

CHAPTER 2: Numbers of Various Sorts

\mathbb{N} : natural numbers, simplest numbers of all, "counting numbers", 1, 2, 3, ...

most basic property: mathematical induction

↓
doesn't include 0

$P(x)$: property P holds for a number x

mathem. induction:

$P(x)$ is true for all $x \in \mathbb{N}$ if

(1) $P(1)$ is true

(2) $P(k)$ true $\Rightarrow P(k+1)$ true

another formulation:

if A is any collection (or set, a synonym) of natural numbers and

(1) $1 \in A$

(2) $k \in A \Rightarrow k+1 \in A$

then A is \mathbb{N} .

* empty collection, null set, \emptyset : set A containing no natural numbers

one more formulation:

Principle of Complete Induction

A set of \mathbb{N} and (1) $1 \in A$

$\Rightarrow A = \mathbb{N}$

(2) $k+1 \in A$ if $1, \dots, k \in A$

this principle is a consequence of the ordinary principle of induction

recursive definitions

ex:

$n! = n(n-1) \cdot \dots \cdot 2 \cdot 1$ can be defined as (1) $1! = 1$
(2) $n! = n \cdot (n-1)!$

ex:

$\sum_{i=1}^n a_i$ is defined as (1) $\sum_{i=1}^1 a_i = a_1$
(2) $\sum_{i=1}^n a_i = a_n + \sum_{i=1}^{n-1} a_i$

\mathbb{Z} : Integers

$\dots, -2, -1, 0, 1, 2, \dots$

→ P7 fails: an inverse does not exist for any member of \mathbb{Z}

\mathbb{Q} : Rational Numbers

quotients m/n of integers, $n \neq 0$

→ P1-P2 true

\mathbb{R} : Real Numbers

Rational + Irrational numbers

represented by infinite decimals, e.g. $\sqrt{2}$, π

Notes

→ every $x \in \mathbb{N}$ can be written as $2k$ or $2k+1$ for some $k \in \mathbb{Z}$

←
even

↓
odd