

HW7 due 05/02/2023(Tuesday) in class

1. Regression with tanh activation function (40%)

In this problem, you are asked to replace the sigmodal activation with tanh activation . In the midterm exam, you are asked to show that

$$\tanh'(x) = 1 - \tanh^2(x)$$

Use this formula to obtain new weight update rule and modify a python code in the appendix for fitting sine function $\sin(6x) + N(0,0.1)$ with regression. $N(0,0.1)$ is gaussian noise with zero mean and standard deviation 1. (Or you can download it in eeclass 上課教材 file name regression v3 bp 04-07-23.py)

The neural network structure is specified by

2 input unit (a bias plus one input data)

2 hidden layer units with sigmodal activaion function

1 output layer unit with linear activation function

In the code, I generate 27 sample data points for input data x^t from

$$y = \sin(6 \cdot x^t) + \text{random.gauss}(0, 0.1)$$

(a) (20%) Print out all the weight coefficients.

(b) (20%) Also copy input data and its actual output (output y) evaluated on these 27 data points into excel and plot output y versus input data along with .

2. Bias variance dilemma

In this problem, you are asked to input a target function $\cos(1.5x) + N(0,1)$. $N(0,1)$ is gaussian noise with zero mean and standard deviation 1. The input $\{X^t\}$ is from a uniform distribution in the interval $[-2, 2]$. For example, you can use python code as follows

$$x = \text{random.uniform}(-2, 2)$$

$$y = \cos(1.5 \cdot x) + \text{random.gauss}(0, 1)$$

Use the python code to generate 5 data sets. Each dataset consists of 20 datapoints $\{x^t, y^t\}$ $t=1, 2, \dots, 20$. (t is sample index.) Copy these data points into excel.

(a) (20%) For each data set, fit the target function with trend line of 4th order polynomial and display the equation in the graph. In total, you will get five trend line, each correspond to a function

$$g_i(x) = w_0 + w_1x + w_2x^2 + w_3x^3 + w_4x^4. \text{ Record the coefficients } w_i (i=0, \dots, 4) \text{ to find average}$$

of these five function $\bar{g} = \left(\frac{1}{N}\right) \sum_{i=1}^N g_i(x)$ ($N=5$)

(b) (20%) Repeat (a) with 2nd order polynomial and hence $g_i(x) = w_0 + w_1x + w_2x^2$. find

average of these five function $\bar{g} = \left(\frac{1}{N}\right) \sum_{i=1}^N g_i(x)$ ($N=5$)

3. (20%, 10% each) For a bin with two kind of marbles (red and green), we denote the actual probability μ of getting red marble and ν is the fraction of red marbles in a sample of size N . (This means we make drawing N times and after each drawing of a marble, we replenish the bin with the marble with the same color to keep μ constant.) Use binomial distribution for $N=10$ to evaluate.

(a) $P[|\nu - \mu| < \epsilon] \quad \mu = 0.75 \quad \epsilon = 0.06$

We use $P[.]$ to denote the probability.

(a) $P[\nu < 1] \quad \mu = 0.9$

Appendix:

```
import random
from random import randrange
from math import *

def sigmod(x):
    return 1/(1+exp(-x))
w_z1=[0,0]
w_z2=[0,0]
#w_z3=[0,0]

#initialize weight coefficients
for i in range(2):
    w_z1[i]=random.uniform(-0.1,0.1)
    w_z2[i]=random.uniform(-0.1,0.1)
    # w_z3[i]=random.uniform(-0.1,0.1)
v_1=random.uniform(-0.1,0.1)
v_2=random.uniform(-0.1,0.1)
#v_3=random.uniform(-0.1,0.1)
v_0=random.uniform(-0.1,0.1) # adding bias term for v

eta=0.05 #define learning rate

# repurpose the input vector
# the first element is bias unit
# the second is the input x
# target function  $f(x) = \sin 6x$  from Alpaydin's book

# set the rest of element to be zero
x = [ [1,0.0], [1,0.0], [1,0.0], \
      [1,0.0], [1,0.0], [1,0.0], \
      [1,0.0], [1,0.0], [1,0.0], \
      [1,0.0], [1,0.0], [1,0.0], \
```

```

        [1,0.0], [1,0.0], [1,0.0], \
        [1,0.0], [1,0.0], [1,0.0],
        [1,0.0], [1,0.0], [1,0.0], \
        [1,0.0], [1,0.0], [1,0.0], \
        [1,0.0], [1,0.0], [1,0.0]

    ]

# desired output array
r= [0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0, \
    0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0, \
    0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
    ]

for i in range(27):
    x1= random.uniform(-0.5,0.5)
    x[i][1]=x1
    y=sin(6*x1)+random.gauss(0,0.1)
    r[i]=y

for i in range(27000):
    j= randrange(27) #randomly pick sample vector of out of 8
    desiredoutput=r[j]
    sum_w_z1=0
    sum_w_z2=0
    #sum_w_z3=0
    sum_v=0
    for k in range(2):
        sum_w_z1=sum_w_z1+ w_z1[k]*x[j][k]
        sum_w_z2=sum_w_z2+ w_z2[k]*x[j][k]
        #sum_w_z3=sum_w_z3+ w_z3[k]*x[j][k]
    z1_h=sigmod(sum_w_z1)
    z2_h=sigmod(sum_w_z2)
    #z3_h=sigmod(sum_w_z3)
    sum_v=v_1*z1_h+v_2*z2_h+v_0 #keep only two hidden unit

    output_y= sum_v    #use linear unit as output

```

```

#delta rule
# weight update Ethm Alpaydin' pseudo code
# update= learning rate*(Desired output - Actualouput)*input
v_1=v_1-eta*(output_y-desiredoutput)*z1_h
v_2=v_2-eta*(output_y-desiredoutput)*z2_h
#v_3=v_3-eta*(output_y-desiredoutput)*z3_h
v_0=v_0-eta*(output_y-desiredoutput)*1

for m in range(2):
    # weight update Ethm Alpaydin' pseudo code
    # update= learning rate *v*z(1-z)*(Desired output - Actualouput)*input
    w_z1[m]=w_z1[m]-eta*v_1*z1_h*(1-z1_h)*(output_y-
desiredoutput)*x[j][m]
    w_z2[m]=w_z2[m]-eta*v_2*z2_h*(1-z2_h)*(output_y-
desiredoutput)*x[j][m]

for j in range(27):
    desiredoutput = r[j]
    sum_w_z1=0
    sum_w_z2=0
    #sum_w_z3=0
    sum_v=0
    for k in range(2):
        sum_w_z1=sum_w_z1+ w_z1[k]*x[j][k]
        sum_w_z2=sum_w_z2+ w_z2[k]*x[j][k]

    z1_h=sigmoid(sum_w_z1)
    z2_h=sigmoid(sum_w_z2)

    sum_v=v_1*z1_h+v_2*z2_h+v_0
    output_y= sum_v

    print('input x',round(x[j][1],3), 'desiredoutput',
round(desiredoutput,3),'actualouput', round(output_y,3))
# in total, this newtork has 7 coefficient, namely 3 v coefficients and 4 w coefficients
print('v0', round(v_0,3), 'v1', round(v_1,3), 'v2', round(v_2,3))
print('wz1_0_bias', round(w_z1[0],3), 'wz1_1', round(w_z1[1],3), 'wz2_0_bias',
round(w_z2[0],3),'wz2_1', round(w_z2[1],3))

```

