1.

初始化:

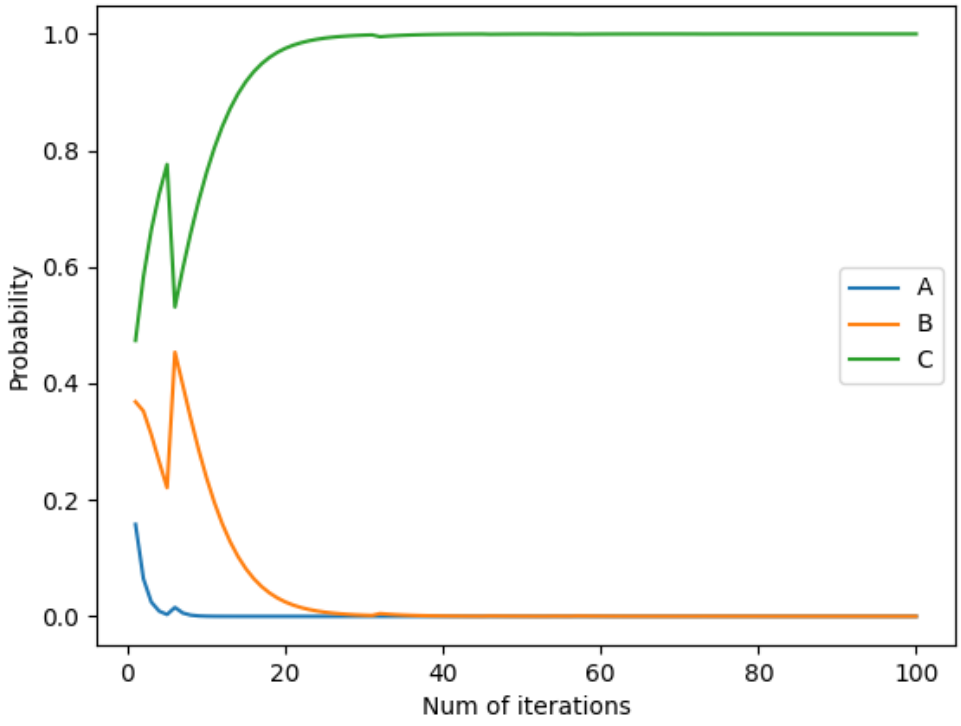
```
import random
import matplotlib.pyplot as plt

yA=[]
yB=[]
yC=[]

priorA=1/3  # P(H=A)
priorB=1/3  # P(H=B)
priorC=1/3  # P(H=C)

Pgreen_A=0.3  # P(green|A)
Pred_A=0.7    # P(red|A)
Pgreen_B=0.7  # P(green|B)
Pred_B=0.3    # P(red|B)
Pgreen_C=0.9  # P(green|C)
Pred_C=0.1    # P(red|C)

# init
P_A_red=0   # P(A|red)
P_A_green=0 # P(A|green)
P_B_red=0   # P(B|red)
P_B_green=0 # P(B|green)
P_C_red=0   # P(C|red)
P_C_green=0 # P(C|green)
```

For loop 計算:

```
iteration=100
for i in range(iteration):
    x=random.uniform(0,1)
    if x>=0 and x<=Pgreen_C:
        P_A_green = Pgreen_A*priorA / (Pgreen_A*priorA + Pgreen_B*priorB + Pgreen_C*priorC)
        P_B_green = Pgreen_B*priorB / (Pgreen_A*priorA + Pgreen_B*priorB + Pgreen_C*priorC)
        P_C_green = Pgreen_C*priorC / (Pgreen_A*priorA + Pgreen_B*priorB + Pgreen_C*priorC)

        priorA=P_A_green
        priorB=P_B_green
        priorC=P_C_green
    else:
        P_A_red = Pred_A*priorA / (Pred_A*priorA + Pred_B*priorB + Pred_C*priorC)
        P_B_red = Pred_B*priorB / (Pred_A*priorA + Pred_B*priorB + Pred_C*priorC)
        P_C_red = Pred_C*priorC / (Pred_A*priorA + Pred_B*priorB + Pred_C*priorC)

        priorA=P_A_red
        priorB=P_B_red
        priorC=P_C_red

    yA.append(priorA)
    yB.append(priorB)
    yC.append(priorC)
```

使用 Matplotlib 畫圖:

```
X=[i+1 for i in range(iteration)]
plt.plot(X, yA, label='A')
plt.plot(X, yB, label='B')
plt.plot(X, yC, label='C')
plt.xlabel('Num of iterations')
plt.ylabel('Probability')
plt.legend(loc='center right')
plt.show()
```



2.

```python
import random
import numpy as np

experiments=10
samples=10**6

def solve(dimension, b):
    for i in range(experiments):
        # init on every experiments
        minW=np.array([])
        minZ=2**31-1

        for j in range(samples):
            w=np.array([random.uniform(0,1) for _ in range(dimension)]) # w = [w1, w2, ..., wi]
            Z=(w-b).T @ (w-b)   # = sum of squares of w-b

            if Z<minZ:
                minW=w
                minZ=Z
        print(f"experiment {i+1}: w = {minW}, Z has min = {minZ}")

solve(3, np.array([3, 1/2, 1/2]))  # <= ans of 2(a).
solve(5, np.array([3, 1/2, 1/2, 1/2, 1/2]))    # <= ans of 2(c).
```

(a)

```
experiment 1: w = [0.9999748  0.48062394 0.46970642], Z has min = 4.001393947592782
experiment 2: w = [0.99989456 0.53597623 0.50788493], Z has min = 4.001778235715958
experiment 3: w = [0.99997424 0.49962645 0.5257034 ], Z has min = 4.000763851210767
experiment 4: w = [0.99989658 0.52443076 0.4854156 ], Z has min = 4.001223253555958
experiment 5: w = [0.99997727 0.49619126 0.53900321], Z has min = 4.001626687384897
experiment 6: w = [0.99992815 0.50001524 0.53244739], Z has min = 4.001340240378512
experiment 7: w = [0.99969452 0.4935292  0.48022849], Z has min = 4.0016548161235095
experiment 8: w = [0.99989644 0.48701354 0.49607637], Z has min = 4.000598288540276
experiment 9: w = [0.99995268 0.51312807 0.48542432], Z has min = 4.000574097172919
experiment 10: w = [0.99996312 0.50488139 0.48719355], Z has min = 4.000335367879495
```

(b)

Yes, w is around [1, 0.5, 0.5] and is quite near corner points.

Z 的幾何意義是 w 到 b 之距離的平方，因此求 min Z 就是 sample w 使得 w 與 b 的距離最近

(c)

```
experiment 1: w = [0.99947562 0.44992954 0.55871642 0.56421734 0.49383559], Z has min = 4.0122143429824595
experiment 2: w = [0.99950396 0.5346672  0.49976225 0.53670365 0.42295067], Z has min = 4.010470052694099
experiment 3: w = [0.99792807 0.56043052 0.44271951 0.46675241 0.49665682], Z has min = 4.0163414865987574
experiment 4: w = [0.9984843  0.45362917 0.46787682 0.48992369 0.58755072], Z has min = 4.017013913521467
experiment 5: w = [0.99828549 0.54223668 0.50877813 0.45219695 0.45568962], Z has min = 4.012970534003914
experiment 6: w = [0.999863   0.4946216  0.5124446  0.47901771 0.49293666], Z has min = 4.001221974333923
experiment 7: w = [0.99898078 0.54182126 0.49038103 0.54340851 0.50837347], Z has min = 4.007873877012792
experiment 8: w = [0.99961978 0.52005032 0.57534213 0.45225733 0.4998701 ], Z has min = 4.009878839523508
experiment 9: w = [0.99957645 0.49965769 0.53317459 0.58669806 0.4449483 ], Z has min = 4.013342306039037
experiment 10: w = [0.99775912 0.467901   0.48436544 0.48091228 0.45731311], Z has min = 4.0124298470953725
```