Assignment 2: Making Your Movie

2021 Fall EECS205002 Linear Algebra

Due: 2021/11/17

Let's make animations. Here are three simple steps.

- Build a model.
- Plan the trajectory.
- Make model moves.

Here is a simple example. For the model, the example built an airplane using three polygons. Each polygon can be represented by a $3 \times n_i$ matrix.

$$P_i = \begin{bmatrix} x_1^{(i)} & x_2^{(i)} & \dots & x_{n_i}^{(i)} \\ y_1^{(i)} & y_2^{(i)} & \dots & y_{n_i}^{(i)} \\ z_1^{(i)} & z_2^{(i)} & \dots & z_{n_i}^{(i)} \end{bmatrix}.$$

The ith column of P_i is a point (x_i, y_i, z_i) in the three dimensional space. To make the later animation easy, we usually build the model centered at $(0, 0, 0)$.

For the trajectory, the airplane flies in a circle, up and down. Generally, we split the movie into N frames. Suppose the displacement in the ith frame is

$$d_i = \begin{bmatrix} d_x^{(i)} \\ d_y^{(i)} \\ d_z^{(i)} \end{bmatrix}.$$

The location of the polygon P_i in the frame j is

$$\begin{bmatrix} x_1^{(i)} + d_x^{(j)} & x_2^{(i)} + d_x^{(j)} & \dots & x_{n_i}^{(i)} + d_x^{(j)} \\ y_1^{(i)} + d_y^{(j)} & y_2^{(i)} + d_y^{(j)} & \dots & y_{n_i}^{(i)} + d_y^{(j)} \\ z_1^{(i)} + d_z^{(j)} & z_2^{(i)} + d_z^{(j)} & \dots & z_{n_i}^{(i)} + d_z^{(j)} \end{bmatrix} = P_i + d_i \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix}.$$

For the movement, the example just uses rotations. As shown in Figure 1, there are three kinds of rotations: roll, yaw, and pitch, which are the rotations along three axes. Those rotations can be represented by matrices. Suppose z-axis is the vertical axis, x-axis is the lateral axis, and y-axis is the longitudinal axis.

$$R_{\text{roll}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}, R_{\text{yaw}} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

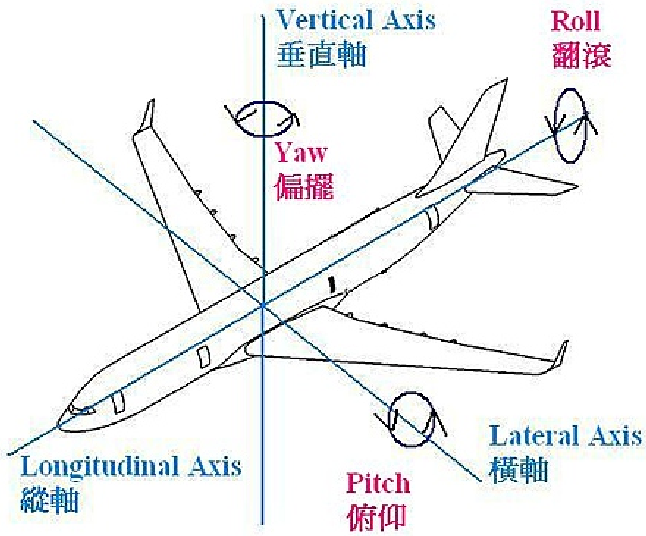

Figure 1: Rotations along three axes.

and

$$R_{\text{pitch}} = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix}.$$

for some angle θ. Those rotation matrices can be applied together by multiplying them,

$$R = R_{\text{yaw}} R_{\text{pitch}} R_{\text{roll}}.$$

But notice that the order matters.

1 Assignments

1. (20%) Design your own model and movie. Your model should have at least 4 polygons, and the length of the movie should be at least 100 frames.
2. (20%) Write a program to animate your model (1), and save it as a GIF file.
   - (20%) **Bonus**: If your animation is good, you will get this extra points or partial of them.
3. (20%) Give an example to explain the effect of applying the rotation matrices (yaw, roll, pitch) in different order. You can use the given example if you cannot finish (1) and (2).
4. (20%) An $n \times n$ matrix A is called *orthogonal* if $A^T A = AA^T = I$.

- Show that all 3×3 rotation matrices are orthogonal matrices. (This result can be applied for any $n \times n$ matrices, but you only need to prove the 3×3 case.)
- Show that any 3×3 matrix A can be expressed as

  $$A = R_{\text{pitch}} R_{\text{yaw}} R_{\text{roll}} U$$

  where U is an upper triangular matrix.

5. (20%) Google what *Gimbal lock* is? Use the given example or your own model to show Gimbal lock, and explain how it happens.

2 Submission

1. Write a report in PDF file that includes the answers of question (1), (3), (4), and (5).
2. The code of (2), (3), (5) should be implemented in the file `implement.py`.
3. The GIF files of (2), (3), and (5).
4. Zip them and submit to eeclass.