

## Homework 2: Optimization and linear algebra using NUMPY due on 03/09/2023 in class

Please install python with NUMPY, matplotlib and random library. Please print out and/or write down the calculation and submit it in class. For coding, you can do it with ANACONDA or install the library separately. There is an instruction on linear algebra for NUMPY.

TA email for emergency:

蕭方凱 [zeus950068@gmail.com](mailto:zeus950068@gmail.com) (my lab location: 工三 210)

### HW1 problem

1. (30%) Use Monte Carlo method to calculate volume of a hyper sphere of radius one . Mathematically, it is the volume integral

$$\int_{x_1^2+x_2^2+x_3^2+x_4^2+x_5^2 \leq 1} dV$$

In other world, this volume is bounded by

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 \leq 1$$

Please use run the experiment for 10 times and for each experiment use number of sample  $N=10^6$  . In the end, please give the average of the volume with standard deviation.

Note The average  $\bar{y}$  and standard deviation  $\sigma$  for a set of data point  $y_i$  ( $i=1, 2, 3, \dots, n$ ) is defined as

$$\text{Average of } y \quad \bar{y} = \frac{\sum_{i=1}^n y_i}{n}$$

$$\text{Standard Deviation} \quad \sigma = \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}}$$

In excel, you can use STDEV to calculate if you copy the datapoint to excel.

2. (30%) Linear programming. For the linear program defined by the following constraints and objective function Z

$$0 \leq x \leq 5$$

$$0 \leq y \leq 10$$

$$9x + 4y \leq 49$$

$$Z = x + y$$

Find out the

- (a) Plot the region defined by the linear inequality. (10%)
- (b) Find out all the corner points for this region. (10%)
- (c) Find out the point that maximizes Z and the value for maximal Z. (10%)

3.(40%) Use NUMPY for matrix manipulation

Define a matrix in the following form

$$X = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}$$

and a vector  $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

using

$$(x_1, y_1) = (0, 0.5) \quad (x_2, y_2) = (1, 2.5) \quad (x_3, y_3) = (3, 12.5)$$

(1) Compute the matrix  $X^T X$  (20%)

(2) Compute the vector  $w = (X^T X)^{-1} X^T y$  (20%)

You need to input these value into the matrix and vector.

Note: there is annoying bug in python. You need to define the column vector y using

```
y=np.array([[y1],[y2],[y3]])
```

If you define it as

```
y=np.array([y1,y2,y3])
```

and you do transpose of y. y will not become a column vector. You will run into trouble.