CS 763: Problem Set: Due: 10:00 PM, 18-Feb

- Please write (only if true) the honor code. If you used any source (person or thing) explicitly state it.
- Important: This is an INDIVIDUAL assignment.
- Always provide a brief explanation. (The length of the explanation required has been forecasted with the amount of space provided.)
- Submit following files in folder name lab03_roll_XX :
  1. readme.txt (case sensitive name). This text file contains identifying information, honor code, links to references used
  2. ReflectionEssay.pdf is optional but a brief one would be nice.
  3. lab03_roll_XX.pdf (includes all solutions).
  4. All relevant tex source (and images only if necessary). No other junk files, please.

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1. State whether or not the following points are the same and explain why.

   (a) $A[2, -1, 3]$, $B[4, -2, 6]$ (b) $A[\sqrt{2}/2, -1, 0]$, $B[1, -\sqrt{2}, 0]$

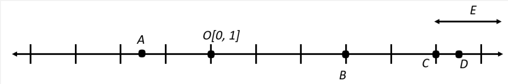
   - In terms of the coordinate geometry the given pair of points are distinct in both the cases.
   - While if we consider the points in a projective plane then the given pair of points are same as they lie on a single ray originating from the origin. This is true for both the cases a and b.

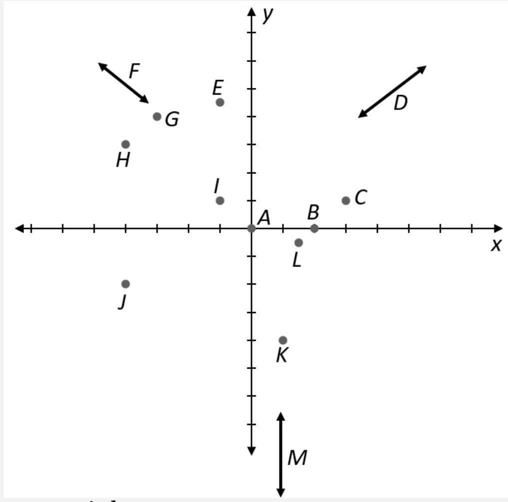
2. In projective three-space, what are the standard homogeneous coordinates of (a) the origin and (b) ideal points determined by the intersections of the extensions of the coordinate axes and the ideal plane?
   In projective three-space the standard homogeneous coordinates of,

   (a) the origin is represented by (0, 0, 1).

   (b) the horizontal and vertical directions are represented by points (1,0,0) and (0,1,0) respectively.

3. Write standard homogeneous coordinates for the points specified in uppercase characters. (Use left and right to distinguish.)





- left
  - $O[0,1]$
  - $A[-0.5,1]$
  - $B[4,1]$
  - $C[6,1]$
  - $D[6.5,1]$
  - $E[1,0]$
- right
  - $A[0,0,1]$
  - $B[0,2,1]$
  - $C[3,1,1]$
  - $D[1,1,0]$
  - $E[-1,4.5,1]$
  - $F[-1,1,0]$
  - $G[-3,4,1]$
  - $H[-4,3,1]$
  - $I[-1,1,1]$
  - $J[-4,-2,1]$
  - $K[1,-4,1]$
  - $L[1.5,-0.5,1]$
  - $M[0,1,0]$

4. Which of the following points lie on the line $3p_1 - 2p_2 + 5p_3 = 0$? Why?

   (a) $A[1,1,2]$ (b) $B[4,1,-2]$

   The point $p = [x,y,z,1]$ lies on the line $l = [a,b,c,d]$ iff $p^T l = l^T p = 0$. Therefore, $B[4,1,-2]$ lies on the given line.

5. Write the coordinates of the lines that are the extensions to the projective plane of the following Euclidean lines.

(a) $3x + 2y = 6$

(b) $4x + 5y + 7 = 0$

The coordinates of the lines that are the extensions to the projective plane of the given Euclidean lines are,

(a) $[3, 2, -6]$

(b) $[4, 5, 7]$

6. Sketch each line in the projective plane whose equation is given.

(a) $2p_1 + 3p_2 + 5p_3 = 0$

(b) $3p_1 - 2p_2 - p_3 = 0$

7. In each of the following cases, sketch the line determined by the two given points; then find the equation of the line.

(a) $A[3, 1, 2]$, $B[1, 2, -1]$

(b) $A[2, 1, 3]$, $B[1, 2, 0]$

8. Find the standard homogeneous coordinates of the point of intersection for each pair of lines.

(a) $p_1 + p_2 - 2p_3 = 0, 3p_1 + p_2 + 4p_3 = 0$ (b) $p_1 + p_2 = 0, 4p_1 - 2p_2 + p_3 = 0$

The point of intersection p of two lines u_1 and u_2 can be calculated by,

$$p = u_1 \times u_2$$

Hence the homogenous coordinates of the point of intersection for the given pair of lines are,

(a) $[-3, 5, 1]$

(b) $[\frac{-1}{6}, \frac{1}{6}, 1]$

9. Determine which of the following sets of three points are collinear.

(a) $A[1, 2, 1]$, $B[0, 1, 3]$, $[2, 1, 1]$ (b) $A[1, 2, 3]$, $B[2, 4, 3]$, $[1, 2, -2]$

To determine whether three points A, B, C are collinear, it would suffice to check if the determinant of the 3×3 matrix containing the points is zero.

$$\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = 8 \neq 0 \qquad \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 3 \\ 1 & 2 & -2 \end{vmatrix} = 0$$

Hence, the set of points $A[1, 2, 3]$, $B[2, 4, 3]$, $[1, 2, -2]$ is collinear.

10. Determine which of the following sets of three lines meet in a point.

(a) $l[1, 0, 1], m[1, 1, 0], n[0, 1, -1]$ (b) $l[1, 0, -1], m[1, -2, 1], n[3, -2, -1]$

To determine whether a set of three lines l, m, n meet at a point, it would suffice to check if the determinant of the matrix $\begin{bmatrix} l & m & n \end{bmatrix}$ is zero.

$$\begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 0 \qquad \begin{vmatrix} 1 & 0 & -1 \\ 1 & -2 & 1 \\ 3 & -2 & -1 \end{vmatrix} = 0$$

Accordingly, both the given set of three lines meet in a point.