



**İSTANBUL TECHNICAL UNIVERSITY  
INFORMATICS INSTITUTE**

**2019-2020 FALL  
BBL501E PROBABILITY AND STOCHASTIC PROCESSES  
FINAL EXAMINATION: TAKE-HOME PART**

**Assigned: 26 December 2019**

**Due: 04 January 2020, 09:00**

*I, the undersigned, pledge to uphold the ethical regulations and practices observed at İstanbul Technical University, and hereby declare that I am submitting my own work, and I have not cheated or plagiarized the work of others, nor have I given away my own work to other students enrolled in this class. I understand that collaboration beyond exchanging ideas and methodologies will constitute cheating, and that I will be referred to the disciplinary board of the university in case such an offense is identified.*

Name: \_\_\_\_\_

Signature: \_\_\_\_\_

- Print this document and write your answers on it. Make sure your answers are legible, and submit the document neatly organized and stapled.
- Use a pen for the cover page, but a pencil for the answers.

*Please leave this grading table empty.*

Q1	Q2	Total
50	50	100

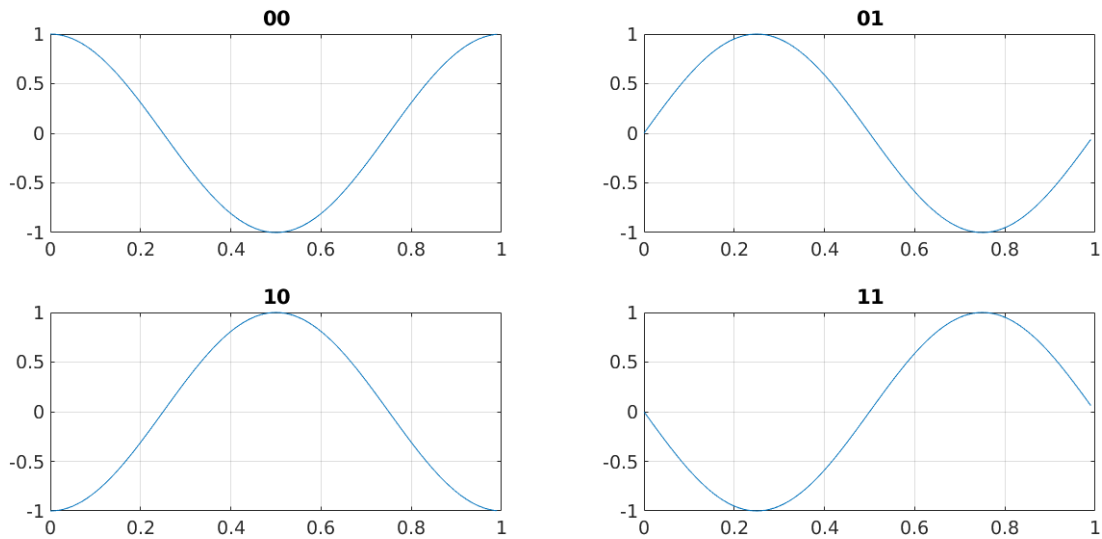
**Question 1.** Two kids, Ali and Ahmed, are playing a game on the street. They observe the cars passing through the street, and Ali earns a point when a black car passes, whereas Ahmed earns a point when a white car passes. The first to collect 5 points wins. Assume that the color of the cars in this city is given as follows (source: [https://en.wikipedia.org/wiki/Car\\_colour\\_popularity](https://en.wikipedia.org/wiki/Car_colour_popularity)):

White	24%	Silver	14%	Brown	6%
Black	23%	Blue	8%	Green	1%
Grey	15%	Red	6%	Others	3%

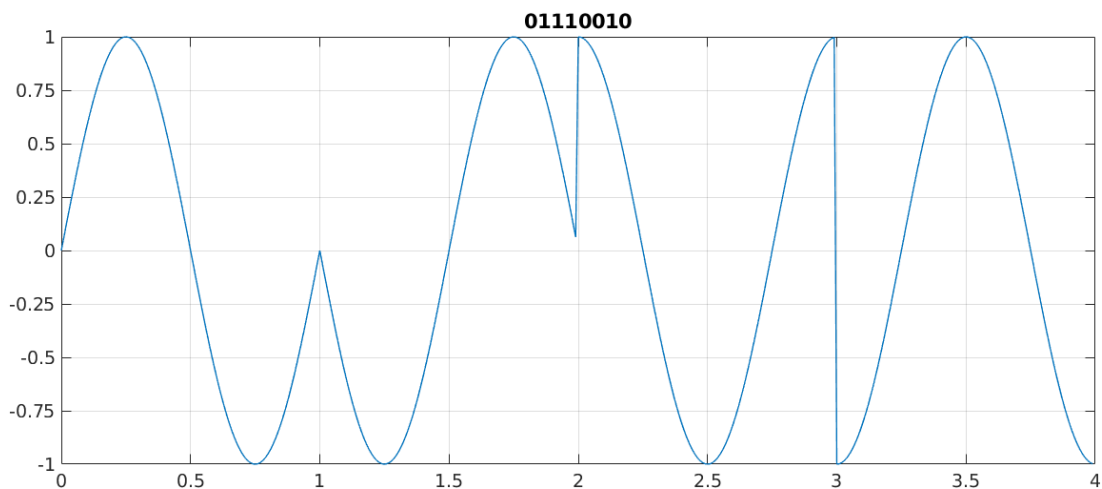
Assume that each passing car is independent of the others. Model this game as a discrete-time Markov chain to find the following. (Attach any computer code you use.)

- (a) Probability that Ali wins 5–2.                      (b) Probability that Ali wins.  
(c) Probability that Ali wins given the first two cars are White and Brown.

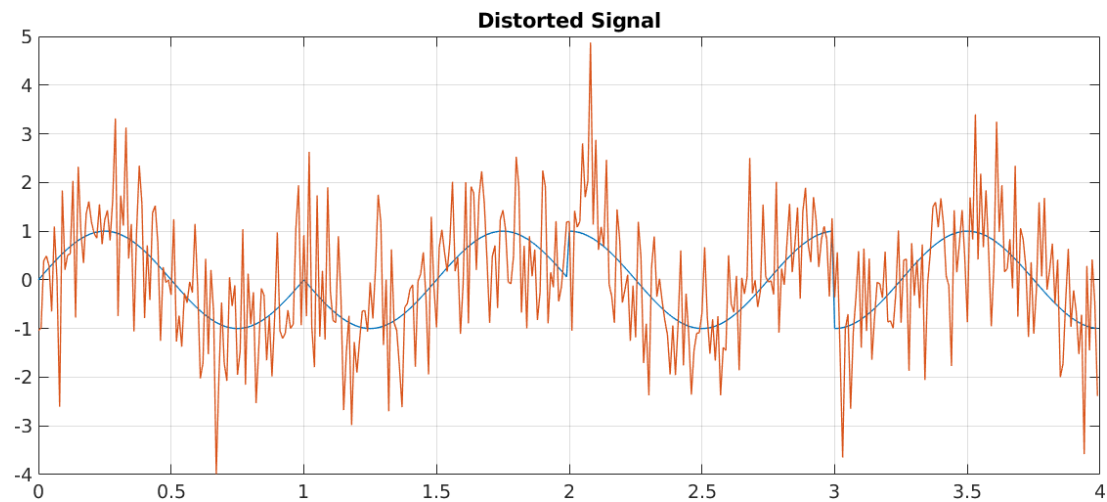
**Question 2.** Consider a wireless digital communication system in which every two bits (called a symbol) are transmitted together using a different waveform. The waveforms for each symbol, which are sinusoid signals with varying phases, are given below. (For the interested, this is a simple QPSK scheme.)



For example, the binary stream 01110010 would be transmitted as follows:



We will assume that there is Gaussian noise in the channel with zero mean and  $\sigma^2$  variance. Therefore, the signal at the receiver is distorted. Below, a sample received signal for  $\sigma = 1$  is given along with the transmitted signal.



The data points for the four waveforms as well as a distorted signal (obtained with  $\sigma = 2$ ) is provided in Ninova. For those who can use Matlab, the file `q2.mat` includes all five arrays. Otherwise, you can use the `txt` files. Determine the binary stream that this data represents. For this task, you need to find which symbol is most similar to each equal-size partition in the given data. Clearly explain your methodology, and attach any computer code you use. Finally, extract the text message carried in the given data, where 8-bit ASCII encoding (for more information: <https://en.wikipedia.org/wiki/ASCII>) was used.