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Introduction to computational modelling: Theory

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Outline & learning aims

1. What is a model & why should I use modelling?
2. Steps of a modelling project
3. Example: algorithm for (pro-environmental) effort-based decisions
4. From algorithm to choices
5. Model fitting & comparison
6. Simulations and robustness checks

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What is a model?

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Computational modelling

Computational models:

- are algorithms or equations that formalise the relationship between theorised variables
- can make predictions about how individuals perform a task
- quantify the variables or processes that guide behaviour

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Why model?

Why model?

1. **Latent variables** – cognitive and neural processes are hidden
2. **Complexity** – how behaviour changes over time
3. **Individual differences** – subject-specific parameters
4. **Data reduction** – single numbers or parameters integrate features
5. **Computational neurology** – understanding disrupted computations
6. **Parametric modulators** – neural processes are not binary

How to model?

Stages of a modelling project

1. **Identify** an algorithm that could explain behaviour
2. **Simulate** data to check model robustness and behavioural effects
3. **Design** and run study optimised for modelling
4. **Fit** models to participants' data
5. **Compare** models and select the "best"
6. **Analyse** parameters and / or neural data

Cohen et al. (2017) *Nature Neuroscience*

Algorithms for effort-based decision-making

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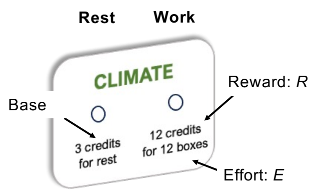
Exerting effort for the environment

Choices to exert physical effort for the climate and a control condition

Prosocial Effort for the Environment Task (PEET)



The *Value* of an option = *Reward* - *Effort*

$V_{(t)} = R_{(t)} - E_{(t)}$

Cutler*, Contreras-Huerta*, Todorova, Nitschke, Michalaki, Koppel, Gkinopoulos, Vogel, Lamm, Västfjäll, Tsakiris, Apps† & Lockwood†

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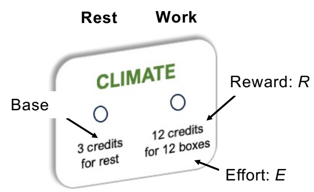
Exerting effort for the environment

Choices to exert physical effort for the climate and a control condition

Prosocial Effort for the Environment Task (PEET)

24 trials: 12 for climate, 12 for food
effort (4 levels 2-5; 50-95% maximum boxes) and
reward (3 levels 2-4; 4 / 12 / 20 credits) varied independently



The ***Subjective*** *Value* of an option = *Reward* – (*Effort* * ***discounting***)

$SV_{(t)} = R_{(t)} - (E_{(t)} * K)$

Cutler*, Contreras-Huerta*, Todorova, Nitschke, Michalaki, Koppel, Gkinopoulos, Vogel, Lamm, Västfjäll, Tsakiris, Apps† & Lockwood†

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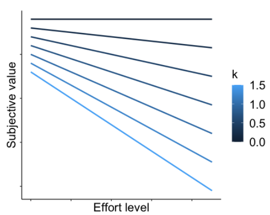
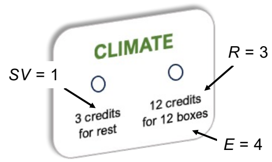
Varying parameter values

Increasing K decreases subjective value

$SV_{(t)} = R_{(t)} - (E_{(t)} * K)$



effort (4 levels 2-5; 50-95% maximum boxes) and reward (3 levels 2-4; 4 / 12 / 20 credits) varied independently



$K = 0.1$: $SV = 3 - (4 * 0.1) = 2.6$

$K = 0.5$: $SV = 3 - (4 * 0.5) = 1$

$K = 1$: $SV = 3 - (4 * 1) = -1$

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From value to choice probabilities

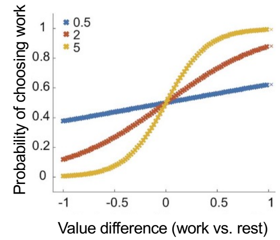
Inverse temperature or decision consistency parameter

Softmax function

What is the probability of choosing work, given the SV of each option?

$$P_{work(t)} = \frac{\exp(SV_{work(t)} * \beta)}{\exp(SV_{work(t)} * \beta) + \exp(SV_{rest(t)} * \beta)}$$

$SV_{work(t)} = R_{(t)} - (E_{(t)} * K)$ $\quad SV_{rest} = 1$



Lockwood & Klein-Flügge (2020) *SCAN*

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Simulating data and fitting parameters to participants' choices

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Choice probabilities to simulations and fitting

Finding the parameter values that best explain the participant data

$K = 0.1$, $\beta = 2$, $R_{(t)} = 3$, $E_{(t)} = 4$

$SV_{work(t)} = 3 - (4 * 0.1) = 2.6$

$$P_{work(t)} = \frac{\exp(5.2)}{\exp(5.2) + \exp(2)} = 0.96$$

Simulate choices

[Generate random number 0-1]

$Choice_{(t)}$ = work if $P_{work(t)}$ > random number

$Choice_{(t)}$ = rest if $P_{work(t)}$ < random number

Fit participant choices

If $Choice_{(t)}$ == work,

$P_{chosen(t)} = P_{work(t)} = 0.96$

If $Choice_{(t)}$ == rest,

$P_{chosen(t)} = 1 - P_{work(t)} = 0.04$

Maximise model evidence by **minimising** negative log-evidence:

Negative log-likelihood = -sum(log(P_{chosen})) = 77.25

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Choice probabilities to simulations and fitting

Finding the parameter values that best explain the participant data

$K = 0.1$, $\beta = 2$, $R_{(t)} = 3$, $E_{(t)} = 4$

$SV_{work(t)} = 3 - (4 * 0.1) = 2.6$

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Fit participant choices

If $Choice_{(t)}$ == work,

$P_{chosen(t)} = P_{work(t)} = 0.96$

If $Choice_{(t)}$ == rest,

$P_{chosen(t)} = 1 - P_{work(t)} = 0.04$

Maximise model evidence by **minimising** negative log-evidence:

Negative log-likelihood = -sum(log(P_{chosen})) = 77.25

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Choice probabilities to simulations and fitting

Finding the parameter values that best explain the participant data

$K = 0.75$, $\beta = 1.5$, $R_{(t)} = 3$, $E_{(t)} = 4$

$SV_{work(t)} = 3 - (4 * 0.75) = 0$

$$P_{work(t)} = \frac{\exp(0)}{\exp(0) + \exp(1.5)} = 0.18$$

Fit participant choices

If $Choice_{(t)}$ == work,

$P_{chosen(t)} = P_{work(t)} = 0.18$

If $Choice_{(t)}$ == rest,

$P_{chosen(t)} = 1 - P_{work(t)} = 0.82$

Maximise model evidence by **minimising** negative log-evidence:

Negative log-likelihood = -sum(log(P_{chosen})) = 4.76

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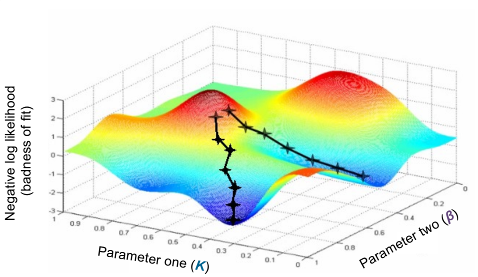
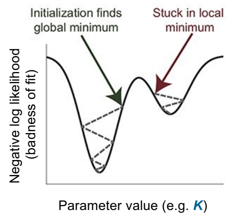
Fitting models to estimate parameters

Finding the parameter values that best explain the participant data





Images by Miriam Klein-Flügge & Patricia Lockwood

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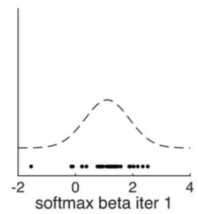
Fitting models to estimate parameters

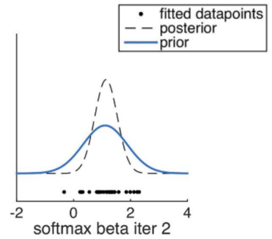
Finding the parameter values that best explain the participant data

Maximum likelihood estimation

Negative log-likelihood = -sum(log(P_{chosen}))



"Hierarchical" expectation maximisation / maximum a posteriori approach



Lockwood & Klein-Flügge (2020) *SCAN*

Huys et al., (2011, 2012) *PLOS Computational Biology*

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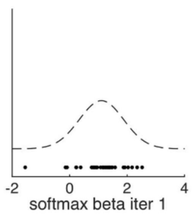
Fitting models to estimate parameters

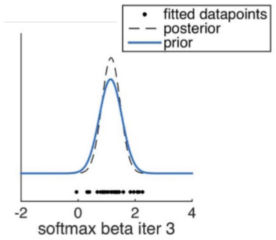
Finding the parameter values that best explain the participant data

Maximum likelihood estimation

Negative log-likelihood = -sum(log(P_{chosen}))



"Hierarchical" expectation maximisation / maximum a posteriori approach



Lockwood & Klein-Flügge (2020) *SCAN*

Huys et al., (2011, 2012) *PLOS Computational Biology*

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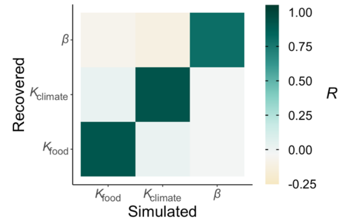
Parameter recovery

Fitting parameters to simulated choices

1. Simulate choices with known true parameters
2. Fit "participant" choices
3. Calculate correlations between true and fitted parameter values

- *Cover full range of potential parameter values*
- *Fine to have more "participants" than participants*



Cutler*, Contreras-Huerta* et al. (preprint)

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Comparing multiple models and selecting the best one

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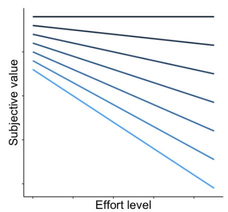
Effort discounting model space

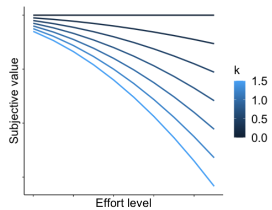
Three possible shapes to capture the discount function

Linear discounting



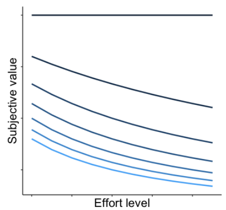
$SV_{(t)} = R_{(t)} - (\boldsymbol{K} * E_{(t)})$

Parabolic discounting



$SV_{(t)} = R_{(t)} - (\boldsymbol{K} * E^2_{(t)})$

Hyperbolic discounting



$SV_{(t)} = \frac{R_{(t)}}{1 + (\boldsymbol{K} * E_{(t)})}$

Hartmann et al., (2013) *Behav. Processes*; Lockwood et al., (2022) *Current Biology*

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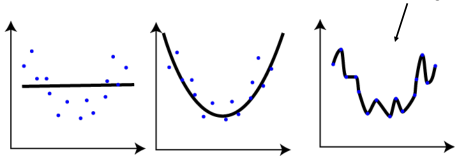
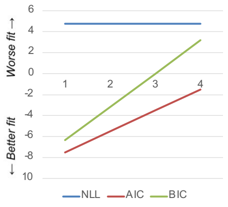
More parameters?

Balancing fit with model complexity



$1K$ 1β $2K\{K_{climate}, K_{food}$ $2\beta\{\beta_{climate}, \beta_{food}$

Fit criteria



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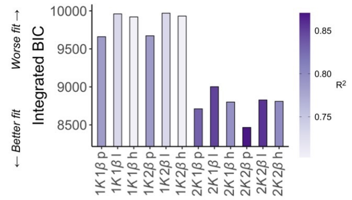
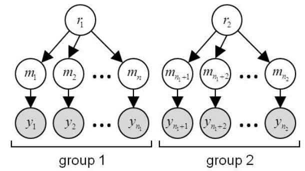
Model selection and fit criteria

Choosing the model that best fits the data



$2K\{K_{self}, K_{other}$ $2\beta\{\beta_{self}, \beta_{other}$

Random-effects analysis between groups



https://mbb-team.github.io/VBA-toolbox/wiki/BMS-for-group-studies/

Lockwood*, Cutler* et al. (2024) *Nature Human Behaviour*

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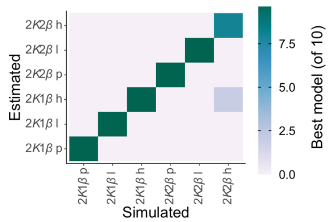
Model identifiability

Testing the robustness of the model selection process

1. Simulate choices with each model in turn
2. Fit all models and select the best
3. Quantify how many times the real simulated model was chosen as best
   - *Range of parameter values at least covering real fitted values*
   - *Have a similar number of "participants" as participants*



Cutler*, Contreras-Huerta* et al. (preprint)

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Comparing parameters between individuals or groups

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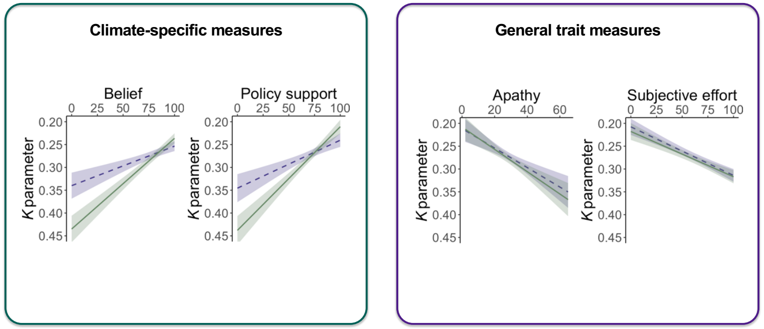
Pro-environmental motivation is climate specific

Climate beliefs and policy support relate to motivation for the environment



Cutler*, Contreras-Huerta* et al. (preprint)

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Summary

Algorithms can integrate task features e.g. effort discounting models

Value calculated using algorithm into choice probability via softmax

Probabilities can be used for simulating data

Parameter values that best fit choices found by minimising error

Model space varying how many parameters, discounting shape…

Select best model based on fit criterion that penalises complexity

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Afternoon practical

Optional preparation:

- Go to: github.com/SDN-lab/csc2024
- Download resources
- [Install MATLAB if you want to try it but don't already have it]
- Charge your laptop during lunch in the link

Website with no download required: sdn-lab.github.io/csc_app

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