CHAPTER 2: Numbers of Various Sorts

$\mathbb{N}$: natural numbers

most basic property: mathematical induction

$P(x)$: property P holds for a number x

mathem. induction:

$P(x)$ is true for all $x \in \mathbb{N}$ if

(1) $P(1)$ is true
(2) $P(k)$ true $\Rightarrow P(k+1)$ true

another formulation:

If A is any collection (or set, a synonym) of natural numbers and

(1) 1 in A
(2) k in $A \Rightarrow k+1$ in A

then A is $\mathbb{N}$.

* empty collection, null set, $\varnothing$: set A containing no natural numbers

one more formulation:
Principle of Complete Induction

A set of $\mathbb{N}$ and (1) 1 in A $\Rightarrow A = \mathbb{N}$
(2) $k+1$ in A if $1, \ldots, k$ in A

this principle is a consequence of the ordinary principle of induction

recursive definitions

ex:
$n! = n(n-1) \cdot \ldots \cdot 2 \cdot 1$ can be defined as (1) $1! = 1$
(2) $n! = n \cdot (n-1)!$

ex:
$\sum_{i=1}^{n} a_i$ is defined as (1) $\sum_{i=1}^{1} a_i = a_1$
(2) $\sum_{i=1}^{n} a_i = a_n + \sum_{i=1}^{n-1} a_i$

$\mathbb{Z}$: Integers

$\ldots, -2, -1, 0, 1, 2, \ldots$

→ P7 fails: an inverse does not exist for any member of $\mathbb{Z}$

$\mathbb{Q}$: Rational Numbers

quotients m/n of integers, $n \neq 0$

→ P1-P12 true

$\mathbb{R}$: Real Numbers

Rational + $\underbrace{\text{Irrational numbers}}_{\text{represented by infinite decimals, e.g. } \sqrt{2}, \pi}$

Notes

→ every $x \in \mathbb{N}$ can be written as $2k$ or $2k+1$ for some $k \in \mathbb{Z}$

$2k$ → even, $2k+1$ → odd