Chapter 3 - Functions

Provisional Def: Function is a rule that assigns to each of certain real numbers, some other real number.

Polynomial Fn f is polyn. if $\forall a_0, \dots, a_n \in \mathbb{R}$ s.t. $f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$

highest power: degree of f

Rational Fns $\frac{p}{q}$, p, q polynomial fns, q not always zero

Combining Fns to produce new fns

f, g fns. Define fn $f+g$ as $(f+g)(x) = f(x) + g(x)$

$\rightarrow$ domain $f+g$ = domain $f \cap$ domain g

Similarly, define $f \cdot g$, f/g, $c \cdot g$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$(c \cdot g)(x) = c \cdot g(x)$ (special case of $f \cdot g$ w/ $f(x) = c$)

constant fn

Some results

$(f \cdot g) \cdot h = f \cdot (g \cdot h)$

$((f \cdot g) \cdot h)(x) = (f \cdot g)(x) \cdot h(x) = (f(x) \cdot g(x)) \cdot h(x) = f(x) \cdot g(x) \cdot h(x)$
$= f(x) \cdot (g(x) \cdot h(x)) = f \cdot (g \cdot h)(x)$

Identity fn: $I(x) = x$

$\rightarrow f(x) = \frac{x + x^2 + x\sin^2 x}{x\sin x + x\sin^2 x} = \frac{I + I \cdot I + I \cdot \sin \cdot \sin}{I \cdot \sin + I \cdot \sin \cdot \sin}$

Define composition fn $f \circ g$

$(f \circ g)(x) = f(g(x))$

domain$(f \circ g) = \{x : x \text{ in dom } g,\ g(x) \text{ in dom } f\}$

$f = I \cdot I$, $g = \sin$ $\Rightarrow$ $(f \circ g)(x) = I(\sin x) \cdot I(\sin x) = \sin^2 x$
$(g \circ f)(x) = \sin(I(x) \cdot I(x)) = \sin(x^2)$

Composition is associative $(f \circ g) \circ h = f \circ (g \circ h)$

$((f \circ g) \circ h)(x) = (f \circ g)(h(x)) = f(g(h(x)))$
$(f \circ (g \circ h))(x) = f((g \circ h)(x)) = f(g(h(x)))$

notation

$x \longrightarrow x^2$

the function $f(x) = x^2$

Def (Function) fn is collection of pairs of numbers w/ following property: (a,b), (a,c) both in the collection then $b = c$.

Def (Domain) domain of fn f is set of all a for which there is some b such that (a,b) is in f.
If a is in domain of f, by the def. of fn, there is a unique b such that (a,b) in f. This unique b is denoted $f(a)$.