

2.2 下列函数在何处可导? 何处不可导? 何处解析? 何处不解析?

$$(1) f(z) = \bar{z} \cdot z^2;$$

$$\begin{aligned} \text{解 } f(z) &= \bar{z} \cdot z^2 = \bar{z} \cdot z \cdot z = |z|^2 \cdot z \\ &= (x^2 + y^2)(x + iy) \\ &= x(x^2 + y^2) + iy(x^2 + y^2), \end{aligned}$$

$$\text{这里 } u(x, y) = x(x^2 + y^2), v(x, y) = y(x^2 + y^2).$$

$$u_x = x^2 + y^2 + 2x^2, \quad v_y = x^2 + y^2 + 2y^2,$$

$$u_y = 2xy,$$

$$v_x = 2xy.$$

要  $u_x = v_y, u_y = -v_x$ , 当且仅当  $x = y = 0$ , 而  $u_x, u_y, v_x, v_y$  均连续, 故  $f(z) = \bar{z} \cdot z^2$  仅在  $z = 0$  处可导, 处处不解析.

$$(2) f(z) = x^2 + iy^2;$$

解 这里  $u = x^2, v = y^2$ ,

连续, 但  $u_x = v_y, u_y = -v_x$  仅在  $x = y$  处成立, 故  $f(z)$  仅在  $x = y$  上可导, 处处不解析.

