UNIT - III

1. A beam 3 m long, simply supported at its ends, is carrying appoint load at its center. If the slope at the ends is 1^0, find the deflection at the mid span of the beam. [A/M-15]

   The deflection at the MID SPAN is given by:
   $y_B = WL^3/ 3EI$
   $y_B = (25000 \times 3000^3) / (3 \times 2.1 \times 10^5 \times 10^8)$
   y_B = 10.71 mm
   The deflection at the free end is given by:
   $y_B = WL^3/ 3EI$
   $y_B = (25000 \times 3000^3) / (3 \times 2.1 \times 10^5 \times 10^8)$
   y_B = 10.71 mm

2. Define: conjugate beam. [A/M-15]

   Conjugate beam is an imaginary beam of length equal to that of original beam but for which load diagram is M/EI diagram.

3. Write the maximum value of deflection for a simply supported beam of constant EI, span L carrying central concentration loud W. [N/D-16]

   The deflection at the centre of a simply supported beam carrying a point load at the centre is given by: $y_c = -(WL^3/ 48EI)$

4. Where the maximum deflection will occur in a simply supported beam loaded with UDL of w kN/m run? [N/D-16]

   The deflection at the centre of a simply supported beam carrying a point load at the centre is given by: $y_c = -(WL^3/ 48EI)$

5. What are the advantages of Macaulay's method over double integration method for beam deflection analysis? [N/D-15]

   Macaulay's method is used in finding slope and deflection at any point of a beam. The points used in this method are:
   a) Brackets are to be integrated as a whole
   b) Constants are written after the first term
   c) The section, for which BM is to be found, should be taken in the last part

6. What is meant by propped cantilever? [N/D-15]

A cantilever which has an additional support at the free end is termed as propped cantilever.

7. What are the methods for finding out the slope and deflection at a section? (N/D -14)

The important methods used for finding out the slope and deflection at a section in a loaded beam are:

a) Double integration method
b) Moment area method
c) Macaulay's method

The first two methods are suitable for a single load, whereas the last one is suitable for several loads.

8. What is a conjugate beam? (A/M-15)

Conjugate beam is an imaginary beam of length equal to that of original beam but for which load diagram is M/EI diagram.

9. A cantilever of length 4 m carriesa uniformly varying load of zero at the free end and 50 kN at the fixed end. If $I = 10^8\ mm^4$ and $E = 2.1 \times 10^5\ N/mm^2$, find the deflection at the free end. (A/M-13)

The deflection at the free end is given by:

$y_B = WL^4 / 30EI$

$y_B = (50 \times 4000^4) / (30 \times 2 \times 10^5 \times 10^3)$

$y_B = 21.33$ mm

10. Write an expression for deflection by moment area method. (A/M-10)

The shear stress at a fiber in a section of a beam is given by:

$y = Ax / EI$

where A is area of BM diagram between A and B
and x is distance of CG of area from B

PART B

1. (a) A beam AB of span 7m ios simply supported at its ends A and B. it carries appoint load of 10kN at a distance of 3m from the ends A and a UDL of 6 kN/m over rigid half span length. Determine (i) the maximum deflection in the beam and (ii) slopes at the ends. Take EI= 10000 kN-m^2. [A/M-13,15],[N/D-

Solution. Span of the beam, $l = 4\text{m}$, $E = 200 \times 10^6\ \text{kN/m}^2$, $I = 20 \times 10^{-6}\ \text{m}^4$

To calculate reaction at B taking moments about A, we get

$$R_B \times 4 = 20 \times 1 + 10 \times 2\left(\frac{2}{2} + 1 + 1\right) = 80$$

$\therefore$ $R_B = 20$ kN

Also, $R_A + R_B = 20 + 10 \times 2 = 40$

$\therefore$ $R_A = 20$ kN

Using Macaulay's method consider any section XX at a distance x from the end A; the bending moment at the section XX,

$$M_x = EI\frac{d^2y}{dx^2} = 20x \,|\, -20(x-1) \,|\, -\frac{10(x-2)^2}{2} \quad \text{...(i)}$$

Integrating, we get

$$EI\frac{dy}{dx} = 10x^2 + C_1 \,|\, -10(x-1)^2 \,|\, -\frac{5}{3}(x-2)^3 \quad \text{...(ii)}$$

Integrating agian, we get

$$EIy = \frac{10}{3}x^3 + C_1x + C_2 \,\Big|\, -\frac{10}{3}(x-1)^3 - \frac{5}{12}(x-2)^4 \quad \text{...(iii)}$$

When, $x = 0, y = 0$

$\therefore$ $C_2 = 0;$

When, $x = 4\text{m}, y = 0$

$$\therefore \quad 0 = \frac{10}{3} \times 4^3 + 4C_1 - \frac{10}{3}(4-1)^3 - \frac{5}{12}(4-2)^4$$

$$= 213.33 + 4\,C_1 - 90 - 6.67$$

$\therefore$ $C_1 = -29.16$

Hence, the slope and deflection equations are

$$EI\frac{dy}{dx} = 10x^2 - 29.16 \,\Big|\, -10(x-1)^2 - 5/3(x-2)^3 \quad \text{...Slope equation}$$

and, $$EIy = \frac{10}{3}x^3 - 29.16x \,\Big|\, -\frac{10}{3}(x-1)^3 \,\Big|\, -\frac{5}{12}(x-2)^4 \quad \text{...Deflection equation}$$

(i) Deflection at C, y_C :

Putting $x = 2\text{m}$ in the deflection equation, we get

$$EI\,y_C = \frac{10}{3} \times 2^3 - 29.16 \times 2 - \frac{10}{3}(2-1)^3$$

$$= 26.67 - 58.32 - 3.33 = -34.98$$

$$\therefore \quad y_C = -\frac{34.98}{EI} = -\frac{34.98}{200 \times 10^6 \times 20 \times 10^{-6}} \times 10^3\ \text{mm} = -8.74\ \text{mm}$$

Hence, $y_C = 8.74$ **mm** (*downward*) **(Ans.)**

(ii) Maximum deflection, y_{max} :

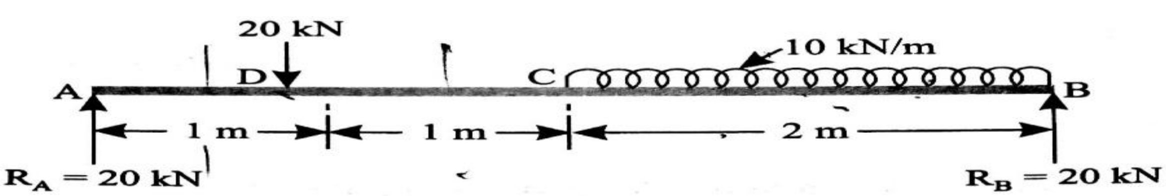
The maximum deflection will be very near to the mid-point C. Let us assume that it occurs in the section between D and C.

For maximum deflection equating the slope at the section to zero, we get

$$EI\frac{dy}{dx} = 10x^2 - 29.16 - 10(x - 1^2) = 0$$

or, $$10x^2 - 29.16 - 10\,(x^2 - 2x + 1) = 0$$

or, $$10x^2 - 29.16 - 10x^2 + 20x - 10 = 0$$



or, $$x = \frac{39.16}{20} = 1.958\,\text{m}$$

Putting the value of x in the deflection equation, we get

$$EI\,y_{max} = \frac{10}{3} \times (1.958)^3 - 29.16 \times 1.958 - \frac{10}{3}(1.958 - 1)^3$$

$$= 25.02 - 57.09 - 2.93 = -35$$

$$\therefore \quad y_{max} = -\frac{35}{EI} = -\frac{35}{200 \times 10^6 \times 20 \times 10^{-6}} \times 10^3\ \text{mm} = -8.75\,\text{mm}$$

i.e. $\quad y_{max}$ **= 8.75 mm (*downward*) (Ans.)**

(*iii*) Slope at the end A, θ_A :

Putting $x = 0$ in the slope equation, we get

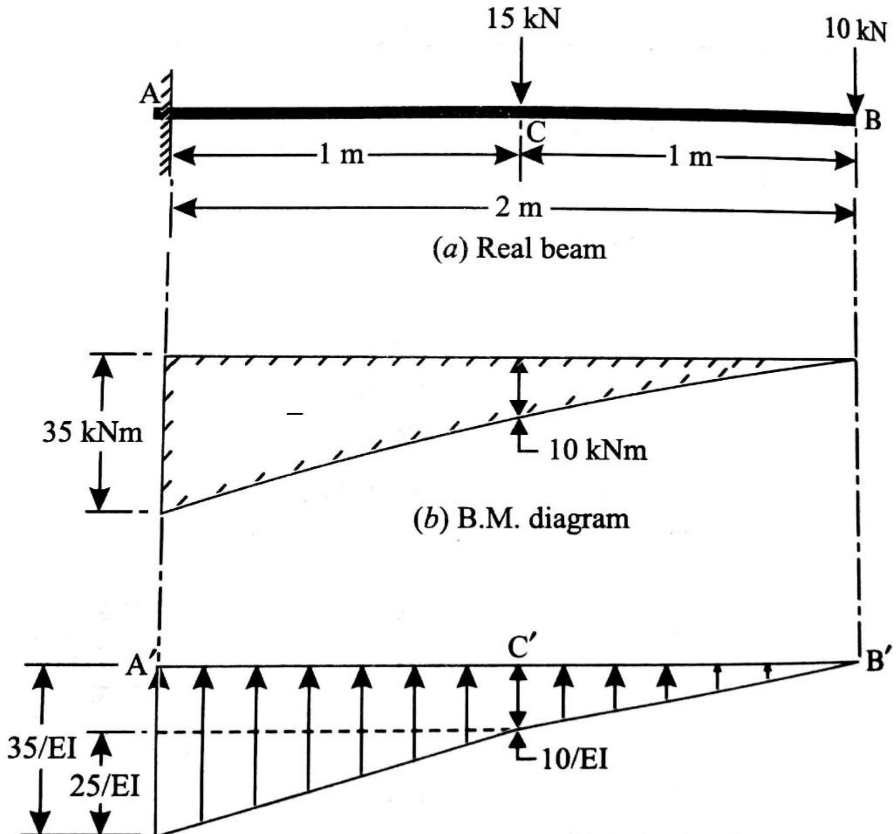
$$EI\frac{dy}{dx} = -29.16$$

$$\therefore \quad \theta_A = \frac{dy}{dx} = -\frac{29.16}{EI} = -\frac{29.16}{200 \times 10^6 \times 20 \times 10^{-6}} = -0.00729\,\text{radian}$$

$$= -0.00729 \times \frac{180}{\pi} = -0.417°$$

Hence $\quad \theta_A$ **= –0.417° (Ans.)**

2. (b) a cantilever of length 'L' is carrying a load of W at the free end and another load of 2 W at its mid span. Determine the slope and deflection of the cantilever at tehe free end using conjucate beam method. Take the flexural rigidity for the half length from fixed end as twice that of the remaining length. [A/M-15], [N/D-15]



(c) Conjugate beam with M/EI loading

The deflection at the free end B, y_B = The B.M. at B' (conjugate beam)

$$\therefore \quad y_B = -\left[\frac{10}{EI}\times 1\times\left(\frac{1}{2}+1\right)+\frac{1}{2}\times 1\times\frac{25}{EI}\times\left(\frac{2}{3}\times 1+1\right)+\frac{1}{2}\times 1\times\frac{10}{EI}\times\frac{2}{3}\right]$$

$$= -\frac{1}{EI}\left(15+\frac{125}{6}+\frac{10}{3}\right) = -\frac{235}{6EI}$$

$$= -\frac{235}{6\times 200\times 10^{6}\times 15\times 10^{-6}}\times 10^{3} = -13\cdot 055 \text{ mm}$$

Hence, *deflection at the free end* = **13·055 mm** (*downward*) **(Ans.)**

3. (a) A SSB of span 6m carries UDL 5 kN/m over a length of 3m extending from left end. Calculate deflection at mid- span. E= $2x10^5$ N/mm2, I= 6.2 x 10^6 mm^4 [N/D-15, 16], [M/J-14]

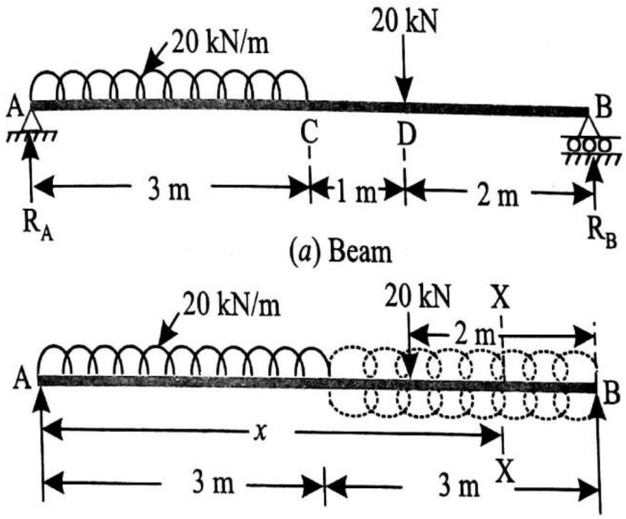
Solut

To find reaction R_B taking moments about A, we have

$$R_B \times 6 = 20 \times 3 \times \frac{3}{2} + 20 \times 4 = 90 + 80 = 170 \qquad \therefore\ R_B = 28{\cdot}33 \text{ kN}$$

and,
$$R_A = (20 \times 3 + 20) - 28{\cdot}33 = 51{\cdot}67 \text{ kN}$$

On part CB of the given beam assume a U.D.L. @ 20 kN/m applied both from above and below so that both these added loads neutralise each other and the net effect remains unchanged [Fig. 8·85 (b)].



(a) Beam

(b) Beam with modified loading

Fig. 8.85

Consider a section XX at a distance x from the support A.

$$M_x = EI\,\frac{d^2y}{dx^2} = 51{\cdot}67x - 10x^2 - 20\,(x-4) + 10\,(x-3)^2 \qquad \text{...}(i)$$

By integrating the above equation successively, we get

$$EI\,\frac{dy}{dx} = \frac{51{\cdot}67x^2}{2} - \frac{10x^3}{3} - 10\,(x-4)^2 + \frac{10\,(x-3)^3}{3} + C_1 \qquad \text{...}(ii)$$

and,
$$EIy = \frac{51{\cdot}67x^3}{6} - \frac{10x^4}{12} - \frac{10\,(x-4)^3}{3} + \frac{10\,(x-3)^4}{12} + C_1x + C_2 \qquad \text{...}(iii)$$

(where, C_1 and C_2 are the constants of integration)

When, $x = 0,\ y = 0 \qquad \therefore\ C_2 = 0$

When, $x = 6 \text{ m},\ y = 0$

$$\therefore \quad 0 = \frac{51 \cdot 67 \times 6^3}{6} - \frac{10 \times 6^4}{12} - \frac{10\,(6-4)^3}{3} + \frac{10\,(6-3)^4}{12} + 6C_1$$

$$= 1860 - 1080 - 26{\cdot}67 + 67{\cdot}5 + 6C_1$$

$$\therefore \quad C_1 = -\,136{\cdot}8$$

Hence the slope and deflection equations are:

$$EI\,\frac{dy}{dx} = \frac{51 \cdot 67x^2}{2} - \frac{10x^3}{3} - 10\,(x-4)^2 + \frac{10\,(x-3)^3}{3} - 136 \cdot 8 \quad \textit{...Slope equation}$$

$$EIy = \frac{51 \cdot 67x^3}{6} - \frac{10x^4}{12} - \left[\frac{10\,(x-4)^3}{3}\right] + \left[\frac{10\,(x-3)^4}{12}\right] - 136 \cdot 8x \quad \textit{...Deflection equation}$$

Deflection at C, y_C :

Deflection y_C at C, where $x = 3$ m (neglecting square brackets in which the terms become –ve) is

$$EIy_C = \frac{51 \cdot 67 \times 3^3}{6} - \frac{10 \times 3^4}{12} - 136 \cdot 8 \times 3 = 232 \cdot 5 - 67 \cdot 5 - 410 \cdot 4 = -\,245 \cdot 4$$

$$\therefore \quad y_C = -\,\frac{245 \cdot 4}{EI} = -\,\frac{245 \cdot 4}{200 \times 10^6 \times 20 \times 10^{-5}} \times 10^3 \text{ mm} = -\,6 \cdot 135 \text{ mm}$$

i.e. $\quad$ y_C **= 6·135 mm** (*downward*) **(Ans.)**

Deflection at D, y_D :

To obtain deflection y_D at D put $x = 4$ m in the deflection equation.

$$EIy_D = \frac{51 \cdot 67 \times (4)^3}{6} - \frac{10 \times (4)^4}{12} + \frac{10\,(4-3)^4}{12} - 136 \cdot 8 \times 4$$

$$= 551{\cdot}15 - 213{\cdot}33 + 0{\cdot}833 - 547{\cdot}2 = -\,208{\cdot}55$$

$$\therefore \quad y_D = -\,\frac{208 \cdot 55}{EI} = -\,\frac{208 \cdot 55}{200 \times 10^6 \times 20 \times 10^{-5}} \times 10^3 \text{ mm} = -\,5 \cdot 21 \text{ mm}$$

Hence, $\quad$ y_D **= 5·21 mm** (*downward*) **(Ans.)**

4. (b) A cantilever beam 4m long carries a load of 500 kN at a distance of 2m from the free end, and a load of W at the free end. If the deflection at the free end is 25mm, calculate the magnitude of the load W, and the slope at the free end. E= 200kN/mm^2. I= 5x 10^7 mm^4.

[N/D-14, 16]

Length, $L = 3\text{ m} = 3000\text{ mm}$

Load at free end, $W_1 = 2\text{ kN} = 2000\text{ N}$

Load at a distance one m from free end,

$W_2 = 4\text{ kN} = 4000\text{ N}$

Distance AC, $a = 2\text{ m} = 2000\text{ mm}$

Value of $E = 2 \times 10^5\text{ N/mm}^2$

Value of $I = 10^8\text{ mm}^4$

Let y_1 = Deflection at the free end due to load 2 kN alone

y_2 = Deflection at the free end due to load 4 kN alone.

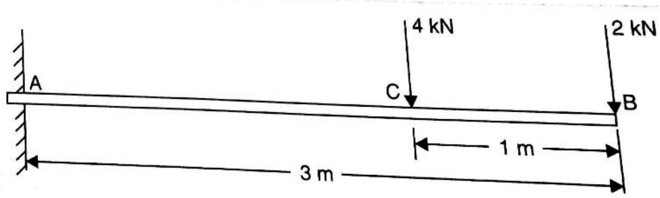


Fig. 13.6

Downward deflection due to load 2 kN alone at the free end is given by equation (13.2 A) as

$$y_1 = \frac{WL^3}{3EI} = \frac{2000 \times 3000^3}{3 \times 2 \times 10^5 \times 10^8} = 0.9\text{ mm}.$$

Downward deflection at the free end due to load 4 kN (*i.e.*, 4000 N) alone at a distance 2 m from fixed end is given by (13.4) as

$$y_2 = \frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(L - a)$$

$$= \frac{4000 \times 2000^3}{3 \times 2 \times 10^5 \times 10^8} + \frac{4000 \times 2000^2}{2 \times 2 \times 10^5 \times 10^8}(3000 - 2000)$$

$$= 0.54 + 0.40 = 0.94\text{ mm}$$

$\therefore$ **Total deflection at the free end**

$$= y_1 + y_2$$

$$= 0.9 + 0.94 = \mathbf{1.84\text{ mm}}. \quad \text{Ans.}$$

5. A beam of length 6 m is simply supported at its ends and carries two point loads of 48 kN and 40 kN at a distance of 1 m and 3 m respectively from the left support. Find

(i) Deflection under each load

(ii) Maximum deflection

(iii) The point at which the maximum deflection occurs.

Take E = 2 10 N/mm , 5 2 · I = 6 4 85·10 mm. [M/J-10,12], [N/D-09]

Length, $L = 4$ m

Value of $E = 2 \times 10^5$ N/mm^2 $= 2 \times 10^5 \times 10^6$ N/m^2

$= 2 \times 10^5 \times 10^3$ kN/m^2 $= 2 \times 10^8$ kN/m^2

Value of $I = 10^8$ mm^4 $= \dfrac{10^8}{10^{12}}$ m^4 $= 10^{-4}$ m^4.

As the load on the beam is symmetrical as shown in Fig. 14.4 (*a*), the reactions R_A and R_B will be equal to 3 kN.

Now B.M. at A and B are zero.

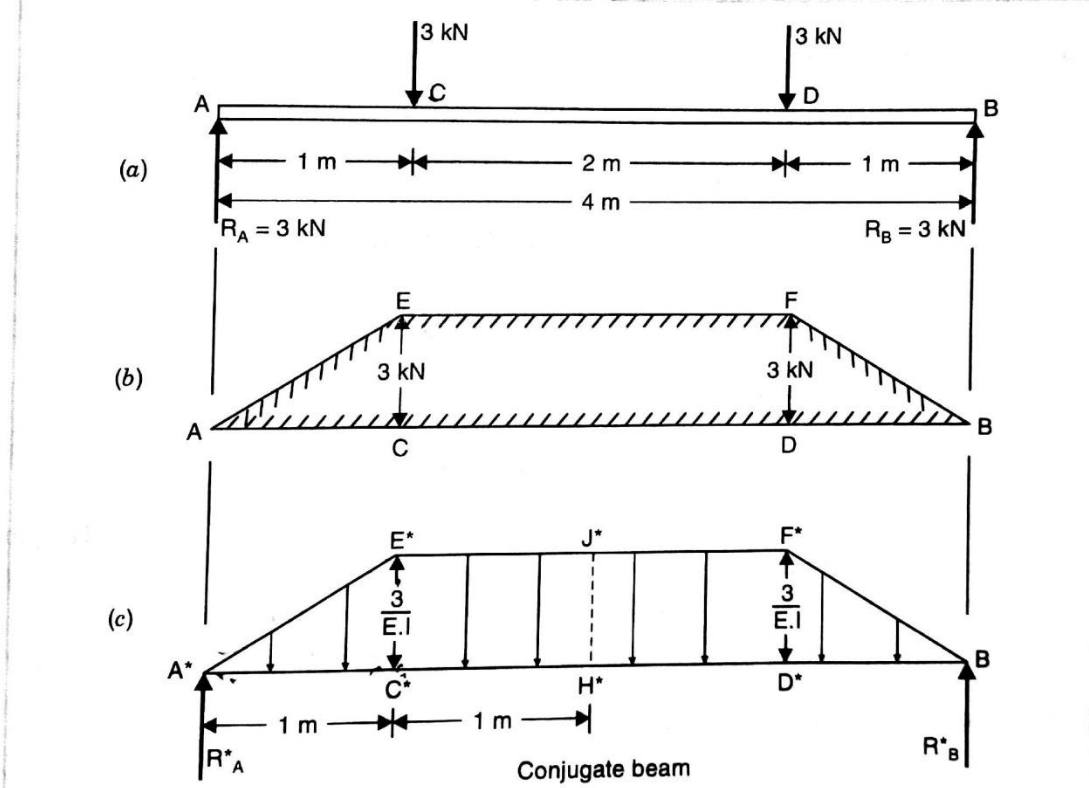


Fig. 14.4

B.M. at $C = R_A \times 1 = 3 \times 1 = 3$ kNm

B.M. at $D = R_B \times 1 = 3 \times 1 = 3$ kNm

Now B.M. diagram can be drawn as shown in Fig. 14.4 (*b*).

Now by dividing the B.M. at any section by EI, we can construct the conjugate beam as shown in Fig. 14.4 (*c*). The loading are shown on the conjugate beam.

Let $R_A^* =$ Reaction at A^* for the conjugate beam and

$R_B^* =$ Reaction at B^* for conjugate beam

[redacted]

The loading on the conjugate beam is symmetrical

$\therefore$ $R_A^* = R_B^*$ = Half of total load on conjugate beam

$$= \frac{1}{2} \text{ [Area of trapezoidal } A^*B^*F^*E^*]$$

$$= \frac{1}{2}\left[\frac{(E^*F^* + A^*B^*)}{2} \times E^*C^*\right]$$

$$= \frac{1}{2}\left[\frac{(2+4)}{2} \times \frac{3}{EI}\right] = \frac{4.5}{EI}$$

(i) Slope at each end and under each load

Let θ_A = Slope at A for the given beam *i.e.*, $\left(\frac{dy}{dx}\right)$ at A

θ_B = Slope at B for the given beam

θ_C = Slope at C for the given beam and

θ_D = Slope at D for the given beam

Then according to conjugate beam method,

θ_A = Shear force at A^* for conjugate beam = R_A^*

$$= \frac{4.5}{EI} = \frac{4.5}{2 \times 10^8 \times 10^{-4}} = \mathbf{0.000225\ rad.} \quad \text{Ans.}$$

$$\theta_B = R_B^* = \frac{4.5}{EI} = \mathbf{0.000225\ rad.} \quad \text{Ans.}$$

θ_C = Shear force at C^* for conjugate beam

= R_A^* – Total load $A^*C^*E^*$

$$= \frac{4.5}{EI} - \frac{1}{2} \times 1 \times \frac{3}{EI} = \frac{3}{EI}$$

$$= \frac{3}{2 \times 10^8 \times 10^{-4}} = \mathbf{0.00015\ rad.} \quad \text{Ans.}$$

Similarly, θ_D = **0.00015 rad.** Ans. (By symmetry)

(ii) Deflection under each load

Due to symmetry, the deflection under each load will be equal

Let y_C = Deflection at C for the given beam and

y_D = Deflection at D for the given beam.

Now according to conjugate beam method,

y_C = B.M. at C^* for conjugate beam

= R_A^* × 1.0 – (Load $A^*C^*E^*$) × Distance of C.G. of $A^*C^*E^*$ from C^*

$$= \frac{4.5}{EI} \times 1 - \left(\frac{1}{2} \times 1 \times \frac{3}{EI}\right) \times \frac{1}{3}$$

$$= \frac{4.5}{EI} - \frac{0.5}{EI} = \frac{4.0}{EI}$$

$$= \frac{4}{2 \times 10^8 \times 10^{-4}} \text{ m} = \frac{4 \times 1000}{2 \times 10^4} \text{ mm}$$

= **0.2 mm.** Ans.

Also y_D = 0.2 mm.

Deflection at the centre of the beam

= B.M. at the centre of the conjugate beam

= $R_A^* \times 2.0$ – Load $A^*C^*E^*$

× Distance of C.G. of $A^*C^*E^*$ from the centre of beam

– Load $C^*H^*J^*E^*$

× Distance of C.G. of $C^*H^*J^*E^*$ from the centre of bea

$$= \frac{4.5}{EI} \times 2.0 - \left(\frac{1}{2} \times 1 \times \frac{3}{EI}\right) \times \left(1 + \frac{1}{3}\right) - \left(1 \times \frac{3}{EI}\right) \times \frac{1}{2}$$

$$= \frac{9}{EI} - \frac{2}{EI} - \frac{1.5}{EI} = \frac{6.5}{EI}$$

$$= \frac{6.5}{2 \times 10^8 \times 10^{-4}} \text{ m} = \frac{6.5 \times 1000}{2 \times 10^4} \text{ mm}$$

= **0.325 mm. Ans.**