HW8 due 05/11/2023 in class

1. Projection of a dataset on a two dimension plane using PCA (50%)

(a) (25%)First, Sketch the noiseless deterministic version of the trajectory in 3D space:

$0<t<2*pi$

$X1(t)=(t/6.28)*\cos t$

$X2(t)=(t/6.28)*\sin t$

$X3(t) = 0$

You can simply sketch it in a paper or using graphic software. No need for python coding. Now using google colab, modify the code to plot the 2 principal components in 2D plane for a parametrized trajectory defined as

$\theta \sim U(0,2pi)$, i.e., from a uniform distribution from 0 and 2pi.

$X1=(\theta/6.28)*\cos \theta + 0.1* N(0,1)$

$X2=(\theta/6.28)*\sin \theta+0.1*N(0,1)$

$X3= 0.1*N(0,1)$

N(0,1) is the Gaussian noise with zero mean and standard deviation 1. Use total N= 1000 in your plot.

(b) (25%) First, sketch the noiseless deterministic version of the trajectory in 3D space:

$0<t<2*pi$

$X1(t)=(t/6.28)*\cos t$

$X2(t)=(t/6.28)*\sin t$

$X3(t) = t$

You can simply sketch it in a paper or using graphic software. No need for python coding. Using this sketch to determine the proper scale for the principal component plot. Use 2 principal component and plot it in 2D as in the example code.

Now switch to the trajectory with noise

$\theta \sim U(0,2pi)$,

$X1=(\theta/6.28)*\cos \theta + 0.1* N(0,1)$

$X2=(\theta/6.28)*\sin \theta+0.1*N(0,1)$

$X3= \theta*1 + 0.1*N(0,1)$

k means toy python code

2. (30%) In this homework, I give you the example code for a two cluster python.

The initial setting is two well-separated Gaussian.

Please set the two clusters to be two Gaussian

x1=random.gauss(0,0.6)

x2=random.gauss(0,0.6)

y1=1.0+random.gauss(0,0.6)

y2=1.0+random.gauss(0,0.6)

(a)(20%) Modify the code. Use N=60 sample (30 sample for each Gaussian cluster) and run for 10 iteration. Plot the sample point in excel using scatter plot along with final estimated cluster center.

(b) (10%) Write down the equation for decision boundary using the estimated cluster center. (it is a line.)

3. (20%) Lagrange multiplier

Suppose we want to minimize the function

$$f(x,y) = x^2 + 2y$$

subject to the constraint

$$g(x,y) = 3x + 2y + 1 = 0$$

Using the method of Lagrange multiplier to find the solution $(\hat{x}, \hat{y})$ and corresponding Lagrange multiplier λ such that $f(\hat{x}, \hat{y})$ is minimized. (Hint: you need to introduce langrange multiplier λ and take partial derivatives.)