CHAPTER 2: Numbers of Various Sorts

N: natural numbers, simplest numbers of all, "counting numbers", 1, 2, 3, ...
↓ doesn't include 0

most basic property: mathematical induction

P(x): property P holds for a number x

mathem. induction:

P(x) is true for all $x \in N$ if

(1) P(1) is true
(2) P(k) true $\Rightarrow$ P(k+1) true

another formulation:

If A is any collection (or set, a synonym) of natural numbers and

(1) 1 in A
(2) k in A $\Rightarrow$ k+1 in A

then A is N.

* empty collection, null set, ϕ: set A containing no natural numbers

one more formulation:
Principle of Complete Induction

A set of N and (1) 1 in A
(2) k+1 in A if 1, ..., k in A
$\Rightarrow A = N$

this principle is a consequence of the ordinary principle of induction

recursive definitions

ex:
$n! = n(n-1)\cdot \cdots \cdot 2 \cdot 1$ can be defined as (1) $1! = 1$
(2) $n! = n \cdot (n-1)!$

ex:
$\sum_{i=1}^{n} a_i$ is defined as (1) $\sum_{i=1}^{1} a_i = a_1$

(2) $\sum_{i=1}^{n} a_i = a_n + \sum_{i=1}^{n-1} a_i$

$\mathbb{Z}$: Integers

$\ldots, -2, -1, 0, 1, 2, \ldots$

→ P7 fails: an inverse does not exist for any member of $\mathbb{Z}$

$\mathbb{Q}$: Rational Numbers

quotients m/n of integers, $n \neq 0$

→ P1-P12 true

$\mathbb{R}$: Real Numbers

Rational + Irrational numbers

represented by infinite decimals, e.g. $\sqrt{2}, \pi$

Notes

→ every $x \in \mathbb{N}$ can be written as $2k$ or $2k+1$ for some $k \in \mathbb{Z}$

$2k$: even

$2k+1$: odd