

A robust day-ahead scheduling strategy for EV charging stations in unbalanced distribution grid

Bakul Kandpal ^{a,*}, Parikshit Pareek ^b, Ashu Verma ^a

^a Department of Energy Science and Engineering, IIT Delhi, India

^b School of Electrical and Electronics Engineering, NTU, Singapore

ARTICLE INFO

Article history:

Received 20 October 2021

Received in revised form

9 February 2022

Accepted 12 March 2022

Available online 16 March 2022

Keywords:

Phase load unbalance

Integrated demand response

Electric vehicles

Optimization under uncertainty

ABSTRACT

Uncontrolled electric vehicle (EV) charging is known to have an adverse impact on power networks. In particular, the inherent problem of phase load unbalance (PLU), due to uneven loading of phases can be substantially increased due to uncoordinated EV charging demands. Increased unbalance would cause sub-optimal operation of the distribution grid with higher thermal losses and increased network congestion. In this paper, a day-ahead EV scheduling strategy to mitigate unbalance is proposed, by controlling single-phase charging demand of EVs with vehicle to grid (V2G) power transfer. In addition, the EVs are also scheduled as per a price-based demand response program. The aim of this work is to use multi-objective lexicographic ordering to achieve economic benefit by controlling EV charging/discharging rates as per dynamic electricity prices, while simultaneously shift EV power consumption among phases to reduce unbalance. Further, uncertainty handling optimization techniques are presented to hedge against any randomness in prices and phase-wise loads. The proposed methods are simulated for a community charging station connected to the unbalanced grid. It is found that the active power consumption among phases can be balanced almost entirely using continuous charging/discharging rates from EVs. Moreover, up to 30% reduction in charging costs can be achieved while maintaining a balanced three-phase system.

© 2022 Elsevier Ltd. All rights reserved.

1. Introduction

In recent years the change in distribution system operation has taken quick advance. Flexible electricity market pricing, increased stochastic loads such as electric vehicles (EVs) and higher penetration of local generation sources such as photovoltaics (PVs) makes the distribution network more complex. EVs are seen to gain importance due to their environmental benefits. However, unpredictable charging demands of EVs creates challenges in their integration to the grid [1]. Further, stringent power quality requirements and robust power system operational performance requirements are posing new challenges to distribution system operators. In addition, consumer awareness regarding grid tariffs and available methods of energy procurement is also increasing.

With such challenges in perspective, several smart scheduling algorithms for EVs are proposed in literature. In Ref. [2], authors propose to reduce the power procurement at peak times using

vehicle to grid (V2G) technology. Similarly, in Ref. [3], authors minimize the voltage deviations using EV battery storage, while in Ref. [4], the authors propose an EV scheduling algorithm to reduce distribution system losses. Moreover, in few articles, day-ahead scheduling algorithms are preferred to make bids in the day-ahead energy markets. On these lines, authors in Ref. [5] propose a day ahead algorithm to reduce operation costs for an energy hub, which includes a charging station. Finally, EVs are also shown to be useful in energy balancing, by using EV batteries to absorb uncertainties of distributed energy resources [6].

A major issue in the low-voltage distribution networks is of unbalance. In the distribution network, unbalance is a common, the major cause of which is the uneven spread of single-phase loads across the three-phases [7]. Unbalance is known to be worsened by high penetration of single phase PVs on a three phase network [8]. Similarly, single-phase charging of EVs is also known to create unbalanced power consumption among the three phases [9]. It is further concluded in Ref. [10] that even with low penetration of EVs in a community, uncoordinated EV charging can severely increase unbalance of loads.

* Corresponding author.

E-mail address: esz178542@ces.iitd.ac.in (B. Kandpal).

Under unbalanced condition, increased current flowing in maximum loaded phase will lead to higher power losses [11], also affecting the transformers life adversely [12]. Moreover, phase unbalance can reduce the available transfer capacity of feeders and transformers compared to balanced grid situation. Thus, unbalance causes poor operating performance of the grid, power quality problems, monetary losses, and also reduces the utilization of assets for the distribution system operator (DSO) [13].

To mitigate unbalance, authors in Ref. [14] proposed to balance active power flow in each phase using PV inverters and EV chargers. In Ref. [15], authors propose a common DC bus for EV chargers and PV converters, which facilitate power transfer within different phases, thus reducing unbalance. However, in such works, the additional infrastructure requirement is quite large, and they assume a three-phase charging infrastructure for EVs which is expensive and still less widespread [16]. Most EVs can easily be charged to required levels using single phase charging equipments, specially when parking duration is long such as at home or workplace. Thus, it is motivating to develop methodologies to reduce unbalance solely using single phase EV charging infrastructure. To this purpose, authors in Ref. [17] use a switching circuit to shift single-phase household loads to balance the three-phases. However, the authors do not provide a detailed algorithm to optimize the phase-wise power injections for unbalance reduction.

In contrast to shifting of household, it would be advantageous to use non-critical loads such as EVs to balance the grid. However, it can be hard to frame economic incentives to quantify the benefits of unbalance reduction. Therefore, it would be a challenge to motivate EV owners for a typical demand response (DR) program focused solely on unbalance reduction. Thus, in this work, a secondary objective of tariff-based EV charging cost reduction is proposed. This would reduce the charging costs for EV owners and further motivate their participation in the DR program.

In [18,19], authors show that daily tariff plans and ancillary service markets can significantly reduce energy costs to EVs owners. Moreover, a price-based scheduling program is proposed in Ref. [20] to monetize parked EVs under time-varying electricity tariffs. In Ref. [21], authors propose cost-saving EV scheduling methods considering battery degradation, which would improve EV owners enthusiasm to participate in microgrid dispatch. Using a multi-objective algorithm, it is shown in Ref. [22] that charging costs to EVs can be minimized along with reduction in voltage deviations.

Finally, it is also known that DR programs are often prone to uncertainties, especially due to the forecasting errors in the data. In particular, the electricity prices and loads in the distribution system may vary unpredictably in real-time. Thus, it is important to consider parameter uncertainty to keep an adequate margin of error in day-ahead DR programs. To this purpose, authors in Ref. [23] use stochastic programming to find optimal bidding schedules in day-ahead markets under uncertain grid prices. In Ref. [24], the authors integrate uncertain EV mobility into their scheduling algorithm causing increased reliability of EV demand satisfaction. Similarly in Ref. [25], the uncertain EV mobility is taken care of by Monte Carlo simulations. A price-based robust scheduling strategy for combined heat and power units under temperature uncertainty is proposed in Ref. [26].

This paper aims to give scheduling algorithms that simultaneously reduce phase load unbalance (PLU) and charging costs to EVs. The optimal schedules for EVs are found day-ahead, knowing the arrival/departure times and state of charge (SOC) requirement of EVs the next day. The EVs are envisioned to shift their demand among phases using a switching circuit as proposed in Ref. [17]. The charging demands are also shifted in time, considering the energy prices, including discharging EV batteries when prices are high.

Moreover, two uncertain parameters are considered in the modelling. The house loads of the distribution network are considered uncertain and modelled as a Gaussian distribution about their mean forecasted value. Moreover, the grid prices for the next day are also considered uncertain, which can vary within pre-defined limits. Thus, in this work, the maximum benefit of both objectives is achieved using subsequent lexicographic ordering. This is in contrast to works such as [5,27], where the EVs are scheduled based on several objectives in conjunction, which divides the benefit of optimal EV charging coordination over several objectives. In particular, authors in Ref. [5] use a weighted sum method which gives partial preference to each of their proposed objectives in their EV scheduling algorithm. This would further create a challenge of choosing optimal weights for every objective and thus, the scheduling framework would also become sensitive to parameter uncertainty. Moreover, in Ref. [25], the authors propose using different charging modes for EVs depending upon the time-urgency of the EV user. This will adversely impact the grid operation as higher charging demand from EVs is known to worsen grid stability issues including unbalance.

In this work, the challenge of unbalance due to single phase EV charging is mitigated. **It is shown that near optimal unbalance reduction can be achieved by the method of phase shifting in coherence with any secondary objective.** That is, PLU can be reduced to near zero while also ensuring the EV users are allowed to choose optimal time-slots for charging/discharging their EV batteries based on day-ahead broadcasted grid prices. Moreover, in contrast to previous works such as [6], all important parameters such as grid electricity tariffs and consumer loads are considered uncertain in modelling of optimization framework. This ensures that the day-ahead charging decisions are resilient to parameter uncertainty in the optimization models.

The contribution of this work can thus be summarized as:

- Development of day-ahead scheduling strategy for PLU reduction in low voltage residential grid, using phase shifting of time-flexible EV loads to balance the phase-wise active power flows. The EV batteries are allowed vehicle-to-grid (V2G) action to supply power to their respective phase, thus, further mitigating uneven phase loading.
- Development of an economic welfare energy arbitrage strategy to schedule EVs based on dynamic electricity tariffs. Moreover, the aim is to achieve maximum reduction in PLU while simultaneously incurring the least charging cost to all EVs, through an ϵ -constraint based multi-objective EV scheduling model.
- To build upon the multi-objective model and develop a tractable robust optimization model to reduce adverse impact of grid price uncertainty on EV charging costs, while also providing a correlation based stochastic model for mitigating impacts of load consumption uncertainties on unbalance reduction.

The remainder of the paper is organized as follows. In section 2, the individual formulations for PLU reduction, charging cost reduction and the multi-objective framework is proposed. In section 3, the details of stochastic as well as robust optimization model for PLU and cost reduction are given. In section 4, the detailed results of the performed numerical simulations are discussed and in section 5, the paper is finally concluded.

2. Problem formulation

The algorithmic framework followed in this work is explained in detail in this subsection. Every EV is assumed to be connected through a switching circuit which can shift the EV's power consumption among phases using static transfer switches (STS) [17].

Thus, the EVs are charged using single phase infrastructure, however, every EV has connections available for all three phases for switching. Moreover, the charging infrastructure is considered to be bidirectional, such that EV batteries can supply power locally to the connected bus of the network. A central aggregator is assumed to make the scheduling i.e. charging/discharging decisions on all EVs. The scheduling of EVs is based on two separate objectives of unbalance and energy arbitrage. The entire problem is solved day-ahead, i.e. the aggregator finds optimal schedules for EVs that will connect the next day. It is assumed that EV users will submit their charging requirements and availability to the aggregator a day ahead. Moreover, the aggregator will use day-ahead energy market's broadcasted values of electricity tariffs, while the phase wise residential loads can be reasonably predicted using benchmark methods [28]. However, as day-ahead forecasts can be erroneous, we consider uncertainties in modelling of electricity tariffs and the phase-wise loads. A framework is constructed to reduce any worst-case impact of uncertainties on the final results. The details of the optimization framework for optimal EV scheduling are given in detail in following subsections.

2.1. System modelling

Let the total number of vehicles be defined by the set $\mathcal{N} = \{1, 2, \dots, N\}$, while the timeline of optimization be defined as $\mathcal{T} = \{1, 2, \dots, T\}$. The loads in the distribution system, without considering EVs, are taken to be unbalanced. The per-phase power consumption at the distribution network bus is defined as ψ_t^R , ψ_t^Y and ψ_t^B . It is assumed that the all three phases consume different amount of power, that is, $\psi_t^R \neq \psi_t^Y \neq \psi_t^B$. The average phase load at any time t is then defined as follows,

$$\psi_t^{avg} = \frac{\psi_t^R + \psi_t^Y + \psi_t^B}{3} \quad \forall t \in \mathcal{T}. \quad (1)$$

The variables for EV charging decisions are defined as x_t^n , i . Here, t denotes the hour, n denotes the EV index and i denotes the phase. So x_2^3 , R will be the charging decision for 3rd EV connected to phase R for the 2nd time-slot. Similarly, for discharging, the defined variable is denoted as y_t^n , i .

Both these variables will be zero beyond the optimization timeline, i.e. $x_t^n, i = y_t^n, i = 0, \forall t \notin \mathcal{T}$. Further, only one of x_t^n, i or y_t^n, i can be non-zero at any instance, allowing either only charging or discharging. Finally, each EV charger is assumed to be single phase, therefore the EV can only exchange power with a particular phase at any time. In the following subsections, the details of the proposed EV scheduling strategies are presented.

2.2. EV scheduling strategy I: phase load unbalance minimization

First, we present a strategy to minimize load deviation of all phase from the average load of three phases. The load unbalance among the three phases can be formulated as,

$$\max_{i \in \Omega} \left(\frac{\psi_i - \psi_{avg}}{\psi_{avg}} \right) \quad (2)$$

where, $\Omega = \{R, Y, B\}$ defines the set of three phases. In (2), the maximum deviation of any phase's load from the average ψ_{avg} is computed. Thus, to reduce load unbalance the EVs can be used to consume power with reference to this measure of unbalance. An objective to achieve unbalance reduction can thus be defined as \mathbf{C}_{unb} .

$$\min_{x_t^n, y_t^n} \sum_{t \in \mathcal{T}} \sum_{i \in \Omega} \left(\psi_t^i + \sum_{n \in \mathcal{N}} (x_t^{n,i} - y_t^{n,i}) - \psi_t^{avg} \right)^2 \quad (3)$$

The optimal charging/discharging decision obtained by solving (3) will reduce unbalance, by shifting EV power consumption to lightly loaded phase, while shifting EV discharging to heavily loaded phase.

While solving (3), the constraints on the energy requirements will be as follows,

$$B_c SoC_{des} \leq E_t^n + E_0^n \quad t \geq T \quad (4)$$

$$B_c SoC_{min} \leq E_t^n + E_0^n \leq B_c SoC_{max} \quad \forall t \in \mathcal{T} \quad (5)$$

where the energy flow into an EV battery till time t_k will be,

$$E_{t_k}^n = \sum_{t=1}^{t_k} \left(\eta_c x_t^{n,i} - \frac{y_t^{n,i}}{\eta_d} \right) P^n \Delta t. \quad (6)$$

Here, E_0^n is the initial SoC of the n th EV. While, (4) fulfills SoC to desired level by the last time-slot T , maximum and minimum SoC bounds are enforced by (5). Charging and discharging efficiency of the chargers are denoted by η_c and η_d respectively.

Moreover, a set of Big-M constraints are included, which will allow charging/discharging of EVs on only one phase at a time,

$$x_t^{n,i} \leq \beta_n \cdot M; \quad y_t^{n,i} \leq \beta_n \cdot M; \quad (7)$$

where β_n^i are binary variables and $M \gg 0$. The binary variables are enforced such that only one of them per-phase can take the value 1,

$$\sum_{i \in \Omega} \beta_n^i \leq 1 \quad \forall n \in \mathcal{N} \quad (8)$$

This allows the charging/discharging of EVs to occur from a single phase at any given time-slot.

2.3. EV scheduling strategy II: price based charging

The energy costs incurred by any EV will be decided the dynamic grid based electricity tariff. Cost minimization would then signify charging the EV during cheaper hours, while discharging the EV during expensive hours.

Under a dynamic tariff, the cost minimization objective, denoted as \mathbf{C}_{pr} , can be formulated as,

$$\min_{x_t^n, y_t^n} \sum_{t \in \mathcal{T}} \sum_{i \in \Omega} \sum_{n \in \mathcal{N}} \left(\alpha_t x_t^{n,i} - \alpha_t y_t^{n,i} \right) \Delta t \quad (9)$$

where α_t is the electricity price for the t th time-slot. Minimization of \mathbf{C}_{pr} is performed subject to earlier SOC constraints (4)–(6). The objective function (9) minimizes cost of charging for all EVs, while also maximizes the benefit from V2G discharging operation. In this work, prices are considered symmetric, i.e. the price per unit (kWh) for charging, is equal to price per unit for discharging, for all time-slots.

2.4. EV scheduling strategy III: multi-objective optimization model

Solving the individual objectives \mathbf{C}_{unb} and \mathbf{C}_{pr} , we aim to obtain reduced unbalance and charging costs for EVs respectively. For increased participation, the EV owners would prioritize on reducing their charging costs i.e. minimizing \mathbf{C}_{pr} , while for efficient

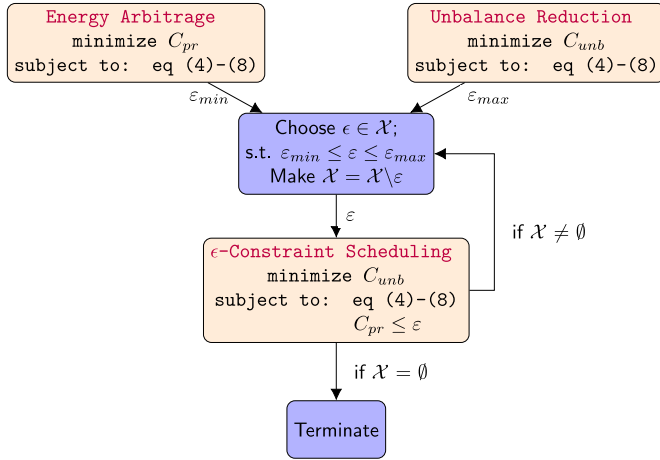


Fig. 1. Flowchart for ϵ -constraint based multi-objective unbalance and cost reduction.

operation of the distribution system, minimizing unbalance i.e. C_{unb} should be preferred individually. In this section, a **multi-objective framework** is modelled where the objectives are solved in succession, using an ϵ constraint method as shown in flowchart of Fig. 1.

Initially, a primary objective is chosen to be minimized i.e. $\min f_1$, while keeping the secondary objective in constraint, i.e. $f_2 \leq \epsilon$. The value of ϵ is chosen from a range of values, which is forced to be within a range. In this work, unbalance objective i.e. C_{unb} is chosen as the primary objective, while keeping the charging cost i.e. C_{pr} to be constrained below ϵ . To find the value of ϵ , a-priori, the two objectives are solved individually. Using the EV schedules obtained from C_{unb} and C_{pr} , the charging cost to EVs incurred through both objectives is computed, namely ϵ_{max} and ϵ_{min} respectively. Thereby, a set X is created, where each $\epsilon \in X$ is chosen such that $\epsilon_{min} \leq \epsilon \leq \epsilon_{max}$. Finally, the unbalance objective is re-optimized while constraining the value of $C_{pr} \leq \epsilon$. This will allow the proposed algorithm to find the optimal phase for shifting the EV charging demands, while also keep the energy costs to EVs below the specified value ϵ . As the value of ϵ is varied, the unbalance reduction under varying charging cost will be achieved. The pseudocode of the optimization framework is given in Algorithm 1.

Algorithm 1
Deterministic EV Scheduling.

Input: PEVs plugin information

- 1: Minimize C_{pr} subject to (4)-(8)
- 2: Create a set X of charging costs using grid points

ϵ -Constraint/Lexicographic ordering:

for $i = 1, 2, \dots, S$ **do**

- 3: Choose $\epsilon_i \in X$
- 4: Minimize C_{unb}
subject to,
a) $C_{pr} \leq \epsilon_i$
b) SOC constraints of (4)-(8)

end

Return: Scheduling set-points for EVs

2.5. Pareto-efficient solutions of multi-objective model

Pareto-efficiency or pareto-optimality of an obtained solution denotes a balance between the two objectives. It is sufficient for a solution \mathbf{x}^* to be pareto optimal if there does not exist any other solution \mathbf{x} such that,

$$f_k(\mathbf{x}) \leq f_k(\mathbf{x}^*) \quad \forall k \quad (10)$$

where, f_k denotes the k th objective, and such that for at least one objective k ,

$$f_k(\mathbf{x}) < f_k(\mathbf{x}^*) \quad (11)$$

The ϵ -constraint method in Algorithm 1 produces pareto-optimal solutions for any choice of epsilon (ϵ) [29]. This denotes that the obtained solution maintains a balance between the two objective of unbalance and charging cost reduction, for any value of ϵ . That is, unbalance can not be reduced further without increasing the combined charging cost of EVs, and vice-versa. Moreover, if the ϵ is fixed to be the minimum value obtained by solving price-based scheduling, the algorithm becomes a lexicographic ordering based optimization [30].

Remark: As electricity tariff is the same for every phase, optimal solution of C_{pr} will not depend upon phase wise EV power consumption. In contrast, PLU reduction objective C_{unb} will strictly depend upon phase wise power consumption of EVs. Therefore, prioritizing phase-wise transfer of EV power consumption would not impact the cost of charging incurred by an EV. Unbalance reduction can thus be achieved as an additional benefit without compromising on the consumer focused objective of price-based EV scheduling i.e. energy arbitrage. The effectiveness of ϵ -constraint method on simultaneously reducing the proposed objectives is also discussed with numerical simulations in section 4.1.

3. Handling uncertainty in loads and price

It is important to note that the grid prices used in formulating C_{pr} and phase loads in formulating C_{unb} , will have uncertainties in their prediction. Thus, to handle possible impact of uncertainties on Algorithm 1, a stochastic as well as a robust method are detailed in this section.

The loads in the grid are considered uncertain, and are modelled using Gaussian distribution [31]. However, as the phases are unbalanced, the three phase's loads are modelled separately. It is further assumed that there exists a correlation among the phase loads. That is, any change in loads of a particular phase would imply correlated change in other phase-loads too.

By contrast, to model uncertainty in grid prices, a robust optimization method is utilized. It is assumed that electricity prices may randomly vary inside an interval, and only the limits of the interval are known. The individual details of the stochastic and interval based robust modelling of uncertainties is given in separate subsections.

3.1. Stochastic modelling of phase loads

The uncertain phase loads are modelled using multivariate normal distribution function. Using an unpredictable stochastic noise, the random loads of i th phase can be written as,

$$\psi_t^i = \psi_{t,0}^i (1 + u) \quad (12)$$

where, $\psi_{t,0}$ are the forecasted loads, while u is a vector which denotes random load fluctuations. It is assumed that the forecasted loads for any time-slot denote the mean value of the particular phase loads i.e.,

$$\mathbb{E}(\psi_t^i) = \psi_{t,0}^i \quad (13)$$

Further, a covariance among phase loads is considered. It is discussed in Ref. [32] that unpredictable weather, temperature etc. have a non-zero correlation with the power consumption of consumers. Thus, the weather/temperature changes may bring sudden changes to consumer loads in the grid. However, it can be assumed that impact of external factors on loads would be nearly similar in each phase.

Thus, the covariance between the phase loads is modelled using a covariance matrix, $\text{Cov}(\psi^R, \psi^Y, \psi^B)$. It is assumed that there is no time-correlation in the loads, such that future loads are independent of the present loading scenarios. Hence, a covariance matrix is generated individually for each time slot. The structure of such a matrix would be,

$$\text{Cov}(\psi^R, \psi^Y, \psi^B) = \begin{bmatrix} \sigma_{\psi^R, \psi^R} & \sigma_{\psi^R, \psi^Y} & \sigma_{\psi^R, \psi^B} \\ \sigma_{\psi^Y, \psi^R} & \sigma_{\psi^Y, \psi^Y} & \sigma_{\psi^Y, \psi^B} \\ \sigma_{\psi^B, \psi^R} & \sigma_{\psi^B, \psi^Y} & \sigma_{\psi^B, \psi^B} \end{bmatrix} \quad (14)$$

where, any element σ_{ψ^i, ψ^j} denotes the covariance between loads of phase i and j . If the value of σ_{ψ^i, ψ^j} is positive, then load increase in i th phase would imply proportional increase in load of j th phase, and vice-versa. The choice for the covariance matrix will decide the scenario generation of three phase loads. It is assumed that there is a positive correlation among phase loads i.e. $\sigma_{\psi^i, \psi^j} > 0$, for all three phases. It can be noticed that matrix $\text{Cov} \in \mathbf{S}^+$, i.e. would be positive definite as well as symmetric.

The values of σ_{ψ^i, ψ^j} would decide the scenario generation for future loads. The maximum covariance between any two phases can be found using variance of individual phase's distribution function. By Cauchy-Schwarz inequality, the maximum covariance among two phases will be limited as, $\sigma_{\psi^i, \psi^j} \leq \sqrt{\sigma(\psi^i)} \sqrt{\sigma(\psi^j)}$, where $\sigma(\psi^i)$ is the variance of i th phase load's Gaussian distribution. Thus, the choice of covariance matrix elements are kept as follows,

$$0 < \sigma_{\psi^i, \psi^j} \leq \sqrt{\sigma(\psi^i)} \sqrt{\sigma(\psi^j)} \quad (15)$$

Using the covariance matrix and forecasted values of the phase loads, a distribution function can be defined, namely $\mathcal{N}(\mu, \text{Cov})$, which can generate several scenarios of phase-wise loads. The probability of occurrence of any chosen scenario can be computed using the cumulative distribution function of $\mathcal{N}(\mu, \text{Cov})$. Thus, a set φ , of scenarios will be generated, for the day-ahead modelling of phase-wise loads in the grid. Every scenario $s \in \varphi$ would have a corresponding probability of occurrence Π_s . Under scenario based optimization, the unbalance reduction objective function, denoted as $\mathbf{C}_{\text{unb}}^{\text{stoc}}$, will be as follows,

$$\sum_{s \in \varphi} \Pi_s \sum_{t \in \mathcal{T}} \sum_{i \in \Omega} \left(\psi_{t,s}^i + \sum_{n \in \mathcal{N}} (x_t^{n,i} - y_t^{n,i}) - \psi_{t,s}^{\text{avg}} \right)^2 \quad (16)$$

where, ψ_t , s^i and ψ_t, s^{avg} are phase load and average load for scenario s . Solving (16) will give the optimal EV scheduling signals considering several different scenarios of loads.

3.2. Robust modelling for real time prices

The day-ahead forecasted prices can vary in real time markets the next day. Thus, an uncertain electricity price is defined as $\tilde{\alpha}_t$, which can vary around the forecasted value α_t , in the interval $[\alpha_t - \hat{\alpha}_t, \alpha_t + \hat{\alpha}_t]$. Thus, the maximum variation in the prices from forecasted values can be of $\pm \hat{\alpha}_t$. It is assumed that EV aggregator knows only $\hat{\alpha}_t$, i.e. only the upper and lower limit of the uncertain price interval are known day-ahead, defining the interval in which the prices may vary. The uncertain prices can be further re-modelled as,

$$\tilde{\alpha}_t = \alpha_t + \eta_t \hat{\alpha}_t \quad (17)$$

where, η_t is a random variable allowed to vary inside $[-1, 1]$.

The energy arbitrage objective of EVs with uncertain grid prices can now be formulated as,

$$\sum_{t \in \mathcal{T}} \sum_{i \in \Omega} \sum_{n \in \mathcal{N}} (\tilde{\alpha}_t x_t^{n,i} - \tilde{\alpha}_t y_t^{n,i}) \cdot \Delta t \quad (18)$$

where, $\tilde{\alpha}_t$ is the uncertain price. Using (17), the formulation of (18) can be re-written as,

$$\sum_{t \in \mathcal{T}} \sum_{i \in \Omega} \sum_{n \in \mathcal{N}} [(\alpha_t + \eta_t \hat{\alpha}_t) x_t^{n,i} - (\alpha_t + \eta_t \hat{\alpha}_t) y_t^{n,i}] \Delta t \quad (19)$$

or equivalently,

$$\sum_{t \in \mathcal{T}} \sum_{i \in \Omega} \sum_{n \in \mathcal{N}} [(\alpha_t + \eta_t \hat{\alpha}_t) \cdot (x_t^{n,i} - y_t^{n,i})] \cdot \Delta t \quad (20)$$

The robust realization for (20) can now be formulated as follows (see Ref. [33], Section 2.2),

$$\sum_{t \in \mathcal{T}} \sum_{i \in \Omega} \sum_{n \in \mathcal{N}} \alpha_t (x_t^{n,i} - y_t^{n,i}) \cdot \Delta t + \sum_{t \in \mathcal{T}} \sum_{i \in \Omega} \sum_{n \in \mathcal{N}} \hat{\alpha}_t \cdot |x_t^{n,i} - y_t^{n,i}| \cdot \Delta t \quad (21)$$

where the second term is added to capture the worst-case charging cost to EVs due to random variation in price.

The absolute values in the objective (21) can be replaced by auxiliary variables $s_t^{\eta_i}$, i , to transform the objective into a tractable

linear program (LP) as follows,

$$\sum_{t \in T} \sum_{i \in \Omega} \sum_{n \in \mathcal{N}} \alpha_t (x_t^{n,i} - y_t^{n,i}) \cdot \Delta t + \hat{\alpha}_t \cdot s_t^{n,i} \cdot \Delta t \quad (22)$$

with additional constraints,

$$-s_t^{n,i} \leq (x_t^{n,i} - y_t^{n,i}) \leq s_t^{n,i} \quad (23)$$

Minimizing the LP objective of (22), denoted as \mathbf{C}_{pr}^{rob} , will produce EV scheduling solutions resilient against the price based uncertainties. That is, the charging/discharging of EVs would now be risk-averse to random fluctuations in the electricity tariff.

Algorithm 2

EV Scheduling Under Uncertainty.

Input: PEVs plugin information

- 1: Minimize \mathbf{C}_{pr}^{rob} of (22), subject to (4)-(8) & (23)
- 2: Create a set \mathcal{X} of charging costs using grid points
- 3: Use $\mathcal{N}(\mu, \text{Cov})$ to create scenario set ϕ
- 4: Compute probabilities Π_s for scenarios in set ϕ .

←Constraint/Lexicographic ordering:

for $i = 1, 2, \dots, S$ **do**

- 5: Choose $\varepsilon_i \in \mathcal{X}$
- 6: Minimize \mathbf{C}_{unb}^{stoc}
subject to,
a) $\mathbf{C}_{pr}^{rob} \leq \varepsilon_i$
b) SOC constraints of (4)-(8)

end

Return: Scheduling set-points for PEVs

In Algorithm 2, the previously defined deterministic objectives are replaced by their robust and stochastic counterparts. Moreover, for stochastic objective of unbalance reduction, first a scenario set is generated with corresponding probabilities. The generated scenario set is then updated using k -means reduction technique [34], to remove highly similar scenarios (see Fig. 2).

The cost reduction objective is solved considering worst case

uncertainty in prices using the robust formulation. Thus, the output of Algorithm 2 provides EV scheduling signals which are robust against any randomness in price or phase loads.

4. Results and discussion

The proposed algorithm framework is modelled in MATLAB 2020b using Yalmip and solved through MOSEK 9.2. The phase wise loads are taken from the supplementary forecasted data provided in Ref. [35]. The initial state-of-charge (SoC) at the arrival of EVs is picked up from a Gaussian distribution \mathcal{N} (40%, 4%). Similarly the desired SoC for EVs is picked randomly from a distribution \mathcal{N} (75%, 4%). The EVs are considered to be connected at the charging station

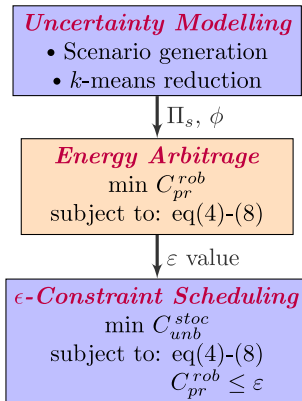


Fig. 2. Flowchart for uncertainty based EV scheduling.

from hours 10:00–15:00. Moreover, both charging and discharging efficiency is taken to be $\eta^c = \eta^d = 0.95$. The values for SoC_{max} and SoC_{min} for EVs are taken to be 100% and 20% respectively. Finally, a total of 20 EVs are considered connected at the charging station, with all EVs having a battery capacity of 40 kWh. The day-ahead electricity tariff is taken from market data available at IEX [36].

4.1. Reduction in unbalance and charging costs

The original loads of the phases are considered uncontrolled household power demand. The three phase load profile is shown in Fig. 3, which shows a large difference in the power demand among phases, due to the different demand of individual households. The power demand of EVs can be included to the household loads to analyze the impact on unbalance of loads. If the power consumption of EVs is optimally shared among phases, the obtained unbalance reduction is seen in Fig. 4. As the EVs are allowed to discharge, the higher loaded phase such as phase – Y at 14th hour are supplied power to reduce grid consumption. Moreover, the charging of EVs is done on lightly loaded phases. Thus, the three phase loads are seen to be equivalently loaded at all hours. Such significant reduction is also possible because EV charging is considered to be continuous, that is, EVs are allowed to consume/supply power at any rate in the range [−7.4, 7.4] kW. This helps in balancing the loads precisely to any amount required.

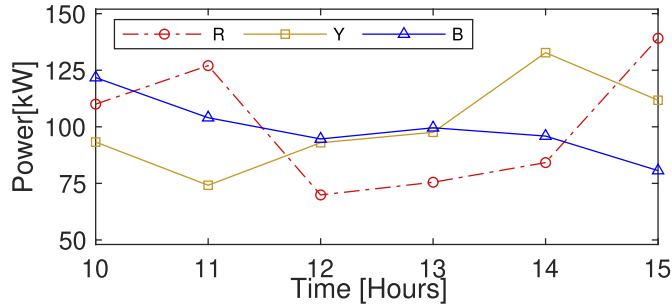


Fig. 3. Unoptimized three-phase load without EVs.

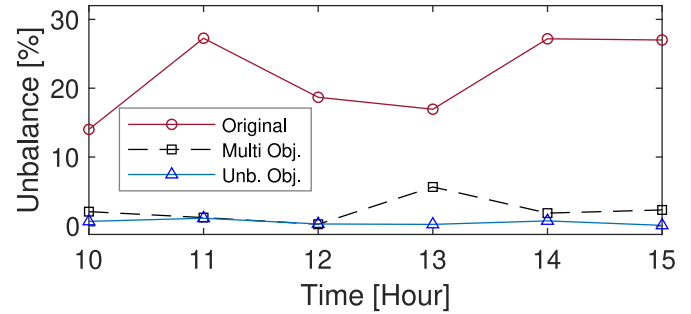


Fig. 5. Hourly load unbalance for different optimization strategies.

Fig. 5 shows the load unbalance in percent for the entire timeline of 10:00–15:00. Load unbalance here is referred to as the maximum deviation of any phase's active power consumption from the average of the three phases. The load unbalance due to household loads, without EVs, is labelled as 'Original'. Moreover, load unbalance that is seen after unbalance reduction using phase shifting of EVs is labelled as 'Unb. Obj.'. It can be noticed that reduction is very effective and unbalance remains near zero, as also seen in Fig. 4. Thus, single phase EV charging/discharging infrastructure can efficiently balance loads at any typical bus, by optimal transfer of EV power demands within phases.

If least charging cost needs to be attained along with unbalance reduction, then power consumption of EVs would also depend upon the electricity tariff. In Fig. 5, 'Multi obj.' denotes the load unbalance achieved after shifting of EVs among phases hourly, while also maintaining the minimum charging cost. It shows effectiveness of lexicographic ordering on the two proposed objectives. It can be noticed that inclusion of additional cost constraint does not majorly impact the potential unbalance reduction. This demonstrates the earlier assertion that focus on unbalance reduction has little impact on optimality of secondary objectives. It can be noticed in Fig. 5, that only the 13th hour unbalance reaches near to 5%, and remains negligible in other hours. Thus, unbalance reduction, as an objective, can be simultaneously achieved while EVs are simultaneously scheduled based on energy arbitrage.

Fig. 6 shows the impact of using ϵ -constraint scheduling on unbalance and charging costs. While, the charging cost is constrained within different values, the possible unbalance reduction is observed. If the charging cost for all EVs is to be kept at least possible value, i.e. INR 592, the unbalance can reach higher than 2%. However, it should be noted that if charging costs are allowed to be increased marginally, the maximum load unbalance is reduced significantly. Moreover, the charging cost are seen to go as high as INR 777 when the unbalance objective is individually minimized.

The value of $\Delta unb/\Delta cost$ in Fig. 6 denotes the variation in PLU

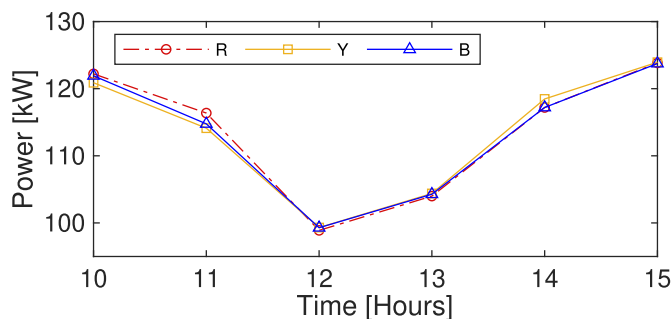


Fig. 4. Unbalance reduction in phase loads.

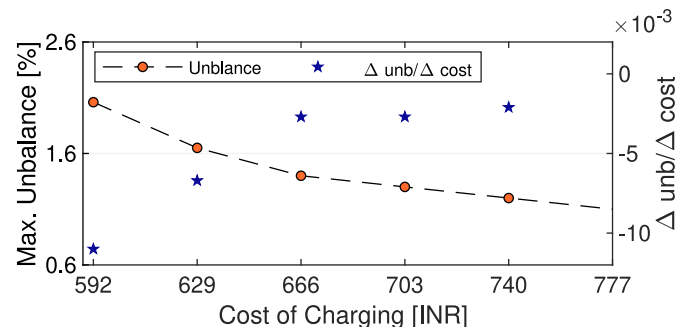


Fig. 6. Maximum load unbalance vs. charging costs of all EVs.

with change in charging costs. It can be noticed that $\Delta unb/\Delta cost$ remains in the range of $-2e-3$ and $-11e-3$. The negative sign denotes that reducing cost of charging for EVs would increase the unbalance of loads. However, such low values signify a low rate of change of one objective with change in another, i.e. the two objectives are decoupled. Thus, charging cost to EVs can be minimized with no significant increase in unbalance of loads.

Fig. 7 shows the effect of varying the EV penetration on PLU. As number of EVs grow, their impact on loading of the grid increases. It can be noticed that with only 5 connected EVs, shifting of EV charging demand is unable to provide substantial reduction in unbalance, as the maximum unbalance remains at a value of 7.8%. However, as EVs increase, their impact on phase balancing also improves. Thereby, when the number of connected EVs increase to 10, the unbalance is seen to be reduced to 1.08%. It can also be noticed that increasing the number of connected EVs beyond a threshold does little to improve the performance of unbalance reduction. That is, the number of EVs to be included in the DR program could be chosen optimally by the EV aggregator. This also depicts the effectiveness of the proposed methodology, where using only few EVs, the desired result of a balanced three phase system is sufficiently achieved.

Fig. 8 (a) shows the power injection at each phase by charging/discharging of EVs. The negative power denotes the discharging of EVs to supply power at the particular phase. It can be noticed that power consumption at each phase is different for any hour. This signifies that few EVs are shifted among phases every hour to maintain balanced loads. This uneven power injection by the single phase EV chargers balances the loads of the grid. In addition, the number of EVs that require shifting among phase is shown in Fig. 8 (b). Out of the total 20 EVs connected, for most hours, around 10 EVs are shifted among phases to reduce unbalance. This signifies that phase-shifting the power consumption of few EVs is needed to minimize unbalance, thereby saving switching losses of the STS.

Fig. 9 shows the SOC of a typical EV for both objectives

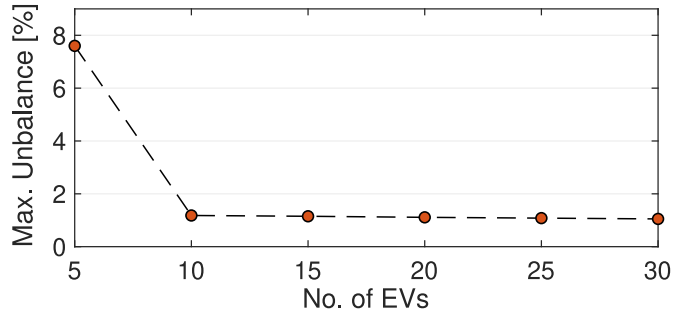


Fig. 7. Maximum load unbalance vs. number of connected EVs.

separately. It can be noticed that if unbalance reduction is to be achieved, the EVs undergo less number of charging/discharging cycles. This shows that merely shifting EVs among phases is sufficient to reduce unbalance. Hence, there is little requirement to modify EV power consumption with respect to time to balance the grid. In contrast, for cost reduction, the EVs are repeatedly charged at lower prices and subsequently discharged at higher prices, to make economic benefit. Thus, in comparison the proposed phase-shifting of EVs will not increase charging cycles of EV batteries, thereby not causing degradation of batteries.

Fig. 10 shows a comparison between the results of proposed methods with the results obtained by using solely the energy arbitrage strategy with variable EV charging power as proposed in Ref. [25]. Fig. 10 (a) shows the maximum load deviation among phases from the median while using solely the arbitrage strategy to charge/discharge EVs. It can be noticed that as the rating of the single-phase EV charger or EVSE is increased from 3.6 kW to 7.4 kW, the unbalance rises. This is because as charging demand of EVs is allowed to become higher, the deviation in phase-loads also increases. Thus, with increased EV charging demand at low price hours, the load deviation is seen to be higher which would further imply higher PLU. However, using the proposed multi-objective framework the phase-wise loads are balanced as deviations are reduced as seen in Fig. 10 (b). Moreover, in Fig. 10 (c), it can be seen

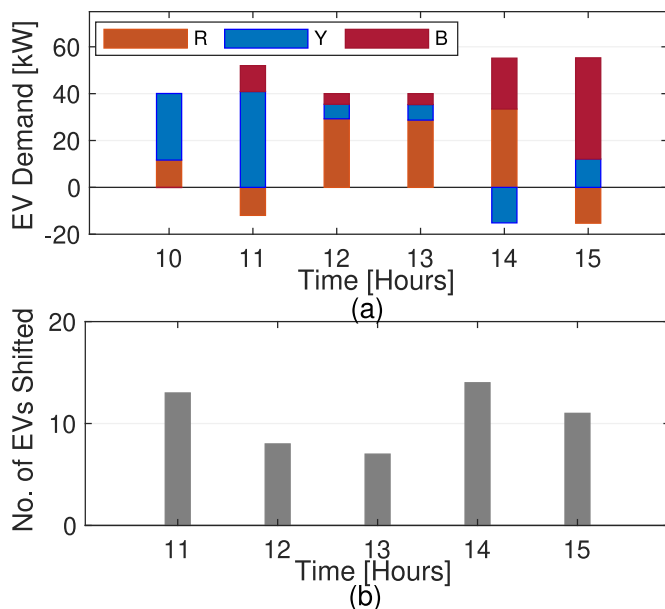


Fig. 8. Power demand of EVs for unbalance reduction. (a) Phase wise EV demand (b) Number of EVs shifted among phases each hour.

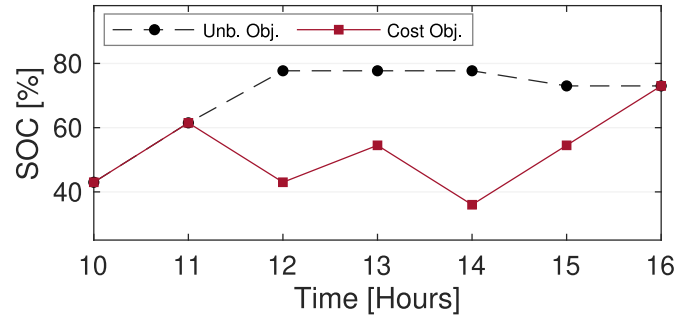


Fig. 9. SOC of a typical EV while minimizing different objectives.

that charging costs to EVs using the conventional energy arbitrage strategy is comparable to those incurred using the proposed multi-objective framework. That is, the proposed method is shown to reduce PLU with flexible time-based charging of EVs to minimize their energy costs towards the least possible value.

4.2. EV scheduling under uncertain loads

For unbalance reduction, several loading scenarios are considered to handle future uncertainty. The different loading scenarios generated for a typical hour are shown in Fig. 11 (b). The covariance

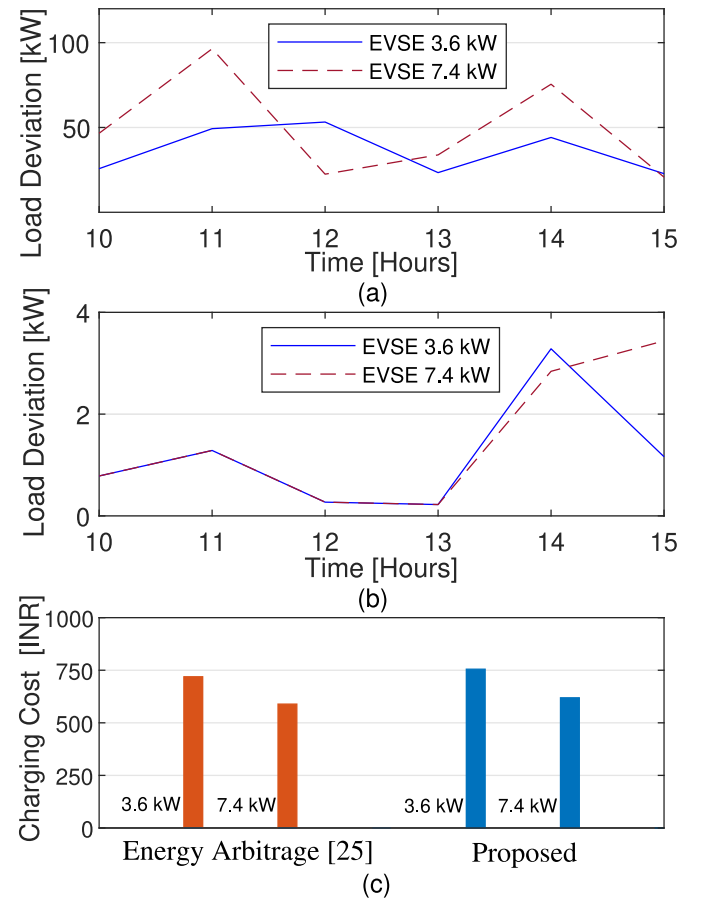


Fig. 10. Comparison of proposed work with conventional energy arbitrage strategy in Ref. [25]. (a) Maximum phase-load deviation from the median using only energy arbitrage. (b) Maximum phase-load deviation from the median using proposed multi-objective framework. (c) Cumulative charging cost to EVs using the proposed and the arbitrage strategy under varying EVSE power rating.

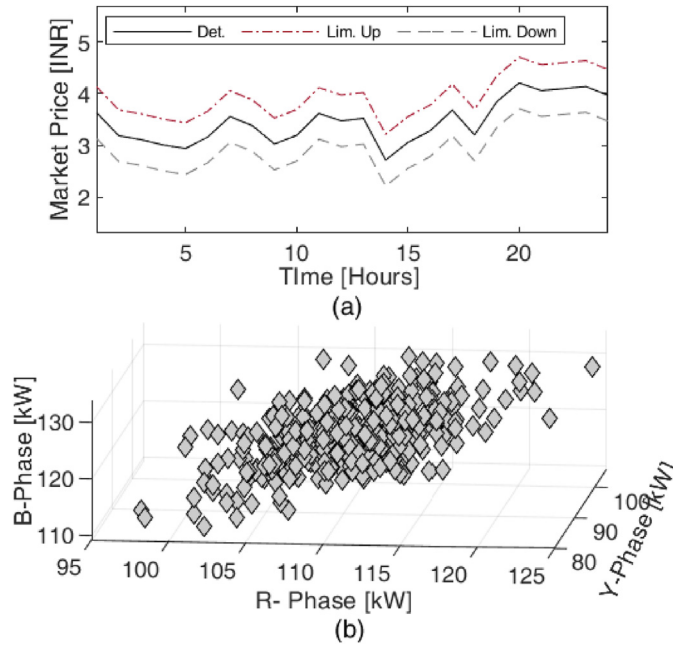


Fig. 11. Uncertainty modelling. (a) Hourly electricity price with upper and lower limits of uncertainty. (b) Scenarios generated for three phase loads using positive covariance.

between any two phases is positive, thus, when with increase in load of any phase, the other phase's loads also moderately increase. The forecasted values for load of all phases are the mean values found at the centre of each axis.

Table 1 compares the expected value of the unbalance objective, using deterministic i.e. C_{unb} , and stochastic i.e. C_{unb}^{stoc} objective. The expected value from both solution considering different loading scenarios is computed. It can be noticed that expected value of deterministic scheduling is always greater than that of stochastic. Higher expected value denotes that under several scenarios, the deterministic objective will do worse on average. This is because the stochastic objective is solved considering uncertainty, and thus already minimizes unbalance over different three-phase loading scenarios. The optimal schedule received by solving C_{unb}^{stoc} is thus more robust to stochastic variation of loads.

4.3. EV scheduling under uncertain grid prices

The deterministic prices as well as lower and higher limit for uncertainty, with $\hat{\alpha}_t = 0.5$, are shown in Fig. 11 (a). For day-ahead optimization, the prices are allowed to vary in a particular range of values. To determine the impact of uncertain prices on EV charging costs, the solution of robust objective C_{pr}^{rob} is compared with that of deterministic optimization objective C_{pr} in Fig. 12.

The solution of C_{pr} is shown in Fig. 12 (a). It shows that the cheaper hours 10th, 12th, 14th and 15th are used for charging the EVs, while the expensive hours 11th and 13th, the EVs are discharged. This power consumption from EVs is considering the

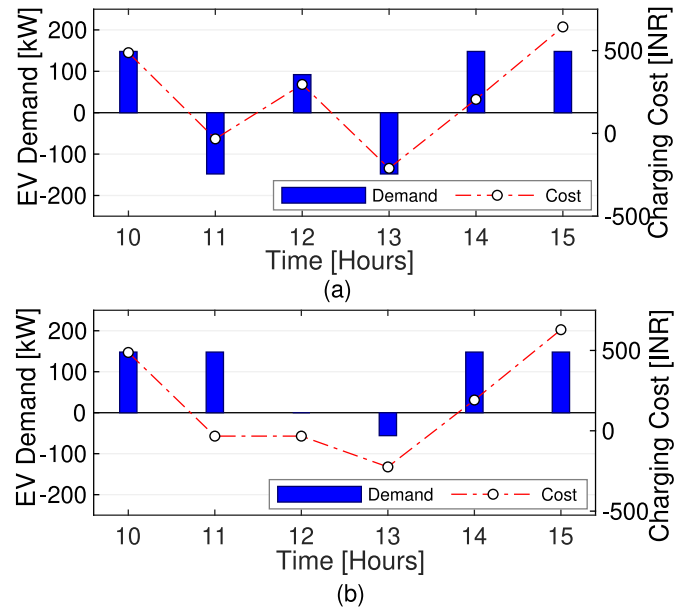


Fig. 12. Charging cost and combined EVs Demand. (a) Deterministic. (b) Robust.

deterministic/forecasted prices. In Fig. 12 (b) the robust schedule considering uncertainty in prices is shown. It can be noticed that in Fig. 12 (b) there is no charging at 12th hour and less discharging at 13th hour. In earlier deterministic case of Fig. 12 (a), the 12th hour was used to charge EV batteries, and subsequently 13th hour was used to discharge them, thus, making a profit. This is possible because in the forecasted prices, the 12th hour is cheaper than the 13th, allowing an opportunity for energy arbitrage. However, considering uncertain intervals of prices, the 12th hour may become more expensive than the lowest price possible at 13th hour. Thus, using robust day-ahead optimization, the EVs are not scheduled in hours 12:00–13:00. This saves EVs from any monetary losses due to uncertainty of prices the next day. The charging costs for both deterministic and robust scheduling under uncertain pricing is also shown in Fig. 12. It can be noticed that the final charging cost of all EVs, at end of 15th hour, is INR 643 for deterministic case, while INR 629 using robust optimization technique. This shows that energy arbitrage can result in economic losses if price uncertainty is not accounted in the EV scheduling process.

Finally, the monetary losses caused by increasing uncertainty of grid prices are given in Table 2. A higher radius of uncertainty will signify higher level of randomness in the electricity prices. Thus, with low radius of uncertainty $\hat{\alpha}_t$ of 0.1, the charging cost computed with robust and deterministic algorithms have difference of 2.23%. However, as the uncertainty radius increases to 0.25, the difference is more than doubled to 7.96%. Further, it can be noticed that the charging costs obtained using deterministic scheduling get exponentially worse as uncertainty increases. Thus, scheduling EVs based on deterministic prices can fare very poorly for any level of uncertainty.

Table 1
Objective Values Using Proposed Stochastic vs. Deterministic Scheduling.

No. of scenarios	Expected objective value (stochastic)	Expected objective value (deterministic)
50	3.20×10^3	3.21×10^3
75	3.21×10^3	3.22×10^3
100	3.216×10^3	3.224×10^3

Table 2
Percent [%] Cost Savings using Proposed Robust Scheduling with Increasing Uncertainty.

Uncertainty Radius of Price ($\hat{\alpha}_t$)	Charging Cost (robust)	Charging Cost (deterministic)	Difference (%)
0.1	658	672	2.23
0.25	739	799	7.96
0.5	802	1006	25.4

5. Conclusion

This paper presents algorithms that focus on mitigating unbalance by transferring EV power consumption among phases. The proposed methods are tested on an arbitrary bus of a network with a high degree of load unbalance. It is observed that load unbalance can be reduced to near zero using optimal charging/discharging schedules of EVs. This is possible with EVs having a continuous range of charging/discharging power. Moreover, using ordering of objective with epsilon constraints, unbalance reduction technique is combined with the alternate objective of energy arbitrage. It is observed that EVs can be simultaneously scheduled to reduce unbalance and also make economic profits by modifying their power consumption based on varying tariffs. Finally, the demand response algorithms are adjusted to include the impact of uncertain electricity prices and loads. Results demonstrate that impacts of worst-case uncertainty can be avoided in day-ahead EV scheduling using the proposed robust as well as stochastic optimization method. For future work, we wish to extend the given algorithms for three-phase EV charging systems, and use budgeted uncertainties for reducing the conservativeness of robust methods.

Credit author statement

Bakul Kandpal: Conceptualization, Methodology, Software, Writing- Original Draft **Parikshit Pareek:** Conceptualization, Methodology. **Ashu Verma:** Conceptualization, Writing- Reviewing & Editing, Validation, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- [1] Nimalsiri NI, Mediawathe CP, Ratnam EL, Shaw M, Smith DB, Halgamuge SK. A survey of algorithms for distributed charging control of electric vehicles in smart grid. *IEEE Trans Intell Transport Syst* 2019;1–19.
- [2] Mahmud K, Hossain MJ, Ravishankar J. Peak-load management in commercial systems with electric vehicles. *IEEE Syst J* 2019;13(2):1872–82.
- [3] Pirouzi S, Aghaei J, Latify MA, Yousefi GR, Mokryani G. A robust optimization approach for active and reactive power management in smart distribution networks using electric vehicles. *IEEE Syst J* 2018;12(3):2699–710.
- [4] Singh J, Tiwari R. Real power loss minimisation of smart grid with electric vehicles using distribution feeder reconfiguration. *IET GTD* 2019;13(18):4249–61.
- [5] Jordehi AR, Javadi MS, Catalão JP. Day-ahead scheduling of energy hubs with parking lots for electric vehicles considering uncertainties. *Energy* 2021;229:120709.
- [6] Nikoobakht A, Aghaei J, Mokarram MJ, Shafie-khah M, Catalão JP. Adaptive robust co-optimization of wind energy generation, electric vehicle batteries and flexible AC transmission system devices. *Energy* 2021;230:120781.
- [7] Shahnia F, Wolfs P, Ghosh A. Voltage unbalance reduction in low voltage feeders by dynamic switching of residential customers among three phases. In: *IEEE power and energy society general meeting*; 2013. p. 1–5. <https://doi.org/10.1109/PESMG.2013.6672798>.
- [8] Hung DQ, Mishra Y. Impacts of single-phase pv injection on voltage quality in 3-phase 4-wire distribution systems. In: *IEEE PESGM*; 2018.
- [9] Elyasibakhtiari K, Azzouz MA, Rezai EA, El-Saadany EF. Real-time analysis of voltage and current in low-voltage grid due to electric vehicles' charging. In: *IEEE SEGE conference*; 2015. <https://doi.org/10.1109/SEGE.2015.7324607>.
- [10] Calearo L, Thingvad A, Suzuki K, Marinelli M. Grid loading due to ev charging profiles based on pseudo-real driving pattern and user behaviour. *IEEE Trans Transport Electr* 2019.
- [11] Pezesghi H, Wolfs PJ, Ledwich G. Impact of high pv penetration on distribution transformer insulation life. *IEEE Trans Power Deliv* 2014;29(3):1212–20. <https://doi.org/10.1109/TPWRD.2013.2287002>.
- [12] Savaghebi M, Jalilian A, Vasquez JC, Guerrero JM. Secondary control for voltage quality enhancement in microgrids. *IEEE Trans on Smart Grid* 2012;3(4):1893–902. <https://doi.org/10.1109/TSG.2012.2205281>.
- [13] Ma K, Li R, Li F. Quantification of additional asset reinforcement cost from 3-phase imbalance. *IEEE Trans Power Syst* 2015;31(4):2885–91.
- [14] Weckx S, Driesen J. Load balancing with EV chargers and PV inverters in unbalanced distribution grids. *IEEE Trans Sustain Energy* 2015;6(2):635–43. <https://doi.org/10.1109/TSTE.2015.2402834>.
- [15] Zhang J, Cui M, Fang H, He Y. Two novel load-balancing platforms using common DC buses. *IEEE Trans Sustain Energy* 2018;9(3):1099–107. <https://doi.org/10.1109/TSTE.2017.2769471>.
- [16] Das H, Rahman M, Li S, Tan C. Electric vehicles standards, charging infrastructure, and impact on grid integration: a technological review. *Renew Sustain Energy Rev* 2020;120:109618.
- [17] Shahnia F, Wolfs PJ, Ghosh A. Voltage unbalance reduction in low voltage feeders by dynamic switching of residential customers among three phases. *IEEE Trans on Smart Grid* 2014;5(3):1318–27. <https://doi.org/10.1109/TSG.2014.2305752>.
- [18] Lago J, De Ridder F, Vranckx P, De Schutter B. Forecasting day-ahead electricity prices in Europe: the importance of considering market integration. *Appl Energy* 2018;211:890–903.
- [19] Li Y, Li L, Peng C, Zou J. An MPC based optimized control approach for EV-based voltage regulation in distribution grid. *Elec Power Syst Res* 2019;172:152–60.
- [20] Yao L, Lim WH, Tsai TS. A real-time charging scheme for demand response in electric vehicle parking station. *IEEE Trans on Smart Grid* 2017;8(1):52–62. <https://doi.org/10.1109/TSG.2016.2582749>.
- [21] Nie Q, Zhang L, Tong Z, Dai G, Chai J. Cost compensation method for PEVs participating in dynamic economic dispatch based on carbon trading mechanism. *Energy* 2022;239:121704.
- [22] Amiri SS, Jadid S, Saboori H. Multi-objective optimum charging management of electric vehicles through battery swapping stations. *Energy* 2018;165:549–62.
- [23] Wozabal D, Rameseder G. Optimal bidding of a virtual power plant on the Spanish day-ahead and intraday market for electricity. *Eur J Oper Res* 2020;280(2):639–55.
- [24] Sun W, Neumann F, Harrison GP. Robust scheduling of electric vehicle charging in LV distribution networks under uncertainty. *IEEE Trans Ind Appl* 2020;56(5):5785–95.
- [25] Zhou K, Cheng L, Wen L, Lu X, Ding T. A coordinated charging scheduling method for electric vehicles considering different charging demands. *Energy* 2020;213:118882.
- [26] Tan H, Yan W, Ren Z, Wang Q, Mohamed MA. A robust dispatch model for integrated electricity and heat networks considering price-based integrated demand response. *Energy* 2022;239:121875.
- [27] Yin W, Ming Z, Wen T. Scheduling strategy of electric vehicle charging considering different requirements of grid and users. *Energy* 2021;121118.
- [28] Zang H, Xu R, Cheng L, Ding T, Liu L, Wei Z, Sun G. Residential load forecasting based on LSTM fusing self-attention mechanism with pooling. *Energy* 2021;229:120682.
- [29] Aghaei J, Amjadi N, Shayanfar HA. Multi-objective electricity market clearing considering dynamic security by lexicographic optimization and augmented epsilon constraint method. *Appl Soft Comput* 2011;11(4):3846–58.
- [30] Miettinen K. Nonlinear multiobjective optimization, vol. 12. Springer Science & Business Media; 2012.
- [31] Athari MH, Wang Z. Impacts of wind power uncertainty on grid vulnerability to cascading overload failures. *IEEE Trans Sustain Energy* Jan. 2018;9(1):128–37. <https://doi.org/10.1109/TSTE.2017.2718518>.
- [32] Wallnerström CJ, Setréus J, Hilber P, Tong F, Bertling L. Model of capacity

- demand under uncertain weather. In: 2010 IEEE 11th international conference on probabilistic methods applied to power systems; 2010. p. 314–8. <https://doi.org/10.1109/PMAPS.2010.5528841>.
- [33] Bertsimas D, Sim M. The price of robustness. *Oper Res* 2004;52(1):35–53. <https://doi.org/10.1287/opre.1030.0065>.
- [34] Xu D, Chen Z, Yang L. Scenario tree generation approaches using K-means and LP moment matching methods. *J Comput Appl Math* 2012;236(17):4561–79.
- [35] Muratori M. Impact of uncoordinated plug-in electric vehicle charging on residential power demand-supplementary data. Tech. rep. Golden, CO: National Renewable Energy Laboratory-Data (NREL-DATA); 2017.
- [36] Indian energy exchange hourly market data. Available online at: <https://www.iexindia.com/marketdata/areaprice.aspx>.