

Homework 2: Optimization and linear algebra using NUMPY due on 03/09/2023

Please install python with NUMPY, matplotlib and random library. Please print out your result and submit it in class. You can do it with ANACONDA or install the library separately. There is an instruction on linear algebra for NUMPY.

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HW1 problem

1. (30%) Use Monte Carlo method to calculate volume of a hyper sphere of radius one . Mathematically, it is the volume integral

$$\int_{x_1^2+x_2^2+x_3^2+x_4^2+x_5^2 \leq 1} dV$$

In other world, this volume is bounded by

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 \leq 1$$

Please use run the experiment for 10 time and for each experiment use number of sample $N=10^6$. In the end, please give the average of the volume with standard deviation.

Note The average \bar{y} and standard deviation σ for a set of data point y_i ($i=1, 2, 3, \dots, n$) is defined as

$$\text{Average of } y \quad \bar{y} = \frac{\sum_{i=1}^n y_i}{n}$$

$$\text{Standard Deviation} \quad \sigma = \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}}$$

In excel, you can use STDEV to calculate if you copy the datapoint to excel.

Solution You need to multiply the result by 2^5

$$\text{Volume} = 5.26 \pm 0.011866232$$

```

import random
numNeedles=1000000 #use 1 million sample for each run
sphereVolume=0.0
iteration= 10

def throwNeedles(numNeedles):
    inSphere=0
    for Needles in range(1, numNeedles+1):
        x1= random.uniform(0,1)
        x2= random.uniform(0,1)
        x3= random.uniform(0,1)
        x4= random.uniform(0,1)
        x5= random.uniform(0,1)
        if (x1*x1+x2*x2+x3*x3+x4*x4+x5*x5)**0.5 <=1:
            inSphere +=1
    #count needles in one quadrant only and multiply by 32=2**5
    return 32*(inSphere/numNeedles)
for iteration in range(1,11):
    sphereVolume=throwNeedles(numNeedles)
    print('Est =',iteration, sphereVolume)

```

2. (30%) Linear programming. For the linear program defined by the following constraints and objective Z

$$0 \leq x \leq 5$$

$$0 \leq y \leq 10$$

$$9x + 4y \leq 49$$

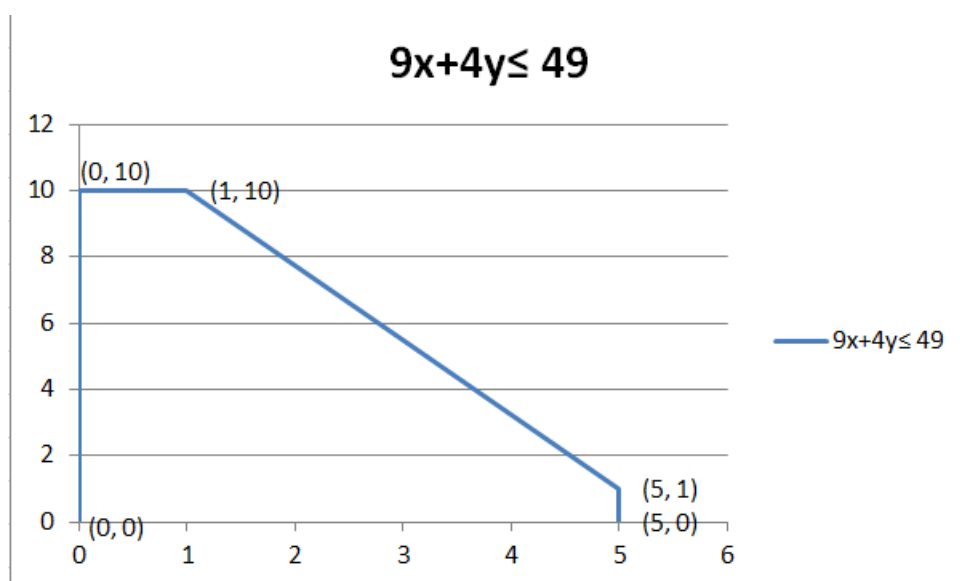
$$Z = x + y$$

Find out the

- Plot the region defined by the linear inequality. (10%)
- Find out all the corner points for this region. (10%)
- Find out the point maximize Z and the value for maximal Z. (10%)

3. solution

(a)



(b)

(0,0)、(5,0)、(5,1)、(1,10)、(0,10)

(c)

$Z=x+y$ 代入點(1,10) 有最大值 $Z=11$

3.(40%) Use NUMPY for matrix manipulation

Define a matrix in the following form

$$X = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}$$

and a vector $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

using

$(x_1, y_1) = (0, 0.5)$ $(x_2, y_2) = (1, 2.5)$ $(x_3, y_3) = (3, 12.5)$

(1) Compute the matrix $X^T X$ (20%)

(2) Compute the vector $w = (X^T X)^{-1} X^T y$ (20%)

You need to input these value into the matrix and vector.

Note: there is annoying bug in python. You need define the column vector y using

`y=np.array([[y1],[y2],[y3]])`

If you define it as

`y=np.array([y1,y2,y3])`

and you do transpose of y. y will not become a column vector. You will run into trouble.

Answer:

$W = (0.5, 1, 1)$

```

# this example is used to calculate 2nd order polynomial regression
# data point are input manually
import numpy as np
x1=0
x2=1
x3=3
y1=0.5
y2=2.5
y3=12.5
# compute data matrix
X=np.array([[1, x1, x1*x1],[1,x2,x2*x2],[1,x3,x3*x3]])
y0=np.array([[y1],[y2],[y3]])
XT=np.transpose(X)
product= np.dot(XT,X)
print(product)
inverse=np.linalg.inv(product)
product2= np.dot(inverse,XT)
w=np.dot(product2,y0)
print( w)

1.1. 11
>>>
= RESTART: /Users/jack/Dropbox/課程規劃/machine learning/Homework/regression 2nd
030123.py
[[ 3  4 10]
 [ 4 10 28]
 [10 28 82]]
[[0.5]
 [1. ]
 [1. ]]

```