1. A chicken lays n eggs. Each egg independently does or doesn't hatch, with probability p of hatching. For each egg that hatches, the chick does or doesn't survive (independently of the other eggs), with probability s of survival. Let $N \leftarrow \text{Bin}(n, p)$ be the number of eggs which hatch, X be the number of chicks which survive, and Y be the number of chicks which hatch but don't survive (so $X + Y = N$). Find the marginal PMF of X, and the joint PMF of X and Y. Are they independent?

$$\begin{aligned}
P(X = i) &= \sum_{j=i}^{n} \binom{j}{i} s^i (1-s)^{j-i} \binom{n}{j} p^j (1-p)^{n-j} \\
&= \sum_{j=i}^{n} \frac{j!}{i!(j-i)!} \frac{n!}{j!(n-j)!} s^i (1-s)^{j-i} p^j (1-p)^{n-j} \\
&= \sum_{j=i}^{n} \frac{n!}{i!(j-i)!(n-j)!} s^i (1-s)^{j-i} p^j (1-p)^{n-j} \\
&= \sum_{r=0}^{n-i} \frac{n!}{i!r!(n-i-r)!} s^i (1-s)^r p^{r+i} (1-p)^{n-i-r} \\
&= \frac{n!}{i!(n-i)!} (ps)^i (1-p)^{n-i} \sum_{r=0}^{n-i} \frac{(n-i)!}{r!(n-i-r)!} (1-s)^r p^r (1-p)^{-r} \\
&= \binom{n}{i} (ps)^i (1-p)^{n-i} \sum_{r=0}^{n-i} \binom{n-i}{r} \left(\frac{(1-s)p}{1-p} \right)^r \\
&= \binom{n}{i} (ps)^i (1-p)^{n-i} \left(1 + \frac{(1-s)p}{1-p} \right)^{n-i} \\
&= \binom{n}{i} (ps)^i (1-p)^{n-i} \left(\frac{1-p+p-ps}{1-p} \right)^{n-i} \\
&= \binom{n}{i} (ps)^i (1-p)^{n-i} \left(\frac{1-ps}{1-p} \right)^{n-i} \\
&= \binom{n}{i} (ps)^i (1-ps)^{n-i} \\
&= \text{Bin}(i|n, ps).
\end{aligned}$$