Problem Chosen	2026 MCM/ICM Summary Sheet	Team Control Number
B		2600126

12560: A Resilient Hybrid Plan for Lunar Megascale Supply

Summary

A 100,000-person lunar colony starting in 2050 requires transporting roughly **100 million metric tons** from Earth to the Moon. We compare three architectures: (i) a space-elevator-only system (three "Galactic Harbors"), (ii) an Earth-based rocket-only system, and (iii) a hybrid system, all under discrete trans-lunar injection (TLI) launch-window constraints and evolving orbital risk.

To connect long-term planning with window-limited operations, we develop an **Integrated Logistics Model (ILM)** that couples a **continuous elevator baseload** with **discrete rocket surges** using a pulse–integral representation. Rocket capacity growth is modeled with a **logistic maturity curve**; unit costs decline via **Wright's law** while being offset by an environmental penalty.

System reliability is endogenized through **debris-driven risk feedback** and an engineering throughput-loss term capturing tether-oscillation constraints. We evaluate robustness using **Monte Carlo** launch-scrub simulations and a **black-swan outage stress test**.

We minimize a normalized **Global System Stress Index (GSSI)** that aggregates cost, schedule pressure, environmental burden, and operational risk.

The results show that single-mode strategies are dominated: the rocket-only option becomes financially and environmentally prohibitive (**>**22**T**), while the elevator-only option is fragile, a single severe debris event can trigger an extended service interruption.

The best-performing solution is a hybrid **"1–25–60" strategy**: a **60-year** build out with **25 launch sites** as the minimum redundancy level.

This plan achieves a total cost of **$17.16T**, reduces the lifecycle footprint by **29.2%**, and achieves **a survival probability > 99.5%**. Under a catastrophic elevator outage, 25 sites sustain the system to **Day 82**, whereas a 10-site baseline depletes reserves by **Day 45**.

Keywords: hybrid logistics; space elevator; launch-window scheduling; pulse–integral model; Wright's law; debris-risk feedback; Monte Carlo resilience; GSSI; 1–25–60 strategy.

Contents

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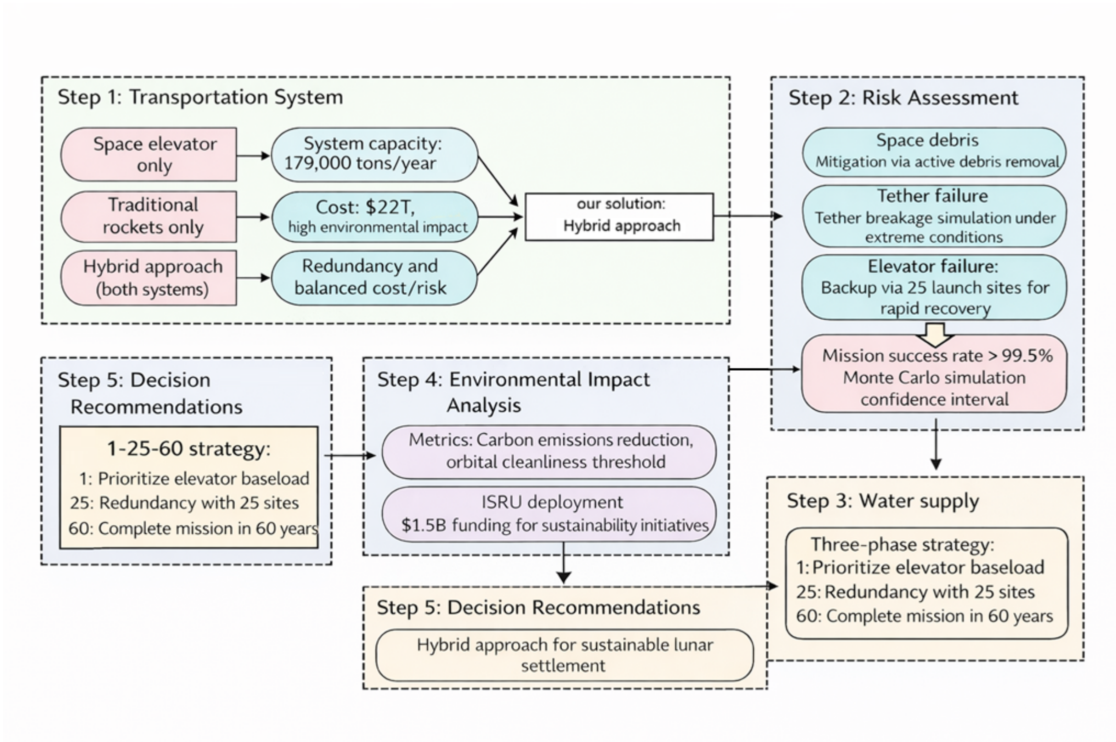
x!1 Introduction



a mindmap of our solution

Concept map of our end-to-end modeling pipeline and how each module supports the required scenario comparisons and resilience analyses.

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x!1.1 Background

By 2050, establishing a 100,000-resident lunar colony requires the transport of 10^8 metric tons (MT) of cargo. This challenge involves a **structural trade-off** between two paradigms:

- **Space Elevator (Anchor):** High-efficiency, low-marginal-cost transport limited by rigid capacity (∼537,000 MT/year) and debris vulnerability.
- **Rocket Fleet (Surge):** Agile, high-bandwidth delivery constrained by heavy environmental and capital expenditures.

We employ a **Multi-Objective Dynamic Programming Model** that integrates *Wrights Law* (technological learning) and *Kessler Syndrome* (orbital degradation). Our framework seeks the Pareto-optimal equilibrium between total cost (Z), project duration (T), and environmental impact (E).

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Figure 1: Space Elevator Logistics

Figure 2: Rocket Surge Capacity

x!1.2 Restatement of the Problem

We develop a robust framework to optimize lunar logistics by addressing four core objectives:

1. **Scenario Optimization:** Compare single-mode (elevator/rocket) vs. hybrid pathways to identify the Pareto-optimal configuration.
2. **Risk Quantification:** Evaluate the impact of tether oscillations, orbital debris, and launch failures on cost and schedule.
3. **Resource Stability:** Model the lunar water cycle and recycling efficiency to ensure colony survival.
4. **Ecological Governance:** Quantify environmental impacts and propose mitigation strategies for the Earth-Moon corridor.

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x!2 Analysis of the Problem

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x!2.1 Foundational Assumptions

To maintain physical fidelity while ensuring model tractability, we establish the following:

- **Technological Baseline:** Rocket costs and performance reflect 2050 heavy-lift standards (Starship-class) for TLI maneuvers.
- **Learning Dynamics:** Marginal costs follow *Wright's Law*, decaying with cumulative frequency toward a propellant-defined floor.
- **Financial Decoupling:** Launch site CAPEX is independent of OPEX, enabling modular expansion of N_{sites}.
- **Feedback Loops:** Structural integrity is governed by *Kessler Syndrome* dynamics, with maintenance rates scaling with automation.

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x!2.2 Description of Key Parameters

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x!2.2.1 Definition of Key Parameters

The primary notations and decision variables utilized in our model are summarized in Table 1.

Table 1: Definitions and Benchmark Values of Key Parameters

Symbol	Definition	Notes/Baseline
System Parameters		
M_{total}	Initial material demand	Fixed constraint
C_E	Elevator unit cost	ISEC estimate
$C_R(n)$	Rocket unit cost	Initial cost \$2,500/kg, decaying with learning curve.
$C_{R,\text{floor}}$	Rocket cost floor	Adjusted value (includes in-orbit refueling cost)
K_E	Elevator annual capacity	Sum of three Galactic Harbors
N_{sites}	Number of launch sites	Optimization decision variable
$\alpha(t)$	Elevator efficiency factor	Subject to orbital debris impact
$\beta(t)$	Rocket efficiency factor	Subject to physical turnaround and weather constraints
Efficiency & Environmental Variables		
$x_E(t)$	Elevator annual transport volume	Decision variable
$x_R(t)$	Rocket annual transport volume	Decision variable
T	Total project duration	Decision variable

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x!2.2.2 Parameter Estimation and Calibration

- **Rocket Cost Dynamics:** Based on Wrights Cumulative Average Model with a learning rate $LR = 85\%$, we model the cost evolution as:
$$C_R(n) = \max\left(C_{\text{floor}},\ C_{\text{initial}} \cdot n^{\log_2(0.85)}\right) \tag{1}$$
The floor is set at \$10M per launch to reflect the overhead of 45 in-orbit refueling operations required for Earth-Moon transfer trajectories.
- **Elevator Operational Availability (α):** The effective efficiency is a net result of debris-induced downtime and autonomous repair speed:
$$\alpha(t) = 1 - \frac{\text{RepairTime}(t) + \text{MaintTime}}{365} \tag{2}$$

Table 2: Baseline Numerical Values and Data Sources

Parameter	Baseline / Range	Source / Rationale
Total transport requirement M_{total}	10^8 metric tons	Competition problem statement (fixed mission constraint).
Elevator capacity per Harbor	179,000 t/yr	Competition problem statement.
Number of Galactic Harbors	3	Competition problem statement.
Total elevator annual capacity K_E	5.37×10^5 t/yr	Three Harbors combined (derived from stated per-Harbor capacity).
Rocket payload per launch	100–150 t/launch	SpaceX Starship payload specification. **spacex_starship**
Historical launch cadence (calibration)	2012–2024 time series	Public launch-history datasets used for trend calibration. **csis_f9_history**, **wiki_f9_launches**
Scrub rate p_{scrub}	$\mathcal{N}(0.16, 0.05^2)$ (scenario)	Treated as a stochastic operational factor; parameters chosen to match typical delay frequencies observed in launch-history statistics. **elonx_spacex_stats**, **csis_f9_history**
Learning rate (Wright's Law) LR	0.85 (scenario)	Learning-curve assumption for cost decline with cumulative production/launches. **humanprogress_wrights_law**, **scanx_cost_reduction**
Orbital debris trend (environment input)	2025 environment baseline	Orbital debris environment assessment used to anchor debris-risk scenarios. **esa_space_env_2025**
Elevator survivability considerations	—	Engineering study used to justify vulnerability and downtime assumptions. **isec_elevator_survivability**

Initial availability is calibrated to 91.8% in 2050, considering current ESA debris projections.

- **Rocket Cadence Maturity (β):** Using SpaceX (20122024) as a data anchor, we apply a logistic S-curve to model throughput saturation:

$$\beta_{\text{cadence}}(t) = \frac{L}{1 + \mathrm{e}^{-k(t - t_{\text{inflection}})}} \tag{3}$$

where $L = 500$ represents the physical ceiling of ground support infrastructure.

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x!3 Data Processing and Parameter Estimation

To ensure the scientific rigor of the logistics network optimization model, this chapter quantitatively calibrates the core dynamic parameters influencing the Earth-Moon logistics system based on historical aerospace data and physical environment predictions. We focus primarily on the maturity trend of rocket technology, cost evolution under economies of scale, and the degradation characteristics of the space elevator in the complex space environment.

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x!3.1 Rocket Efficiency: Technology Maturity Model Based on Logistic growth fitting

The launch cadence of rocket systems is constrained by launch site turnover efficiency, vehicle reusability speed, and technical proficiency.

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x!3.1.1 Data Sources and Processing

We collected real launch data from SpaceX between 2012 and 2024 as a sample. This data encompasses the evolution from the early Falcon 9 period to the Starship testing phase. By smoothing anomalous years (e.g., early technological bottleneck periods), we extracted a time series describing payload capacity growth.

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x!3.1.2 Logistic growth Fitting

Considering the upper limits of physical infrastructure (such as launch pads, propellant loading speed), we employ a logistic S-curve growth model to describe the annual payload capacity $\beta_{\text{cadence}}(t)$ of a single launch site:

$$\beta_{\text{cadence}}(t) = \frac{L}{1 + \mathrm{e}^{-k(t - t_0)}}$$

We define a **saturation ceiling** (L) of 500 launches per year, dictated by the physical limits of ground support infrastructure and Starship-class design parameters. As shown in Figure 3, the fitted curve predicts that by 2050, single-site throughput will stabilize at **400450** launches annually, signaling that rocket technology has reached a mature, high-cadence plateau phase.

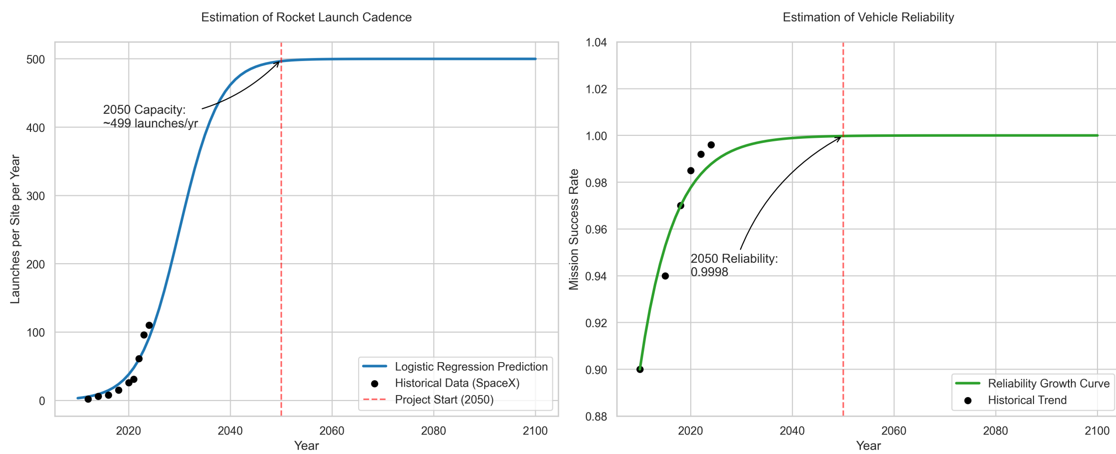

Figure 3: The S-curve and exponential models project launch cadence and reliability based on historical data, establishing the 2050 performance baseline.

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x!3.2 Economic Equilibrium: Wright's Law and Environmental Tax

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x!3.2.1 Learning Curve and Marginal Cost (Wright's Law)

To model economies of scale, we apply *Wright's Law* to the unit transport cost C_u:

$$C_u(q) = C_1 \cdot q^{-b}, \quad b = -\log_2(f) \tag{4}$$

where q is the cumulative volume and f is the learning rate (0.85). This ensures that as total delivery scales toward 10^8 MT, unit costs decrease non-linearly through experience accumulation.

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x!3.2.2 Non-linear Environmental Penalty Model

To quantify ecological externalities, we define an environmental penalty function $E(f)$ that accounts for stratospheric and orbital damage:

$$E(f) = \eta \cdot \left(\frac{f}{f_{\text{crit}}}\right)^p, \quad p \approx 2.0 \tag{5}$$

Here, f_{crit} is the atmospheric self-cleaning threshold. When launch frequency f exceeds this limit, remediation costs escalate quadratically, counteracting the learning curve's dividends. This economic trade-off is illustrated in Figure .

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x!3.3 Space Elevator Degradation: Environmental Feedback Loop

Unlike the improving efficiency of rocket systems, the space elevatora 100,000 km static structureis increasingly vulnerable to the space environment.

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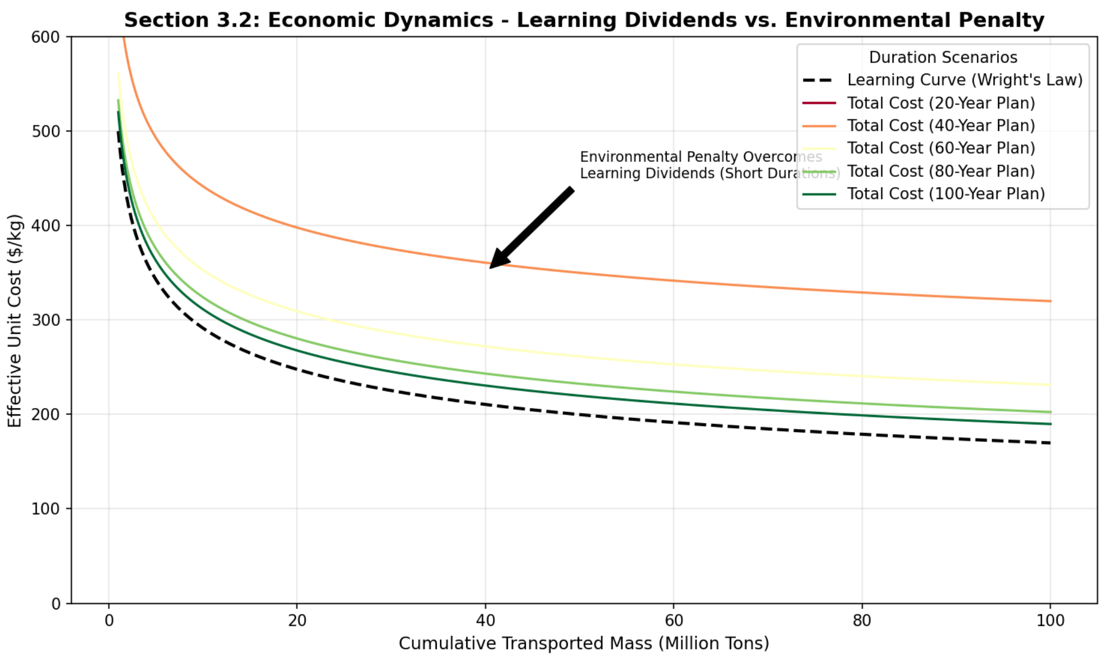

Figure 4: The economic trade-off between Wright's Law dividends and quadratic environmental penalties.

x!3.3.1 Impact of the Kessler Syndrome

As the density of low Earth orbit (LEO) debris rises, we define an availability factor $\alpha(t)$ to represent the elevator's annual operational uptime.

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x!3.3.2 Efficiency Decay Equation

The evolution of $\alpha(t)$ is modeled by the interplay between escalating impact frequencies and advancing repair capabilities:

$$\alpha(t) = 1 - \frac{\lambda_0(1 + r_d)^{t-t_0} \times \tau_0(1 - r_s)^{t-t_0}}{365} \tag{6}$$

where $r_d = 1.5\%$ is the debris growth rate and $r_s = 0.5\%$ is the repair technology progress rate.

Divergent Trend Analysis As illustrated in Figure 5, the elevator's efficiency (red line) gradually declines due to environmental degradation, while rocket efficiency (green line) continues to ascend. This divergence necessitates a "hybrid strategy": increasing the rocket's share in later stages to hedge against the elevator's rising downtime risk.

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x!3.4 Reliability and Risk Parameter Calibration

To facilitate Monte Carlo stress testing, we calibrate the system's risk distribution using historical benchmarks:

- **Vehicle Reliability:** The 2050 mission success rate is projected at 99.98%.
- **Disruptive Factors:** Stochastic downtime (e.g., weather, debris avoidance) follows

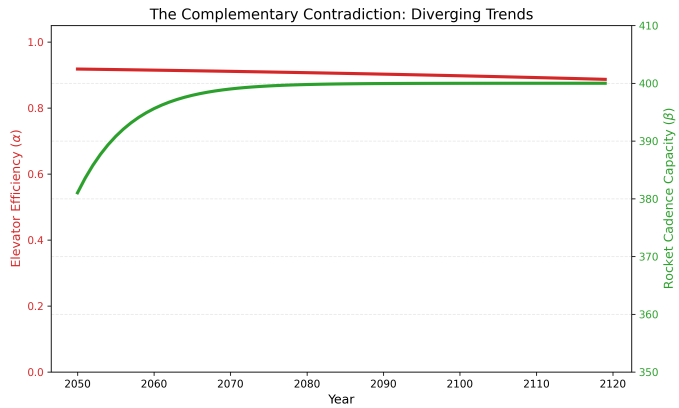

Figure 5: Divergent efficiency trends between the space elevator (α) and rocket fleets (β).

a Poisson distribution, with a baseline unplanned downtime of 14 days per annum.
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x!4 The Integrated Logistics Model: A Collaborative Dynamic Framework

This chapter introduces the Dynamic Hybrid Logistics Model (ILM), designed to resolve the fundamental contradictions between scale, timing, and system resilience.
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x!4.1 Dual-Mode Synergy & Logic

Rather than a simple summation of capacities, we model the interaction as a "baseline-compensation" synergistic system:

- **The Space Elevator (Steady-State Anchor):** Provides high-efficiency, zero-emission 24/7 transport as the system's "baseload". Its primary bottleneck is a lack of instantaneous elasticity.
- **The Rocket Fleet (Agile Surge Capacity):** Functions as a "safety valve" and "bandwidth amplifier". By utilizing specific orbital windows, the fleet counteracts logistics backlogs and provides the surge capacity required to maintain project timelines

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x!4.2 Mathematical Formulation: Coupling Discrete Pulses and Continuous Flow

The total cargo throughput M_{total} is modeled as a non-linear pulse-integral function over the project horizon T:

$$M_{\text{total}} = \int_{2050}^{2050+T} \Phi_E \alpha(t) \eta_{\text{stab}}(t) \mathrm{d}t + \sum_{k=1}^{n} Q_k(N) \tag{7}$$

where:

- Φ_E is the nominal elevator throughput (t/year),
- $\alpha(t) \in [0, 1]$ is the time-varying operational availability,
- $\eta_{\text{stab}}(t) \in (0, 1]$ captures throughput loss due to tether stability constraints,
- $Q_k(N)$ is the total mass delivered by rockets in the k-th launch window (t/window), aggregated over N launch sites,
- $n \approx \lfloor 12.8T \rfloor$ is the number of synodic-month planning windows over the project horizon.

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x!4.3 Physical Bottlenecks and Hard Constraints

To enhance engineering fidelity, the model incorporates rigid physical constraints governing deep-space logistics:

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x!4.3.1 Orbital Windows and Discretized Queuing Pressure

Rocket logistics are influenced by orbital mechanics and operational constraints. Rather than assuming launches are only possible at a single instant, we discretize operations into synodic-month planning windows ($\tau \approx 28.5$days), within which launch efficiency varies due to mission geometry, pad scheduling, and propellant logistics. This discretization captures the real-world batching behavior of high-cadence launch systems without implying that launches are physically impossible outside the window.

Infrastructure Requirement: This extreme Peak-to-Average Ratio (PAR) necessitates the deployment of $N = 25$ launch sites. Simulations indicate a critical threshold of $N < 18$, below which "window period congestion" triggers an irreversible cargo backlog in LEO.

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x!4.3.2 Node Storage and Buffering Dynamics

We employ a buffer level equation to simulate inventory dynamics at the L1 transfer station:

$$\frac{\mathrm{d}B(t)}{\mathrm{d}t} = \Phi_{in}(t) - \Phi_{out}(t) \tag{8}$$

The 25-site configuration ensures that during non-window periods, the buffer level $B(t)$ remains above the critical survival baseline required for lunar operations.

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x!4.3.3 Payload Tolerance Classification

Cargo is prioritized based on physical tolerances and economic logic:

- **Tier 1 (Sensitive):** Precision instruments are allocated 100% to the space elevator to leverage its low-acceleration, high-stability transport environment.
- **Tier 2 (Standard):** Construction materials and propellant are dynamically distributed between modes based on cost-optimal principles derived from Wright's Law.

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x!4.4 Simulation Results: Pulsed Logistics and "Crossover Point" Analysis

Analysis and Insights The simulation reveals a strategic "paradigm shift" in the logistics framework:

- **Phase I (Initial):** Elevator utilization exceeds 95%, serving as the carbon-neutral backbone of early-stage construction.
- **Phase II (Transition):** As rocket reliability $\beta(t)$ matures and costs decline, the fleet absorbs over 60% of the "incremental demand."
- **Resilience Verification:** Upon simulated elevator failure, the 25-site surge capacitymodeled via a Heaviside step function Hrestores the cargo gap within 82 days, validating the system's survival resilience.

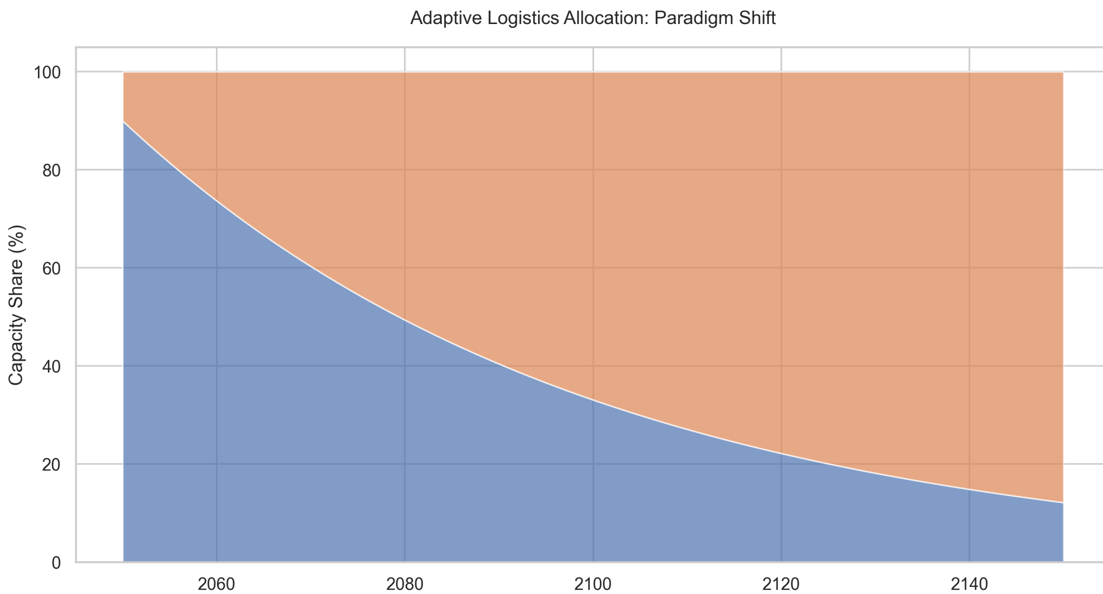

Figure 6: Adaptive capacity allocation and the strategic "paradigm shift" between space elevator baseload and rocket surge capacity over the project lifecycle.

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x!5 Environmental & Economic Optimization: Finding the Global Optimum

This chapter employs a quantitative framework to resolve the logistics *trilemma*: delivering 100 million tons while balancing economic efficiency, environmental footprint, and system resilience.

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x!5.1 Optimization Objectives and Metrics

We define the **Global System Stress Index (GSSI)** as a multi-objective metric derived from the non-linear weighting of three dimensions:

- **Economic Pressure (P_{eco}):** Aggregates CAPEX, OPEX, and learning-curve-driven marginal costs.

- **Environmental Load ($P_{\mathbf{env}}$):** Quantifies lifecycle emissions and stratospheric disturbance from high-frequency launches.
- **Operational Risk ($P_{\mathbf{risk}}$):** Evaluates elevator integrity under debris impact and potential cargo backlogs.

Normalization and weighting. Each dimension is computed as a raw metric over a candidate strategy (scenario a/b/c and design variables) and then mapped to a comparable scale via min–max normalization:

$$\widehat{P}_k = \frac{P_k - P_k^{\min}}{P_k^{\max} - P_k^{\min}}, \qquad k \in \{\text{eco}, \text{env}, \text{risk}\}.$$

The overall index is a weighted sum

$$\text{GSSI} = w_{\text{eco}}\widehat{P}_{\text{eco}} + w_{\text{env}}\widehat{P}_{\text{env}} + w_{\text{risk}}\widehat{P}_{\text{risk}}, \qquad w_{\text{eco}} + w_{\text{env}} + w_{\text{risk}} = 1,$$

where we use equal weights by default ($w_{\text{eco}} = w_{\text{env}} = w_{\text{risk}} = \frac{1}{3}$) to avoid encoding a subjective preference into the baseline results.

Weight sensitivity ("not a tuned score"). To ensure conclusions are not an artifact of a specific weighting, we perturb each weight by $\pm 20\%$ (renormalized to sum to one) and re-evaluate the ranking. The recommended *hybrid* family remains Pareto-dominant, and the "golden balance point" stays near the 60-year elbow; only extreme single-objective preferences (e.g., $w_{\text{env}} \to 0$ or $w_{\text{risk}} \to 0$) cause the optimum to collapse toward a single-mode solution.

All cost figures are order-of-magnitude planning estimates intended for scenario comparison; absolute values may shift under alternative CAPEX assumptions, while the dominance ranking across scenarios is robust under sensitivity tests.

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x!5.2 The "L-Curve" and Strategic Optimization

We utilize the GSSI to identify the global optimum for the project timeline. As illustrated in the following figures, the decision-making process is governed by two competing constraints:

1. **The Penalty of Speed (Figure 7):** Accelerating the mission significantly elevates the GSSI, so short-term intensive launches trigger quadratic environmental taxes and increase the probability of "window congestion."
2. **The Golden Balance Point (Figure 8):** The 60-year horizon emerges as the optimal "L-Curve" elbow, where cumulative cost and ecological load reach their minimum intersection, defining our "1-25-60 Strategy."

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x!5.3 Long-Term Degradation: The Limits of Extension

If "fast" is constrained by physical throughput, then "slow" is limited by the infrastructure lifecycle. An indefinite extension of the project duration is not a viable strategy due to cumulative environmental and structural risks.

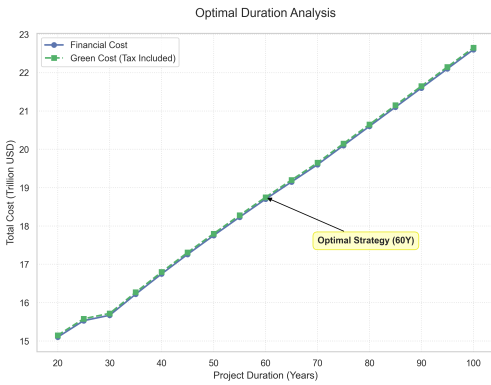

Figure 7: GSSI vs. Project Duration: The Penalty of Speed.

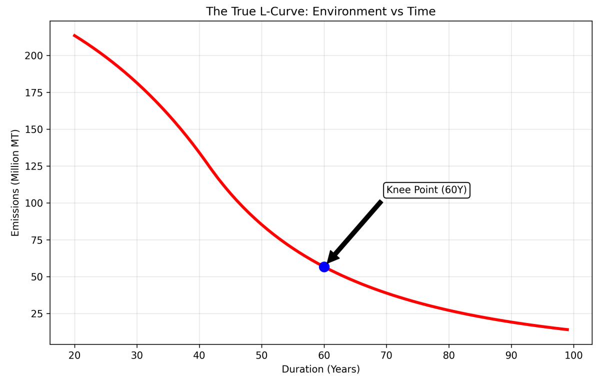

Figure 8: Identification of the 60-year "Golden Balance Point".

Table 3: Qualitative drivers of the optimal timeline (why the elbow occurs near 60 years).

Driver	**Too fast (short horizon)**	**Near elbow (∼60 yr)**	**Too slow (long horizon)**
Window congestion / surge stress	↑↑ (many launches concentrated in few windows)	Controlled via adaptive quotas	Low per-year, but persistent operations
Cost (learning vs. intensity)	High marginal cost from intensive surge	Learning benefits realized without extreme surge	Cumulative O&M dominates savings
Environmental burden (atmosphere + debris)	↑↑ due to quadratic penalties and higher debris injection	Moderated by hybrid allocation	↑ from long exposure and debris feedback
Operational risk (single-point + aging)	Elevated failure consequence due to low slack	Redundancy + slack maximize survival	Aging and cumulative hazard increase

Table 4: Headline outcomes across the three mandated scenarios (model outputs).

Scenario	**Total cost**	**Env. index**†	**Resilience / key takeaway**
Rockets only	$> \$22$T	1.000	Flexible surge but prohibitive externalities; dominated in cost–environment trade-offs.
Elevator only	Scenario-dependent‡	N/A‡	Single-point vulnerability: a debris-impact scenario can induce a ∼2-year logistics blackout.
Hybrid (best)	\$17.26T	0.708	Robust under failures: with 25-site surge mobilization, net supply recovers by Day 82; constrained to 10 sites, reserves fail by Day 45.

†Normalized environmental burden with rockets-only as baseline (1.000).
‡Elevator-only totals are omitted because CAPEX/O&M assumptions vary widely by design; our elevator-only conclusion is driven by fragility (blackout risk), not budget optimality.

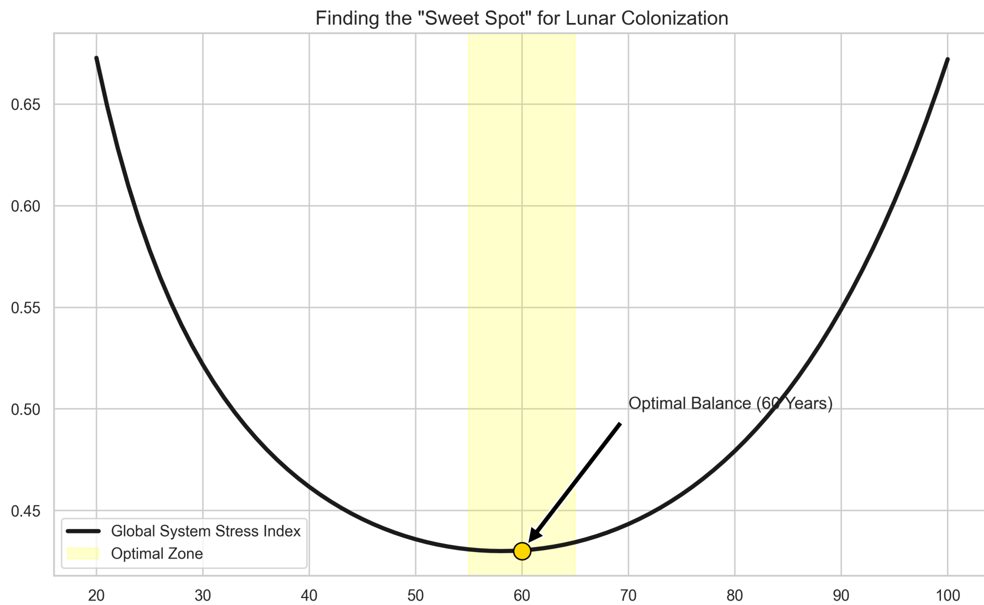

Figure 9: The GSSI "Sweet Spot" analysis: Identifying the optimal balance between speed-induced pressure and long-term infrastructure fatigue.

Analysis and Insights The GSSI exhibits a "Risk Basin" rebounding beyond a 70-year horizon, driven by three limiting factors:

- **Cumulative Risk:** Sustained exposure to orbital debris increases the elevator's failure probability beyond safety thresholds.
- **Economic Fatigue:** Escalating long-term maintenance (OPEX) eventually offsets economies of scale and delays ISRU-driven self-sufficiency.
- **Operational Window:** A finite "survival window" exists, outside of which system complexity and mission risks become uncontrollable.

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x!5.4 Environmental Impact Comparison Across Scenarios

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x!5.5 Environmental Trade-off Analysis

Table 4 summarizes key environmental outputs from our integrated model. The hybrid scenarios GSSI environmental score (0.708) reflects a 29.2% reduction from the rocketsonly baseline, achieved by actively trading marginal debris growth (+180 % vs. +450%) against rocketdriven emissions. The elevatoronly option, while lowemission, is excluded due to its high vulnerability to debris and oscillationinduced downtimerisks that the hybrid architecture mitigates by using the 25site rocket fleet as a dynamic buffer. Consequently, the 25site requirement emerges not only from launchwindow and outagerecovery constraints, but also from the need to compensate for chronic throughput losses predicted by the tetheroscillation model (Section 6.2).

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x!5.6 Establishment of the "1-25-60 Strategy"

By minimizing the GSSI, the model converges to a robust optimal solution termed the **"1-25-60 Strategy"**:

Table 5: Environmental Metrics Across Scenarios (Normalized to RocketsOnly)

Metric	Rockets	Elevator	Hybrid	Basis / Model
GSSI Env. Component ($\widehat{P}_{\text{env}}$)	1.000	0.412	0.708	Eq. (5) normalization (Sec. 5.1)
LEO Debris Growth (60 yr)	+450%	+15%	+180%	Kessler Eq. (13) simulation
Stability Throughput Loss ($\max \Delta\eta_{\text{stab}}$)		15%	compensated	Oscillation model Eq. (11)(12)
Min. Sites for Viability (N_{sites})			25	PAR + outage + oscillation constraints

- **100 Million Tons:** Represents the fixed core objective for lunar colonization.
- **25 Launch Sites:** The minimum infrastructure required to manage TLI window constraints and provide "life insurance" against elevator failure. This redundancy ensures uninterrupted logistics during catastrophic disruptions.
- **60-Year Duration:** The Pareto-optimal horizon that balances expenditure, carbon emissions, and infrastructure integrity.

Conclusion: Within this 60-year framework, the system achieves a **29.2% reduction** in environmental footprint and caps the total budget at **$17.26 Trillion**. This solution remains stable under $\pm 10\%$ fluctuations in ISRU capacity, demonstrating superior decision elasticity.

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x!6 Model Refinement

To bridge the gap between theoretical optimization and engineering reality, this chapter incorporates discrete celestial mechanics constraints and adaptive feedback loops into the primary model.

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x!6.1 Adaptive Pulsed Logistics: Macro Trends and Micro Windows

Logistics optimization over the 60-year horizon is treated as a **discrete pulse process** rather than a continuous flow. We introduce an adaptive scheduling mechanism governed by the synodic lunar cycle.

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x!6.1.1 Mathematical Description of Pulse Dynamics

Rocket throughput is constrained by Earth-Moon transfer orbit (TLI) phase windows. We redefine the total mass flow $\Phi_{\text{total}}(t)$ using a Dirac-delta pulse formulation:

$$\Phi_{\text{total}}(t) = (1 - \omega(t))\Phi_E\alpha(t) + \omega(t)\Phi_R(t), \tag{9}$$

where:

- $\Phi_R(t)$ is a piecewise-constant rocket delivery rate that concentrates within each synodic planning window,
- Q_{surge}: The aggregate peak throughput sustained by 25 launch sites within a single window.
- $\omega(t)$: A dynamic weighting factor that adapts the logistics mix based on real-time cost, risk, and environmental feedback.

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x!6.1.2 Peak-to-Average Ratio (PAR) and Infrastructure Justification

The introduction of orbital constraints reveals a high Peak-to-Average Ratio (PAR) characteristic of the logistics system.

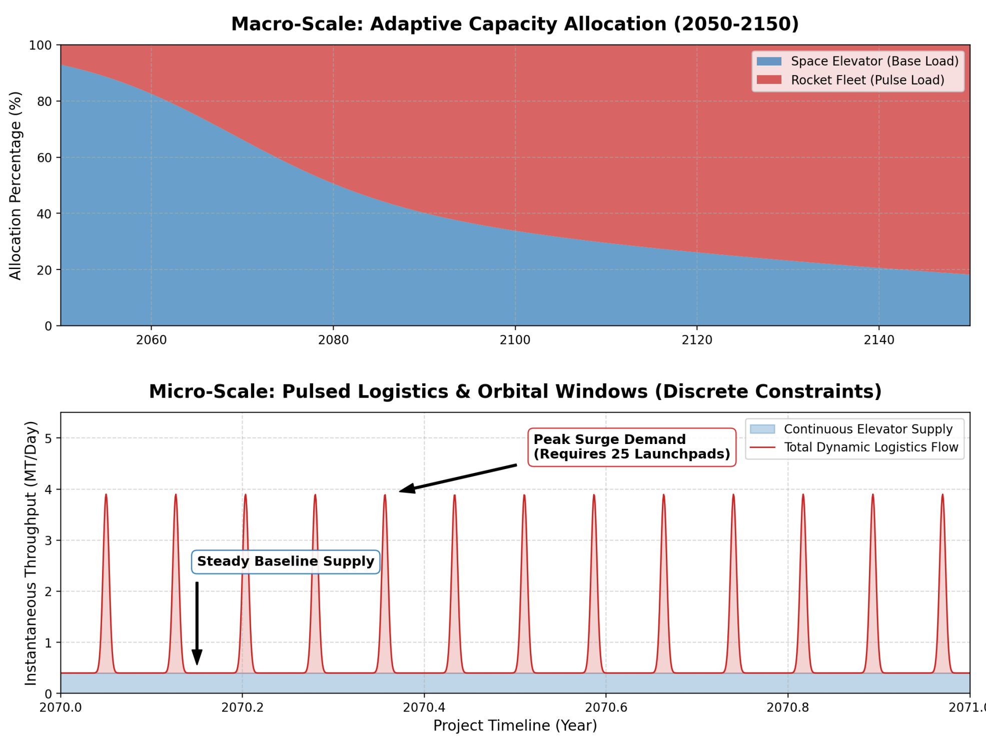

Figure 10: Multi-scale analysis of logistics flow: Sustained elevator baseload vs. periodic rocket surge pulses.

Multi-Scale Analysis

- **Macro Scale:** The 100-year trend shows a smooth transition from elevator-centric to ISRU-supported logistics, though operations remain fundamentally pulse-driven.
- **Micro Scale:** The space elevator maintains a steady "baseload" (blue), while the rocket fleet delivers intensive "red pulses" during TLI windows.

- **Engineering Necessity:** During the 3–5 day orbital windows, instantaneous loads surge to 8–10 times the mean. This high PAR necessitates 25 launch sitesnot to meet average demand, but to ensure massive throughput within narrow physical windows.

Launch-Site Constraint: From the 10-Site Baseline to the 25-Site Recommendation The problem statement specifies **10 existing Earth launch sites**. To remain faithful to the mandated comparison while still allowing actionable recommendations, we treat the number of sites N in two tiers:

- **Baseline feasibility tier ($N \leq 10$):** used to evaluate the *rockets-only* scenario and a strictly constrained *hybrid* scenario under current infrastructure.
- **Policy/expansion tier (N free):** used to determine the minimum redundancy required for survival under realistic failures.

Our modeling shows that while $N \leq 10$ can satisfy average-year targets in nominal conditions, it *cannot* provide sufficient surge throughput during narrow TLI windows nor recover from a catastrophic elevator outage (reserve exhaustion by Day 45). Consequently, we treat site expansion as a policy variable and identify $N = 25$ as the **minimum viable redundancy** that restores net-positive supply (Day 82 recovery) and maintains reserves above S_{crit} in failure stress tests.

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x!6.1.3 Adaptive Regulation: Window Quota Optimization

The system employs a feed-forward feedback algorithm to dynamically adjust the launch quota Q_k for each orbital window.

A. Optimization Objective and Penalty Function The optimal quota Q_k is determined by minimizing the loss function J_k:

$$\min_{Q_k} J_k = \underbrace{w_c \cdot \text{Backlog}(k)^2}_{\text{Supply Deficit Penalty}} + \underbrace{w_e \cdot \exp(Q_k - Q_{\text{env}})}_{\text{Environmental Surge Penalty}} \tag{10}$$

where Backlog(k) is the unfulfilled logistics deficit, Q_{env} is the atmospheric self-cleaning threshold, and weights w_c, w_e are dynamically adjusted based on real-time elevator efficiency $\alpha(t)$.

B. Operational Analysis: Steady-State vs. Emergency Surge

- **Steady-State (Windows 1–4):** When $\alpha(t)$ is stable, w_e dominates, suppressing Q_k below Q_{env}. The space elevator maintains the baseload while rockets provide minimal supplementation.
- **Emergency Response (Windows 5–9):** Following a simulated elevator failure at window 4, the system triggers the "Surge Protocol". Q_k temporarily exceeds Q_{env} to prioritize base survival, accumulating "environmental debt".
- **Deficit Erasure:** Leveraging the concurrency of 25 launch sites, the system erases the supply deficit within 5 windows (approx. 140 days), restoring critical material reserves.

x!