of poverty as it is more sensitive to a transfer at the lower end of the distribution of the poor than at the upper end.

6. **Foster-Greer-Thorbecke(FGT) Poverty Index:** This is family of poverty indices and is given by the following formulation:

$$FGT_\alpha = \frac{1}{N}\sum_{i=1}^{H}\left(\frac{z-y_i}{z}\right)^\alpha$$

where z is the poverty threshold, N is the number of people in the economy, H is the number of poor, y_i is the income of person i [17].
By substituting different values for α the FGT index reduces to the indices as given in table 2.1.

FGT_α	Formula	Name
FGT_0	$\frac{H}{N}$	Headcount ratio
FGT_1	$\frac{1}{N}\sum_{i=1}^{H}\left(\frac{z-y_i}{z}\right)$	Poverty gap index

Table 2.1: Different FGT Formulations

When $\alpha = 2$,

$$FGT_\alpha = \frac{1}{N}\sum_{i=1}^{H}\left(\frac{z-y_i}{z}\right)^2$$

FGT_2 is the most widely used formulation in development economics as it gives greater weight to those that fall far below the poverty line than those that are closer to it [17].

While these indexes are helpful one way or another, they are static. Hence, they don't capture the poverty trap resulting from those agents who reside in the unstable equilibrium region of the s curve as in figure 2.1, who fall back to the 'Poverty Zone' amidst a shock. Using these indices are hence not wise in situations like a pandemic as they cannot project an extreme increase in poverty.

Taking poverty as mere lack of money, Universal Basic Income (UBI) and stimulus packages were experimentally tried among groups of people in different countries. While some of them succeeded, the majority failed. The Alaska Permanent Fund Dividend program (PFD), a form of UBI, helped to reduce the poverty rate among the Alaskan native seniors by 59 percent during the period (1990-2015) of study. On the other hand, the poverty rate increased dramatically among the Alaskan native children and middle age group [18]. Further, this scheme failed to reduce the rate of 'deep poverty' which is defined as *"living in a household with a total cash income below 50 percent of its poverty threshold"* by the U.S. Census Bureau [19] [18]. Similarly, evidence from the 2017-18 Finland experiment showed that, while UBI helped people to remain happy, stress-free, and increased trust in other people and social institutions, they failed to help people secure jobs or improve their life by making them financially independent [20].

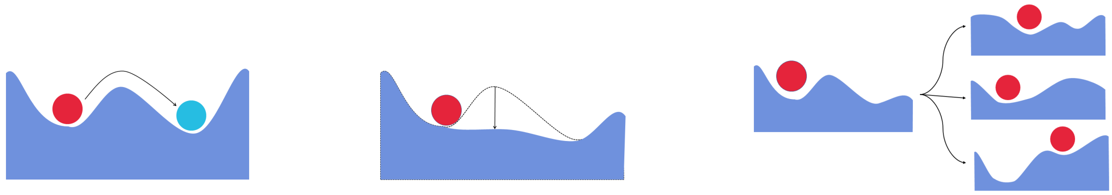
Evaluating the success of UBI and similar programs, which treat poverty as a single-dimensional problem is crucial now more than ever. Many countries including Spain and Germany are now resorting to new basic income experiments in response to the COVID'19 pandemic [21].

From the examples of Alaska and Finland, it is evident that overlooking non-financial factors that might be aiding poverty could be a reason why people were in poverty traps. It is important to take into account social-ecological relationships [1], education and health, access to power and resources, weather and natural resources, social connections, etc [2], of a person or household or community for studying poverty traps to create effective poverty alleviation policies. Hence a multi-dimensional approach leading to a multi-dimensional poverty trap model is crucial in realizing the causal mechanisms of the poverty trap.

The examples of supplemental income (UBI) in Finland and Alaska, help to understand the importance of treating poverty as a multi-dimensional problem qualitatively. The interactions with the beneficiaries of UBI, their behavior during the benefit period, etc aided in arriving at this conclusion. However, there is a lack of analytical foundation necessary to create a policy response [22]. Hence, the challenge here is to develop a multi-dimensional poverty trap model that is best suited to a given situation with a solid analytical foundation.

2.2 Resilience Thinking

Stephane Hallegatte in his study published by World Bank in 2014 defines economic resilience as *"the ability of an economy as a whole to cope, recover from and reconstruct after a shock"* [23]. The faster an economy overcomes the shock the more resilient the economy is. Simultaneously, it is regularly used to allude to the 'economic resilience' of individual families or firms, and their capacity to adapt to or recuperate from a shock and adjust to changing monetary conditions in the more extensive economy [24]. In this context, it is very helpful to identify how and whom did a shock to the economy affect, i.e. the distributional impacts of a shock. This in turn helps in developing better plans to help those in need and also to avoid these shocks in the future. There are many other definitions of economic resilience depending on the context in which it is defined. However, the basic idea remains the same. All these definitions treat resilience as a numerical outcome that can be measured. This helps the humanitarian and development programs in collecting and monitoring data from the world's most vulnerable regions [25]. At an operational level, 'resilience' would offer the first thorough means of monitoring development resilience in the world's most vulnerable places. On a scientific level, it would help interdisciplinary and comparative research for designing and evaluating interventions and strategies [25]. But, this approach dangerously simplifies and neglects the complex intertwining dynamics that characterize the situation [26]. However, the concepts of resilience thinking and poverty trap go hand in hand owing to the common conceptual basis of attractors, thresholds, and other nonlinear dynamics that are shared by poverty trap models and resilience thinking [27]. This in turn would help us understand the occurrence and sustenance of poverty traps [1].



(A) Regime Shift: Crossing the Barrier

(B) Regime Shift: Lowering the Barrier

(C) Transformation

FIGURE 2.2: Application of Resilience Thinking [1]

Steven et.al, discusses how the resilience thinking concepts of *'regime shifts'*- either pushing the poor over the barrier via external asset input (figure 2.2(A)) or lowering the barrier via a change in practice so that the poor can cross over the barrier by themselves(figure 2.2(B)) - and *'transformation'* -re-configuring the system in novel ways(figure 2.2(C))- when applied on social-ecological interactions, helped in studying poverty traps and develop effective poverty alleviation policies [1]. Barret et.al in their study named, 'Poverty Traps and Social Protection', provide remarkable examples of the application of the resilience thinking concepts [22]. Unlike earlier studies [28] that treated poverty as a single dimensional problem, Steven et.al and Barret et.al in their respective studies, regard poverty as a multi-dimensional problem [1][22]. Steven et.al apply the resilience thinking concepts to develop a multi-dimensional poverty trap model in the backdrop of rural agricultural communities [1]. Since the model was based on rural agricultural landscape they considered natural capital, physical capital and cultural capital as their multiple dimensions. In this context natural capital incorporates the fertility of the soil, weather etc and physical capital includes assets such as machinery that can increase the economic outcome and cultural capital represents the traditional knowledge passed down from one generation to other,heritage etc [1].

Barret et.al in their multi-dimensional poverty trap model incorporates human capital which they define as 'the innate ability that includes the physical stature, cognitive development, and the level of education with which a person enter adulthood' along with financial capital. They applied the concepts of 'regime shift' in two contexts namely, 'social protection' - individuals adjusting their behavior in response to poverty alleviation methods in presence of a shock, and, 'social relief'- individuals doesn't change their behavior in response to the policies implemented to support individuals in the presence of a shock. Barret et.al gives an example of implementing 'crossing barrier' behavior of regime shift in their study titled 'Poverty Traps and Social Protection'. The 'Need-Based Assistance' where each individual is given financial support to attain a higher income stage was implemented as a 'social relief' policy and 'safety nets' - used to prevent agents from falling below a threshold and 'cargo net' - money transfers made to lift people above threshold, were implemented as 'social protection' policy [9]. Their one of a kind model has been an inspiration for the model developed during the course of this thesis project.

How and which capital and hence dimension must be included in the multi-dimensional model depends on the scenario under study. Hence, an understanding of the Community Capitals Framework is essential and is discussed in section 2.3.

2.3 Community Capitals Framework

Poverty, as discussed in section 2.1 should be taken into account as a multi-dimensional problem. But, what are the different dimensions of poverty and how can we identify them? Although this is highly dependent on each scenario that is being studied, Emery and Flora define the *Community Capitals Framework* (CCF) which helps in mapping strategies and studying the impacts of different capitals on a community's well-being [2]. CCF presents a holistic approach to identify the various interacting factors of a community by means of seven capitals [2]. They are:

1. Natural Capital: The natural assets of a place, the geography, the weather, natural resources, etc.
2. Cultural Capital: The tradition, language, culture, and heritage that determines how people/community perceive the world.
3. Human Capital: The skills and abilities of a person- education, health, intellect, etc, that influences the earning capacity of an individual.
4. Social Capital: The positive or negative connections that a person or community makes within themselves or with others.
5. Political Capital: The access to forms of political power- government, and similar agencies, that people/communities utilize to voice out their needs and opinions.
6. Financial Capital: The money that is present within the community or accumulated on the path of progress, and which can be utilized for investing in activities for a secure future of the community.
7. Built Capital: The existing infrastructure or new infrastructure that is constructed to support activities that help in the progress and betterment of the community.

Flora and Emily, argue how these seven capitals are intertwined such that any changes to one of the seven can positively or negatively impact the other capitals, thereby impacting the community [2]. A pictorial depiction of the relationships among six of the seven capitals is given in figure 2.3. This is the stock and flow diagram developed from the case study of HomeTown Competitiveness (HTC) formed in rural Nebraska, to prevent youth migration to cities and to re-develop the community and its economy [2]. As shown in figure 2.3 not all capitals were used in the re-development of this community. Hence this is very much context-dependent.

Depending on the situation, i.e. community that is being studied, the relationship between the various capitals could differ. This in turn could lead to a "spiraling-up" or

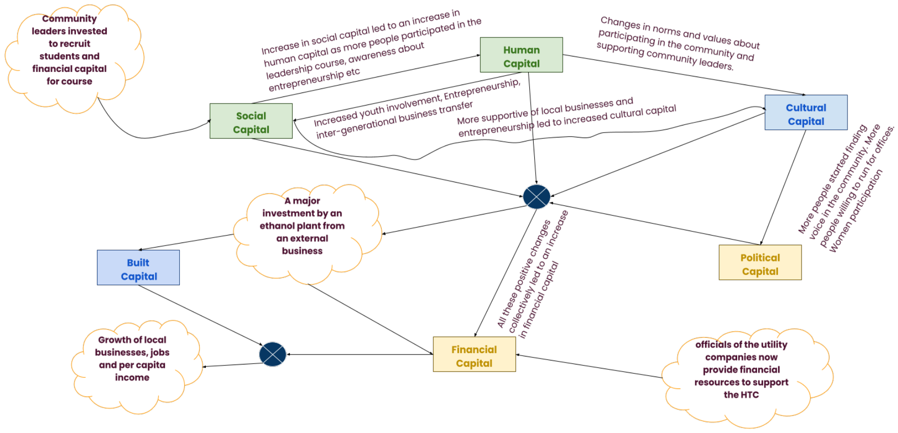


FIGURE 2.3: Flow Among Different Capitals: The Case Study of HomeTown Competitiveness(HTC),Nebraska [2]

"spiraling-down" or "dual effect" depending on the interventions and strategies used on the CCF.

In a study of the dynamics of the community capitals, in the context of 2 rural cities during the North Dakota oil boom in the years 2005-2007, the communities witnessed a dual effect [29]. i.e. " an increase in one capital leads to an overall increase in other capital(s) while at the same time resulting in an overall decrease in another capital(s), or a decrease in one capital leads to an overall decrease in some capital(s) while causing an overall increase or stability in other capital(s)" [29]. While the results of applying the CCF to rural women's empowerment in Ethiopia had some different results. The study identified an upward spiraling effect when investments are made in all six except the built capital and had also resulted in asset accumulation within the capitals [30]. The study also identified that the interaction between the social, human and financial capitals as the key entry point to rural women's empowerment [30]. However, in the process of rebuilding a community in rural Nebraska, Emily and Flora identified social capital as the key entry point resulting in the spiraling up of other capitals [2]. Hence, it is extremely important to identify how the various capitals interact and what effect that could result in, for each community or household which is the subject of study.

All the examples discussed above applied CCF at the level of a community. The global Multi-Dimensional Poverty Index(MPI) of the Oxford Poverty and Human Development Initiative(OPHI) for the year 2020, introduced 10 indicators based on 3 dimensions of poverty at an individual level [31]. The 3 dimensions of poverty introduced by OPHI are health, education, and living standards; and each of the 10 indicators belongs to any one of these three poverty dimensions. The dataset curated through this process is internationally comparable and is from over 100 participating countries [31]. The advantage of this global MPI is that each of the 10 indicators can be considered belonging to one of the seven capitals of CCF but on an individual level. For example, 'years of schooling' and 'school attendance', the 2 indicators of the education dimension can be said as belonging to the human capital of the CCF. Similarly, 'sanitation', one of the indicators defined by the living standards dimension can be considered as the built capital.

This project intends to study, the interventions at the social and human capital at an individual (agent) level on an artificially created society. Although understanding the interactions between all the seven capitals are important to develop a generalized model, it is beyond the scope of this project. Besides, as mentioned previously, the interactions and the circumstances leading to the poverty trap are unique to each situation.

2.4 Poverty Trap Models

Barret et.al (2013), argues that poverty traps can be generated by a variety of structural mechanisms. They could be found at the individual or household level, while some operate at the community, regional, or national scales. Some are single equilibrium mechanisms while others are multiple equilibrium mechanisms [22]. A single equilibrium poverty trap occurs when a person or community remains poor forever, such that there is no other stable state. Whereas multiple equilibrium poverty traps exist when an individual or community fluctuates between the states of being poor or non-poor, or if there exist multiple stable states [22]. There are one-dimensional or multi-dimensional poverty trap models depending on the number of dimensions of poverty that are being studied.

This section aims to present the different types of multi-dimensional multiple equilibrium poverty trap models that exist in the literature and to introduce the base model- Boltzmann Wealth Model- upon which the resulting multi-dimensional poverty trap model of this project is founded. Among the many existing models, we can have a broad categorization as, System Dynamics Models and Agent-Based Models. These models will be studied in detail in the following sections.

2.4.1 System Dynamics Modelling

System Dynamics Modelling is a problem-oriented modeling method, which helps to systematically answer the 'what-if' questions [32]. This method is used to study the non-linear behavior of complex systems such as climate and weather patterns, military movements, healthcare systems, etc. The main advantage of this approach is that it helps the policymakers get a better idea of how their system behaves in response to the intended changes. This in turn will help them understand the consequences of their decisions and changes [32]. In this type of modeling the focus will be upon the systemic factors rather than on individuals.

Although other models incorporate the Community Capitals Framework (CCF) in applying resilience thinking concepts in social-ecological modeling, none of them were intended to make poverty trap models or poverty alleviation strategies. For instance, Padowski et.al used the concepts of CCF and resilience in incorporating the social system dynamics at the Yakima River Basin, Columbia, for Food-Energy-Water (FEW) resilience and Sustainability modeling [33]. They intended to bring together social scientists and engineers, to develop social-ecological resilience models, in response to the changing climatic conditions, while taking into account the social, cultural, political, and economic issues of the Yakima River Basin. They also aimed to adopt innovations while avoiding any unintended negative consequences [33].

Steven et.al, developed a multi-dimensional, process-based, dynamical, multiple equilibrium poverty trap model in the premise of rural agricultural communities [1]. They considered Natural Capital, Physical Capital, and Cultural Capital as the dimensions for their multi-dimensional multiple equilibrium poverty trap model. The choice of the capitals is justified owing to the rural agricultural scenario that formed the subject of study. As mentioned above the models developed by Steven et.al, are unique in that they apply the resilience thinking concepts of regime shift and transformation to develop poverty alleviation methods on their multi-dimensional multiple equilibrium poverty trap model [1]. The poverty trap models developed by Steven et.al, is summarized in table 2.2.

2.4.2 Agent-Based Modelling

Agent-based models are used to simulate the actions and interactions of autonomous agents to understand the behavior of a system and what governs its outcomes. The model formulated by Steven et.al, studied the poverty trap formation at a rural community level. They also mention how an exogenous shock like flood affects the already weak natural capital in the case of Intensification Trap Model [1]. This in turn led to the adoption of methods that make the natural capital less susceptible to flood at the community level [1]. Zining Yang's POFED(Politics, Fertility and Economic Development) dynamic agent-based model studies the relationship among politics, economic and demography change at both micro and macro levels [34]. He combined system dynamics modeling with agent-based modeling. The dynamics resulting from the change in political environment, which often has inter-generational impacts are captured with a system

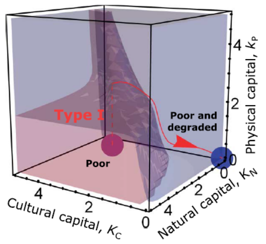
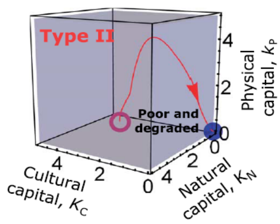
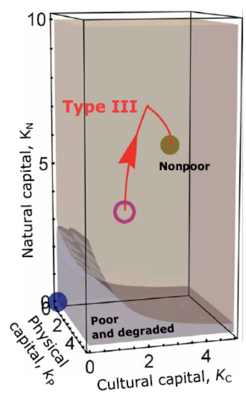
Resilience Thinking Concepts Used		
Regime Shift		Transformation
Type I	Type II	Type III
Pushing over the barrier	Lowering the barrier	Transforming the System
Eg: Asset inputs such as money, farming equipment, or novel seeds	Eg: Improved market access, women's saving group, community-based natural resource management group, public education programs etc	Eg: Modern farming techniques with traditional cultivation practices
Intensification Trap Model		**Transformative Pathway**
$\frac{dk_P}{dt} = s(k_P)E(K_N)f(k_P) - (\delta_P + \eta)k_P$ $\frac{dK_N}{dt} = T(K_C)G(K_N)L(k_P) - \delta_N K_N$ $\frac{dK_C}{dt} = P(K_N) - \delta_C K_C$ K_N = Natural Capital K_C = Cultural Capital G = Growth of natural capital P = Traditional practices δ_N = natural decay or loss rate of natural capital δ_C = natural decay or loss of cultural capital E = increased production due to natural capital L = reduction in growth of natural capital due to physical capital T = effect of cultural capital on growth of natural capital		$\frac{dk_P}{dt} = s(k_P)E(K_N)f(k_P) - (\delta_P + \eta)k_P$ $\frac{dK_N}{dt} = T(K_C)G(K_N)L_2(k_P) - \delta_N K_N$ $\frac{dK_C}{dt} = P(K_N) - \delta_C K_C$ L_2 = modified form of L with nonzero limit at large k_P
		

TABLE 2.2: Multi-dimensional Multiple Equilibrium Poverty Trap Models of Steven et.al [1]

dynamics model in an attribute called 'political capacity'. This is combined with the generalizable non-cooperative Prisoner's dilemma game in an agent-based model. The agent-based model captures the interactive political and economic dynamics of an individual. The model results were valuable in understanding how macro level changes constrain or incentivize an individual's behavior at a micro-level, and how the interactions at micro-level shape the macro structures [34]. Similarly, Brinkmann and his colleagues developed an agent-based model to understand the dynamics of social-ecological traps in the South Western Madagascar where the farmers are negatively affected by the unsustainable exploitation of natural resources. The area witnessed a dramatic increase in the land use pressure on resources over the past years. They illustrated a model-driven scenario and devised methods to help the farmers escape this trap [35].

In order to fulfill the objectives of this project, and to study the impact of human and social capital in determining whether a person exists as poor or non-poor, it is essential that the focus is upon an individual rather than on a community. Hence, agent-based modeling approach is used in this project. In scenarios of an exogenous shock like the COVID'19 pandemic, the role of social capital - physical contact, time spend together, the social network, etc- is crucial in determining the spread of infection and its impact. Hence, a micro-simulation method or an agent-based modeling approach is necessary. Using this approach each agent's or individual's behavior is studied to identify the possible poverty trap and thereby suggest strategies to alleviate it.

2.4.2.1 Boltzmann Wealth Model

The agent-based model upon which this project's model is founded is inspired by the Boltzmann Wealth Model. Econo-physicists drew a parallel between Boltzmann's kinetic theory of collisions in gases with pairwise economic transactions between two individuals. This introduced a two-body approach wherein individuals trade and transact money from one individual to another [36]. A statistical model of pairwise money transaction between two agents was thus formulated and presented by many physicists in the 1980s and 90s [36]. However, a turning point was when Yakovenko et.al proved the emergence of Boltzmann- Gibbs distribution, with the help of computer simulations of economic models wherein two agents trade through a locally conserving transaction [37]. Chakraborti and Chakrabarti further studied the effect of introducing a savings propensity factor- the tendency of an individual to save a percentage of the money to themself during the trade [38]. While the equilibrium money distribution of the model developed by Yakovenko et.al is the Boltzmann-Gibbs distribution, the one by Chakraborti and Chakrabarti showed an asymmetric Gaussian-like distribution [37][38]. With the inclusion of the 'saving propensity factor', Chakraborti and Chakrabarti, argue that even

with individual self-interest, the dynamics become co-operative and the agents interact which doesn't happen in the model of Yakovenko et.al [38]. They further argue that this global feature, can be regarded as an illustration of 'self-organization' in the market as proposed by the Father of Economics, Adam Smith [38]. The trade and transaction happening between the agents in this model with saving propensity factor results in an interaction between the agents, thereby introducing social capital in the model.
Barret et.al in their model studied how individual asset accumulation gets affected when changes and differences are induced in the innate ability of an individual which captures the immutable physical stature, cognitive development, and educational attainment with which they enter adulthood, multiple production technologies, and other risk factors [9]. This introduced the human capital in the multi-dimensional poverty trap model they created and studied. Barret et.al, hence went on to study how the innate ability of an individual influence to overcome the poverty trap in the presence of a shock [22]. They studied the impact of a safety net mechanism- adding monetary assistance- on two different types of social behavior. They are, social relief, where an agent doesn't change their behavior in response to the safety net mechanism, and social protection where an agent adapts according to the safety net supplemented to combat the loss occurred due to a shock [22].

2.4.3 Poverty Trap Models: Conclusion

In this project, inspirations are drawn from both the system dynamics model put forth by Steven et.al and the agent-based approach used in the Boltzmann wealth model. The Boltzmann wealth model is kept as the base model and social capital and human capital are added, simultaneously making the necessary changes to the base model. The resilience thinking concepts are then incorporated into the model, to help the trapped individuals to escape the poverty trap.

2.5 Conclusion

This session aimed to introduce the literature that guided the process of putting together this project. By closely studying them it can be concluded that to develop effective poverty alleviation strategies, poverty must be regarded as a multi-dimensional problem. Since each situation and circumstances are unique, it is also important to identify a possible interaction between the different capitals as defined by the Community Capitals Framework (CCF). This is also crucial in realizing effective poverty alleviation strategies. To consider an exogenous shock like the COVID'19 pandemic, it is necessary to examine the effects of interaction at the levels of individuals. Hence, an agent-based approach, based on the Boltzmann Wealth Model is used in this project. The social capital and human capital which are critical in the context of a pandemic are incorporated into the model.

2.5.1 Personal Story

Coming from a middle-class family in a rural village in Kerala, India, I have had my own experiences with poverty. Although we were quite comfortable, I had to see the plight of many families. I would like to share the example of a particular family who had two sources of income - a husband who was a daily wage laborer, and a few goats and cows. They couldn't afford any luxuries and lived in a tiny house along with their three children and an alcoholic father. If something unexpected like a health issue, or if something happens to their livestock, etc - an exogenous shock- their life will be disrupted. It then takes ages for them to put their lives back together. However, for the past 5 years, they are doing exceptionally well and they now live in a good house and use a motorbike. There were mainly 2 changes. First, the wife of the family got a job thereby bringing more to the plate. Secondly, their investment in children's education paid off, and their eldest son got a job and helped his father to get a better job. In this personal example discussed it is evident that any increase in the income of the family didn't help them to come out of the poverty trap. The meager income when invested in education, and livestock, and an additional income (wife's) helped them to overcome the barriers and in the long run, making them capable to even afford luxuries.

This anecdotal example demonstrates how investments in human capital -education and social capital - securing jobs for father and mother through communication with the external world, eventually led to an increase in the financial and built capital. This also clearly shows that poverty is a multi-dimensional problem. To capture this complex phenomenon of the poverty trap and exploring it empirically is the challenge of this project.

Chapter 3

Model Development

Developing a multi-dimensional poverty trap model with human capital, social capital and financial capital as the three dimensions is the major objective of this thesis project.A step-by-step approach was necessary to closely analyze the results of the model in development at each stage to attain the goal of this project. This process is summarized in the flowchart given in figure 3.1.

This section aims to present the step-by-step approach that led to the development of the final model of this project. Hence each subsection of this section will discuss the different models that were developed at each stage, the conclusions drawn and the improvements done on them to proceed to the next stage of the model development.

3.1 The Base Model: Boltzmann Wealth Model

The famous Boltzmann Distribution is also known by the name Gibbs distribution of the statistical mechanics and mathematics gives the probability that a system will be in a certain state as a function of that state's energy and temperature. That is,

$$p_i \propto e^{\frac{-\epsilon_i}{kT}}$$

where p_i is the probability of the system to be in state i, ϵ_i is the amount of energy possessed by the system at that state, T is the temperature and k is the Boltzmann constant. Similarly, Boltzmann's kinetic theory of collisions in gases shows how the total energy is conserved in perfectly elastic collisions of atoms and molecules. Suppose an atom with energy ϵ_1 collides with another atom that has energy ϵ_2 in a perfectly elastic fashion, such that, some amount of energy, say $\Delta\epsilon$, is transferred between them. This interaction between the atoms can be summarized as shown in figure 3.2. As shown in figure 3.2

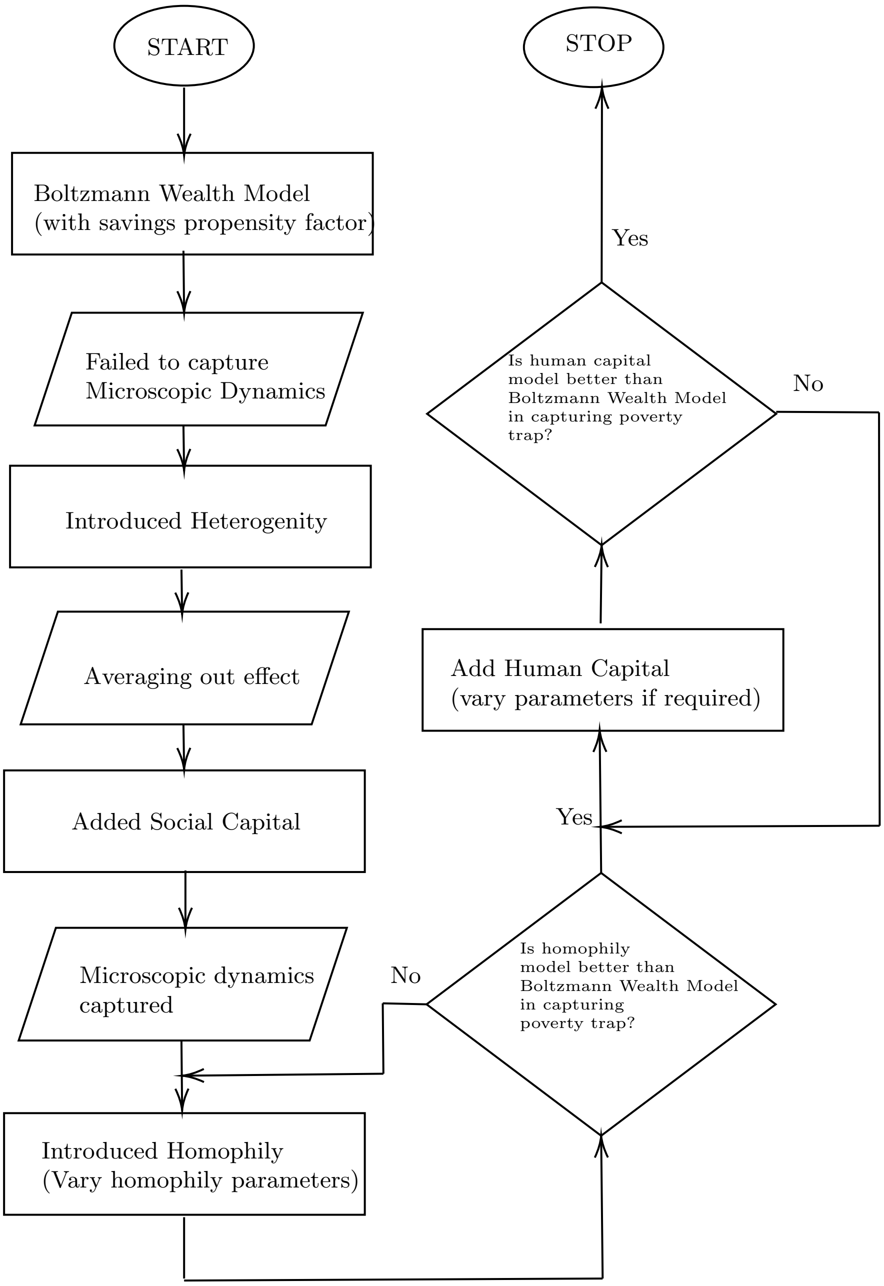


FIGURE 3.1: Model Development

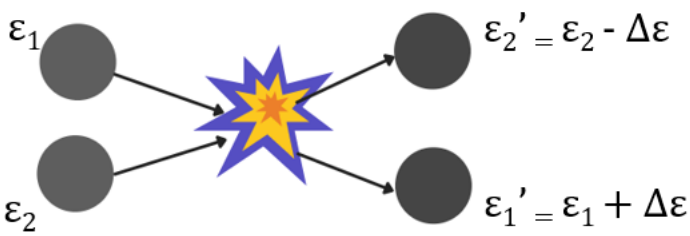


FIGURE 3.2: Perfectly Elastic Collision of Atoms

the total energy before and after the collision remained to be $\epsilon_1 + \epsilon_2$ with $\Delta\epsilon$ energy transaction happening between the two atoms. Econo-physicists drew a parallel between the elastic collision between atoms and molecules with a pairwise economic transaction between two individuals in an economy. This application of statistical physics's concepts in economics has revolutionized the way problems of economic inequality are studied and analyzed. Yakovenko and Dragulescu made a revolutionizing discovery that for any arbitrary and random sharing but locally conserving money transactions between any two agents in a market, the money distribution goes to the equilibrium Boltzmann Gibbs distribution of statistical mechanics [36]. As a result a new formulation replacing the 'energy' term of Boltzmann's kinetic theory of gases with 'amount of money' could be formed. Thus, for any two individuals in a given economy with their wealth m_1 and m_2 respectively transact and exchange Δm amount of money, the total amount of money before and after the transaction remains the same.

One of the major findings of the study led by Yakovenko and Dragulescu was the identification of the 'temperature' equivalent of the Boltzmann equation as the 'average amount of money per agent' in its economics counterpart [36]. This resulted in the development of the Boltzmann Wealth Model. This model obeyed all the laws of the Boltzmann-Gibbs distribution. However, the base model of this study was inspired by Chakraborti and Chakrabarti, who introduced the concept of 'savings propensity factor - λ' [38]. In this model, all agents are assigned the same savings propensity factor λ, such that when they trade they save λ percentage of what they have for themselves and allow only the remaining amount to be involved in the trade. Introducing λ changed how the model behaved. For instance, the multiplicative property i.e. $P(m_1 m_2) = P(m_1)P(m_2)$ doesn't remain valid. This could be due to the introduction of the self-interest of an individual causing the money dynamics to be cooperative. This further brings about a global ordering [5]. These observations of Chakraborti and Chakrabarti could be verified in this project as well.

3.1.1 Model Description

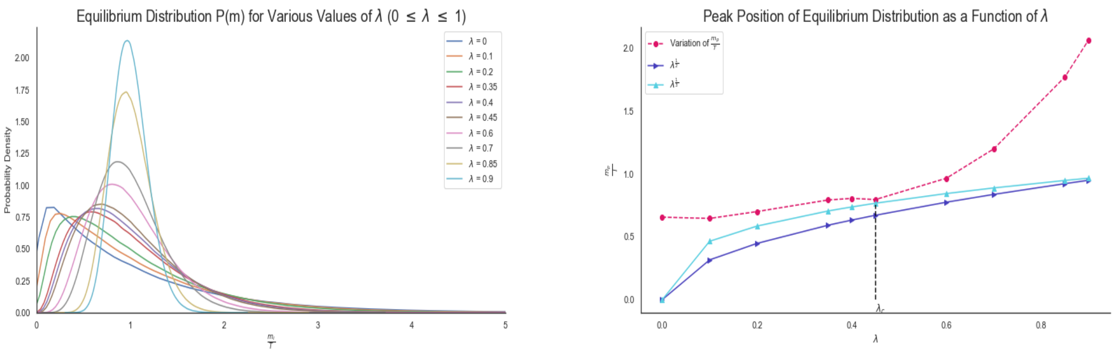
In the agent-based Boltzmann Wealth Model with savings propensity factor λ, an economy of 500 well-mixed agents were placed on a mesa grid [39] and were allowed to trade for 2500 time steps. While trading each agent saves a fraction, λ, of their money m_i for themselves. The trade between 2 agents i and j can be represented by the following equations.

$$
\begin{aligned}
& m_i \rightarrow m_i^{'} \\
& m_j \rightarrow m_j^{'} \\
where, \quad & m_i^{'} = m_i - \Delta m \\
& m_j^{'} = m_j + \Delta m \\
and, \quad & \Delta m = (1 - \lambda)[m_i - \epsilon(m_i + m_j)]
\end{aligned}
$$

The amount of money possessed by each agent, m_i at the time i, and the average money per agent - temperature, mT_i are recorded at every time step. These values were then used to draw insights from the model.

3.1.2 Observations and Results

The model was run for different values of savings propensity factor λ such that $0 \leq \lambda \leq 1$. The equilibrium money distribution for various λ values when plotted resulted in figure 3.3(A). The plot in figure 3.3(A) could show that the equilibrium distribution is robust and does not depend on the initial money distribution in the market. Another observation that could be made, although the plots are not presented here is that changing the number of agents N or changing the initial value of money m_1 for the agents does not result in a different distribution than what is already shown in figure 3.3(A). These observations are in line with that of the Boltzmann Wealth Model with savings propensity factor by Chakraborti and Chakrabarti [38]. Another key finding from this model



(A) Equilibrium Money Distribution

(B) Crossover Point at $\lambda = 0.45$

FIGURE 3.3: Major Observations From Boltzmann Wealth Model

is the identification of a crossover point (λ_c). In this economy, even when an individual trades with self-interest (λ), the emergence of a global ordering as mentioned above, cannot be neglected. As a result, the entire market ends up with the most probable money - maximum money an agent possess while in equilibrium. For any random agent, the variation of this most probable money as a function of λ was plotted as shown in figure 3.3(B). Looking at the pink line in figure 3.3(B), a sudden increase can be found at a certain point. Before this point the most probable money varies closely like $\lambda^{\frac{1}{2}}$ and after this point the plot varies closely along the lines of $\lambda^{\frac{1}{3}}$ as shown in figure 3.3(B). This point where the most probable money changes its behavior when plotted against λ is called the crossover point. Another feature associated with the crossover point was that the probability that an agent with zero money first disappeared at this point. This aspect of the crossover point was not verified in this project. The crossover point (λ_c) savings propensity value was hence used as the model's savings propensity factor for generating further insights.

The time series analysis of the Boltzmann Wealth Model with savings propensity value $\lambda = 0.45$ that was run for 2500 time steps resulted in plots as shown in figures 3.4(A) and (CT) for two random agents. Similarly, the wealth distribution for the same agents as shown in figures 3.4(B) and (D). It could be observed that both time series money data and the wealth distribution for both the agents resemble very much. This could be attributed to the emergence of global behavior, such that the agents re-configure their wealth to fit into the global distribution.

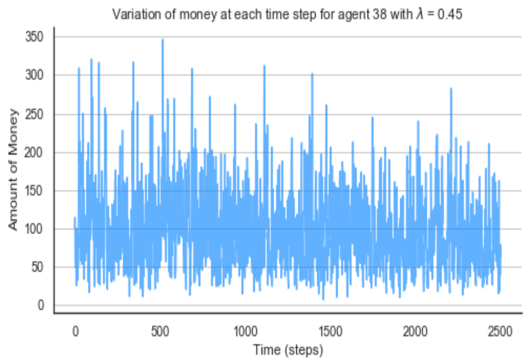
This observation leads to another observation that the agent-based model has failed to capture the dynamics at an individual level as the model behavior points to a collective population-level dynamics. Hence, it can be concluded that there is coarse-graining in the Boltzmann Wealth Model with saving propensity such that the model is incapable of capturing the microscopic dynamics.

This conclusion is further supported by the lack of observance of the 'S- curve' when the amount of money at time t is plotted against that at time t+1. The capacity curves in figure 3.5(A) and (B) doesn't properly fit the data to give a 'S-curve'. The goodness of fit for agents 38 and 128 can be summarized using a 2 samples Kolmogorov Smirnov test as shown in table 3.1. With the level of significance(α) kept at 0.05, the results of

	Agent 38	Agent 128
p -value	0.0002291	8.55874 x 10^{-07}
KS statistic	0.173913	0.220735

TABLE 3.1: Goodness of Fit of Capacity Curves

the goodness of fit can be accepted and hence the conclusion is validated.



(A) Time Series Analysis of Money Data

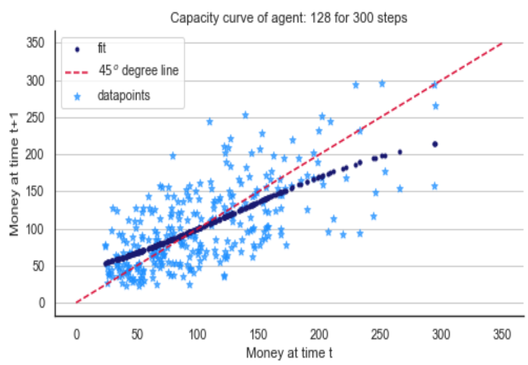
(B) Distribution of Money

(C) Time Series Analysis of Money Data

(D) Distribution of Money

FIGURE 3.4: Time Series Analysis and Wealth Distribution of Random Agents of Boltzmann Wealth Model with $\lambda = 0.45$

(A) Capacity Curve of Agent 38



(B) Capacity Curve of Agent 128

FIGURE 3.5: Capacity Curves of Random Agents of Boltzmann Wealth Model with $\lambda = 0.45$

3.1.3 Conclusion

The Boltzmann Wealth Model with savings propensity factor showed that the equilibrium money distribution is not dependent on the initial money distribution. The distribution remained the same with the changing values of the number of agents (N) and the average money per agent (T), for the different values of λ. The crossover point (λ_c), where the probability of agents with zero money first disappears is approximated to be 0.45 and this aligned with the results of Chakraborti and Chakrabarti [38]. The problem of coarse-graining was evident from the results of the model, as all agents exhibited the same behavior. This is due to the emergence of a global order even when an agent is behaving out of self-interest during the trading process. As a result, the agent-based Boltzmann Wealth Model with saving propensity factor has failed to capture the microscopic dynamics of the system and shows behavior at a population level.

3.2 Adding Heterogeneity

The Boltzmann Wealth Model with the same savings propensity factor λ for all agents exhibited coarse-graining and failed to capture the microscopic dynamics. As the first step to resolve this, heterogeneous agents - agents with different savings propensity factor was introduced in the existing model.

3.2.1 Model Description

The basic settings of the existing Boltzmann Wealth Model were retained in this model. i.e, 500 agents with an initial capital m_i of 100 were placed on the mesa grid. They were allowed to trade such that each agent had their own savings propensity factor λ_i randomly drawn from a uniform distribution that has an upper bound of 0.9 and a lower bound of 0.1. The choice of the limit is justified as when λ_i is 0 then the model becomes the original Boltzmann Wealth Model with no savings propensity factor. Similarly, if λ_i is 1, then the agent would be saving the entire money for themselves and hence won't be an active participant in the economy. The transaction between 2 agents say, i and j with wealth m_i and m_j, savings propensity factor λ_i and λ_j respectively happens according to the following equations.

$$
\begin{aligned}
& m_i \rightarrow m_i^{'} \\
& m_j \rightarrow m_j^{'} \\
where, \quad & m_i^{'} = m_i - \Delta m \\
& m_j^{'} = m_j + \Delta m \\
and, \quad & \Delta m = (1 - \lambda_i)[m_i - \epsilon(m_i + m_j)]
\end{aligned}
$$

The model was run for 2500 time steps and the amount of money possessed by each agent, m_i at the time i and the average money per agent - temperature, mT_i is recorded at every time step. These values were then used to draw insights from the model.

3.2.2 Observations and Results

The equilibrium wealth distribution of this model is as shown in figure 3.6.

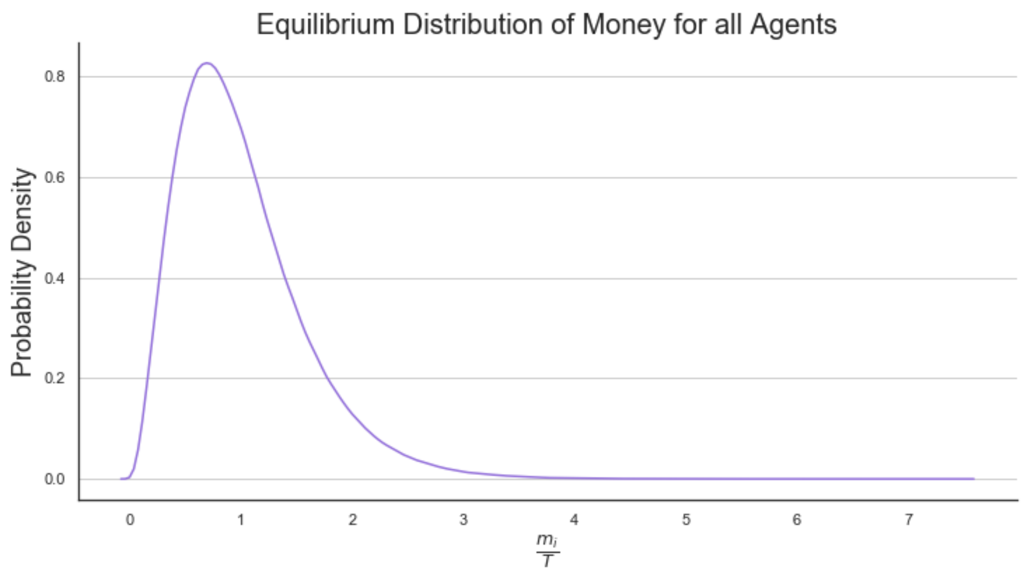
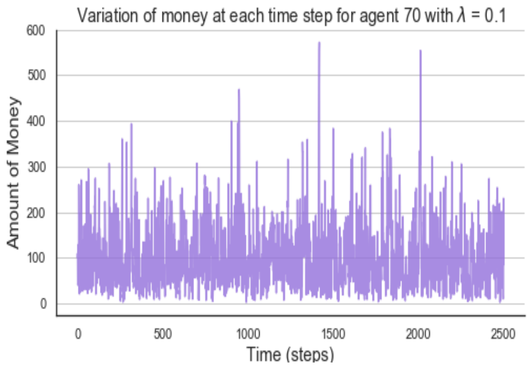


FIGURE 3.6: Equilibrium Wealth Distribution of Boltzmann Wealth Model with Heterogeneous λ

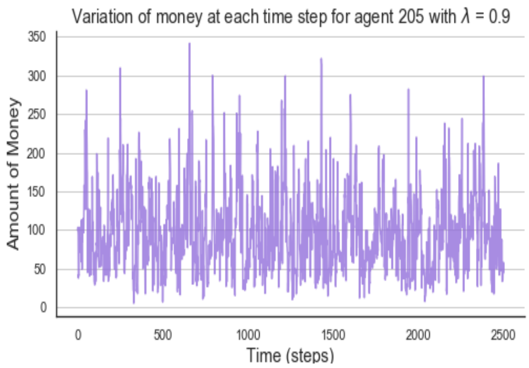
The equilibrium wealth distribution of figure 3.6 shows homogeneity in the amount of money possessed by the agents. At first glance, it can be observed that heterogeneity is not attained in this model as expected. Hence, the time-series data of random agents are plotted to gain insight into their at an individual level. The time-series data and the corresponding wealth distribution of random agents with varying savings propensity values are as shown in figure 3.7. Each plot in figure 3.7 corresponds to different savings propensity factor such that when $\lambda = 0.9$ as in 3.7(C) and (D), it is expected that the agent would save more of their money and hence there should be a heterogeneous distribution. However, all the plots in figure 3.7 look the same and are no different from the plots of the Boltzmann Wealth Model with savings propensity factor $\lambda = 0.45$ as in figure 3.4. As a result, it could be observed that there is no heterogeneity in this model.

A possible reason for this behavior could be that the agents in this model were allowed to communicate/trade with each other with no restriction, irrespective of the different savings propensity factors and different income levels. i.e. there is an inherent assumption



(A) Time Series Analysis of Money Data

(B) Distribution of Money



(C) Time Series Analysis of Money Data

(D) Distribution of Money

FIGURE 3.7: Time Series Analysis and Wealth Distribution of Random Heterogeneous Agents of Boltzmann Wealth Model

that the population is well-mixed and all agents are allowed to trade and transact with everyone. As a result, the heterogeneity effect is averaged out to make any difference notable difference.

3.2.3 Conclusion

The desired results are not obtained from the model built at this stage. The results from the Boltzmann Wealth Model with savings propensity factor λ and the Boltzmann Wealth Model with heterogeneous agents closely resemble each other. Hence, the model has once again failed to capture the microscopic dynamics due to the assumption that the population is well-mixed and all agents are allowed to trade and transact with all other agents irrespective of the difference in the fractions of money each agent saves for themselves (λ_i). This leads to an averaging of the total wealth and hence no heterogeneity is visible in the results.

3.3 Adding Social Capital

One of the major limitations of the heterogeneity model was the assumption of a well-mixed population and letting all agents communicate with each other irrespective of differences in their wealth. In real life, no person interacts with all others. Hence, we translated the entire model built in the mesa grid to a network where each agent has contacts with few other agents depending on the situation. This process introduced the 'social capital' in the model which captures the interaction amongst the agents. This process changed the existing one-dimensional model into a two-dimensional model.

3.3.1 Model Description

The new model from now on will be addressed as the Network Model. Albert Barabasi Network was chosen to make this model, for its close resemblance with a real-world network. Mesa grid was replaced with NetworkGrid from the python mesa package. This setup was retained as a placeholder for an agent in the network system. However, all the functionalities are implemented based on the current state of the Albert Barabasi Network.

500 agents were placed in an Albert Barabasi Network implemented using the networkx package of Python. The barabasi_albert_graph() has 2 parameters - number of nodes (n) and number of edges to attach from a new node to existing nodes (m) [40]. The agents were regarded as the nodes and the network is grown by attaching new nodes with m edges, which are preferentially attached to existing nodes with a high degree. In the Network Model 'm' had to be at least 10 because, when a smaller number of edges were added the model didn't exhibit the desired behavior and had the same wealth value for all the agents for all time steps. This could be because most of the agents existed alone as a result of preferential attachment to existing nodes with a higher degree at the time of initialization.

The model was run for a total of 2500 time steps and trade between the immediate neighbors in the network was permitted. This ensured that an agent was not interacting with everyone but with a selected group in their vicinity which remained the same throughout the execution.

3.3.2 Observations and Results

The overall equilibrium wealth distribution of all agents during the entire time steps is as shown in figure 3.8. Figure 3.8 appears to be quite confusing. It shows that a majority of

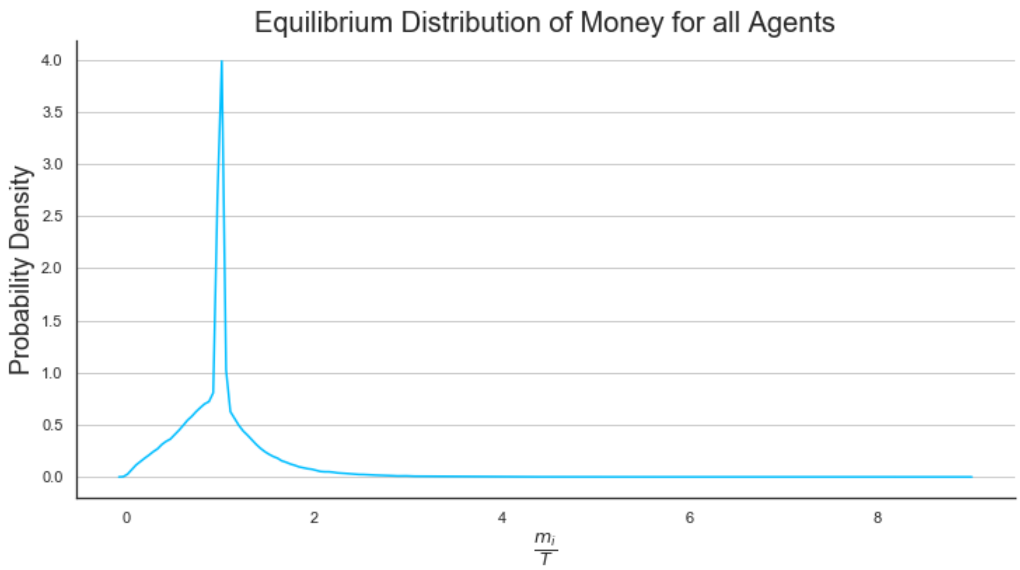
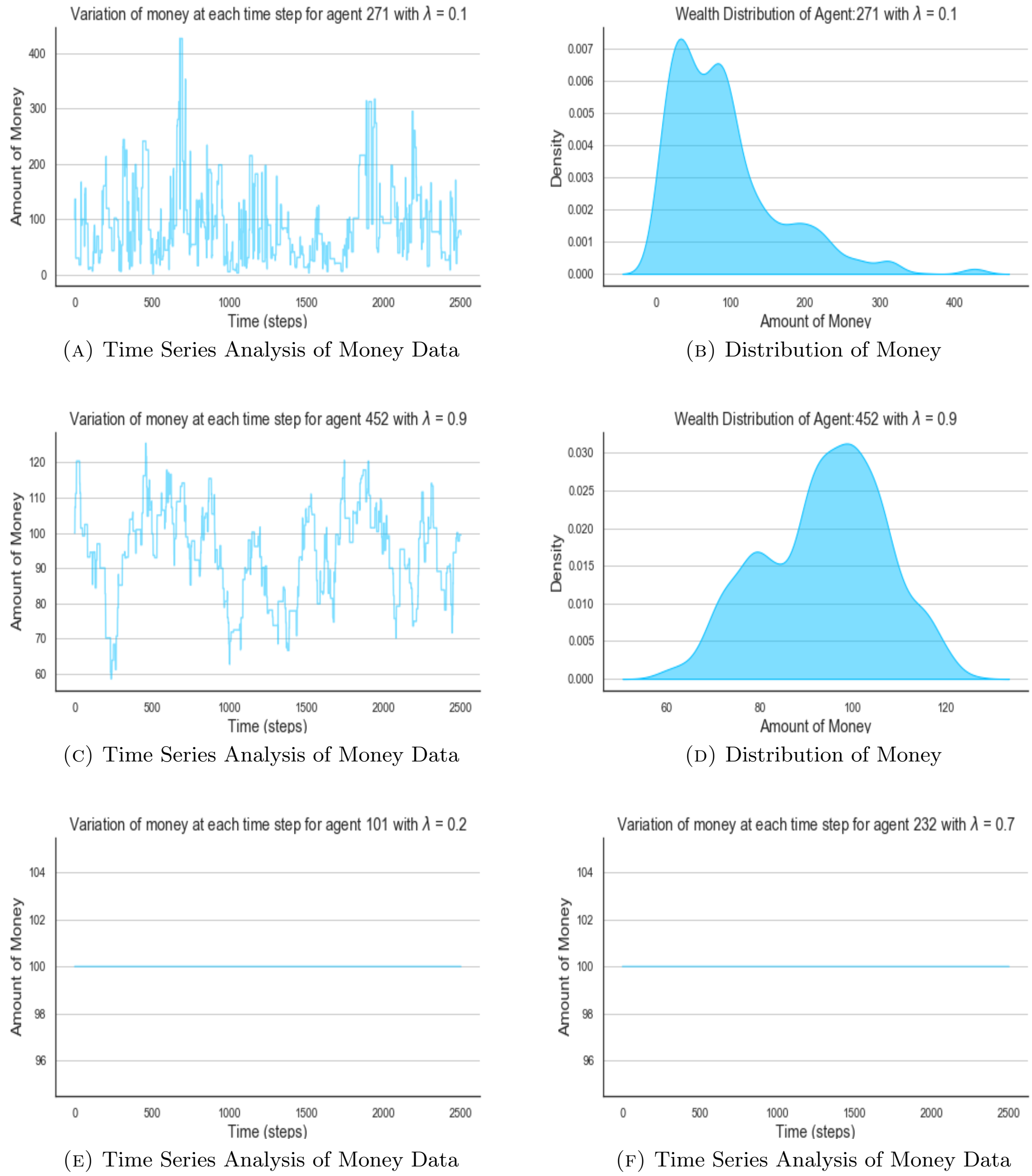


FIGURE 3.8: Equilibrium Wealth Distribution of Network Model

agents possessed the same amount of money throughout the 2500 time steps. As a result, the individual behavior had to be looked into. Figure 3.9 shows the time series data and the distribution of wealth of random agents with different savings propensity values. (E) and (F) of figure 3.9 shows the time series data alone for two random agents. It can be observed that these agents possessed the initial amount of money throughout the entire run and have not participated in trade. Many such lone agents never had the chance to trade and contribute to the economy even though they possessed a considerable amount of money. This could be attributed to the initial preferential attachment of nodes to existing nodes with a higher degree, which might have left some nodes with no links to other nodes. As a result, these nodes existed without any neighbors and hence never participated in trade. Hence the equilibrium probability distribution in figure 3.8 can be justified. Simultaneously, as figure 3.9 (A), (B), (C), and (D), shows the network induced interaction effect had a significant impact on the economy. Many agents exhibited multi-modal wealth distributions indicating the possession of different amounts of money for longer periods of time. Hence, Network model was able to introduce some amount of heterogeneity among the agents.

3.3.3 Conclusion

The Network Model could resolve the problem of the lack of heterogeneity of the Boltzmann Wealth Model. However, there are many agents in this model who do not participate in trade due to the limitations of the model itself. This has impacted the overall wealth distribution and is not a desirable result.



(A) Time Series Analysis of Money Data

(B) Distribution of Money

(C) Time Series Analysis of Money Data

(D) Distribution of Money

(E) Time Series Analysis of Money Data

(F) Time Series Analysis of Money Data

FIGURE 3.9: Time Series Analysis and Wealth Distribution of Random Agents of Network Model

3.4 Introducing Homophily

In the Network Model, one of the major limitations was that the network was stagnant which put the lone agents at disadvantage by not letting them participate in the economy. So, in this stage, the focus was mainly to create a network that evolves and changes in time based on the economic stature of an agent. The concept of homophily was hence introduced to the Network Model. Homophily can be formally defined as ‘the propensity of similar agents to connect to each other' [41]. This is one of the most basic social processes that may be observed in both human and animal communities [41].

3.4.1 Model Description

This model will be addressed as Homophily Model from now on. The mesa NetworkGrid was removed in this model which earlier served as a placeholder for agents. Albert Barabasi Network was used here as well due to the reasons explained in section 3.3.1. Homophily was introduced in this model by means of edge weight assigned to the links between two agents in the network. The edge weight varies between 0 and 1 with 0 indicating no similarity and 1 indicating a strong similarity. The initial edge weight between agents is assumed to be 1. In order to make the network mobile and active three additional functionalities were introduced in this model. They are:

1. Global Attachment: Global attachment happens at the model level where 2 agents are chosen randomly and a link is established between them if no link is present already. In the case of an already existing link, another agent with whom no link is present is chosen[42].

2. Local Attachment: 2 agents are randomly chosen at a given time step and if no link is present between these agents, then a link is formed between them. In the unlikely event of an already existing link between the agents, a maximum of 5 attempts is made to find a new agent, without an existing link between them. A limit of 5 was necessary in order to avoid being stuck in an infinite loop in case of a complete network. Local Attachment happens as many times as the number of agents in the network.

3. Link Deletion: Links between random agents are deleted and if no link is present, then another agent is chosen such that there exists an edge between the two agents. Link deletion also happens as many times as the number of agents in the network [42].

Since these processes helped to keep the network active, the value of 'm' - the number of edges to attach from a new node to existing nodes - is chosen as 1 at the time of initialization, to let the network evolve on its own.

As mentioned above, edge weight is the measure of homophily between any 2 agents. Although edge weight is 1 at the time of initialization, it changes during the processes of global attachment, local attachment, and trade according to the Fermi Dirac distribution as given below [41].

$$w_{ij} = \frac{1}{1 + e^{a(m(x_i,x_j)-b)}}$$

where w_{ij} is the edge weight of the link between agent i and agent j, a is the homophily parameter, $m(x_i, x_j)$ is the difference between the amount of money possessed by agents i and j respectively and b is the characteristic distance between the nodes in embedding space where the weight becomes $\frac{1}{2}$. Fermi-Dirac distribution was chosen to compute the edge weights because, among all the possible edge weights that meet the constraints, the ones computed using Fermi-Dirac will give the most unbiased values for the properties of the network.

The method by which the agents trade remained the same as in the case of the Network Model, where each agent traded with their neighbors. However, in the Homophily Model, some amount of stochasticity was introduced in the trading process. i.e. trade between any 2 agents is permitted only when the edge weight is greater than or equal to a random value generated between 0 and 1. This random value is different at each time step. After the transaction, the edge weight of the links between the agents involved and their corresponding neighbors had to be updated due to the changes in the wealth. This model was also run for a total of 2500 time steps.

3.4.2 Observations and Results

The overall wealth distribution of the Homophily Model that was run for 2500 time steps is as shown in figure 3.10. The time-series data and the wealth distribution of random agents with different savings propensity values are as shown in figure 3.11. Figures 3.10 and 3.11 of the Homophily Model looks quite confusing as it shows no difference from it's Boltzmann Wealth Model counterparts in figures 3.3 and 3.4 respectively.

Since the Network Model showed a significant difference from the Boltzmann Wealth Model, this behavior of the Homophily Model, which is supposed to be a better version of the Network Model doesn't look promising. So, it became necessary to look deeper into the data generated from both models and not limit to the time series data analysis and the wealth distribution.

3.4.2.1 Comparing Boltzmann Wealth Model and Homophily Model

1. **Wealth Distribution:** Although many people are delinquent in reporting their earnings to government bodies, many countries have mandated to report all the assets of a deceased for inheritance tax. In the year 2003, based on the assets

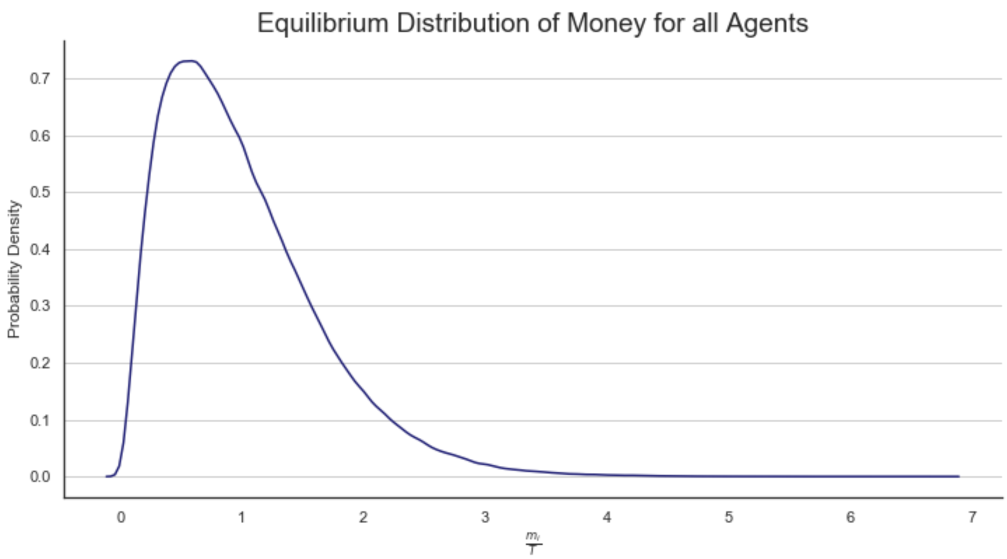
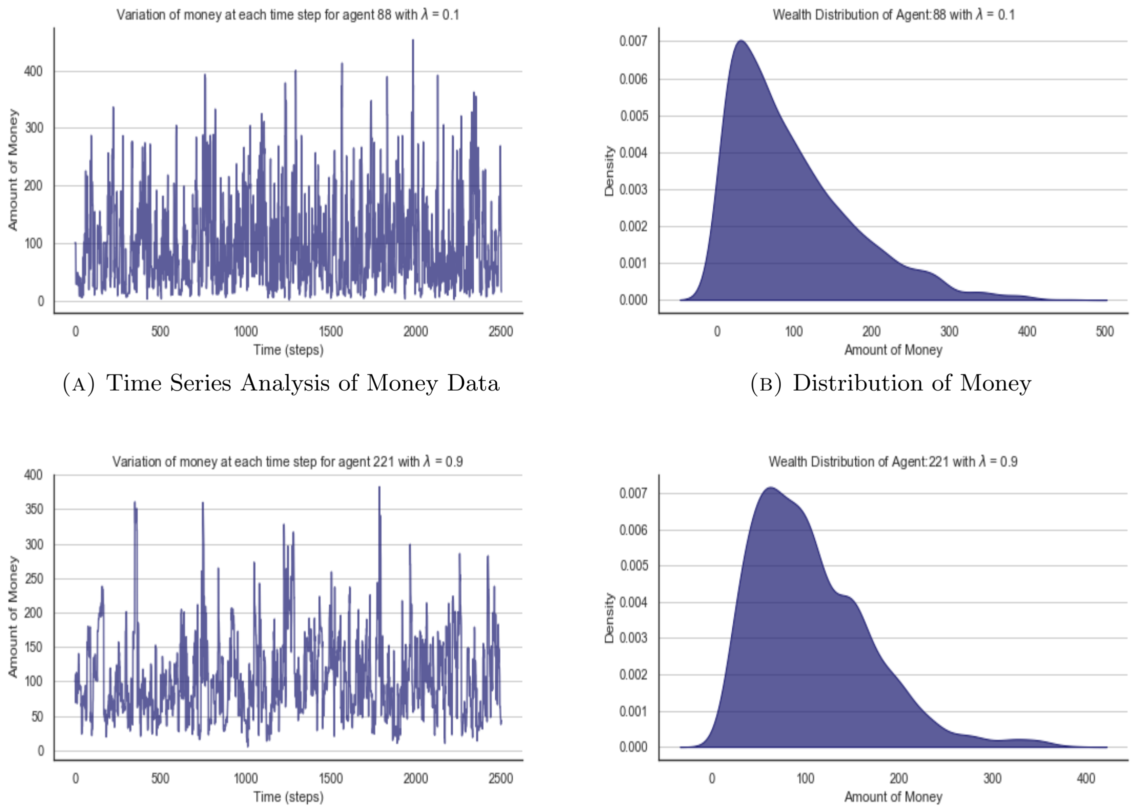


FIGURE 3.10: Equilibrium Wealth Distribution of Homophily Model



(A) Time Series Analysis of Money Data

(B) Distribution of Money

(C) Time Series Analysis of Money Data

(D) Distribution of Money

FIGURE 3.11: Time Series Analysis and Wealth Distribution of Random Heterogeneous Agents of Homophily Model

of the dead reported to the Inland Revenue, tax agency of the United Kingdom, the wealth distribution of the entire population could be reconstructed after an adjustment procedure based on the age, gender, and other characteristics of the deceased [36]. Based on this empirical evidence it was observed that the lower part of the wealth distribution, for 90 percentile i.e the majority of the population exhibited Maxwell-Boltzmann distribution. The remaining 10 percentile who were the richest showed a Pareto Power-law distribution [36]. The Boltzmann Wealth Model and the Homophily Model showed a similar behavior as shown in figures 3.12 and 3.13.

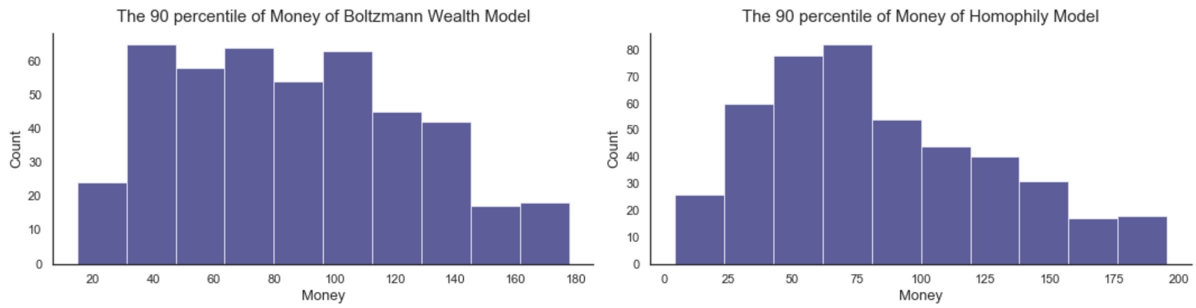


FIGURE 3.12: The 90 Percentile of Equilibrium Wealth Distribution

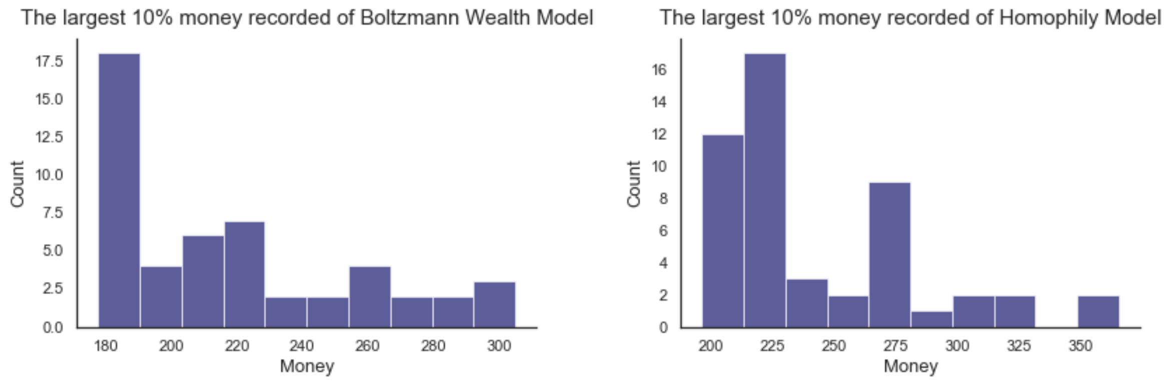


FIGURE 3.13: The Largest 10 Percentile of Equilibrium Wealth Distribution

The data derived from both models were fit to Maxwell Boltzmann distribution and Pareto Power-law distribution for the 90 percentile wealth distribution and largest 10 percentile wealth distribution respectively. The Maxwell Boltzmann distribution's fit on the 90 percentile of the wealth distribution is as shown in figure 3.14. From this figure, it can be observed that the data from both models fit the Maxwell-Boltzmann distribution quite well. So the goodness of fit had to be measured, to compare the two models. Here, the log-likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) of the fit are measured and are summarized as shown in table 3.2.

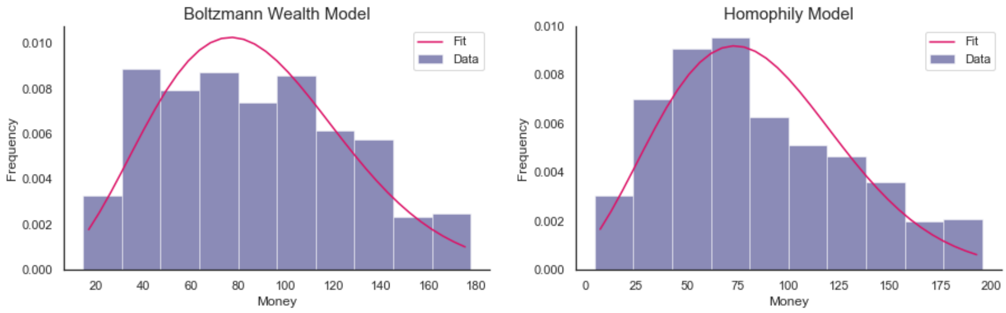


FIGURE 3.14: Fitting Maxwell-Boltzmann Distribution on 90 Percentile Wealth Distribution

	Boltzmann Wealth Model	Homophily Model
Log-likelihood	-4618.2985	-4614.7272
AIC	9240.5972	9233.4543
BIC	9248.8157	9241.6728

TABLE 3.2: Goodness of Fit of Maxwell Boltzmann Distribution

The log-likelihood of the Boltzmann Wealth Model is smaller than that of the Homophily Model as listed in table 3.2. Similarly, the AIC and BIC of the Homophily Model have a lower value than the Boltzmann Wealth Model. Hence it can be concluded that the data derived from the Homophily Model gives a better fit with the Maxwell - Boltzmann distribution even though by small margins as shown in table 3.2.

The largest 10 percentile of the wealth distribution which exhibited the Pareto-Power-law were fitted with Pareto distribution as shown in figure 3.15.

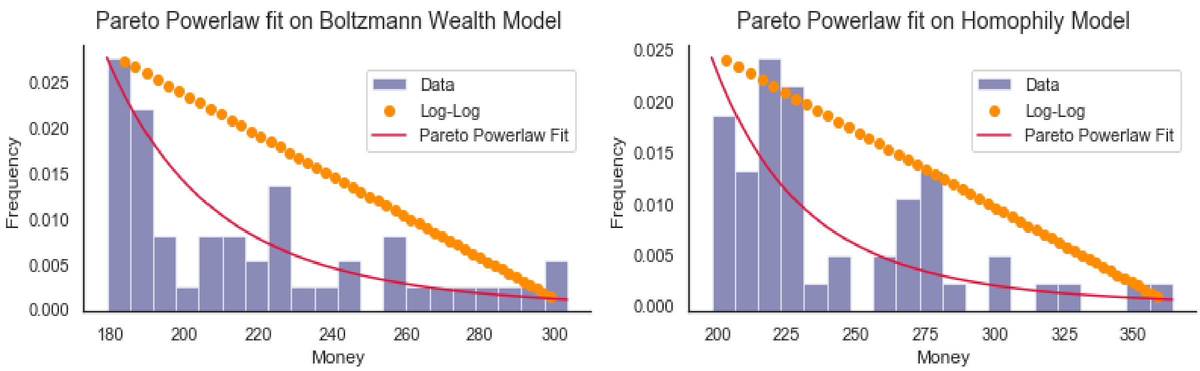


FIGURE 3.15: Fitting Pareto-Power law Distribution to the Largest 10 Percentile of Wealth Distribution

Further, the goodness of fit of the data with the Pareto-Power law was checked with the two-sample Kolmogorov-Smirnov test. The results are summarized in table 3.3.

	Boltzmann Wealth Model	Homophily Model
p-value	1.39 x 10^{-5}	4.8 x 10^{-6}
KS Statistic	0.48	0.5

Table 3.3: Goodness of Fit of Pareto-Power law Distribution

The Boltzmann Wealth Model and the Homophily Model both gave a p-value lower than the level of significance kept at 0.05 as shown in table 3.3. It can be concluded that both the largest 10 percentile wealth data from both models gave a good fit with the Pareto-Power-law and hence comparison between the two cannot be made based on this data.

2. **Analysing the Spread:** The spread of wealth was analyzed by plotting the average amount of money possessed by the agents over certain first and last time steps. The figure 3.16 shows the amount of money in possession with different agents averaged over 20, 50, 75, 200, 500, and 700 first and last time steps. The Homophily model showed a greater spread i.e. less randomness due to selective communication. However, the average money remained the same due to the conservation of money in the trade. Hence, from the analysis of the spread, it can be concluded that Homophily Model gives a better result and hence a better model than the original Boltzmann Wealth Model due to its close resemblance to real-world behavior.

3. **Agents Living in Poverty:** The concept of poverty trap - i.e agents staying in perpetual poverty, was applied to both models to produce a comparison as to which model captures the poverty trap among agents more efficiently.

   A 10 percentile threshold was defined for both models such that the agents staying below this threshold value for consecutive time steps are said to be trapped in poverty. The sum of these consecutive time steps for each agent who stayed in the poverty trap was found and distribution is plotted from those values. This is shown in figure 3.17. From this figure, it is evident that the total number of consecutive time steps for any agent who stayed below the threshold is maximum for the Homophily Model. A tabular summary of some of the insights drawn by

	Boltzmann Wealth Model	Homophily Model
Threshold	40.492	30.212
Maximum Total Time Steps*	153	472
Largest Time Step*	13	37
		*consecutive time steps spend below threshold

Table 3.4: Summary on Poverty Trap

comparing the occurrence of poverty traps in both models is given in table 3.4.

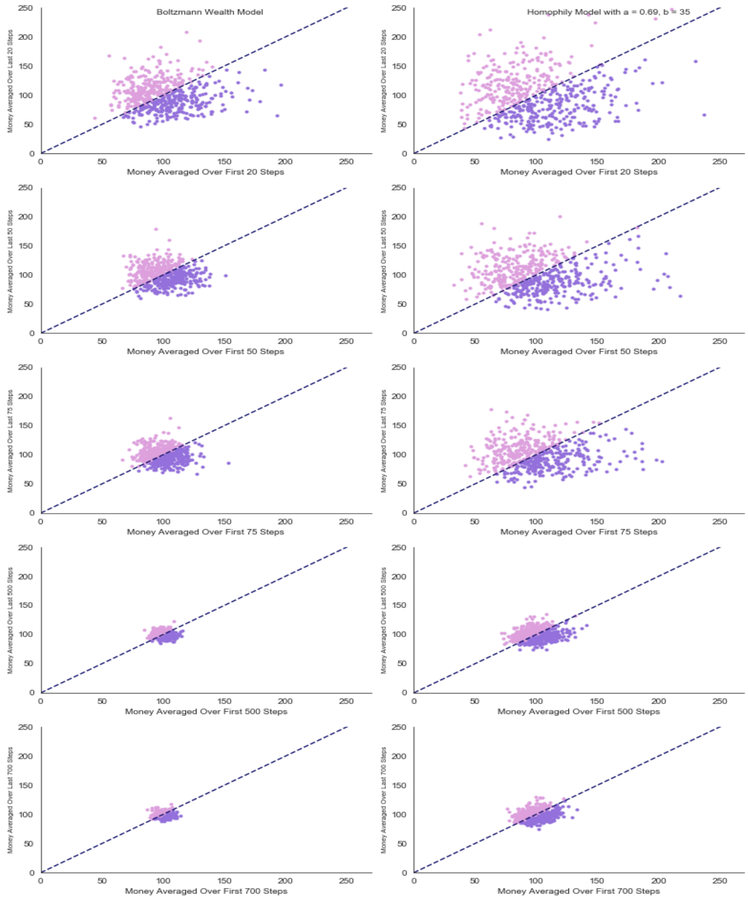


Figure 3.16: Analysis of Spread

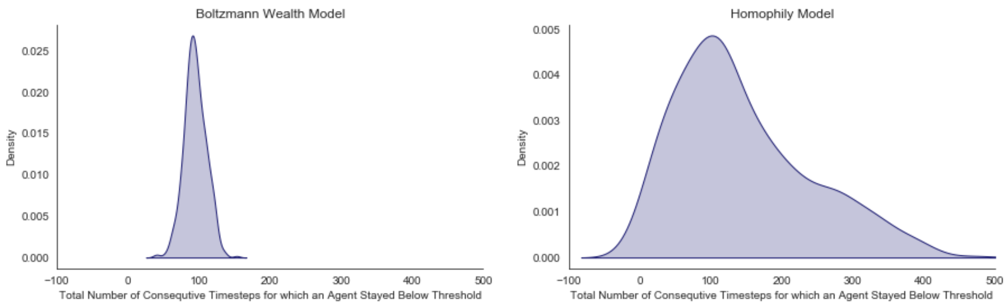


Figure 3.17: Distribution of Total Number of Poverty Traps

From figure 3.17 and table 3.4, it can be observed that Homophily Model could capture the poverty trap more efficiently than the Boltzmann Wealth Model.

3.4.3 Conclusion

The addition of ‘Social Capital’ and thereby translating the one-dimensional poverty trap model into a two-dimensional model has resulted in a better model. The Homophily Model is superior to the Boltzmann Wealth Model. This could be attributed to its close resemblance to a real-world scenario, the similarity of the model data behavior to that of the empirical data collected by the UK Inland Tax Authority, reduced randomness in the money distribution, and the capturing of more numbers of poverty trap occurrence among the agents. Hence, it could be concluded that the step-by-step approach in making the two-dimensional poverty trap model has resulted in a preferable and superior model.

3.5 Adding Human Capital

Human capital is defined as the ability of an individual to fend for themselves. It is represented by a numeric value that captures the immutable physical stature, cognitive development, and educational attainment with which they enter adulthood and hence the economy [9]. In this stage, human capital was incorporated into the Homophily Model to make the three-dimensional final model of this project. This model will be addressed as the Human Capital Model henceforth. This stage of the project was hugely inspired by the multi-dimensional poverty trap model created by Christopher Barret and his colleagues [9].

3.5.1 Model Description

As mentioned above the human capital- α was incorporated by using a numerical value. This value was drawn from a normal distribution with a mean of 1.08 and a variance of 0.074. The concept of money/wealth m_i was discarded and was replaced by 'stock of capital -k_i'. Both 'money' and 'stock of capital' are essentially the same except that in the Human Capital Model, an agent is capable of generating an income given the stock of capital k_i and the corresponding human capital α_i. The value of the stock of capital k_i is drawn from a uniform distribution that has an upper bound of 0.1 and a lower bound of 10. The values for human capital α_i and stock of capital k_i are taken the same as in the study by Barret et.al [9]. Unlike the previous models where all agents had the same initial amount of money(m_0), in this model all agents have different amounts of initial stock capital (k_0).

An agent with a stock of capital k_i and human capital α_i at time step i can generate income $f(\alpha_i, k_i)$ such that they choose between a high technology or a low technology depending on the critical level income. i.e.

$$f(\alpha_i, k_i) \begin{cases} f_L(\alpha_i, k_i) = \alpha_i k_i^{\gamma_L} \\ f_H(\alpha_i, k_i) = \alpha_i k_i^{\gamma_H} - E \end{cases} \tag{3.1}$$

such that γ_L and γ_H are the parameters corresponding to lower and higher technology with $0 < \gamma_L < \gamma_H < 1$, and E is the fixed cost associated with the high technology. An agent prefers investment in higher technology when their stock capital k_i and human capital α_i allows them to earn an income such that $f_H(\alpha_i, k_i) \geq f_L(\alpha_i, k_i)$. i.e. more the income an agent has the more expensive things they can buy.

At every time step 't', an agent 'i' has a consumption of c_i and investment i_i such that the sum of these should be less than or equal to the income that an agent had earned for that period. i.e.

$$c_i + i_i \leq f(\alpha_i, k_i)$$

The stock capital for the next time step t+1, is given by the following equation.

$$k_{i(t+1)} = \theta_t[i_t + (1-\delta)k_{it}]$$

The unknown terms in the equation will be explained shortly. Suppose the values for θ_t and δ_t and k_{it}, the initial capital, are known beforehand. Then the value for investment i_t had to be found to solve for the next step stock of capital $k_{i(t+1)}$.

This problem, hence can be formulated as an optimization problem as shown below:

$$\begin{aligned} max \;\; E_\tau \sum_{t=\tau}^{\infty} \beta^{t-1} u(c_t) \\ such\ that \;\; c_t + i_t \leq f(\alpha_i, k_{it}) \\ k_{i(t+1)} = \theta_t[i_t + (1-\delta)k_{it}] \end{aligned}$$

where E_τ is the expectation at the start of the period τ, β is the time discount factor and $u(c_t)$ is the utility function defined over consumption c_t and is given as:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

δ is the asset depreciation rate, and θ is the shock factor such that $\theta \in [0, 1]$.

When $\theta = 1$ then there is no shock and when $\theta < 1$ then there is a negative shock that destroys the assets.

The investment rule in $k_{i(t+1)}$ would become the policy function of the Bellman equation in the presence of asset shocks and is given as:

$$V(k_t) \equiv \max_{i_t}\{u(f(\alpha, k_t) - i_t) + \beta E[V(k_{t+1}|k_t, i_t)]\}$$
$$where E[V(k_{t+1}|k_t, i_t)] = \int V(\theta_t[i_t + (1-\delta)k_t])d\Omega(\theta_t)$$

where $\Omega(.)$ is the cumulative density function of θ_t.

The optimization problem can be summarized as:
given stock of capital k and human capital α what should be the maximum investment an agent can make in the presence of asset shocks.
This is solved with the help of the notes of Eric Sims on 'Neoclassical Growth Models'[3].

The equation is solved using first-order Lagrangian and is shown below:

$$L = \sum_{t=\tau}^{\infty} \beta^{t-1} E_\tau \Big[\frac{c_t^{1-\sigma} - 1}{1-\sigma} + \lambda_t(\theta_t(\alpha k_t^\gamma - c_t + (1-\delta)k_t) - k_{t+1})\Big]$$

The first order conditions are : $\frac{\partial L}{\partial c_t} = 0$, $\frac{\partial L}{\partial \lambda_t} = 0$ and $\frac{\partial L}{\partial k_{t+1}} = 0$

$\frac{\partial L}{\partial c_t} = 0$

$$\begin{aligned} &\implies \beta^{t-1} E_\tau \Big[(1-\sigma)\frac{c_t^{(1-\sigma-1)}}{1-\sigma} - \lambda_t \theta_t\Big] = 0 \\ &\implies c_t^{-\sigma} = \lambda_t \theta_t \end{aligned} \tag{3.2}$$

$\frac{\partial L}{\partial \lambda_t} = 0$

$$\begin{aligned} &\implies \theta_t\Big(\alpha k_t^\gamma - c_t + (1-\delta)k_t\Big) - k_{t+1} = 0 \\ &\implies k_{t+1} = \theta_t\Big(\alpha k_t^\gamma - c_t + (1-\delta)k_t\Big) \end{aligned} \tag{3.3}$$

$\frac{\partial L}{\partial k_{t+1}} = 0$

$$\begin{aligned} &\implies \beta^{t-1} E_\tau \Big[\lambda_{t+1}\theta_{t+1}(\alpha\gamma k_{t+1}^{\gamma-1} + (1-\delta)) - \lambda_t\Big] = 0 \\ &\implies \lambda_t = \beta^{t-1} E_t \lambda_{t+1}\theta_{t+1}(\alpha\gamma k_{t+1}^{\gamma-1} + (1-\delta)) \end{aligned} \tag{3.4}$$

τ can be replaced as 't' as both denotes a given time period. From equation 3.2,

$$c_t^{-\sigma} = \lambda_t \theta_t$$
$$\implies \lambda_{t+1} = \frac{c_{t+1}^{-\sigma}}{\theta_{t+1}}$$

Substituting for λ_{t+1} in equation 3.4

$$\begin{aligned} \lambda_t &= \beta^{t-1} E_t \frac{c_{t+1}^{-\sigma}}{\theta_{t+1}} \theta_{t+1} \Big[\alpha\gamma k_{t+1}^{\gamma-1} + (1-\delta) \Big] \\ &= \beta^{t-1} E_t c_{t+1}^{-\sigma} \Big[\alpha\gamma k_{t+1}^{\gamma-1} + (1-\delta) \Big] \end{aligned}$$

Substituting for λ_t in equation 3.2,

$$\begin{aligned} c_t^{-\sigma} &= \lambda_t \theta_t \\ &= \beta^{t-1} E_t c_{t+1}^{-\sigma} \Big[\alpha\gamma k_{t+1}^{\gamma-1} + (1-\delta) \Big] \end{aligned} \tag{3.5}$$

By substituting for k_{t+1} from equation 3.3, in equation 3.5 to solve for c_{t+1} gives,

$$\begin{aligned} c_{t+1}^{-\sigma} &= \frac{c_t^{-\sigma}}{\beta^{t-1} E_t \theta_t \Big[\alpha\gamma k_{t+1}^{\gamma-1} + (1-\delta) \Big]} \\ c_{t+1}^{\sigma} &= c_t^{\sigma} \beta^{t-1} E_t \theta_t \Big[\alpha\gamma k_{t+1}^{\gamma-1} + (1-\delta) \Big] \\ \implies c_{t+1} &= c_t \Big(\beta^{t-1} E_t \theta_t \Big[\alpha\gamma k_{t+1}^{\gamma-1} + (1-\delta) \Big] \Big)^{\frac{1}{\sigma}} \end{aligned} \tag{3.6}$$

From equations 3.3 and 3.5, the stock of capital and the consumption for the next time step can be determined.

These equations hence will be used in the model to simulate an economy of 500 agents. All the agents will enter the economy with an initial stock of capital k_0 such that it is different for each agent. As explained previously the initial k_0 is drawn from a uniform distribution with a lower and upper bound of 0.1 and 10 respectively. The stock of capital(k_t) changes according to 3.3 as time progresses. The γ term of equation 3.3 will be either γ^L or γ^H depending on the type of technology the agent invests in during that time step. This is determined with the equations in 3.1, where an agent chooses high technology only when $f_H(\alpha_i, k_{it}) \geq f_L(\alpha_i, k_{it}$. All agents have a unique savings propensity value λ and are randomly generated such that $0 < \lambda < 1$. This value remains constant throughout the agent's lifetime. Similarly, all agents have a unique human capital value α drawn from a normal distribution with a mean of 1.08 and a variance of 0.074, such that this value remains the same during an agent's lifetime.

All other features that were introduced in the Homophily Model were retained in the Human Capital Model, to incorporate 'Social Capital' in this model. Thus the model is translated into a three-dimensional model, with the dimensions being the social capital, human capital, and financial capital.

Determining the initial consumption c_{i0} of an agent 'i', was the most challenging part of all in this project. For this, again, Eric Sim's notes on Neoclassical Growth Model were used and the method is heavily drawn from it [3].

The optimal point is where the $\frac{k_{t+1}}{k_t} = 1$ and $\frac{c_{t+1}}{c_t} = 1$ isoclines meet as shown in figure 3.18. The $\frac{k_{t+1}}{k_t} = 1$ isocline is found from the capital accumulation equation in 3.3 and by the assumption $k_{t+1} = k_t$, i.e next step stock of capital = current step stock of capital. Hence,

$$\begin{aligned} k_t &= \theta_t\Big(\alpha k_t^{\gamma} - c_t + (1-\delta)k_t\Big) \\ c_t &= \frac{\theta_t \alpha k_t^{\gamma} + \Big[\theta_t(1-\delta) - 1\Big]k_t}{\theta_t} \\ \implies c_t &= \alpha k_t^{\gamma} + (1-\delta)k_t - \frac{k_t}{\theta_t} \end{aligned} \tag{3.7}$$

With the equation in 3.7, the consumption at time t can be found. This equation is used to initialize c_{0i} of an agent i. As mentioned above the optimum value would be where the two isoclines intersect. So, stock of capital(k) and consumption(c) would have a tendency to attain the optimum value as shown by the arrows pointing to the optimal point in figure 3.18. i.e When k is low, c cannot be higher and when k is high, c cannot be too low. This set bounds on the value of consumption c_t based on the value of the stock of capital(k_t). This is called bounded rationality, as the optimum consumption(c_t) values are determined based on a boundary rather than exact values. In the model, this optimal condition had to be met in the subsequent time steps and is implemented with the help of the slope.

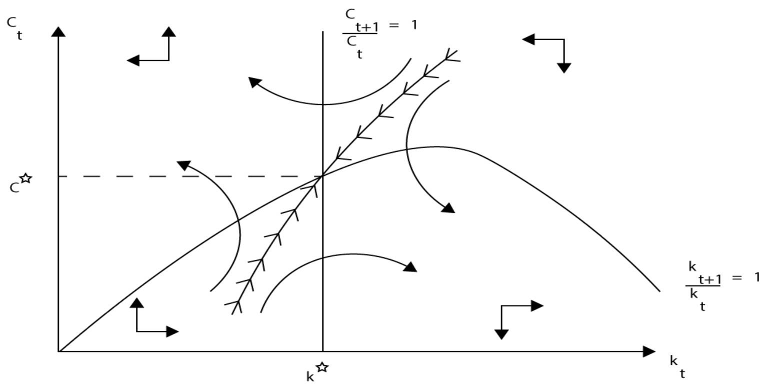


FIGURE 3.18: The Optimum Values for c_t and k_t based on Bounded Rationality [3]

The slope of the equation 3.7 is given as:

$$\frac{dc_t}{dk_t} = \gamma\alpha k_t^{\gamma-1} + (1-\delta) - \frac{1}{\theta_t} \tag{3.8}$$

The slope leads to 2 conditions and they are:

1. Slope is positive: i.e. $\frac{dc_t}{dk_t} > 0 \implies k_t$ is a small value. Hence c_t should also be a small value. These points are depicted by the arrows pointing to the optimal point in the first quadrant of figure 3.18.

2. Slope is negative: i.e. $\frac{dc_t}{dk_t} < 0 \implies k_t$ is a large value. Hence c_t can be a large value. These points are depicted by the arrows pointing to the optimal point in the fourth quadrant of figure 3.18.

Hence the processes involved in the Human Capital Model can be summarized as below:

1. The stock of capital k, human capital α, and savings propensity factorλ are initialized for all agents.

2. Using the initial values for k and α, the initial income is found. Depending on the income each agent chooses the type of technology that they would invest in.

3. The initial value for consumption c can hence be determined according to equation 3.7 and γ will be replaced by γ_H or γ_L depending on the type of technology.

4. In the income updating process, the next time step stock of capital(k_{t+1}) is calculated using the equation in 3.3.

5. Once the next time step income is computed the optimum consumption is calculated using the isocline equation 3.7, and the slope at the corresponding k_t is found using equation 3.8. This helps to identify in what quadrant the corresponding c_t must be located and, hence, remain as close as possible to the optimum value.

6. If the computed c_t is lower than the isocline consumption value and the stock of capital k_t, then the computed c_t can be accepted. Otherwise, the computed c_t will be replaced by the isocline consumption. To avoid the same consumption value occurring in the subsequent time steps, some amount stochasticity is introduced in the isocline consumption by subtracting a random value between 0 and 1, subject to the condition that it does not exceed the stock of capital k_t or does not result in a negative c_t.

7. The processes of money transaction, local attachment, and link deletion of the Homophily Model also happen in the Human Capital Model without any change in any of the methods.

8. These processes are repeated as many time steps as needed for a specific number of agents.

Parameter	Value
Homophily Parameter(a)	0.69
Characteristic Distance (b)	35
Fixed Cost E	0.45
γ_L	0.3
γ_H	0.45
σ	1.5
β	0.95
δ	0.08
θ	0.8

TABLE 3.5: Default Values of Parameters of Human Capital Model

The default values used for the parameters of the Human Capital Model are listed in table 3.5. This model is computationally heavy and hence, time-consuming. So, when explaining each result, changes made to any of the parameters - number of agents, total time steps, will be mentioned.

3.5.2 Micawber Frontier

Barret et.al, in their study 'Poverty Traps and Social Protection', defines Micawber Frontier as the amount of stock of capital(k_t) required by an agent 'i' with innate ability α_i to make a switch from choosing low technology to high technology. This stock of capital is denoted by $\tilde{k}(\alpha)$ and is also known as 'critical capital' [9]. Micawber Frontier as defined by Barret et.al, had to be determined to analyze the effectiveness of the Human Capital Model.

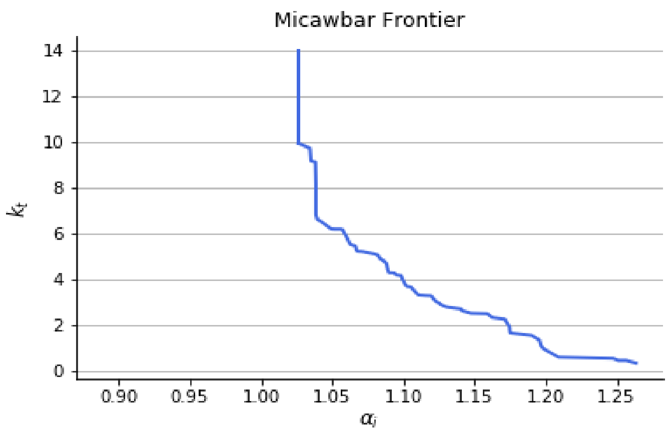


FIGURE 3.19: Micawber Frontier

The Social Capital from the Human Capital Model was removed by deleting the network characteristics such as trade/communication, global attachment, local attachment, and link deletion. The model was run for a total of 100 agents for 100-time steps. The stock of capital (k_t) and the innate ability (α) of those agents who made a switch from low technology to high technology was recorded and plotted as shown in figure 3.19. This gave the Micawber Frontier as determined by Barret and his colleagues. This Frontier although found without the social capital aspect is beneficial in comparing the progress made by agents in terms of their stock of capital (k_t) and innate ability (α_i).

An agent with innate ability (α_i) less than 1.02 would at least need a critical capital($\tilde{k}(\alpha_i)$) of 14 to cross-over the Micawber Frontier and to invest in high technology as shown in figure 3.19. As the innate ability increases, i.e., towards the right side of the frontier of figure 3.19, the critical capital ($\tilde{k}(\alpha_i)$) required to cross-over the Micawber Frontier decreases.

3.5.3 Observations and Results

The following observations are made on the basis of the Human Capital Model executed for 100 time steps for 500 agents.

Unlike the previous models, the results of the Human Capital Model were very different. Each agent exhibited a unique behavior and hence a generalization of the results was difficult. However, certain trends could be identified. One of the common behavior exhibited by all agents was that of a 'dipping point'. This point can be defined as the lowest stock of capital (k_t) attained by an agent in the economy. For a random agent, the dipping point is as shown in figure 3.20. This dipping point occurs within the first 10 time steps of the model run and hence it can be associated with an agent's behavior of getting acquainted with the economy and the processes involved, before realizing what is a best practice to achieve and maintain the maximum possible stock of capital given their innate ability α. i.e. there is a brief 'warm period' before the desired agent behavior appears. The agent's behavior that is exhibited within the occurrence of the tipping point is explained in the next paragraph.

The results of this model were comparable to that of a real-world scenario. Like human beings, the agents in this model exhibited confusion and adaptation behavior. In the initial time steps the agents behave rather erratically with a large amount spent on consumption ($c_t < k_t$), resulting in a reduction of the stock of capital (k_t) in the subsequent time steps. This behavior continues for some time steps - a warm-up period- before finally realizing what is best for them. During the warm-up period, some agents who started by investing in high technology might have already made a flip to low technology

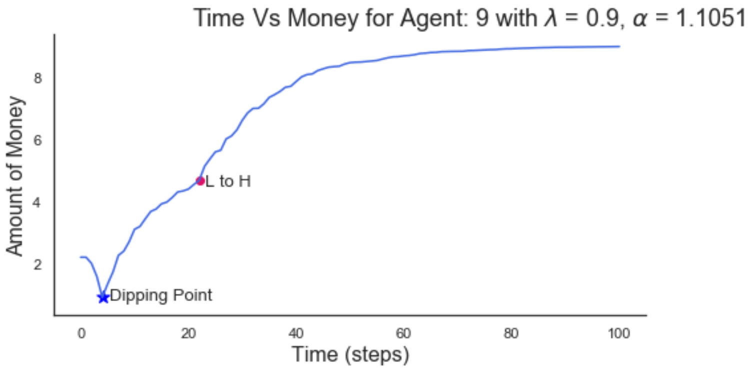
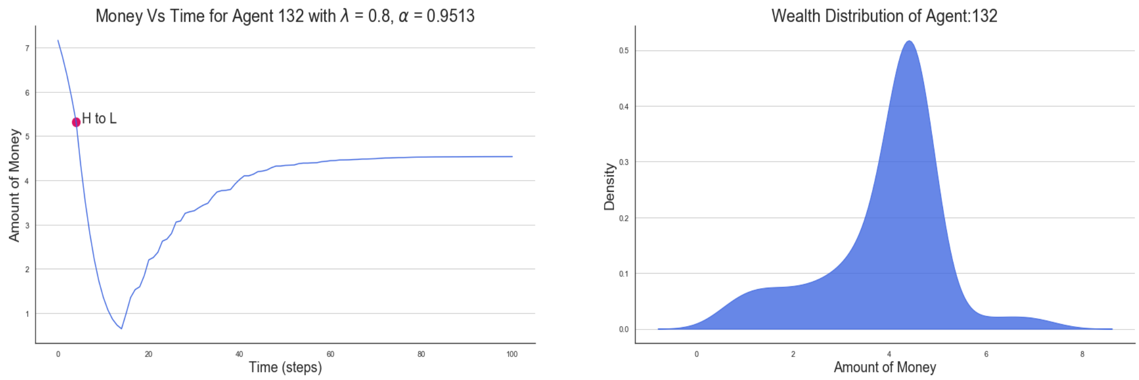


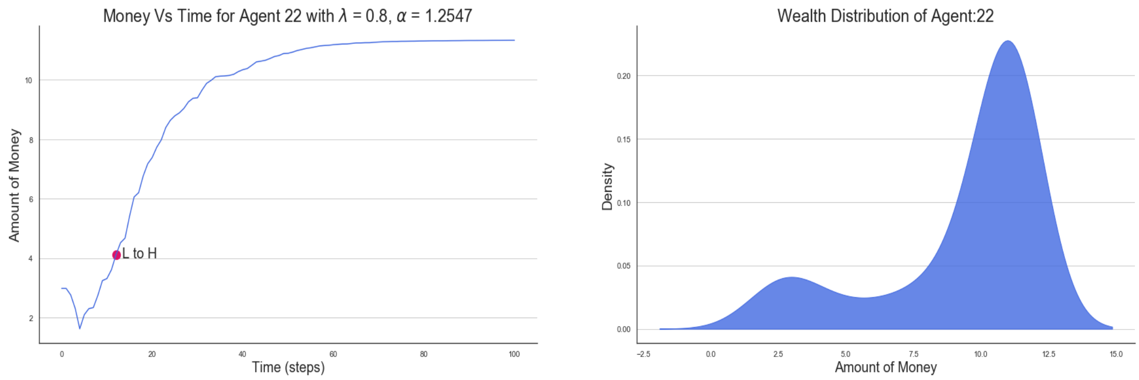
FIGURE 3.20: Dipping Point of a Random Agent in Human Capital Model

due to a reduction in the stock of capital (k_t) or vice versa depending on the stock of capital (k_t), innate ability α and savings propensity factor λ as shown in figure 3.21. The first switch from one technology to another as in figure 3.21 happens before the dipping point for agents with smaller innate ability α who started with a high initial stock of capital(k_0). Whereas, the switch is after the dipping point for agents with higher values for innate ability α irrespective of the initial stock of wealth (k_0) as shown in figure 3.21(B). How quickly the switch occurs is determined by the value of innate ability α. The larger the innate ability α, the faster an agent moves to a higher stock of capital (k_t). This is shown in figures 3.21(B) and (C). Hence, the transition from low technology to high technology or vice versa, is very much dependent on the innate ability α and savings propensity factor λ, of an agent. If the stock of capital(k_t) and the innate ability α is large enough, then the agent might continue using high technology without switching to low technology as shown in figure 3.22. The plots in figure 3.22 features 2 types of agents, one with higher innate ability α as shown in figure 3.22(A) and an agent with lower innate ability α as in figure 3.22(B). Both agents exhibit erratic consumption before settling into a stable stock of capital (k_t) and consumption (c_t). This could be attributed to the behavior during the warm-up period before adapting to the economy. Similarly, some agents can only afford to invest in low technology throughout the execution of the model as shown in figure 3.23. These agents also exhibit confused behavior and erratic consumption, the duration of which is dependent on the innate ability α.

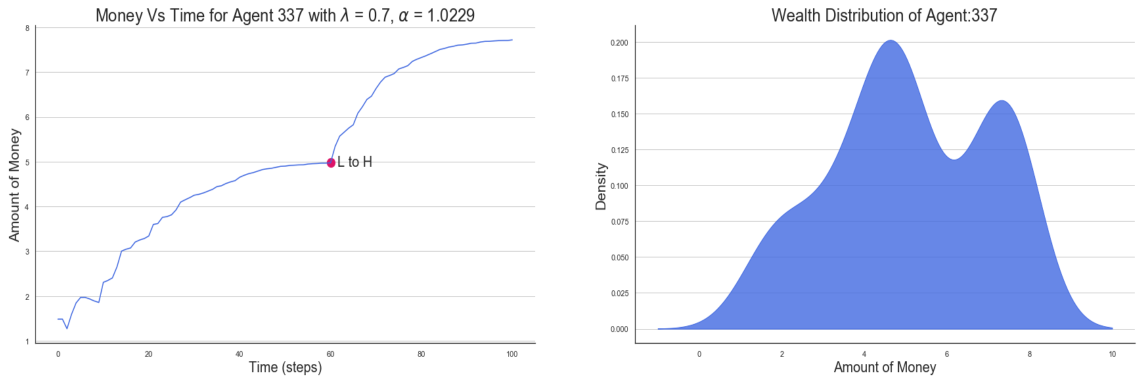
Some time steps before the switch (irrespective of low to high, or high to low), the agent might have already started adapting by changing the consumption. This adaptation happens faster for agents with higher values for innate ability α. This is further enhanced by a higher savings propensity factor λ. Figure 3.20 shows that the agent who started with a low initial stock of capital k_0, but with a higher value for innate ability ($\alpha_9 = 1.1051$) and savings propensity factor ($\lambda_9 = 0.9$), attains a large enough income quite fast to switch from investing in low technology to high technology.



(A) Effect of Large k_0 and λ Overridden by Small α



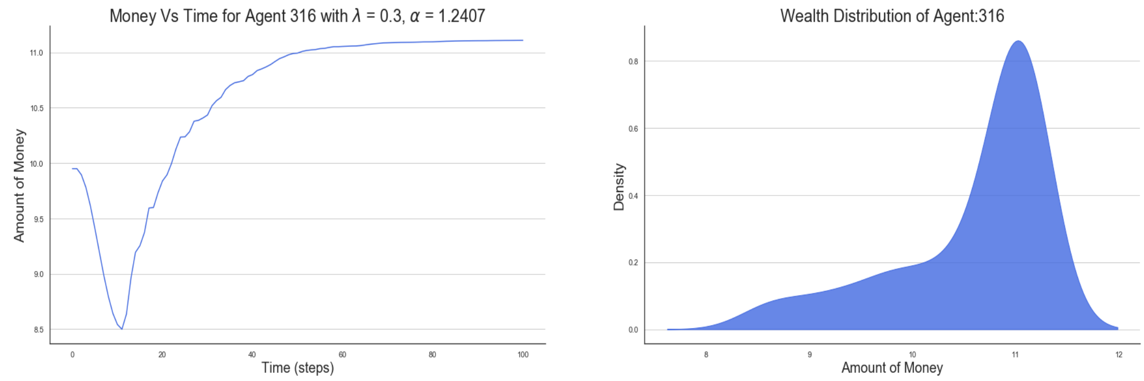
(B) Effect of Small k_0 Overpowered by Large α and λ



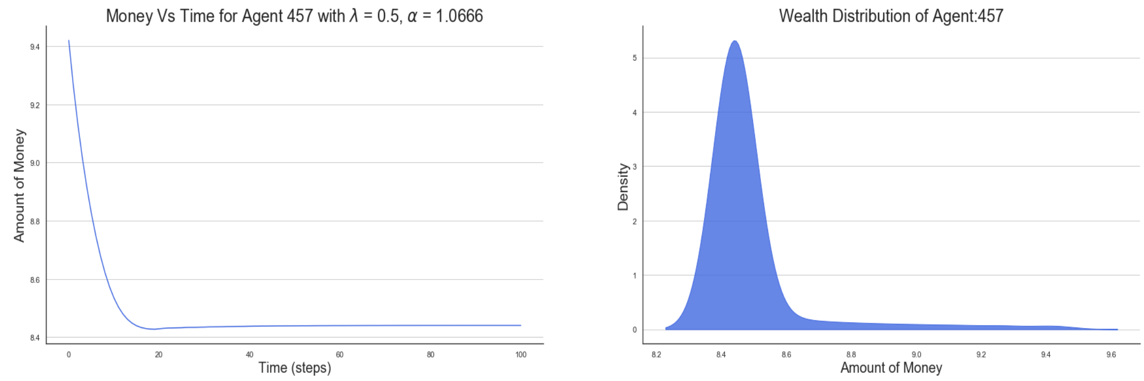
(C) Effect of Small k_0 Overpowered by Large α and λ

FIGURE 3.21: Random Agents Switching to Different Technology

In the case of agents with smaller values for savings propensity factor, λ and higher values for innate ability α, the stock of capital(k_t) increases slowly. Almost all agents exhibited this erratic behavior indicated by the 'dip' as shown in figures 3.20, 3.21, 3.22 and 3.23. After this point, the agents tend to reduce consumption with occasional large consumption, to increase their stock of capital(k_t). This reduction in consumption behavior had already started before this 'dipping point' as explained above. This behavior continues for two or three-time steps after the 'dipping point'. This behavior can be related to human behavior of exploring opportunities and adapting before realizing the



(A)



(B)

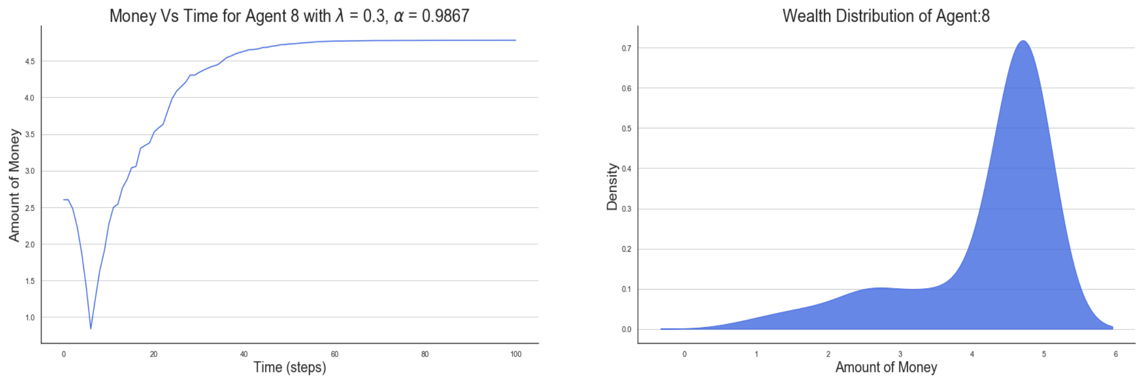
FIGURE 3.22: Random Agents Who Remains Investing in High Technology for 100 Steps

best strategy. After this point, the agents tend to reach a maximum stock of capital (k_t) that can be achieved with their corresponding innate ability, α, and savings propensity factor, λ.

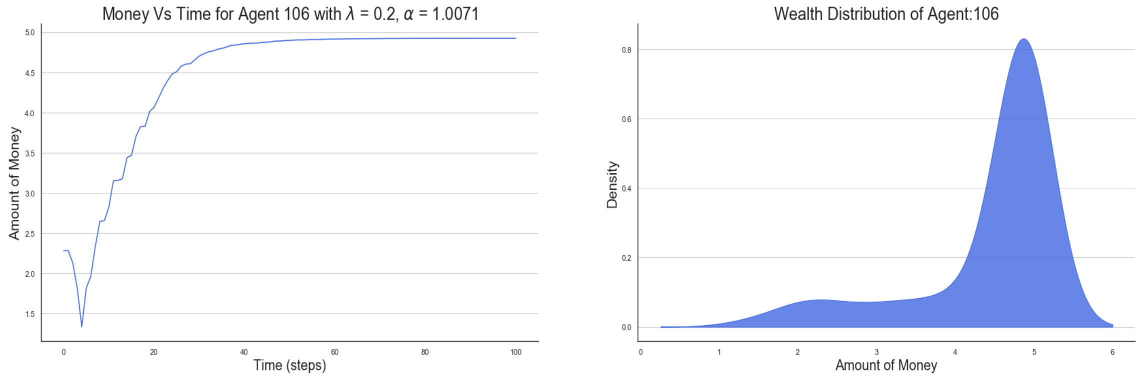
The erratic behavior is again observed before few time steps of the exact time step of the switch if an agent switches from one technology to another as shown in figure 3.24. This dipping point behavior is not prominent enough to result in a low value for the stock of capital (k_t). As explained above the behavior then continues for 2 or 3 time steps before making the switch of investing in a different technology. This second switch is observed only in the case of agents with higher innate ability α.

For those agents who started off with large stock of capital (k_t) but with a lower innate ability (α), the switch from investing in high technology to lower technology is permanent as shown in figure 3.21(A). This behavior is further aided by a low savings propensity factor (λ) for some agents, due to which they cannot attain a higher stock of capital (k_t).

These observations suggest that the agent behavior is dependent on their innate ability α, savings propensity factor λ, and the available stock of capital at that time k_t. However,



(A)



(B)

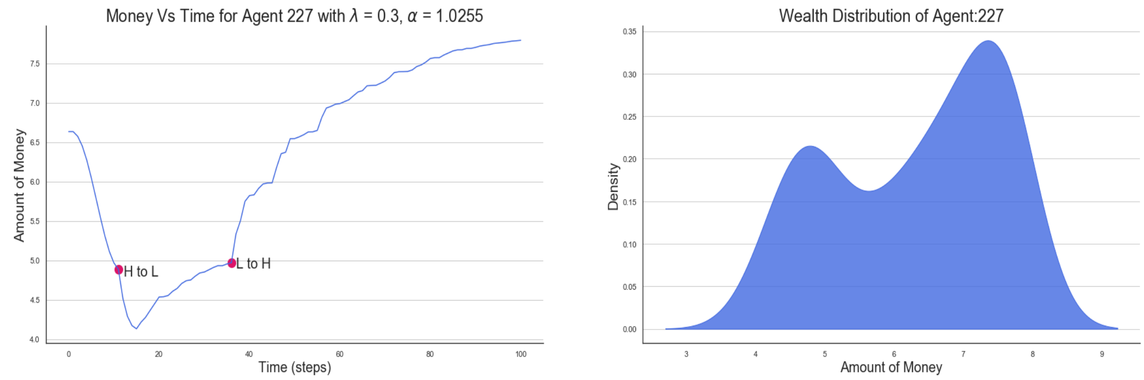
FIGURE 3.23: Random Agents Who Remains Investing in Low Technology for 100 Steps

innate ability α is the most important of the three, as it overrides the effects of savings propensity factor λ and initial stock of capital k_0 as shown in figure 3.21(A).

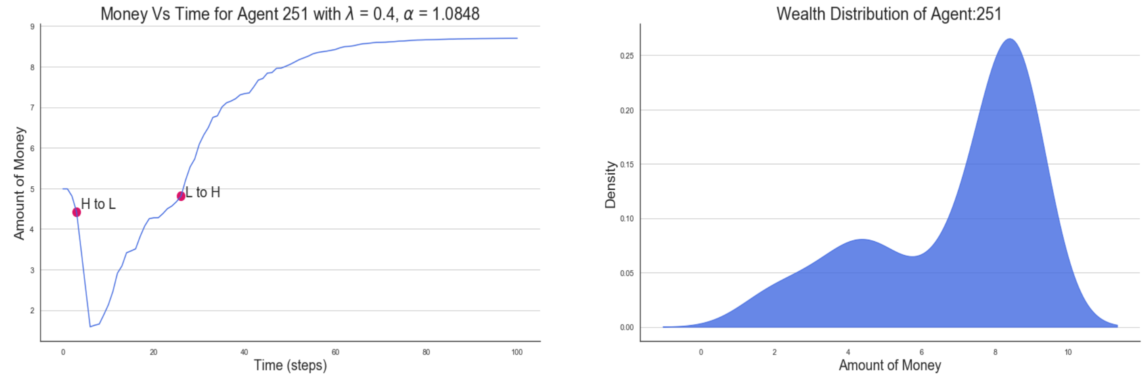
Three general results are also drawn from this Human Capital Model and they are listed below. All three results are drawn from the Human Capital Model of 500 agents that were executed for a period of 100-time steps.

1. **Overall Distribution of k_t:** The overall distribution of the stock of capital is as shown in figure 3.25. As figure 3.25 shows the distribution is no longer a homogeneous distribution but a bi-modal distribution with heterogeneous agents.

2. **Number of Agents in Poverty Trap:** The poverty trap was determined by counting the total number of consecutive time steps for which an agent stayed below a certain threshold stock of capital(k_t). This threshold was set to be at the 10 percentile of the overall distribution of the stock of capital(k_t). The distribution of the number of consecutive time steps for which agents stayed below the 10 percentile threshold is as shown in figure 3.26. The maximum consecutive number



(A)



(B)

FIGURE 3.24: Random Agents Who Switches Technology Investment Twice During 100 Steps

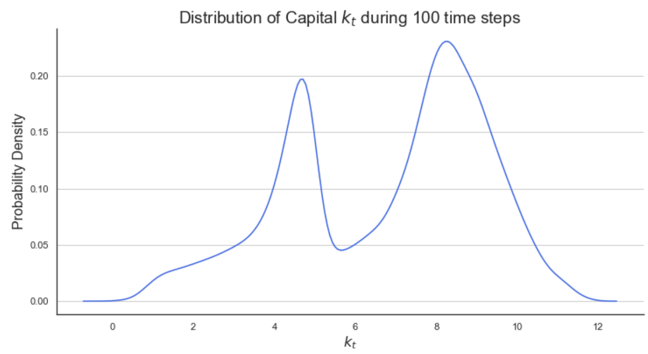


FIGURE 3.25: Distribution of Stock of Capital k_t

of times for which an agent stayed below the 10 percentile threshold is 35 and this agent has an innate ability α of 0.834624, which is the least innate ability α in the Human Capital Model. The distribution as in figure 3.26 is a bi-modal distribution, which indicates that there is a difference in the number of times different agents stayed in the poverty trap.

3. **Number of Switches:** The number of switches made by an agent from one technology to another was calculated. The distribution of the number of switches

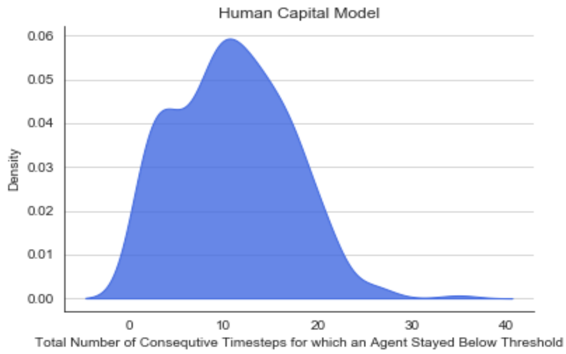


FIGURE 3.26: Distribution of Number of Agents in Poverty Trap for Consecutive Time Steps

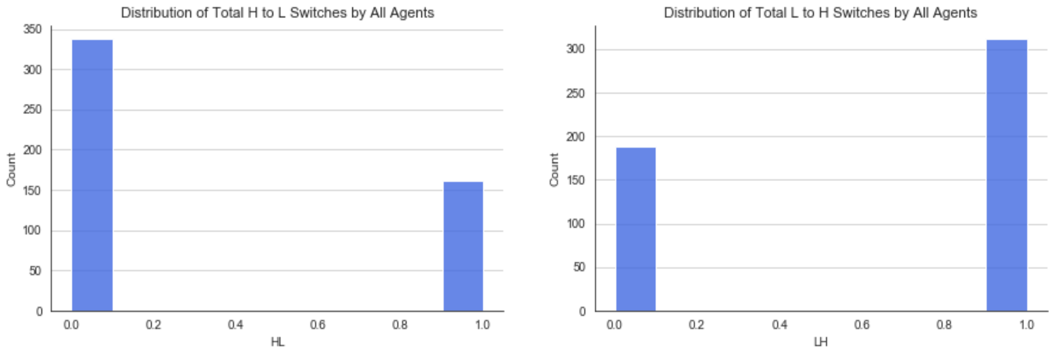


FIGURE 3.27: Distribution of Number of Switches

for both, switches from High to Low technology and Low to High technology is as shown in figure 3.27. As evident from figure 3.27 very few agents made a switch from High to Low technology (162 agents), whereas a large number of agents made a switch from Low to High technology (312 agents). Some agents also continued to remain using High technology (338 agents) and low technology (188 agents) without making a switch to the other technology.

3.5.3.1 Model Progression: Micawber Frontier

Figure 3.28 depicts the progression of the Human Capital Model in terms of the agent's innate ability (α_i) and stock of capital (k_t) at time t. The first plot in figure 3.28 shows the state of economy at time step t = 0. As time progresses some agents accumulate enough capital to cross-over the Micawber Frontier and invests in high technology. At time step t = 50, even agents with low innate ability (α_i) has made a significant improvement in accumulating stock of capital (k_t), whereas, some agents - the agents with higher innate ability(α_i), have already crossed over the Micawber Frontier as shown in figure 3.28. The figure 3.29 zooms in on to time steps 75, 80, 85 and 90, where minus-

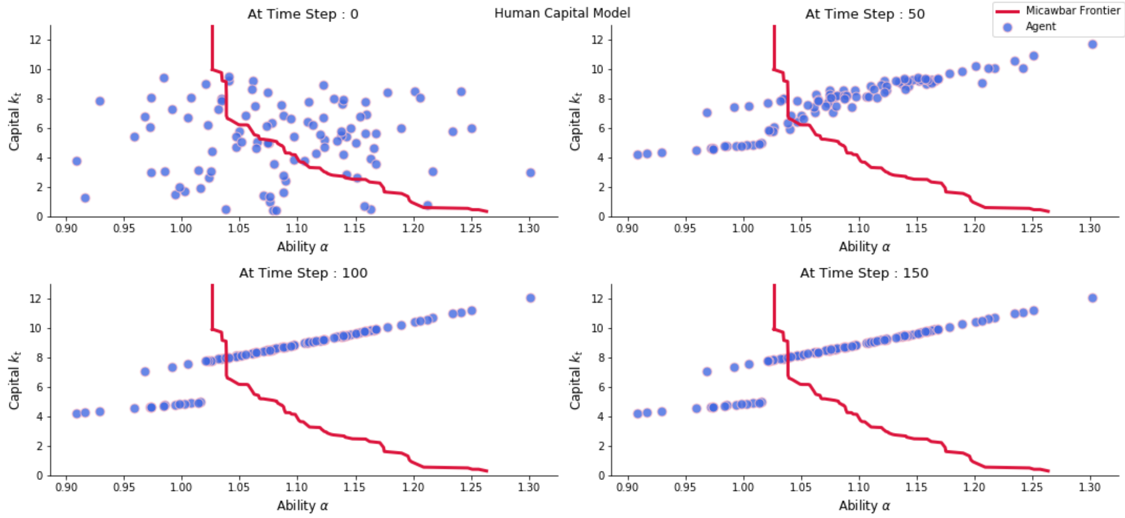


FIGURE 3.28: Progression of Human Capital Model: Agents Crossing the Micawber Frontier

cule changes in the stock of capital (k_t) pushes some agents over the Micawber Frontier, before maintaining an almost stable stock of capital (k_t) by almost all agents from time step 100 on wards as shown in figure 3.28. From figure 3.28 and 3.29, it is evident that

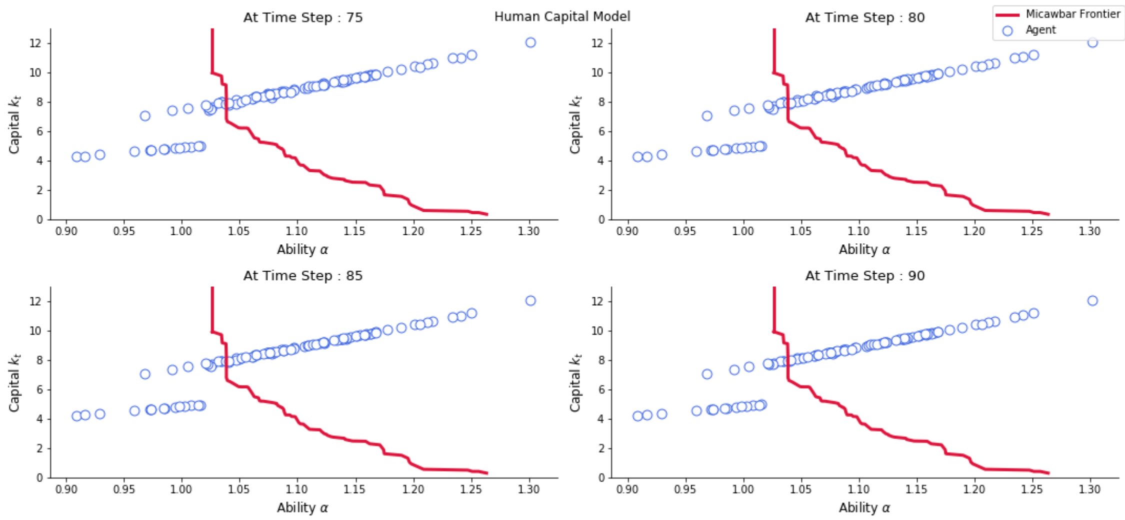


FIGURE 3.29: Zooming in on Time Steps 75 to 90

'innate ability(α_i)' plays a critical role in enabling the agents to cross over the Micawber Frontier irrespective of the fact that the agents have accumulated enough stock of capital(k_t) to make a transition from investing in low to high technology.

In figures 3.28 and 3.29 except for time step 0, there are multiple agents with the same innate ability(α) between $\alpha = 0.96$ and $\alpha = 1.04$ approximately. Consider agents with innate ability(α) ≈ 1.03, who exists with a low and a high stock of capital(k_t). Suppose the agent at the high stock of capital(k_t) suffers a shock and falls back to the level of low capital(k_t), then this agent wouldn't recover unless the innate ability(α) is increased. This behavior could be attributed to 'hysteresis', which is the persistence of an event in the future economy, even after the factors that led to the event have been removed, here exogenous shock. This also result in the poverty trap where are the agents are trapped in poverty without a chance to recover. This shows how important it is to improve the innate ability(α) of agents for a better economy.

3.5.4 Conclusion

The Human Capital Model which captures Financial Capital, Social Capital, and Human Capital is a superior poverty trap model than the conventional one-dimensional model. The concept of 'income' was introduced in this model. Based on the stock of capital(k_t) and innate ability(α_i) the agents earn their income by investing in high or low technology. The choice of technology acted as a differentiator between poor and rich agents. Further, the transition from investing in low to high technology in the absence of Social Capital gave Micawber Frontier, which also differentiates the poor from the rich. The model captured the agents who remained trapped in poverty for continuous time steps by their choice of investment(in high/low technology) and level of capital(k_t). The occurrence of hysteresis could also be captured from the results of this model. The model also identified innate ability(α_i) as one of the important deciding factor of an agent's financial stability. Poverty alleviation strategies could be applied on this model to determine which is the best strategy to lift the poor out of poverty by pushing them over the Micawber Frontier.

3.6 Conclusion

The final model - Human Capital Model- was created by taking a step by step approach - making intermediate models, testing their effectiveness, and making necessary changes before adding new features. The final model simulates the agent behavior in terms of their human, social and financial capital in the presence of an exogenous shock.

Chapter 4

Sensitivity Analysis

The final model, the Human Capital Model incorporates social capital, human capital, and financial capital. While the model results are based on default values for the homophilic and human capital parameters, it is equally important to study how the model output varies in response to the variations in these parameters. Hence sensitivity analysis was done for the homophilic and human capital parameters. Sensitivity analysis helps to evaluate the implementation of the model and to understand the behavior of the system - here, the three-dimensional poverty trap model [43]. The parameters and the corresponding bounds where the model is tested for are listed in the table 4.1.

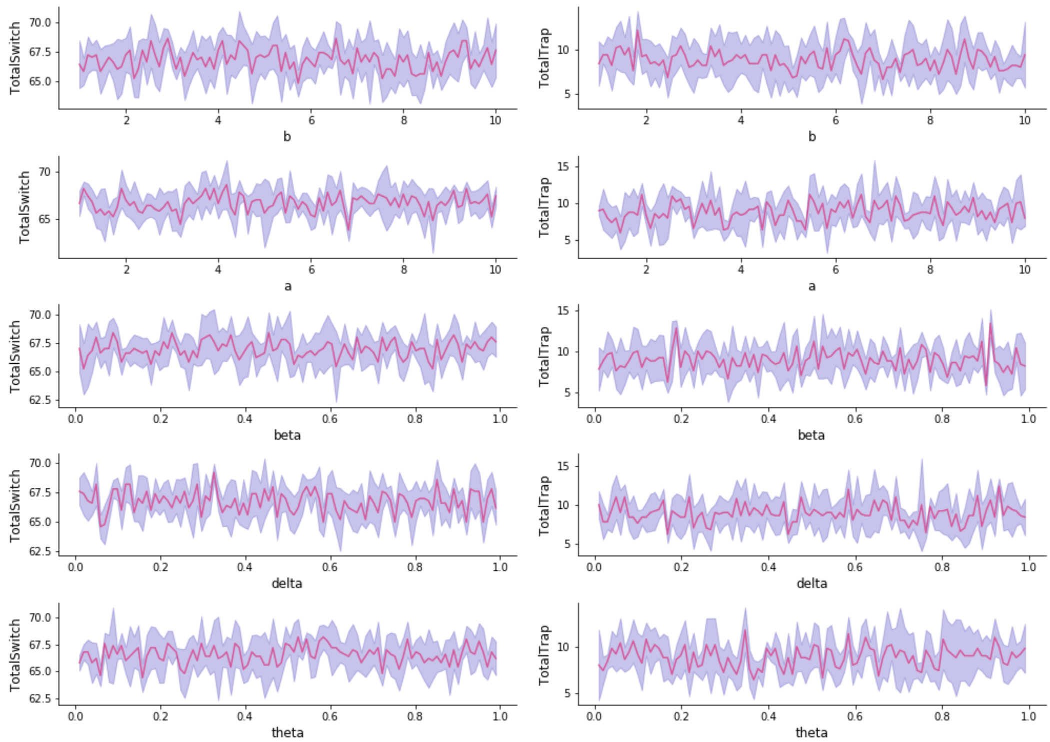
Parameter	Nominal Value	Range
θ	0.8	[0.01,0.9]
β	0.95	[0.01, 0.9]
δ	0.08	[0.01, 0.9]
Homophily Parameter(a)	0.69	[1,10]
Characteristic Distance(b)	35	[1,10]

Table 4.1: Parameters and Sensitivity Analysis Bounds

The shock factor θ varies between 0 and 1, with 0 indicating no shock and the value 1 implies a severe shock. Hence, 100 distinct samples from the bounds [0.01, 0.9] were chosen to study the output variation. The time discount factor β when decreased below 0 would result in negative values for the next step consumption(c_{t+1}), and increasing it above 1 would result in increased values for c_{t+1}. The ultimate goal is to increase the capital accumulation behavior so that the agents can afford to invest in high technology. Hence, the given bounds were chosen for the time discount factor. Similarly, the asset depreciation rate(δ) is chosen to be between [0.01,0.9] as any value less than or equal to 0 would increase the value of the assets possessed by an agent exorbitantly whereas any value greater than or equal to 1 would decrease the value of the assets. So, to keep the value of the assets of an agent reasonable the given bounds were chosen.

The homophilic parameters a and b, the parameters deciding the amount of homophily and the characteristic distance respectively were chosen to be between [1,10]. This range was chosen because of the results of Talaga et.al who found this range performing well for their Social Distance Attachment Model [41].

For the different values of the parameters listed in table 4.1, the model outputs namely, the total number of switches made by all agents from one technology to another, and the total number of agents who stayed in poverty trap for consecutive time steps were measured. Both local and global sensitivity analysis were done for these parameters and their outcomes measured. Due to the limitations in computing power only 100 distinct samples from the given bounds were sampled using Saltelli's sampling scheme. This was done with the help of the python package SALib [44]. Each sample was executed for a total of 100 agents and for 100 time steps in the model.



(A) Total Number of Switches

(B) Total Number of Agents in Poverty Trap

Figure 4.1: Sensitivity Analysis: OFAT

4.1 Local Sensitivity Analysis: OFAT

Local sensitivity analysis is used to understand the influence of a single parameter on the model's output. This was done using OFAT(One Factor At a Time) method. Five replicates per parameter set were used to roughly estimate the spread of the output. In this method, when one parameter is tested in the model, all the other parameters assumed the nominal values as listed in table 4.1.

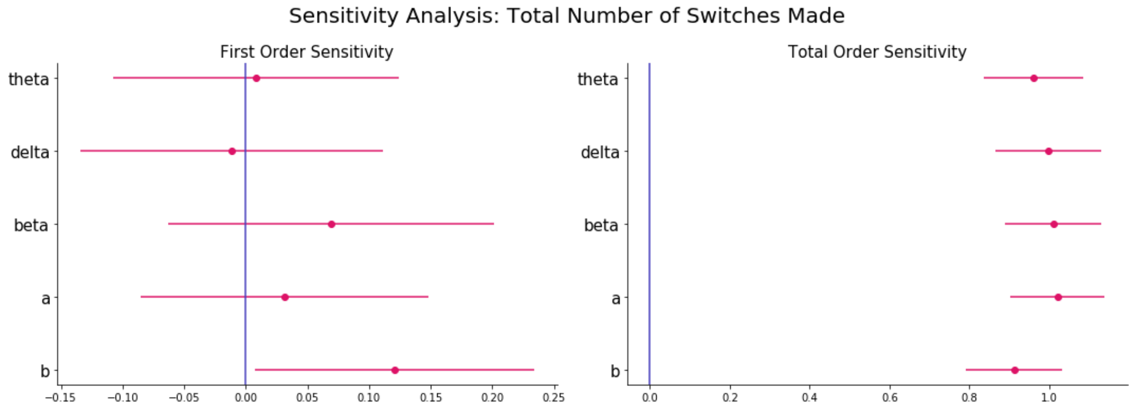
The result of the OFAT sensitivity analysis on the final model for the parameters listed in table 4.1 is shown in figure 4.1. The OFAT analysis doesn't help to make a positive claim regarding the model response to different values of the parameters and in identifying the relationship amongst the various input parameters. As a result, a global sensitivity analysis had to be done.

4.2 Global Sensitivity Analysis: SOBOL

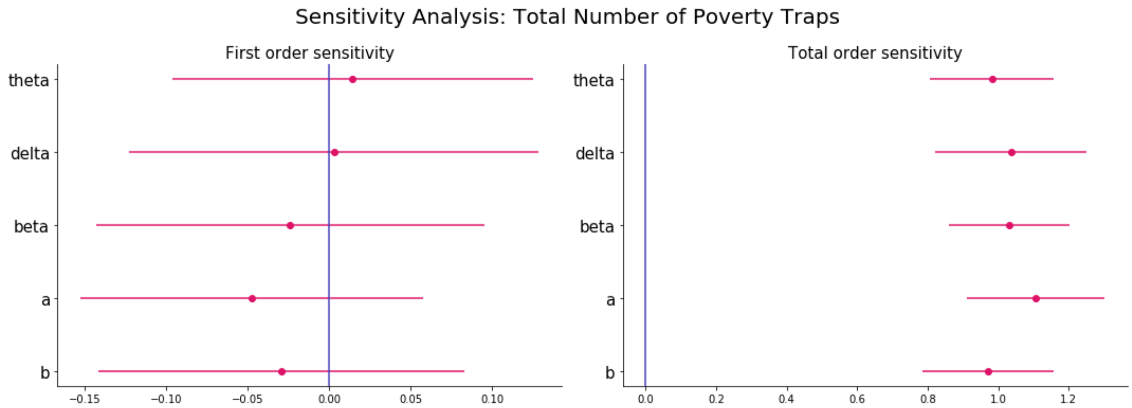
Global sensitivity analysis determines the effects of changes of multiple parameters simultaneously in the model's output. It is done using the SOBOL decomposition method. Figure 4.2 gives the SOBOL sensitivity analysis done for the input parameters of table 4.1 for the model outputs.

In the results of first-order sensitivity, the confidence interval is very large and some of them are in the negative area. The negative values in the sensitivity estimates are due to numerical inaccuracy of the Saltelli method [43]. The wider intervals for all parameters for both output variables for the first-order sensitivity in figure 4.2(A) and (B) shows that the estimates are not accurate. This could be attributed to the smaller sample size of the parameters. As a result, a conclusive remark about the impact of these parameters on the model output cannot be made.

All parameters showed a high total order and indicated a possible interaction amongst the five parameters as shown in the total order sensitivity in figure 4.2. In total order sensitivity, the more influential parameters have wider confidence intervals. Here, the total order sensitivity of both, the number of switches and the number of poverty traps showed asset depreciation rate(δ) as the most influential parameter among the five. In the total order sensitivity for the number of traps, characteristic distance(b) also looked influential apart from the asset depreciation rate(δ). However, an irrefutable claim cannot be made owing to the lack of enough samples.



(A) Total Number of Switches



(B) Total Number of Agents in Poverty Trap

Figure 4.2: Sensitivity Analysis: SOBOL

4.3 Conclusion

A local sensitivity analysis was done using the OFAT method and a global sensitivity analysis was done using the SOBOL decomposition method. The model results based on the input parameters and the range listed in table 4.1 weren't completely reliable. This could be due to the lack of enough samples. However, it can be concluded that all 5 parameters interact among themselves from the results of total order sensitivity. To be decisive about the sensitivity of the model parameters, it is essential to run the model for more time steps with more samples. Nonetheless, this couldn't be done at this point due to the limitations in computing power.

Chapter 5

Experiments

This section aims to present the experiments performed and the corresponding results obtained from the Human Capital Model. The experiments involved applying the resilience thinking concept of pushing individuals to help them 'cross over the barrier'. Among the many methods to achieve this, the experiments of this project focuses on 'Safety Net' and 'Cargo Net' policies. Safety Nets protects the income of an individual from falling below a certain minimum level. The term Cargo Net is coined by Barret in his study titled 'Rural Poverty Dynamics: Development Policy Implications' as *"helping people climb over the threshold or lifting them over the barrier that deems them poor through support and aid"* [45].

In the two experiments that are discussed in the subsequent sections monetary assistance are given to agents to attain the critical capital($\tilde{k}(\alpha)$) of the Micawber Frontier. Depending on the agent's consumption and investment behavior they either eventually cross over the critical capital ($\tilde{k}(\alpha)$) or remain trapped below the Micawber Frontier as shown in the Human Capital Model progression in figure 3.28.

For both experiments it was assumed that there is an external agency that decides and allocates financial assistance to deserving agents. This agency also determines the budget, which is a percentage of the Gross Domestic Product(GDP) of the economy. In the experiments that follows, GDP is taken as the sum of income of all agents at a given time step. In addition, it is also assumed that the budget is available externally and not collected from the economy. The 2 types of budget allocations schemes which are experimented on the Human Capital Model are Need-Based Assistance and Fuzzy Safety Net. Due to the computing power limitations the experiments could be performed in an economy of 100 agents and for a duration of 150 time steps. If the duration is different for any experiment it will be mentioned in the corresponding explanation.

5.1 Need-Based Assistance

The idea of Need-Based Assistance scheme is to support poor agents financially by pushing them over the barrier which deems them poor. In the context of Human Capital Model, the purpose of Need-Based Assistance is to help the agents attain higher incomes to invest in high technology.

5.1.1 Experiment Specifics

The concept of 'Budget(B)' as mentioned above was introduced in this experiment. It was decided and allocated by an agency. Based on the budget, it is given either to help the agents below the money metric line to attain this level or a fraction of the money metric line value. The budget is 7% of GDP at every time step. i.e.

$$B = 7\% GDP_t$$

Increasing the percentage from 7 to a higher number is advantageous as it results in all individuals being pulled out of the poverty trap eventually. However, this number is fixed to be 7 as it is sufficient to close the poverty gap substantially. Money Metric Poverty Line defined by Barret et.al as 'the level of income that a middle ability person, say with $\alpha = 1.12$ would produce in steady-state equilibrium at the low technology' was kept to be at 1.62 [9]. The other parameters of the Human Capital Model remained the same as given in table 3.5. Further, the agents are not aware of this 'Need-Based Assistance' and, hence they do not behave anticipating the assistance. The process involved in this scheme can be summarized as below:

1. At every time step the poverty shortfall - poverty gap - is calculated as below:

$$S = \sum_{y_i < y_p} (y_p - y_i)$$

where y_i is the income of an agent i.

2. If Budget $B > S$, then all poor agents (agents with income $y_i < y_p$) will be provided with wealth such that their income $y_i = y_p$.

3. If Budget $B < S$, then all poor agents ($y_i < y_p$) will be provided wealth such that their income $y_i = \frac{B}{S} y_p$

4. The experiment was set up for an economy of 100 agents and for a duration of 150 time steps.

Need-Based Assistance can be viewed as a 'safety net' as it protects some agents completely against their income falling below the money metric line(y_p) for certain time steps(when $B > S$). It does not act as a safety net for everyone due to the limitations of budget(B).

5.1.2 Observations and Results

The snapshots of the Human Capital Model with the Need-Based Assistance Scheme at time steps t = 0, 50, 100 and 150 is shown in figure 5.1. At time step 50 of figure

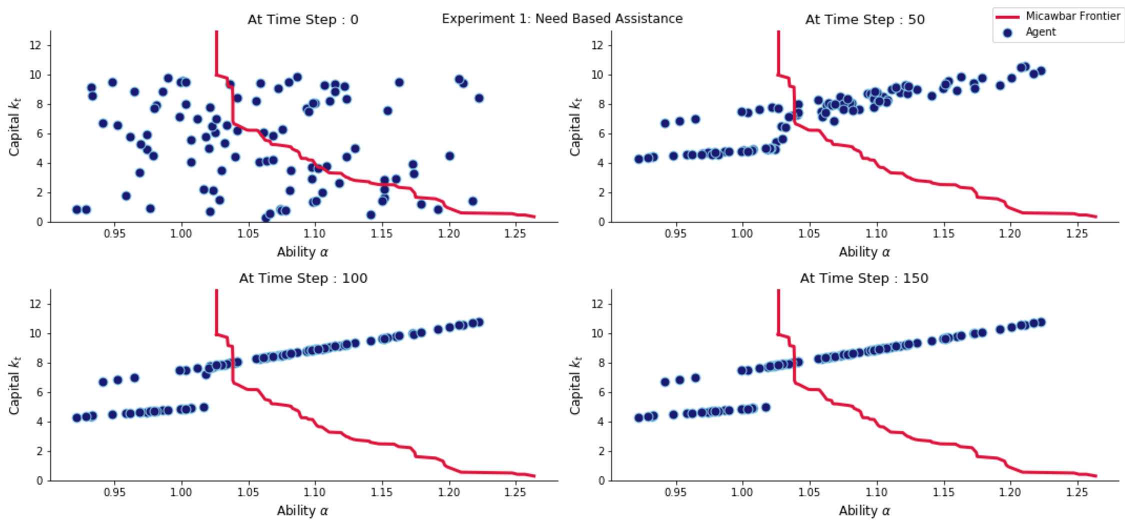


FIGURE 5.1: Need-Based Assistance: Model Progression

5.1, all agents with an innate ability(α_i) greater than 1.05 have already crossed over the Micawber Frontier, no matter how low their initial stock of capital(k_t) was. Whereas for the Human Capital Model without Need-Based Assistance scheme take more time steps to achieve this feat as shown in figure 3.28. The snapshot of models with and without scheme as in figures 5.1 and 3.28 respectively, at time step 150 looks almost identical. This points to the fact that agents with a higher innate ability(α_i) have a better chance of crossing the Micawber Frontier than the ones with low values for innate ability(α_i), irrespective of the increase in the stock of capital(k_t) and subsequent investment in high technology by the latter. The occurrence of hysteresis is also evident in this experiment from figure 5.1. This is not a desirable behavior as it leads to a poverty trap.

Figure 5.2 shows the total number of agents using high and low technology during the lifetime of the Human Capital Model with the Need-Based Assistance scheme. The initial dip and rise for high and low technology respectively indicate a brief warm-up period when the agents are getting acquainted with the system. The distribution of the number of switches made from high to low technology and vice versa by the agents are

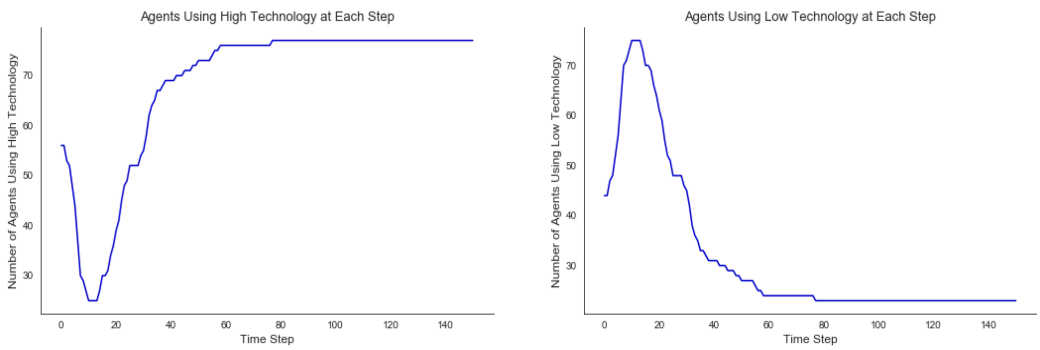


FIGURE 5.2: Variation in the Number of Agents Using High/Low Technology

as shown in figure 5.3. Unlike the Human Capital Model as in figure 3.27, the number of agents making a transition from low technology to high technology is larger in the model with Need-Based Assistance. This is due to the increase in the stock of capital(k_t) possessed by agents under the Need-Based Assistance who previously had a lower stock of capital(k_t).

FIGURE 5.3: Distribution of Number of Switches

The advantage of this scheme is that the poorest agents benefit the most from this scheme such that the increase in the stock of capital (k_t) enables them to invest in high technology which they couldn't previously. However, the monetary assistance is not enough to help them cross over the Micawber Frontier. Investing the entire budget during the lifetime of the model in replenishing the stock of capital(k_t) of the poorest will hinder the budget from being invested in other areas which can improve the innate ability(α_i) of the agents among other things like opportunities creating employment, sanitation etc.

5.1.3 Conclusion

The poorest of all agents benefit the most from the Need-Based Assistance scheme. In the long run, this scheme is not beneficial as the agents with lower innate ability(α_i) values couldn't accumulate a stock of capital(k_t) to invest in high technology. Since the agents are not making their decisions anticipating the monetary support, they tend to behave the same way as in the original Human Capital Model. Budget hence acts as an 'additional stock of capital' which the agents utilize for their consumption. This doesn't help the economy unless the innate ability(α_i)of the agents is increased to make rational decisions in the presence of exogenous shock. The innate ability(α_i) can be increased by making investments in developmental projects that directly contribute to the educational, health, and other facets of individuals. This is not possible when the whole budget is spent on assisting agents during the entire time. Hence, Need-Based Assistance cannot be regarded as an efficient method of supporting poor people as it doesn't change the economy beneficially.

5.2 Fuzzy Safety Net Transfers

Providing protection and monetary assistance indefinitely has 2 disadvantages. First of all, when agents anticipate the agency's financial support they tend to take advantage of it by increasing their consumption(c_t), such that they're always in the category of people who needs support. This kind of behavior is called 'negative moral hazard' [9]. Asset accumulation is a desirable behavior as it leads to an increase in GDP and enables the agents to invest in high technology. This is a positive moral hazard in this context. But the presence of a safety net via the agency's budget results in increased consumption and decreased accumulation of assets as explained above and is not desirable for the growth of an economy. Secondly, more budget allocated to helping people due to their irrational behavior would mean little or no budget for developmental projects like schools, hospitals, etc which would, in turn, facilitate in increasing the innate ability(α_i).

5.2.1 Experiment Specifics

In the original Human Capital Model with the parameter values as listed in table 3.5, agents now consume and accumulate the stock of capital(k_t) anticipating the agency's monetary assistance. Capital(k_t) is accumulated in a way to decrease the negative moral hazards which otherwise would have led to increased consumption(c_t) and decreased capital(k_t) accumulation behavior. This is accomplished by introducing a new transfer scheme for next stage stock of capital(k_{t+1}) and is defined as below [9]:

$$k_{(t+1)} = \begin{cases} k_{(no\ shock)} = \alpha k^{\gamma} - c_t + (1-\delta)k_t \\ k_{(with\ shock)} = \theta_t(\alpha k^{\gamma} - c_t + (1-\delta)k_t) \end{cases}$$

$$k_{(t+1)new} = \begin{cases} \Big(\big[\eta(.) + (1-\eta(.))\theta_t\big]k_{(no\ shock)}\Big)\ if\ k_{(no\ shock)} > \tilde{k}(\alpha_i)\ and\ k_{(with\ shock)} > \xi\tilde{k}(\alpha_i) \\ k_{(with\ shock)}\ otherwise \end{cases}$$

where i is the agent, $\eta(.) \in [0,1]$ and is a function of $k_{(no\ shock)}$ and is given as:

$$\eta = 1 - \frac{1}{\xi\frac{\tilde{k}(\alpha)}{\theta_t} - \tilde{k}(\alpha)} k_{(no\ shock)}$$

ξ is a constant and is set to be 16 such that η stays within the given limit of [0,1]. η controls the agent behavior such that the positive moral hazard increases. This is dependent on the position of the stock of capital(k_t) of the agent with respect to the Micawber Frontier. Hence, η has the following values during the execution of the model.

$$\eta = \begin{cases} 1\ if\ k_{no\ shock} = \tilde{k}(\alpha) \\ 1 - \frac{1}{\xi\frac{\tilde{k}(\alpha)}{\theta_t} - \tilde{k}(\alpha)} k_{(no\ shock)}\ if\ k_{no\ shock} \in \left(\tilde{k}(\alpha), \frac{\xi\tilde{k}(\alpha)}{\theta_t}\right) \\ 0\ if\ k_{no\ shock} = \frac{\xi\tilde{k}(\alpha)}{\theta_t} \end{cases}$$

The Budget(B) is taken to be 7% of the GDP at a given time step due to the reason explained in section 5.1. Budget(B) is transferred to individuals based on the closeness of their stock of capital(k_t) with the Micawber Threshold($\tilde{k}(\alpha)$). Barret et.al, calls the threshold-based money transfers as 'Productive Safety Net(PSN)'. The allocation of budget involved in Fuzzy Safety Net transfer is summarized as below:

1. The sum of difference of the Micawber Threshold($\tilde{k}(\alpha)$) and stock of capital(k_t) of all agents who were recently pushed below $\tilde{k}(\alpha)$ is calculated at a given time step. This sum can be denoted as PSN.
   i.e. PSN = $\sum psn_i$
   where $psn_i = \tilde{k}(\alpha) - k_{(with\ shock)}$, if $k_{(with\ shock)} < \tilde{k}(\alpha)$ and $k_{(no\ shock)} > \tilde{k}(\alpha)$.
2. If the Budget B is greater than this sum, i.e. B > PSN, then all deserving agents are given stock of capital(k_t) equivalent to psn_i so as to help them overcome the disadvantage induced by the exogenous shock.
3. If there is remaining Budget after the Productive Safety Net(PSN) transfers, then agents who are already below Micawber Threshold due to initial low stock of

capital(k_t) or prior mishap that left them restrained from benefiting the PSN, are given a Cargo Net(CN) transfer that lift them over the Micawber Frontier.

4. Cargo Net transfer is calculated the same way Productive Safety Net transfer is calculated, as the sum of difference of the Micawber Threshold($\tilde{k}(\alpha)$) and stock of capital(k_t) for agents who are below Micawber Threshold.
   i.e. CN = $\sum_{\forall\ agents\ i, k_{it} < \tilde{k}(\alpha_i)} \tilde{k}(\alpha_i) - k_{it}$

5. If CN > B + PSN, then a prioritization of budget allocation is done such that, the agents who are closer to the Micawber Frontier are given the priority.

6. If B > PSN + CN, then the remaining budget B - PSN - CN is allocated using the Need-Based Assistance scheme.

To avoid indefinite financial assistance, the fuzzy safety net is made temporary such that the support stops after a predetermined time step, say T_g and it was fixed to be 20. This is done to take advantage of the positive moral hazard of the transfer scheme and to avoid negative moral hazard once the transfer period (T_g) has passed[9]. The model was run for 100 agents and for a period of 150 time steps due to computing power limitations.

5.2.2 Observations and Results

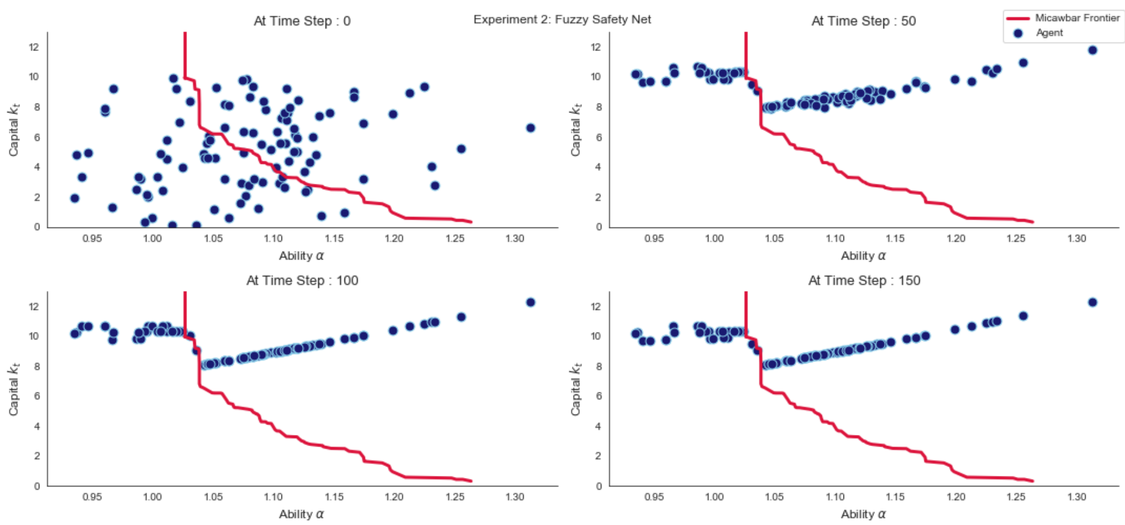


FIGURE 5.4: Fuzzy Safety Net: Model Progression

The snapshots of the Human Capital Model with Fuzzy Safety Net transfers are as shown in figure 5.4. All agents with high innate ability(α_i) who started off with low initial stock of capital(k_t) are already above the Micawber Threshold by time step 50. The snapshots at time steps 100 and 150 doesn't differ very much except for some agents who are below the Micawber Threshold. Since the transfer period T_g is limited to 20, it would be interesting to observe the dynamics closer to T_g. This is shown in figure 5.5. The figure 5.5 shows how the intermediate innate ability(α_i) agents crosses the

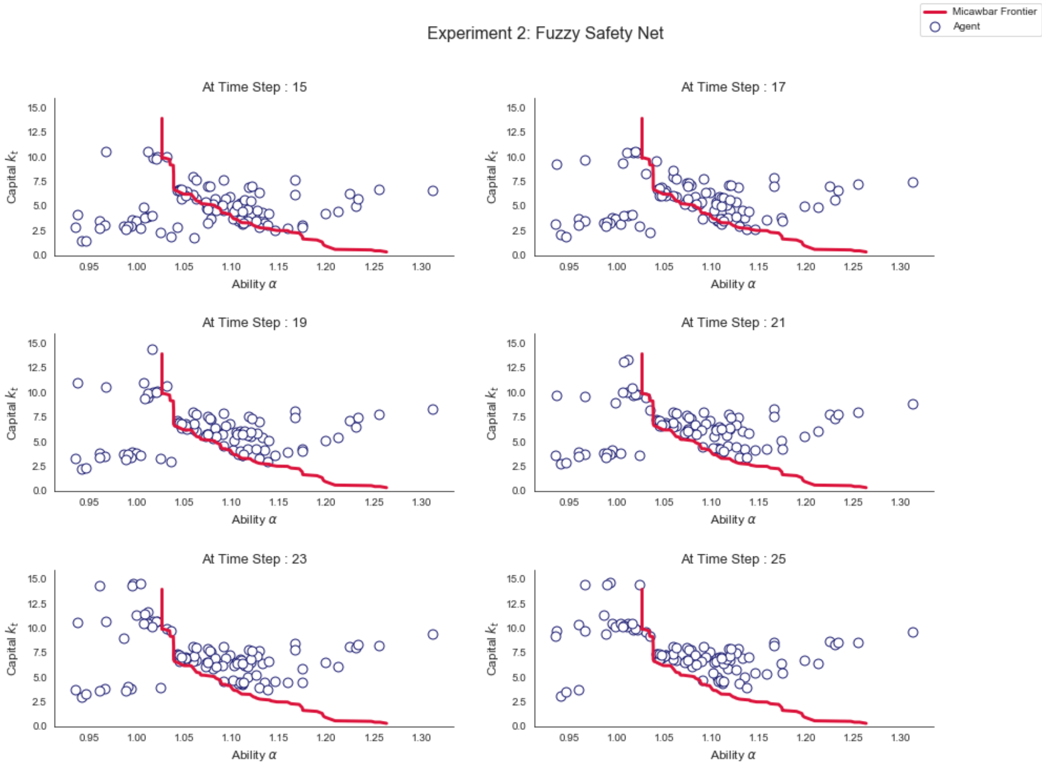


FIGURE 5.5: Zooming on Specific Time Steps

Micawber Frontier during time steps 16, 17 and 19. Interestingly, the agents with lower values for innate ability(α_i) has benefited hugely from this transfer scheme. At time steps 19 and 21 from figure 5.5 some agents have already crossed-over the Micawber Frontier. Keeping figures 5.4 and 5.5 side by side, it can be observed that the agents with low innate ability(α_i) has attained a higher stock of capital(k_t), and eventually crossing the Micawber Threshold. This agents however, in the absence of the Safety Net(time steps 100 and 150 of figure 5.4) had settled to a lower stock of capital(k_t). This could be attributed to their low innate ability(α_i) which otherwise would have aided them in spending and investing their stock of capital(k_t) wisely. At $T_g = 20$, the Safety Net was removed but the benefits of it continued evident from the agent behavior at time steps 21,23 and 25 of figure 5.5 and time steps 50, 100 and 150 of figure 5.4. Hence, the aim of the Fuzzy Safety Net transfer scheme to increase the positive moral hazard resulting in capital(k_t) accumulation behavior, is also achieved. The hysteresis behavior is almost eliminated in this experiment as evident from the model snapshots in figure 5.4 and 5.5.

The figure 5.6 shows the number of agents who are investing in high and low technology during the lifetime of the model. From time step 28 onward,all agents are investing in high technology. i.e. all agents have attained a stock of capital(k_t) which enabled them to afford investment in high technology. Hence, the Fuzzy Safety Net transfer has successfully induced 'capital(k_t) accumulating' behavior among the agents.

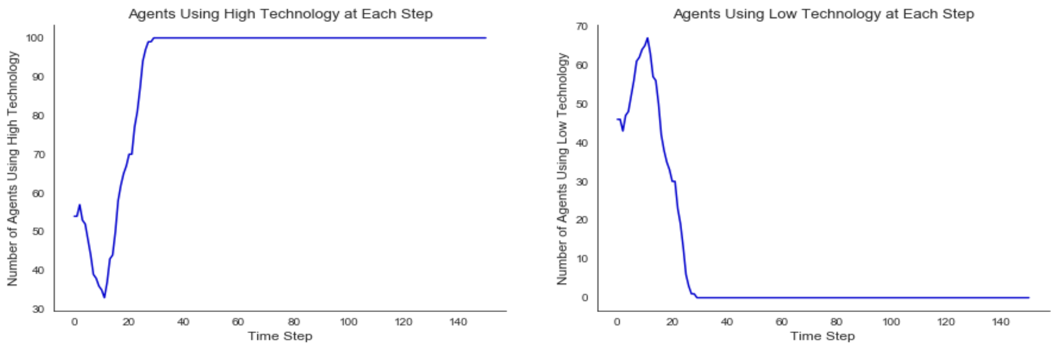


FIGURE 5.6: Variation in the Number of Agents Using High/Low Technology

The distribution of the number of switches made by all agents during the execution of the model is as shown in figure 5.7. Unlike the results from the previous experiment and that from the Human Capital Model, the maximum number of times that an agent switched from one technology to another is 4 for Fuzzy Safety Net transfers.

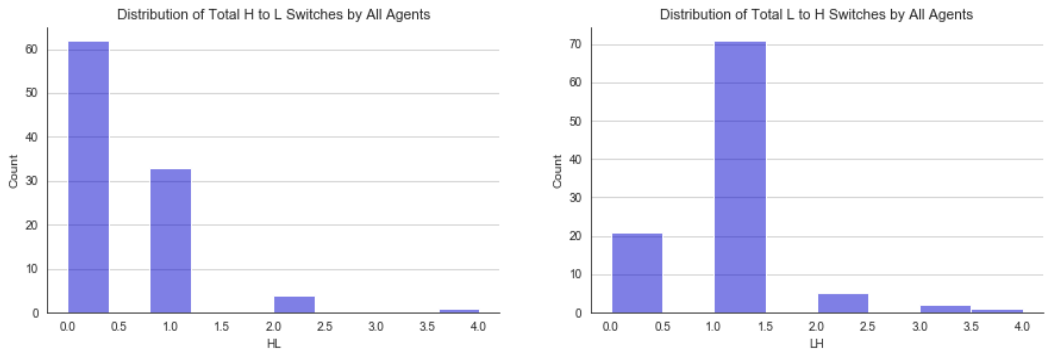


FIGURE 5.7: Distribution of Number of Switches

These switches were mostly made during the transfer period T_g when the agents still received financial support from the agency. The agents with low innate ability(α_i) who were below the Micawber Threshold received monetary assistance frequently. The effect of lower innate ability(α_i) and exogenous shock(θ_t) causes fluctuation in the value of stock of capital(k_t). In addition, there is also a brief warm-up period during which the agents are acquainting with the system. The comparatively larger number of switches can hence be justified.

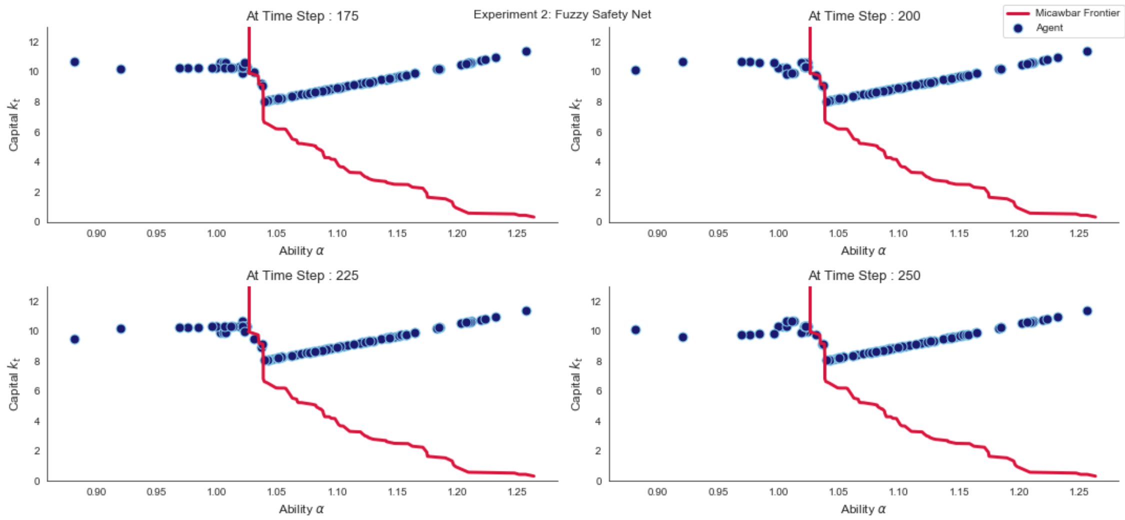


FIGURE 5.8: Fuzzy Safety Net: 250 Steps

The experiment was also done for an additional 100 time steps, i.e. for a total of 250 time steps. The snapshots of model progression of this model during the last couple of time steps are as shown in figure 5.8. The results were identical to that of the model run for 150 time steps, with all agents investing in high technology from time step 28 onward.

5.2.3 Conclusion

The Fuzzy Safety Net was introduced to support the deserving agents for a brief period of time. A limit was set on the transfer period so as to invest the Budget in other developmental activities which would enhance the economy and it's individuals. In order for this to be possible it was essential that the agents be inculcated with the 'capital(k_t) accumulating' positive moral hazard behavior. From the results of the model explained above it can be concluded that this is attained.i.e. there is an increase in the positive moral hazard and the agents have attained and maintained enough stock of capital(k_t) even after the transfer period that lets them invest in high technology.

Chapter 6

Results

The research questions that were addressed in this project were:

1. What are the mechanisms characterizing and consolidating the poverty trap in an urban setting?

2. What are the effects human and social capital has on this process?

These results were attained by:

1. Developing a multi-dimensional agent-based poverty trap model that incorporates the financial, social and human capital aspects of an individual.

2. Applying resilience thinking principles on this model to develop poverty alleviation strategies and find their effectiveness in the presence of an exogenous shock.

6.1 Mechanisms Consolidating the Poverty Trap

From the results of the final three-dimensional model of this project, the Human Capital Model, it was found that poverty trap is prevalent among agents who are on the left side of the Micawber Frontier as in figure 3.28. and hence has smaller chances of coping well in the presence of exogenous shock. As a result, these agents with lower values for innate ability(α) irrespective of their initial stock of capital(k_0) tends to be in a poverty trap in the presence of a shock. Even with low initial values of wealth(stock of capital k_0) and low or almost no supplemental income, the agents with higher innate ability(α) performed better and acquired enough wealth to get out of the poverty trap.

The model also identified the existence of 'hysteresis' wherein an agent with lower innate ability value(α) tend to fall back to a lower stock of capital(k_t) from a higher stock of capital(k_t) and subsequently maintain the lower stock of capital(k_t) in the presence of an exogenous shock. So, it is important to resort to practices that increase the human capital quotient of individuals in this context. This will in turn directly help the agents to get out of the poverty trap.

6.2 Effects of Human and Social Capital

Continuing the explanation in section 6.1, it was further identified that the innate ability - the representation of human capital, played a critical role in consolidating the poverty trap. i.e. agents who are in a hysteresis loop, thereby in poverty trap, could not break free from this trap unless the innate ability of the agent is increased. This result was obtained on a dynamic network where the agents communicate based on their 'likeness - homophily' with the other agent.

6.3 Multi-Dimensional Poverty Trap Model

The final model of this project incorporated financial capital, social capital and human capital, thereby translating the conventional one-dimensional poverty trap model into a three-dimensional model. Agent-Based modeling approach was used so as to capture the impact of social capital and human capital on an individual level. Besides, the impact of exogenous shocks like COVID'19 propagates in the social network as a consequence of an agent-agent interaction.

The incorporation of the Micawber Frontier(figure: 3.19) of the two-dimensional poverty trap model by Barret and his colleagues into this three-dimensional model demarcated the less earning individuals from the rich individuals. This was further asserted by the type of investment made by individuals for adopting high or low technology.

6.4 Application of Resilience Thinking

Resilience Thinking principle of Regime Shift- pushing over the barrier, is applied on the final model to understand its effectiveness in helping agents overcome poverty. Two schemes namely Need-Based Assistance and Fuzzy Safety Net Transfer is experimented.

From the results of the Need-Based Assistance scheme, it could be concluded that providing financial support to agents who doesn't modify their actions anticipating the support scheme doesn't help the economy. The financial support in this context would merely act as an additional stock of capital(k_t) to poor people without changing their present condition. Under this scheme, agents were still trapped in poverty. In addition, the Need-Based Assistance diverts the Budget from being invested on education, health, sanitation etc which directly contributes in improving agents human capital(α).

Fuzzy Safety Net transfer with a limited transfer period proved to be beneficial in inducing a stock accumulation behavior, thereby increasing the positive moral hazard among agents even in the presence of an extreme exogenous shock of 0.8. In this scheme, it was taken that the agents take their decisions (consumption and investment in technology) anticipating the monetary support. Compared to Need-Based Assistance, Fuzzy Safety Net transfer was successful in lifting poor agents out of poverty to a level of high stock of capital(k_t). As a result, even after the transfer period the agents continued to exhibit increasing positive moral hazard enabling them to invest in high technology.

Chapter 7

Conclusion

To study the effects of social interaction and the innate ability of an individual to fend for themselves on financial stability, a multi-dimensional poverty trap model was created. This model incorporated the social capital, human capital, and financial capital of the Community Capitals Framework created by Emily and Flora [2]. Due to the limitations of the availability of data, the model was tested on an artificially created society of individuals using the Mesa grid of python mesa package, and the social network is added using the Albert Barabasi Network of the python networkx package. The model is successful in capturing the poverty trap among individuals. The agents who stayed below the Micawber Frontier are taken as the worst-performing agents and are likely to be in a poverty trap.

The Resilience Thinking principles of Regime Shift - pushing over the barrier when applied on this model as a poverty alleviation strategy yielded different results. It is identified that the Fuzzy Safety Net transfer is a better method than the Need-Based Assistance on this model in the presence of an extreme exogenous shock of 0.8. This is due to the induction of the capital accumulating behavior and increase in the positive moral hazard among the agents in the case of the former. In addition, the short transfer period of the Fuzzy Safety Net encourages the Budget to be invested in other aspects of the society like education, hospitals, disease prevention, sanitation, etc, which would directly impact the capitals of the CCF. However, this is not explored in this project.

Fuzzy Safety Net transfer scheme lifts the majority of the agents out of the poverty trap. The few agents who appear to be still below the Micawber Frontier have a large stock of capital(k_t) that enables them to afford an investment in high technology. These agents can further cross over the Micawber Frontier only by increasing their innate ability(α). This requires a Budget to be spent on areas that benefit the human capital facet of individuals like education, healthcare, etc. Sufficient budget spend on healthcare, disease prevention, sanitation, etc helps the agents overcome the poverty trap in the presence of exogenous shocks from that of a pandemic.

7.1 Future Work

The model validation which couldn't be done due to the unavailability of data could be done in the future once the data is available. The savings propensity factor(λ) and innate ability(α) of an agent are taken from random uniform distribution and normal distribution respectively. These values must be found based on real-world data. For instance, the innate ability(α) that comprises an individual's health, education, cognitive ability, physical stature, etc can be found by creating a composite index using multivariate statistical methods.

In the results of the Human Capital Model and Need-Based Assistance scheme when applied on Human Capital Model, the phenomenon of hysteresis could be observed. This would cause some agents to be in a poverty trap irrespective of the financial assistance provided. So, fixing the cause alone would not change the status quo. This makes it a challenge for policymakers to develop effective poverty alleviation strategies in the presence of exogenous shocks. Hence, finding the area where an investment, be it financial or non-financial, would directly impact the people's condition thereby lifting them out of the poverty trap is critical. In this project, it was found that such agents could be lifted out of the poverty trap only by improving their innate ability. In the future, to develop effective poverty alleviation strategies, policymakers must identify the key area of investment.

In this project, only two poverty alleviation methods that are already existing in literature are studied for their effectiveness. More methods can be created which might even be more effective than the ones already existing.

To make a general model that can be applied in all situations all other dimensions of poverty from the Community Capitals Framework should be built into this model. When other dimensions are included the dynamics will be different and hence more effective poverty alleviation methods can be devised that are supported with empirical evidence.

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