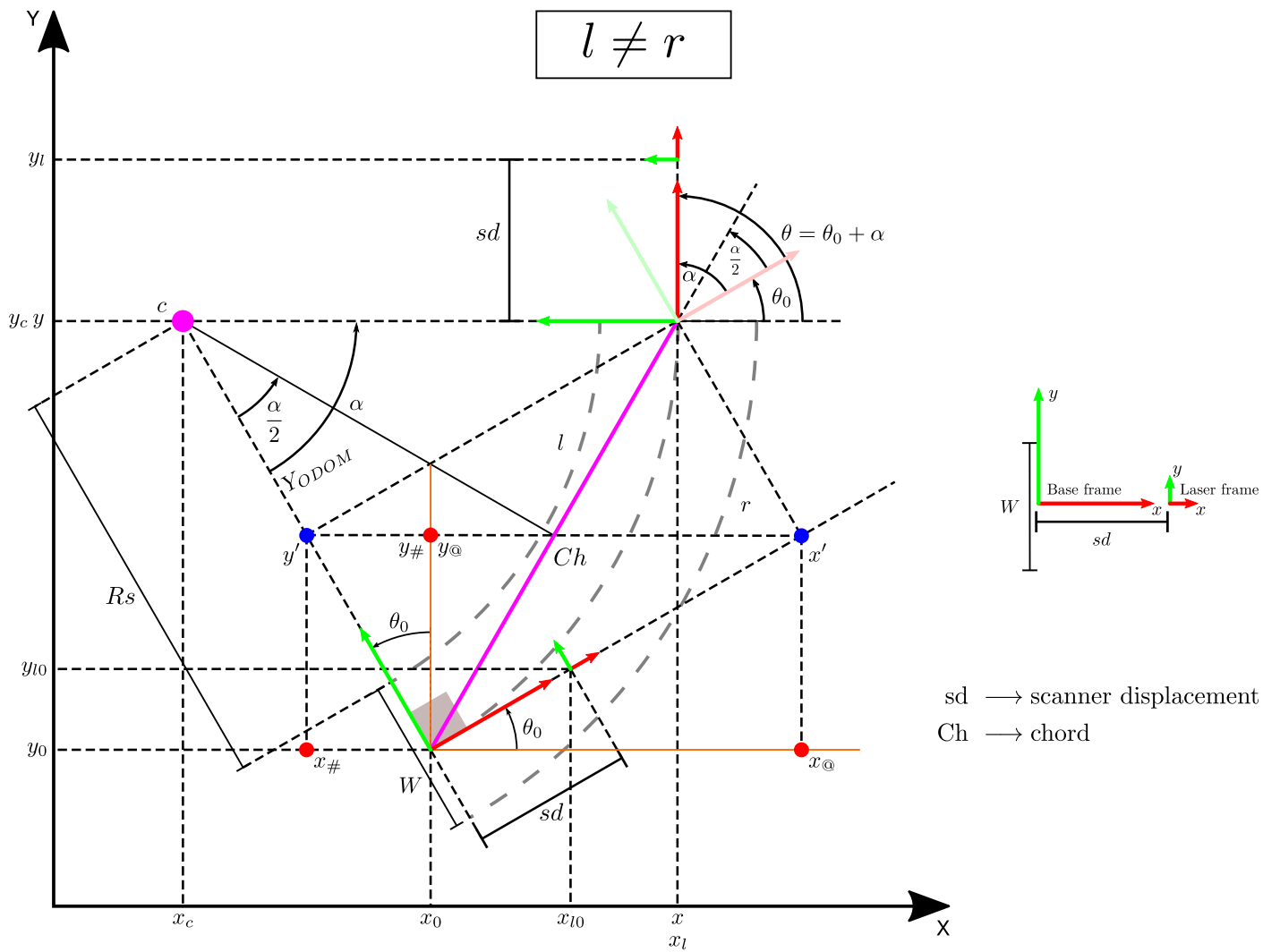


Robot's coordinates in the global reference frame



Given:

the robot's initial pose, (x_0, y_0, θ_0) , and the motion commands, (l, r) .

$$\begin{aligned}r &= (Rs + W) \alpha \\l &= Rs \alpha \\r - l &= W \alpha\end{aligned}$$

$$\begin{aligned}\alpha &= \frac{r - l}{W} \\Rs &= \frac{l}{\alpha}\end{aligned}$$

$$\begin{aligned}x_l &= x + sd \cos(\theta) \\y_l &= y + sd \sin(\theta)\end{aligned}$$

$$\begin{aligned}\cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v\end{aligned}$$

$$\begin{aligned}\cos(u - v) - \cos(u + v) &= 2 \sin(u) \sin(v) \\ \sin(u + v) - \sin(u - v) &= 2 \cos(u) \sin(v)\end{aligned}$$

$$Ch = 2 \left(Rs + \frac{W}{2} \right) \sin \left(\frac{\alpha}{2} \right)$$

$$\begin{aligned}
x &= x_0 + x_{\textcircled{a}} + x_{\#} \\
&= x_0 + x' \cos(\theta_0) + y' \cos(\theta + 90^\circ) \\
&= x_0 + Ch \cos\left(\frac{\alpha}{2}\right) \cos(\theta_0) + Ch \sin\left(\frac{\alpha}{2}\right) \cos(\theta + 90^\circ) \\
&= x_0 + Ch \cos\left(\frac{\alpha}{2}\right) \cos(\theta_0) - Ch \sin\left(\frac{\alpha}{2}\right) \sin(\theta) \\
&= x_0 + Ch \cos\left(\theta_0 + \frac{\alpha}{2}\right) \\
&= x_0 + 2 \left(Rs + \frac{W}{2}\right) \sin\left(\frac{\alpha}{2}\right) \cos\left(\theta_0 + \frac{\alpha}{2}\right)
\end{aligned}$$

$$\begin{aligned}
y &= y_0 + y_{\textcircled{a}} + y_{\#} \\
&= y_0 + x' \sin(\theta_0) + y' \sin(\theta + 90^\circ) \\
&= y_0 + Ch \cos\left(\frac{\alpha}{2}\right) \sin(\theta_0) + Ch \sin\left(\frac{\alpha}{2}\right) \sin(\theta + 90^\circ) \\
&= y_0 + Ch \cos\left(\frac{\alpha}{2}\right) \sin(\theta_0) + Ch \sin\left(\frac{\alpha}{2}\right) \cos(\theta_0) \\
&= y_0 + Ch \sin\left(\theta_0 + \frac{\alpha}{2}\right) \\
&= y_0 + 2 \left(Rs + \frac{W}{2}\right) \sin\left(\frac{\alpha}{2}\right) \sin\left(\theta_0 + \frac{\alpha}{2}\right)
\end{aligned}$$

Another way to calculate the robot's coordinates in the global reference frame:

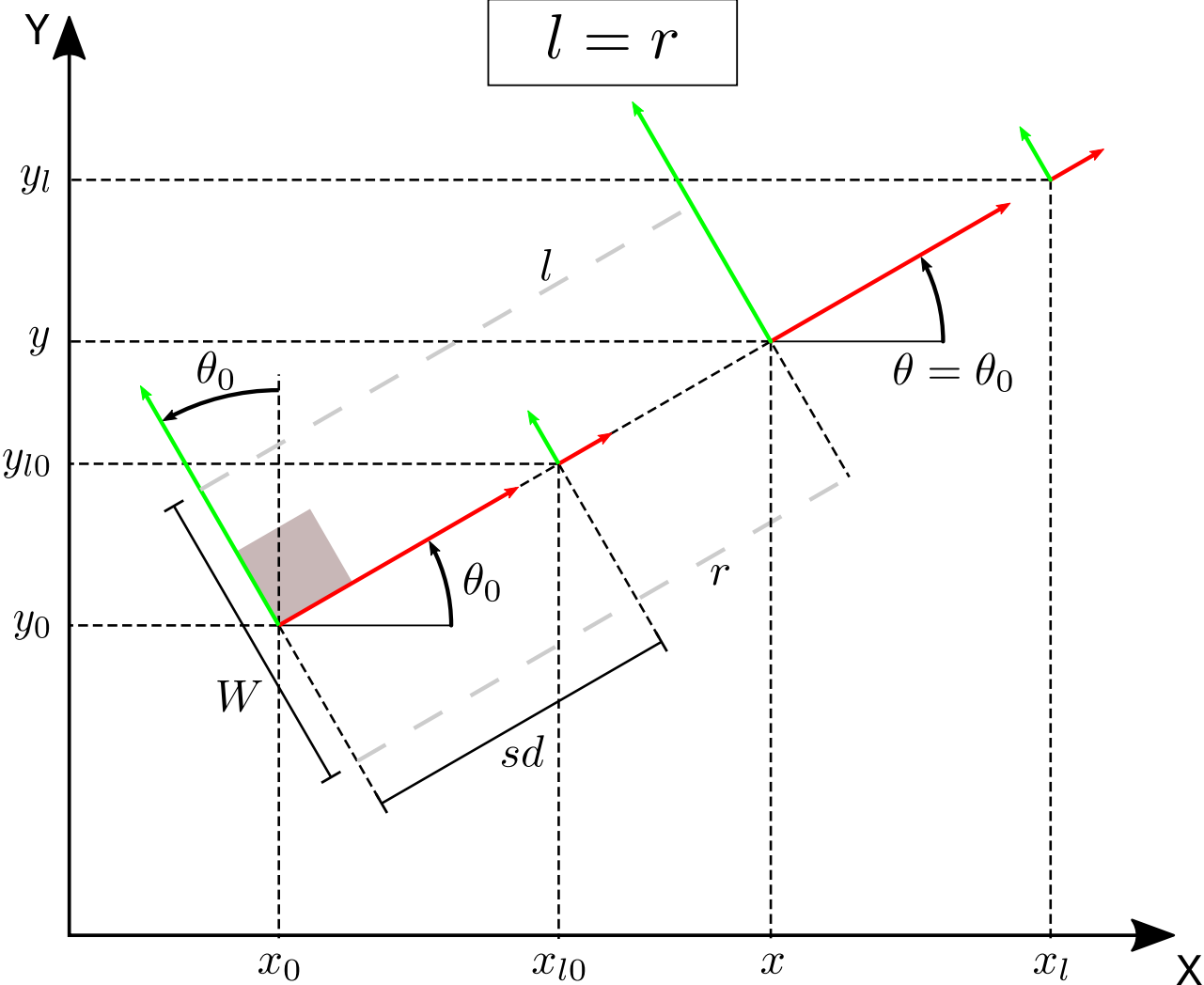
$$\begin{aligned}
 x_c &= x_0 + \left(Rs + \frac{W}{2}\right) \cos(\theta_0 + 90^\circ) \\
 &= x_0 - \left(Rs + \frac{W}{2}\right) \sin(\theta_0) \\
 y_c &= y_0 + \left(Rs + \frac{W}{2}\right) \sin(\theta_0 + 90^\circ) \\
 &= y_0 + \left(Rs + \frac{W}{2}\right) \cos(\theta_0)
 \end{aligned}$$

$$\begin{aligned}
 x_c &= x + \left(Rs + \frac{W}{2}\right) \cos(\theta + 90^\circ) \\
 &= x - \left(Rs + \frac{W}{2}\right) \sin(\theta) \\
 y_c &= y + \left(Rs + \frac{W}{2}\right) \sin(\theta + 90^\circ) \\
 &= y + \left(Rs + \frac{W}{2}\right) \cos(\theta)
 \end{aligned}$$

$$\begin{aligned}
 x &= x_c + \left(Rs + \frac{W}{2}\right) \sin(\theta) \\
 &= x_0 - \left(Rs + \frac{W}{2}\right) \sin(\theta_0) + \left(Rs + \frac{W}{2}\right) \sin(\theta) \\
 &= x_0 + \left(Rs + \frac{W}{2}\right) (\sin(\theta) - \sin(\theta_0)) \\
 &= x_0 + 2 \left(Rs + \frac{W}{2}\right) \cos\left(\theta_0 + \frac{\alpha}{2}\right) \sin\left(\frac{\alpha}{2}\right)
 \end{aligned}$$

$$\begin{aligned}
 y &= y_c - \left(Rs + \frac{W}{2}\right) \cos(\theta) \\
 &= y_0 + \left(Rs + \frac{W}{2}\right) \cos(\theta_0) - \left(Rs + \frac{W}{2}\right) \cos(\theta) \\
 &= y_0 + \left(Rs + \frac{W}{2}\right) (\cos(\theta_0) - \cos(\theta)) \\
 &= y_0 + 2 \left(Rs + \frac{W}{2}\right) \sin\left(\theta_0 + \frac{\alpha}{2}\right) \sin\left(\frac{\alpha}{2}\right)
 \end{aligned}$$

Robot's coordinates in the global reference frame



Given:

the robot's initial pose, (x_0, y_0, θ_0) , and the motion commands, (l, r) .

$$x = x_0 + l \cos(\theta_0)$$

$$y = y_0 + l \sin(\theta_0)$$

$$x_l = x + sd \cos(\theta)$$

$$y_l = y + sd \sin(\theta)$$