HW3. You need to read Bayesian update before you can answer problem 1.

1. (30%)

In this problem, you are asked to add one Bin to the Bayesain update problem.

In total, there are three possible bins with probability

P(red |H=A) =0.7

P(green |H=A) =0.3

P(red |H=B) = 0.3

P(green |H=B) = 0.7

P(red |H=C) = 0.1

P(green |H=C) = 0.9

We start with the assumption that the prior probability is given by

P(H=A) =1/3

P(H=B)= 1/3

P(H=C)= 1/3

Now for simulation, we place Bin C in the black box and start to draw ball from the Bin C. Please run the Bayesian update for 100 iteration and obtain the posterior probability P(H=C | data) (data can be red or green) or equivalently the new prior P(H=C) because you need to reset the prior with posterior and update the prior probability accordingly. Plot it in excel or other grapher software.

2. A random sampling algorithm for quadratic programming

Consider the following quadratic programming problem

Objective function

Z= $(\boldsymbol{w}-\boldsymbol{b})^T(\boldsymbol{w}-\boldsymbol{b})$

The feasible region is cube of dimension d defined by

$0 \leq w_* \leq 1$ i=1,2,3.....d

(a) (30%) First, we set d=3 and consider a 3D problem with :

$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_{2,} \\ w_0 \end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix} 3 \\ 1/2 \\ 1/2 \end{bmatrix}$$

Now randomly and uniformly sample the feasible region and find both *the minimal value of Z* and $\boldsymbol{w}$ such that Z(**w**) is the minimum of Z. Usually we write it as

$$\boldsymbol{w} = \mathrm{argmin}_{\boldsymbol{w}\in;<=>} \boldsymbol{Z}$$

Repeat the experiment for 10 times and use 10^6 sample points for each experiment and for each run, print out both minimal value of Z and the corresponding w. (Print out both the code and result.)

(b) (10%) Is the answer you find in close proximity to the corner point of cube? Corner point (0,0,0) (1,0,0) (0,1,0) (1,1,1) (0,1,0) (1,1,0).....

Can you justify $\boldsymbol{w}$ using geometrical interpretation? (no coding is needed for this problem.)

(c) (30%) Now we change the problem to 5 dimension (d=5). Hence w is a five dimensional vector

$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_{\mathrm{B}} \end{bmatrix}$$

and set

$$\boldsymbol{b} = \begin{bmatrix} 3 \\ 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$$

Now randomly sample the feasible region for 10^6 and find both $\boldsymbol{w}$ such that $Z(\boldsymbol{w})$ is minimum and the minimum of Z.

Repeat the experiment for 10 times and using 10^6 sample point for each experiment and for each run, print out both minimal value of Z and the corresponding w. (Print out both the code and result.)