5　电场力的功　电势

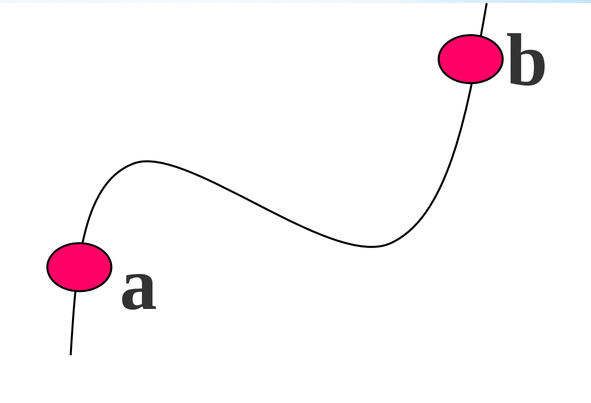
Work done by Electric Force

Electric Potential

1. Work done by Electric Force 静电力作功的特点

The work done by the force is given by

$$W = \int_a^b \vec{F} \cdot \mathrm{d}\vec{r} = \int_a^b F \cos\theta ds$$



We will show that the force on a charged particle in a electric field produced by any combination of charges at rest is always a conservative force field(保守力场）

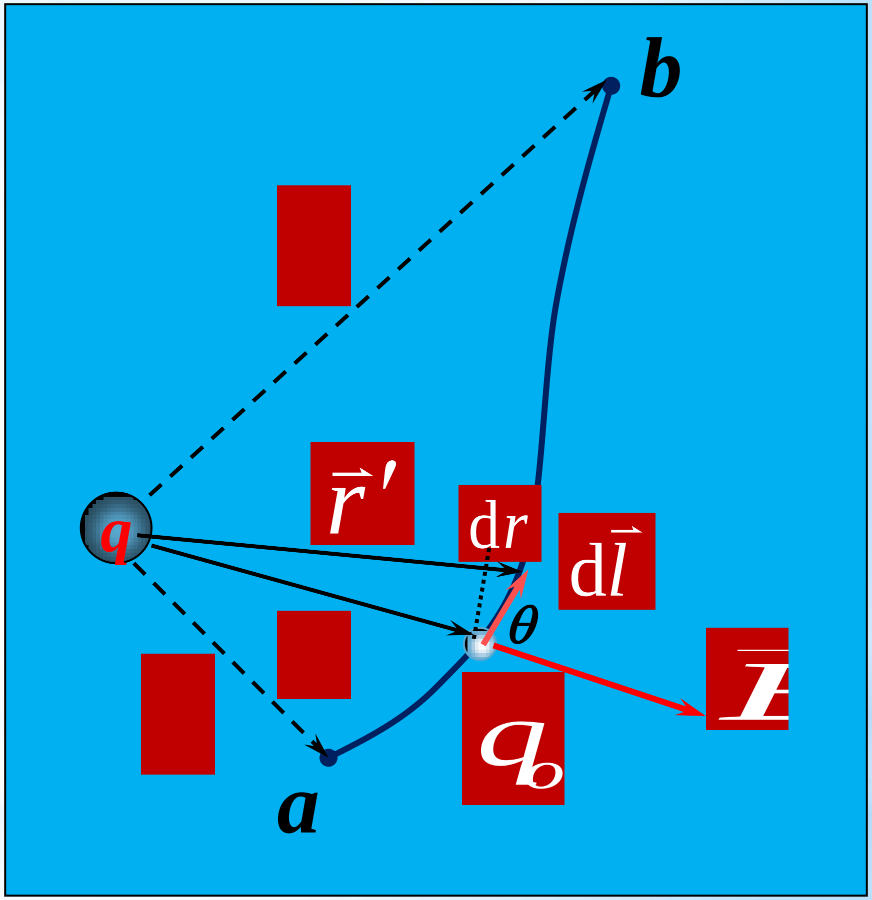
1) Consider the electric field of a point charge. *A* test charge q_0 moves from *a* to *b* in the electric field by the electric force. The work done by the electric force is given by

$$\mathrm{d}W = q_o\vec{E}\cdot\mathrm{d}\vec{l} = q_0 E\cos\theta\,\mathrm{d}l$$

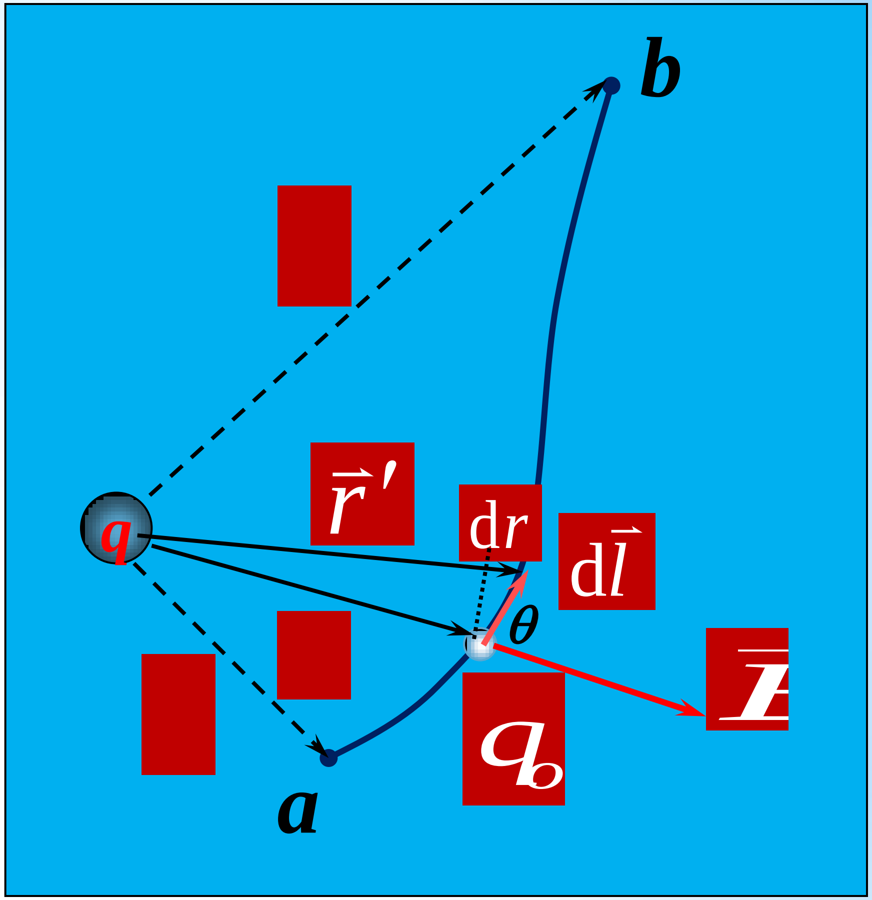
$$\mathrm{d}W = \frac{q_o q}{4\pi\varepsilon_o r^2}\cos\theta\,\mathrm{d}l = \frac{q_o q}{4\pi\varepsilon_o r^2}\mathrm{d}r$$

$$W_{ab} = \int_{r_a}^{r_b}\frac{q_o q}{4\pi\varepsilon_o r^2}\mathrm{d}r$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$



$$W=\frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$$



结论：给定试验电荷在静电场中移动时，电场力所作的功只与试验电荷的起点和终点的位置有关，而与路径无关。

Conclusion: work done by electric field depends only on the initial & final positions, but not on the path.

2) In general, the electric field equals to

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots\cdots$$

The work done on test charge by the electric field $\vec{E}$ is given by

$$W = \int_a^b q_0 \vec{E}.d\vec{r} = \int_a^b q_0 \vec{E}_1.d\vec{r} + \int_a^b q_0 \vec{E}_2.d\vec{r} + \cdots\cdots$$

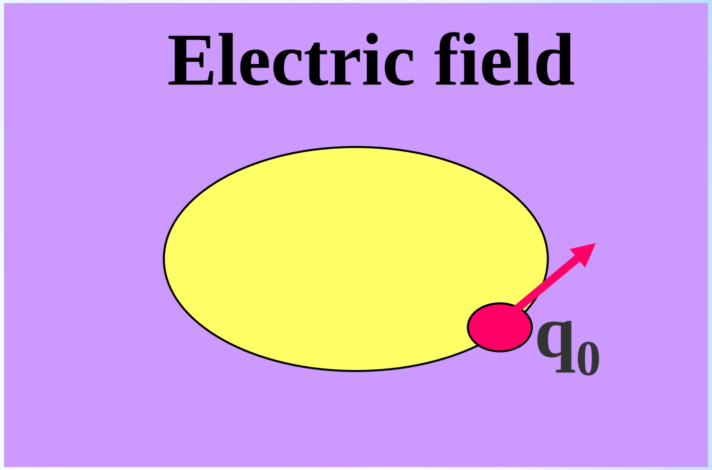
Since the every term in the above formula（公式）depends only on the positions a and b, the sum depends only on the positions a and b. Therefore, the electrostatic force is conservative.

即电场力是保守力。静电场是保守场。

2.Circular theorem of the electrostatic field

静电场的环路定理

If the test charge moves around a closed path, the work by the electric field is zero:



$$W = \oint_{\ell} q_0 \vec{E} \cdot d\vec{\ell} = 0$$

静电场的环路定理：

$$\oint_{\ell} \vec{E} \cdot d\vec{\ell} = 0$$

静电场中电场强度沿任意闭合路径线积分（环流）为零。

3. Electric potential energy and electrical potential 电势能和电势

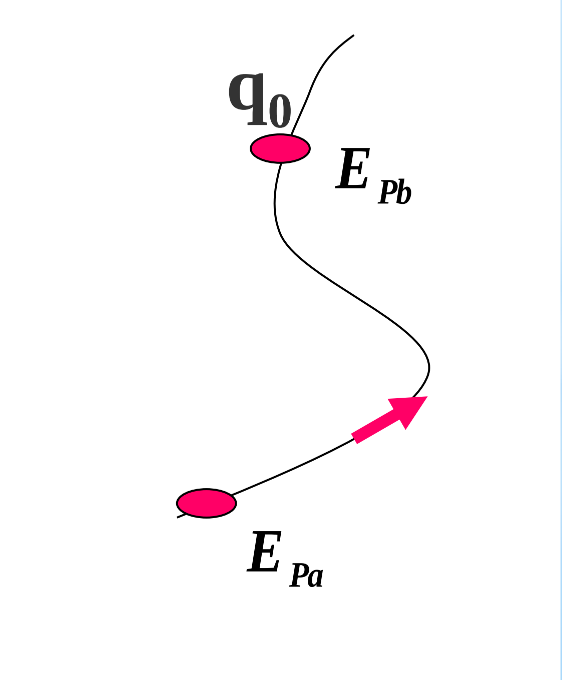
（1） 静电力是保守力，可引入电势能的概念。

（2）定义静电力由 *a* 点→*b* 点作的功 为电势能增量的负值：

$$W_{ab} = q_o \int_a^b \vec{E} \cdot d\vec{l} = -(E_{Pb} - E_{Pa}) = E_{Pa} - E_{Pb}$$

E_{Pa} : the electric potential energy of q_0 at point a.

E_{Pb} : the electric potential energy of q_0 at point b.



$$W_{ab}=\int_a^b q_o\vec{E}\cdot d\vec{l}=E_{pa}-E_{pb}$$

令b点的势能为零（$E_{pb}=0$）

a点的势能： $$\boxed{E_{pa}=\int_a^b q_o\vec{E}\cdot d\vec{l}}$$

结论：试验电荷q_o在空间某处的电势能在数值上就等于将q_o从该处移至势能的零点电场力所作的功。

电势能的零点可以任意选取，习惯上，当场源电荷为有限带电体时，通常把电势能的零点选取在无穷远处。

空间a点的电势能： $$\boxed{E_{pa}=\int_a^\infty q_o\vec{E}\cdot d\vec{l}}$$

注意：

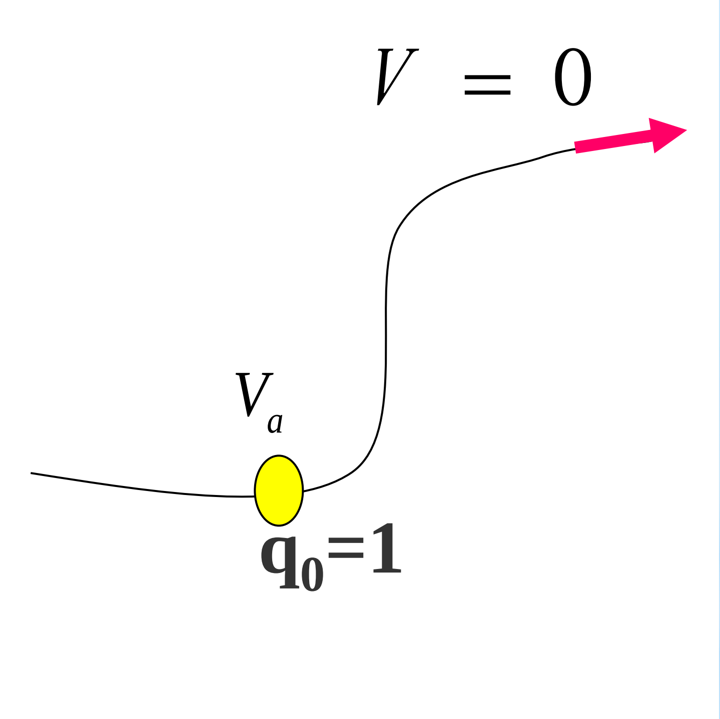
- 电势能为电场和位于电场中的电荷这个系统所共有。
- 电势能是标量，可正可负。

Electrical potential 电势：

$$V_a = \frac{E_{pa}}{q_0} = \int_a^{\text{势能零点}} \vec{E} \cdot d\vec{l}$$

单位：伏特（V= $J \cdot C^{-1}$）

结论： 电场中a点的电势，在数值上等于把单位正电荷从a点移至势能的零点处电场力所作的功。



也等于在a点的单位正电荷所具有的电势能。

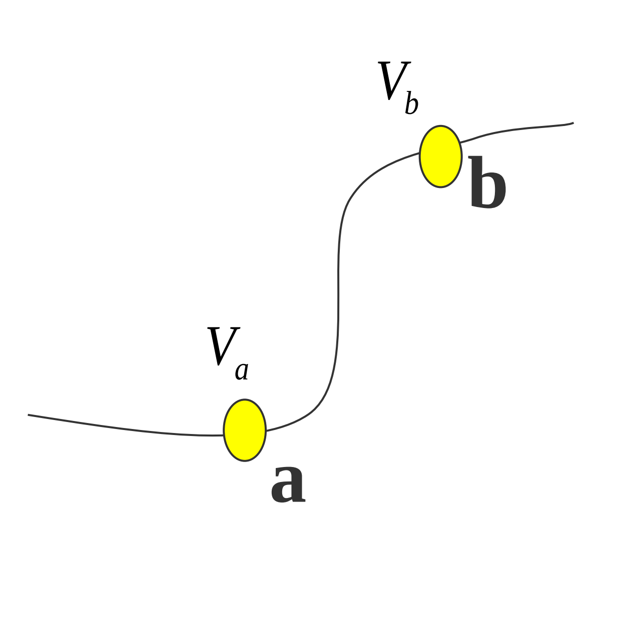
电场中一点的电势其数值与势能零点的选取有关。

The potential difference between point a and point b is

电势差:

$$V_a - V_b = \int_a^{\infty} \vec{E} \cdot d\vec{\ell} - \int_b^{\infty} \vec{E} \cdot d\vec{\ell}$$

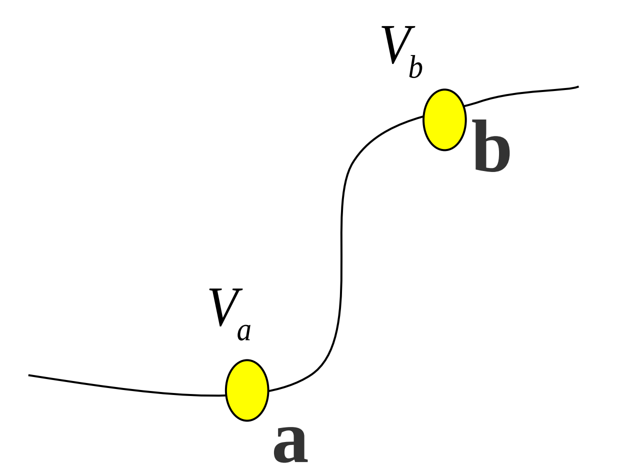
$$= \int_a^{\infty} \vec{E} \cdot d\vec{\ell} + \int_{\infty}^{b} \vec{E} \cdot d\vec{\ell} = \int_a^{b} \vec{E} \cdot d\vec{\ell}$$



结论: 静电场中*a*, *b*两点的电势差，等于将单位正电荷从*a*点移至*b*点电场力所作的功。是绝对的。

In summary: $E_{Pa} = qV_a$

$$W_{a\to b} = -(E_{Pb} - E_{Pa}) = E_{Pa} - E_{Pb}$$
$$= q(V_a - V_b) = q\Delta V$$



- 电势是描述电场性质的物理量（能量）；
- 电势 V 是标量; 而 $\vec{E}$ 指向电势降低的方向;

  仅在电场力作用下, 正电荷的运动：电势高→低;

  仅在电场力作用下,负电荷的运动：电势低→高.
- 电势的零参考点的选取是任意的。

  有限大带电体一般选无穷远为电势零点;

  对无限大带电体的电场，通常选特殊点或线或面上的电势为零.

4. The calculation of the electrical potential
电势的计算

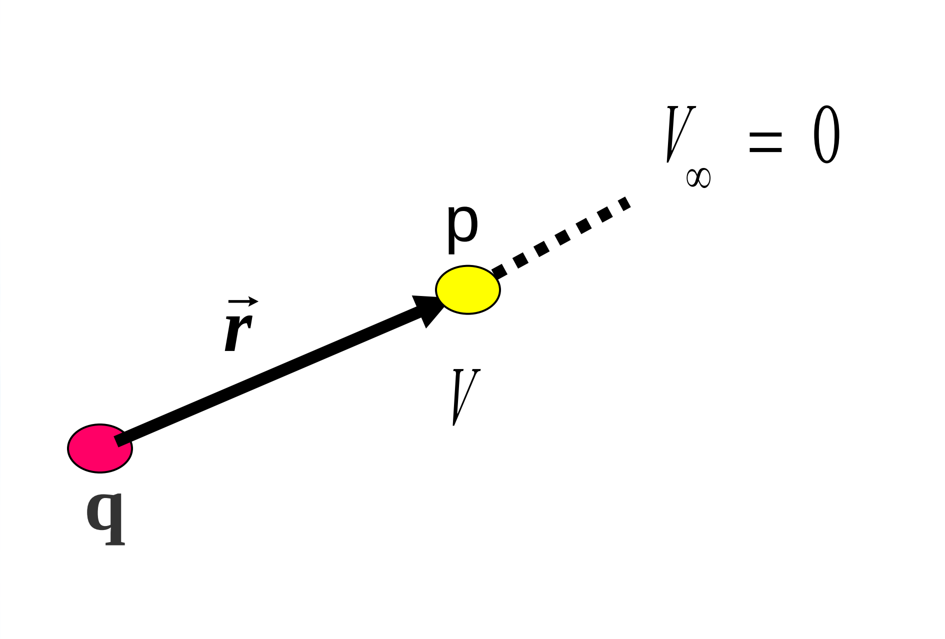
1) The electrical potential of field produced by a point charge q 点电荷的电势：

点电荷电场中的电势：

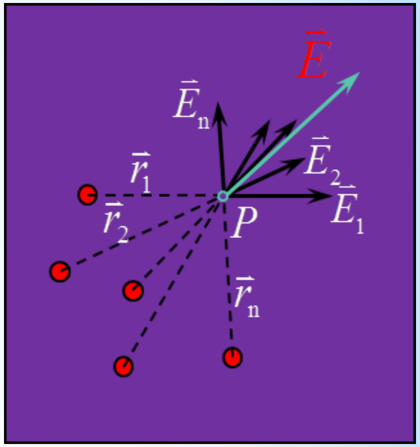
无穷远为电势零点！

$$V_p = \int_p^\infty \vec{E}\cdot \mathrm{d}\vec{l} = \int_r^\infty \frac{q}{4\pi\varepsilon_o r^2}\mathrm{d}r$$

$$= \frac{q}{4\pi\varepsilon_o r}$$



2) The electrical potential of field produced by the any charged system点电荷系电场中的电势：



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n \qquad V_p = \int_p^{\infty} \vec{E} \cdot \mathrm{d}\vec{l}$$

$$V_p = \int_p^{\infty} (\vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n) \cdot \mathrm{d}\vec{l} = \sum_i V_i$$

$$\therefore V = \sum V_i = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i}$$

the superposition principle of electrical potential.

电势叠加原理： 点电荷系电场中任一点的电势，等于各个点电荷单独存在时在该点处的电势之代数和。

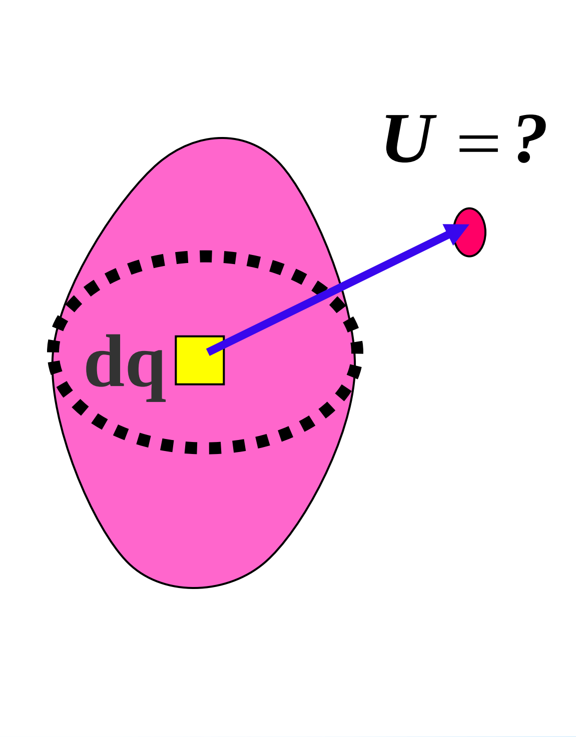
3）连续分布电荷电场中的电势：

$$V=\int_V dV=\int_V \frac{dq}{4\pi\varepsilon_0 r}$$

$$=\int_V \frac{\rho dV}{4\pi\varepsilon_0 r}$$ 体电荷

$$=\int_S \frac{\sigma dS}{4\pi\varepsilon_0 r}$$ 面电荷

$$=\int_\ell \frac{\lambda d\ell}{4\pi\varepsilon_0 r}$$ 线电荷



特点：标量积分！！！

In summary:

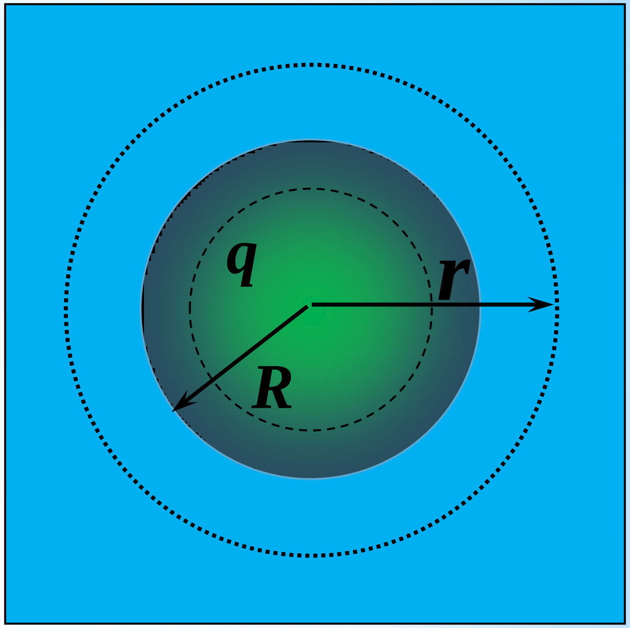
The two methods calculating the electrical potential:

法一. 由电势定义式 $V_p = \int_p^{零点} \vec{E} \cdot d\vec{l}$ 计算P点电势。

法二. 根据点电荷的电势公式和电势迭加原理求电势。

例1. 半径为R的均匀带电球体，带电量为q。求电势分布。

解： $\oint\!\!\oint_S \vec{E}\cdot d\vec{s} = \dfrac{1}{\varepsilon_0}\sum q_i \begin{cases} \vec{E}_{内} = \dfrac{qr}{4\pi\varepsilon_o R^3}\hat{r} & (r<R) \\ \vec{E}_{外} = \dfrac{q}{4\pi\varepsilon_o r^2}\hat{r} & (r>R) \end{cases}$



$(r \geq R)$

$$V_{外} = \int_r^{\infty} \vec{E}_{外}\cdot d\vec{r}$$

$$= \int_r^{\infty} E_{外} dr = \int_r^{\infty} \frac{q}{4\pi\varepsilon_o r^2} dr = \frac{q}{4\pi\varepsilon_o r}$$

$(r < R)$

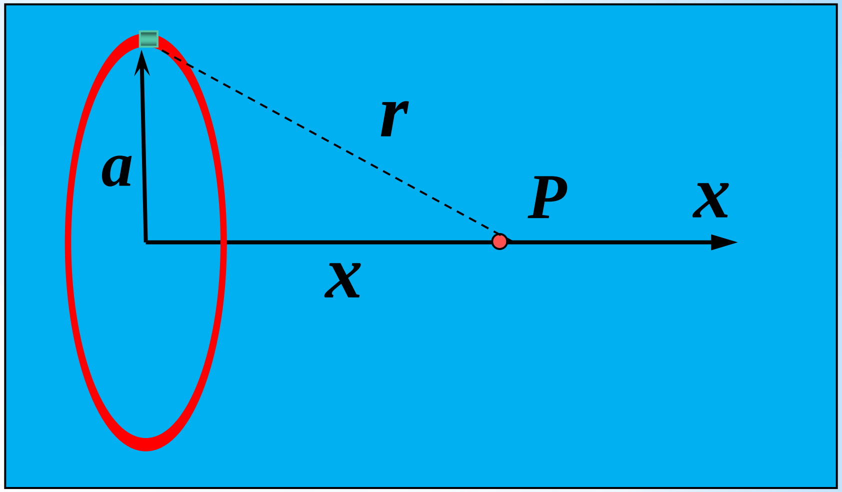
$$V_{内} = \int_r^{\infty} \vec{E}\cdot d\vec{r}$$

$$= \int_r^{R} \vec{E}_{内}\cdot d\vec{r} + \int_R^{\infty} \vec{E}_{外}\cdot d\vec{r} = \int_r^{R} E_{内} dr + \int_R^{\infty} E_{外} dr$$

$$= \int_r^{R} \frac{qr}{4\pi\varepsilon_o R^3} dr + \int_R^{\infty} \frac{q}{4\pi\varepsilon_o r^2} dr$$

$$= \frac{q}{8\pi\varepsilon_o R^3}(R^2 - r^2) + \frac{q}{4\pi\varepsilon_o R} = \frac{q(3R^2 - r^2)}{8\pi\varepsilon_o R^3}$$

例2. 均匀带电圆环，带电量为q，半径为a，求轴线上任意一点的P电势。



解： $$dq = \lambda dl = \frac{q}{2\pi a} dl$$

$$dV = \frac{dq}{4\pi\varepsilon_o r} = \frac{q\,dl}{8\pi^2\varepsilon_o a r}$$

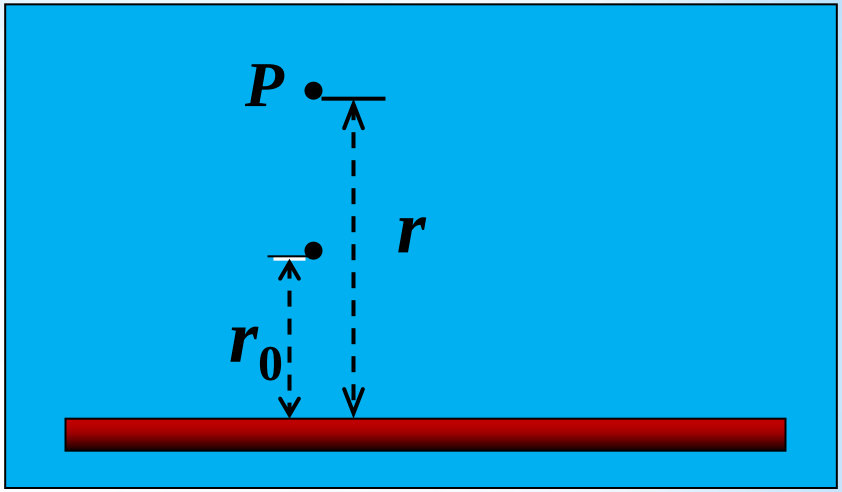
$$V = \int dV = \frac{q}{8\pi^2\varepsilon_o a r}\int_L dl = \frac{q\cdot 2\pi a}{8\pi^2\varepsilon_o a r}$$

$$\therefore V = \frac{q}{4\pi\varepsilon_o r} = \frac{q}{4\pi\varepsilon_o\sqrt{x^2+a^2}}$$

法二： $$E = \frac{1}{4\pi\varepsilon_o}\frac{qx}{(x^2+a^2)^{3/2}}$$

$$V = \int_x^\infty \vec{E}\cdot d\vec{x} = \frac{q}{4\pi\varepsilon_o}\int_x^\infty \frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{q}{4\pi\varepsilon_o\sqrt{x^2+a^2}}$$

例3. 求无限长均匀带电直线外任一点P的电势。(电荷密度λ)



解: $$E=\frac{\lambda}{2\pi\varepsilon_o r}$$

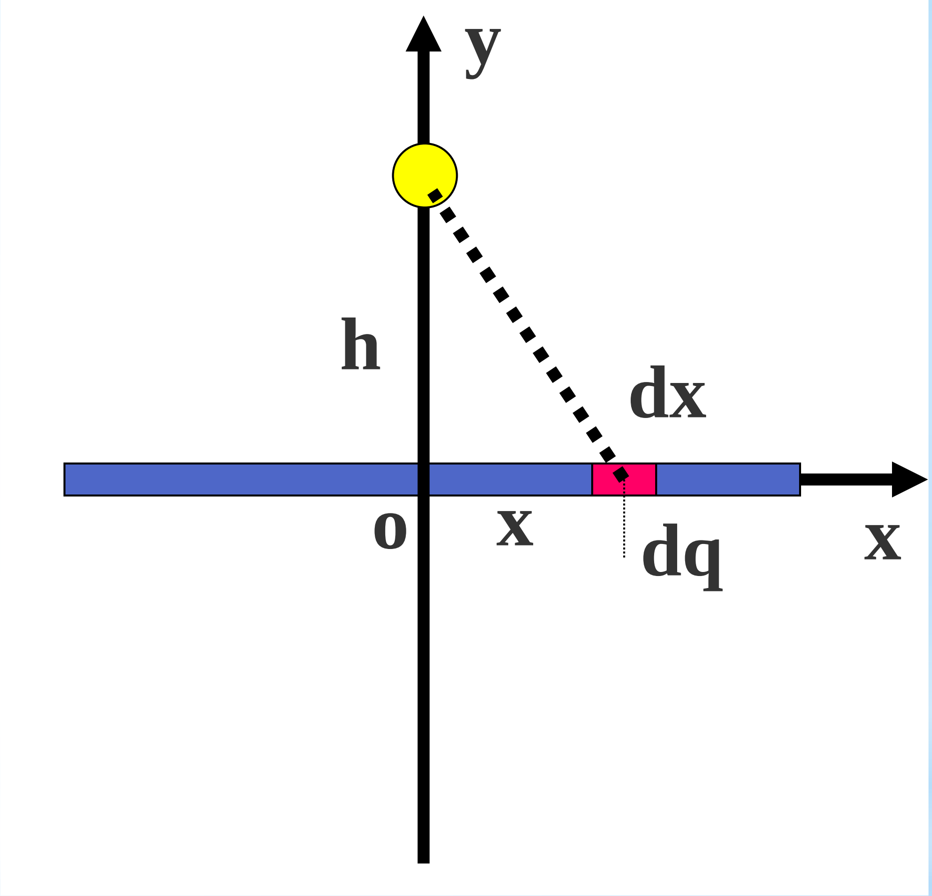
$$V=\int_r^{r_o}\vec{E}\cdot\mathrm{d}\vec{l}=\int_r^{r_o}\frac{\lambda}{2\pi\varepsilon_o r}\mathrm{d}r$$

$$=\frac{\lambda}{2\pi\varepsilon_o}\ln r\Big|_r^{r_o}=\frac{\lambda}{2\pi\varepsilon_o}(\ln r_o-\ln r)=\frac{\lambda}{2\pi\varepsilon_o}\ln\frac{r_o}{r}$$

如果势能零点在 r_o=1m

$$V=\frac{-\lambda}{2\pi\varepsilon_o}\ln r$$

例4:长为L的带电细杆，电荷线密度为λ，求其中垂线上距X轴 h 远处的电势。



解:(1)选无限远处电势为零;

（2）用电势叠加原理，有

$$U=\int_{\ell}\frac{dq}{4\pi\varepsilon_0 r}$$

$$=\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{\lambda \mathrm{d}x}{4\pi\varepsilon_0\left(x^2+h^2\right)^{1/2}}$$

$$=\frac{\lambda}{2\pi\varepsilon_0}ln\left(\frac{L}{2h}+\sqrt{1+\frac{L^2}{4h^2}}\right)$$

例5: 一均匀电场，电场强度 $\vec{E}=4\vec{i}+6\vec{j}$ ，求XY平面内点a（3，2）和点b（1，0）间的电势差（单位均取国际单位制）

解:

$$\Delta U=\int_a^b \vec{E}.\mathrm{d}\vec{\ell}$$

$$=\int_a^b E_x \mathrm{d}x + E_y \mathrm{d}y$$

$$=4\times(1-3)+6\times(0-2)$$

$$=-20(V)$$

作业：**25, 26, 28**