

HW 5 LOGISTIC classifier

Due 03/28/2023 in class

The code from the lecture example is attached in the homework. You can modify the code.

1. Logistic classification for a different set of synthetic data

In the lecture, I gave an example of using synthetic data and feed it to the weight update algorithm and obtain a logistic probability distribution.

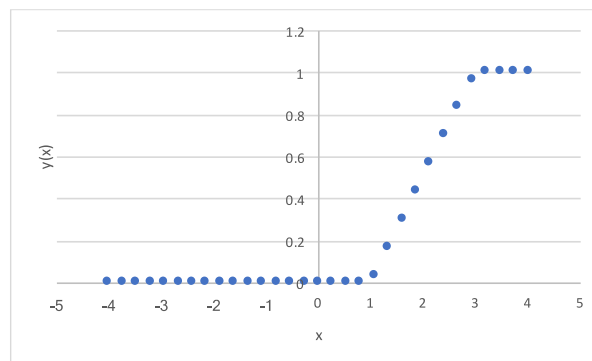
Now we wish to turn to another probability distribution as a new synthetic dataset.

First, let's define a piecewise function $y(x)$ as

$$y(x) = 0 \text{ when } x < 1$$

$$y(x) = 0.5 * (x - 1) \text{ when } 1 < x < 3$$

$$y(x) = 1 \text{ when } x > 3$$



Now generate x^t from $U(-4,4)$, i.e., a uniform distribution from the interval $[-4,4]$

For a given x^t , generate a random number $zeta$ from $U(0,1)$, i.e., a uniform distribution from interval $[0,1]$ and the outcome r^t is determined by

$$r^t = 1 \text{ when } 0 < zeta < y(x^t)$$

$$r^t = 0 \text{ otherwise}$$

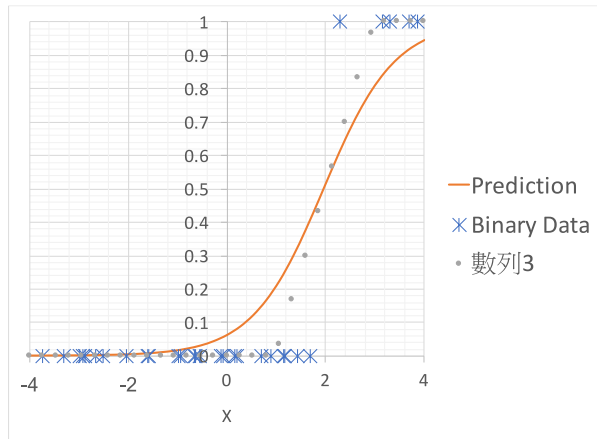
In total, please generate 30 sample for x^t . $t=1,2,\dots,30$ (Note we use Ethem Alpaydian's notation and t is sample index.)

Use this new data set to feed into the weight update algorithm for 100 iteration.

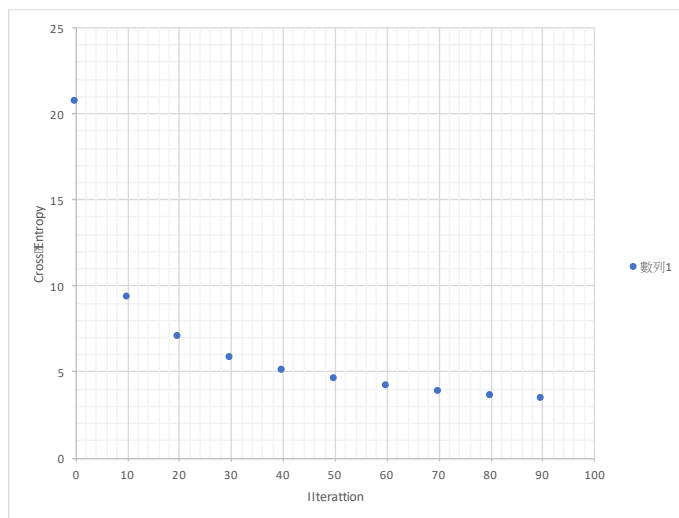
(a) Plot the new data for the interval $[-4,4]$ (15%)

(b) Find the weight coefficient w_1 and w_0 after 100 iteration. Plot the logistic function using w_1 and w_0 . (20%)

(c) Plot the cross entropy versus number of iteration. You can sample the data for every 10 iteration and use the data from 0 iteration to 90 iteration. (15%)



$w_1 = 1.5, w_0 = -2.5$



2. Calculation of decision boundary.

In the lecture, I show you the decision boundary for two bivariate Gaussian distribution is a circle assuming the prior for Gaussian 1 (class 1) and Gaussian 2 (class 2) are equal $P(C_1)=P(C_2)=0.5$. In this problem, I will give you another set of parameters and ask you to use the formula to calculate the radius and location of decision boundary.

$$\|\mathbf{x} - \mathbf{x}_c\|^2 = r^2$$

$$\mathbf{x}_c = \frac{\sigma_2^2 \boldsymbol{\mu}_1 - \sigma_1^2 \boldsymbol{\mu}_2}{\sigma_2^2 - \sigma_1^2}$$

$$r^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_2^2 - \sigma_1^2} \left[\frac{\|\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2\|^2}{\sigma_2^2 - \sigma_1^2} + 4 \ln \left(\frac{\sigma_2}{\sigma_1} \right) \right]$$

For class 1, the mean and variance for Gaussian is given by

$$\boldsymbol{\mu}_1 = (0,0)$$

$$\sigma_1 = 1$$

$$\mu_2 = (3,4)$$

$$\sigma_2 = 3$$

(a) Calculate the center and radius of the decision boundary (a circle)
(30%)

center (-1.5, -2)

radius $\frac{9}{8} \left(\frac{25}{8} + 4 * \ln(3) \right) = 2.91$

(b) Use the python code to calculate the misclassification (error) rate for class 1.
The misclassification rate for class means that the sample is from Gaussian 1 but being classified as Gaussian 2. You can use 1000 sample point from Gaussian 1 and 1000 sample point from Gaussian 2.

Answer 43%

(c) Use the python code to calculate the misclassification (error) rate for class 2.
The misclassification rate for class means that the sample is from Gaussian 2 but being classified as Gaussian 1. This rate is low. For each run, you can use 1000 sample point from Gaussian 1 and 1000 sample point from Gaussian 2. Run the experiment 10 times and give the answer with mean \pm standard deviation.

Answer $3.1\% \pm 5.1 \%$

```

import random

x1_record=[]
x2_record=[]
x1=0.0
x2=0.0
y1=0.0
y2=0.0
count1_error=0
count2_error=0
#decision boundary

for i in range(1000):
    x1=random.gauss(0,1) #create random number gauss (mean, sigma)
    x2=random.gauss(0,1)
    x1_record.append(float(x1))
    x2_record.append(float(x2))
    #print ('x1', round(x1,2), 'x2', round(x2,2))
    if (x1+1.5)**2+(x2+2)**2> 2.91**2 :
        count1_error += 1
    else: pass
print('error rate 1 in %', count1_error)
for j in range (10):
    count2_error=0.0
    for i in range(1000):
        y1=3.0+random.gauss(0,3)
        y2=4.0+random.gauss(0,3)
        #print ('x1', round(y1,2), 'x2', round(y2,2))
        if (y1+1.5)**2+(y2+2)**2< 2.91**2:
            count2_error += 1
        else: pass
    print('error rate 2 in %',j,' ', count2_error)

```

Appendix: code for logistic classification and decision boundary for two gaussian

```
import random
import math
w1_set=1.2
w0_set=0
x=0.0
outcome=0
w1=0.01
w0=0.01
eta=0.01
x_record=[]
r_record=[]

def sigmod(x,w1,w0):
    return 1/(1+ math.exp(-w1*x-w0))
for iteration in range(30):
    x=random.uniform(-2,2)
    zeta=random.uniform(0,1)
    y= sigmod(x,w1_set, w0_set)
    if zeta <= y:
        outcome=1
        x_record.append(float(x))
        r_record.append(int(outcome))
        print( 'iteration', iteration, 'x', round(x,2), ' ', outcome)

    else:
        outcome=0
        x_record.append(float(x))
        r_record.append(int(outcome))
        print( 'iteration', iteration, 'x', round(x,2), ' ', outcome)

epoch=100
sum1=0.0
sum2=0.0

sum_en=0.0
print ('sum1', sum1, 'sum2', sum2)
for iteration2 in range(epoch):
    sum2=0.0
    sum1=0.0
    sum_en=0.0
    for i in range(30):
        y= sigmod(x_record[i],w1,w0)
        sum1= sum1+(r_record[i] -y)*x_record[i]
        sum2=sum2+(r_record[i] -y)
        #computing cross entropy
        sum_en= sum_en-(r_record[i]*math.log(y)+(1-r_record[i])*math.log(1-y))

    w1=w1+eta*sum1
    w0=w0+eta*sum2
    if iteration2 %10==0:
        print( 'iteration', iteration2, 'error', sum_en)
        #print( 'iteration', iteration2, 'w1', round(w1,2), 'w0', round(w0,2))

print('final result', w1, ' ', w0)
```

```
# class 1 data is from Gaussian with mean (0,0) and sigma 1
# class 2 data is from Gaussian with mean (2,0) and sigma 2
# 100 data points are generated
# error rate for class 1 falling outside the circle is calculated
# error rate for class 2 falling inside the circle is calculated
# in the textbook    error rate 1  0.1056 our answer 10%
# in the textbook    error rate 2  0.2642 our answer 26%
```

```
import random

x1_record=[]
x2_record=[]
x1=0.0
x2=0.0
y1=0.0
y2=0.0
count1_error=0
count2_error=0
#decision boundary

for i in range(100):
    x1=random.gauss(0,1) #create random number gauss (mean, sigma)
    x2=random.gauss(0,1)
    x1_record.append(float(x1))
    x2_record.append(float(x2))

    if (x1+0.667)**2+x2**2> 2.34**2 :
        count1_error += 1
    else: pass
print('error rate 1 in %', count1_error)
for i in range(100):
    y1=2.0+random.gauss(0,2)
    y2=0.0+random.gauss(0,2)

    if (y1+0.667)**2+y2**2< 2.34**2:
        count2_error += 1
    else: pass
print('error rate 2 in %', count2_error)
```