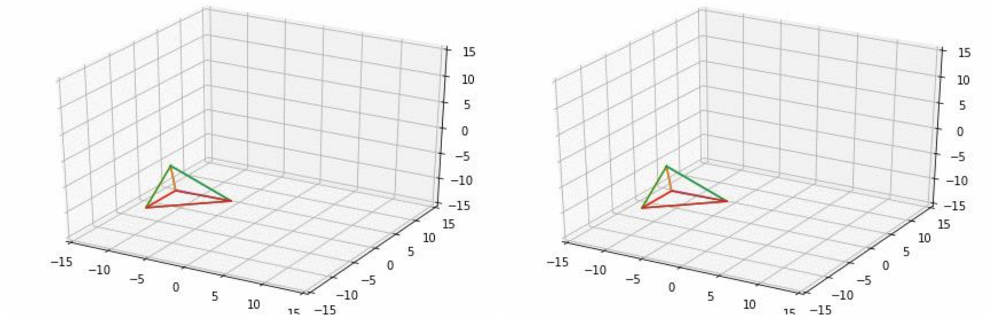
1.

我設計的 model 是一個四面皆為三角形的三角錐，它四個頂點座標分別為(1,0,2), (3,0,0), (0,1,0), (0,-1,0)，它的運動軌跡是沿著左下到右上的一條線來回運動，我將 xdata 設為[0, 0.02, 0.04, …… ,1.98, 2.00, 2.00, 1.98, 1.96, …… ,0.02, 0]，長度為 200，ydata 也是類似的道理，範圍是從-2 到 0 再從 0 到-2，間隔 0.02，所以長度也是 200。

3.



當我交換 yaw,pitch,roll 矩陣的乘積順序後，發現物體旋轉的方向跟結果會不一樣，左圖是按照原本的軌跡在運動(yaw*pitch*roll)，右圖是將乘積順序交換過後所產生的結果(roll*pitch*yaw)。仔細觀察可以發現當三角錐運行到端點(左下、右上角)的時候，左圖的最尖端的頂點會水平移動，而右圖的尖端則是往下運動，原因是因為在三維空間中的旋轉是沒有交換律的，所以旋轉矩陣的乘積順序很重要，以 x,y 軸各旋轉 90 度的例子來說，如果先轉 x 再轉 y，它相乘的結果會是[[0,0,1],[1,0,0],[0,1,0]]；如果順序相反的話，結果則會是[[0,1,0],[0,0,1],[1,0,0]]，由此可見會產生出完全不一樣的矩陣，因此交換 yaw,pitch,roll 的乘積順序會大大影響物體運動軌跡與模式。

4.(1)

4(1) roll : $AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta \\ 0 & \sin\theta\cos\theta-\cos\theta\sin\theta & \sin^2\theta+\cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$A^TA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos^2\theta+\sin^2\theta & -\cos\theta\sin\theta+\sin\theta\cos\theta \\ 0 & -\sin\theta\cos\theta+\cos\theta\sin\theta & \sin^2\theta+\cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow AA^T = A^TA = I \quad \therefore R_{roll}$ is orthogonal

yaw : $AA^T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta & 0 \\ \sin\theta\cos\theta-\sin\theta\cos\theta & \sin^2\theta+\cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$A^TA = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2\theta+\sin^2\theta & -\sin\theta\cos\theta+\sin\theta\cos\theta & 0 \\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \sin^2\theta+\cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow AA^T = A^TA = I \quad \therefore R_{yaw}$ is orthogonal

pitch : $AA^T = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta+\sin^2\theta & 0 & \sin\theta\cos\theta-\sin\theta\cos\theta \\ 0 & 1 & 0 \\ \sin\theta\cos\theta-\sin\theta\cos\theta & 0 & \sin^2\theta+\cos^2\theta \end{bmatrix} = I$

$A^TA = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta+\sin^2\theta & 0 & -\sin\theta\cos\theta+\sin\theta\cos\theta \\ 0 & 1 & 0 \\ -\sin\theta\cos\theta+\sin\theta\cos\theta & 0 & \sin^2\theta+\cos^2\theta \end{bmatrix} = I$

$AA^T = A^TA = I \Rightarrow R_{pitch}$ is orthogonal

4.(2)

先證 orthogonal matrix 的乘積依然是 orthogonal，再利用 QR decomposition 即可得證。

4(2) first we want to prove the product of orthogonal matrix is orthogonal.

pf: let A·B to be 2 orthogonal matrix, so we have $\begin{cases} AA^T = A^TA = I \\ BB^T = B^TB = I \end{cases}$

since $(AB)^TAB = B^TA^TAB = B^{-1}A^{-1}AB = I$

since A·B are orthogonal $\Rightarrow A^T = A^{-1}$ and $B^T = B^{-1}$

and $AB(AB)^T = ABB^TA^T = ABB^{-1}A^{-1} = I$

$\therefore$ AB is an orthogonal matrix

Then we know R_{pitch}、R_{yaw}、R_{roll} are all orthogonal

$\Rightarrow R_{pitch}R_{yaw}R_{roll}$ is an orthogonal matrix

by QR decomposition, Any matrix A can be written as a multiple of orthogonal matrix ($R_{pitch}R_{yaw}R_{roll}$) and an upper triangular matrix U

Hence $A = R_{pitch}R_{yaw}R_{roll}U$ #