The goal of this notebook is to predict GDP in general. I'm training models on all data instead of splitting into training and test because the end goal is to predict the future.

Testing:

- With and without Year label
- For all values versus predicting in the new year, 2021

In [1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

In [4]:
```
df_eurostat = pd.read_csv("https://raw.githubusercontent.com/SDuncan5/Eurostat-Data/ma
df_eurostat = df_eurostat.drop(columns=["Unnamed: 0"])
df_eurostat
```

Out[4]:

	geo	Year	CPI	Immigrants	Population	Housing Index	GDP	emigration	unemployment
0	Austria	2011	93.35	82230.0	8391643.0	81.60	310128.7	51197.0	3.3
1	Austria	2012	95.75	91557.0	8429991.0	87.57	318653.0	51812.0	3.5
2	Austria	2013	97.77	101866.0	8479823.0	92.10	323910.2	54071.0	3.8
3	Austria	2014	99.20	116262.0	8546356.0	95.33	333146.1	53491.0	4.0
4	Austria	2015	100.00	166323.0	8642699.0	100.00	344269.2	56689.0	4.1
...	...	...	...	...	...	...	...	...	...
280	Slovakia	2017	100.90	7188.0	5439232.0	112.99	84669.9	3466.0	5.4
281	Slovakia	2018	103.46	7253.0	5446771.0	121.32	89874.7	3298.0	4.3
282	Slovakia	2019	106.33	7016.0	5454147.0	132.39	94429.7	3384.0	3.8
283	Slovakia	2020	108.47	6775.0	5458827.0	145.06	93444.1	2428.0	4.4
284	Slovakia	2021	111.53	5733.0	5447247.0	154.33	100255.7	3395.0	4.5

285 rows × 18 columns

In [10]:
```
# not per_capita data
X_reg = df_eurostat[["geo", "Year", "CPI", "Immigrants", "Population", "Housing Index"
X_cap = df_eurostat[["geo", "Year", "CPI", "Immigrants_pcap", "Population", "Housing I
gdp = df_eurostat["GDP"]
gdp_cap = df_eurostat["GDP_pcap"]
```

KNN - General

Feature Selection

In [20]:

```
from itertools import chain, combinations

def powerset(iterable):
    features = list(iterable)
    return chain.from_iterable(combinations(features, r) for r in range(len(features)+

reg_features = ["geo", "Year", "CPI", "Immigrants", "Population", "Housing Index", "em

# Creating a power set of all possible regular features to test for the best model
reg_power_set = []
for subset in powerset(reg_features):
    reg_power_set.append(list(subset))

# First value is empty, so remove
reg_power_set = reg_power_set[1:]
len(reg_power_set)
# That's a lot of combinations
```

Out[20]: 2047

In [21]:

```
# Takes ~5 min to run

from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.neighbors import KNeighborsRegressor
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

# define function to calculate estimate of test error for a given feature set
def get_cv_error(features):

  quant_vars = []
  cat_vars = []
  for feature in features:
    if X_reg.dtypes[feature] == "int64" or  X_reg.dtypes[feature] == "float64":
      quant_vars.append(feature)
    else:
      cat_vars.append(feature)

  ct  = make_column_transformer(
      (StandardScaler(), quant_vars),
      (OneHotEncoder(handle_unknown="ignore"), cat_vars),
      remainder="drop"
  )

  pipeline = make_pipeline(
      ct,
      KNeighborsRegressor(n_neighbors=5)
  )
  # errors from cross-validation
  cv_errs = -cross_val_score(pipeline, X=X_reg[features],
                             y=gdp,
                             scoring="neg_mean_squared_error", cv=10)
  # calculate average of the cross-validation errors
  return cv_errs.mean()
```

```
  ["geo", "Year", "CPI", "Immigrants", "Population", "Housing Index", "emigration", "u

# calculate and store errors for different feature sets
errs = pd.Series()
for features in reg_power_set:
  errs[str(features)] = get_cv_error(features)

errs
```

```
<ipython-input-21-6638aa4ed671>:38: FutureWarning: The default dtype for empty Series will be 'object' instead of 'float64' in a future version. Specify a dtype explicitly to silence this warning.
  errs = pd.Series()
```

Out[21]:

```
['geo']
1.734278e+11
['Year']
7.050848e+11
['CPI']
7.353740e+11
['Immigrants']
1.334403e+11
['Population']
9.331704e+10

...
['geo', 'Year', 'CPI', 'Population', 'Housing Index', 'emigration', 'unemployment', 'total_deaths', 'Exports', 'Imports']                    5.754563e+10
['geo', 'Year', 'Immigrants', 'Population', 'Housing Index', 'emigration', 'unemployment', 'total_deaths', 'Exports', 'Imports']           5.874167e+10
['geo', 'CPI', 'Immigrants', 'Population', 'Housing Index', 'emigration', 'unemployment', 'total_deaths', 'Exports', 'Imports']            4.929362e+10
['Year', 'CPI', 'Immigrants', 'Population', 'Housing Index', 'emigration', 'unemployment', 'total_deaths', 'Exports', 'Imports']           6.116426e+10
['geo', 'Year', 'CPI', 'Immigrants', 'Population', 'Housing Index', 'emigration', 'unemployment', 'total_deaths', 'Exports', 'Imports']    5.931857e+10
Length: 2047, dtype: float64
```

In [23]:

```
errs.sort_values()
```

Out[23]:

```
['geo', 'Year', 'Population', 'Imports']                            3.808488e+10
['geo', 'Year', 'CPI', 'Population', 'Imports']                     3.928704e+10
['geo', 'Year', 'Immigrants', 'Population', 'Imports']              4.095578e+10
['geo', 'Year', 'Immigrants', 'Population', 'Exports', 'Imports']   4.100335e+10
['geo', 'Year', 'CPI', 'Population', 'Exports', 'Imports']          4.312268e+10
                                                                        ...
['Year', 'Housing Index']                                           7.774358e+11
['Year', 'CPI', 'Housing Index']                                    7.798444e+11
['Housing Index', 'unemployment']                                   8.081463e+11
['CPI', 'Housing Index']                                            8.128890e+11
['unemployment']                                                    8.393198e+11
Length: 2047, dtype: float64
```

The features with lowest MSE are:

- ['geo', 'Year', 'Population', 'Imports']
- ['geo', 'Year', 'CPI', 'Population', 'Imports']
- ['geo', 'Year', 'Immigrants', 'Population', 'Imports']
- ['geo', 'Year', 'Immigrants', 'Population', 'Exports', 'Imports']

K Selection

In [25]:
```
ct = make_column_transformer(
    (StandardScaler(), ['Year', 'Population', 'Imports']),
    (OneHotEncoder(handle_unknown="ignore"), ['geo']),
    remainder="drop"
)

pipeline = make_pipeline(
    ct,
    KNeighborsRegressor(n_neighbors=5)
)
```

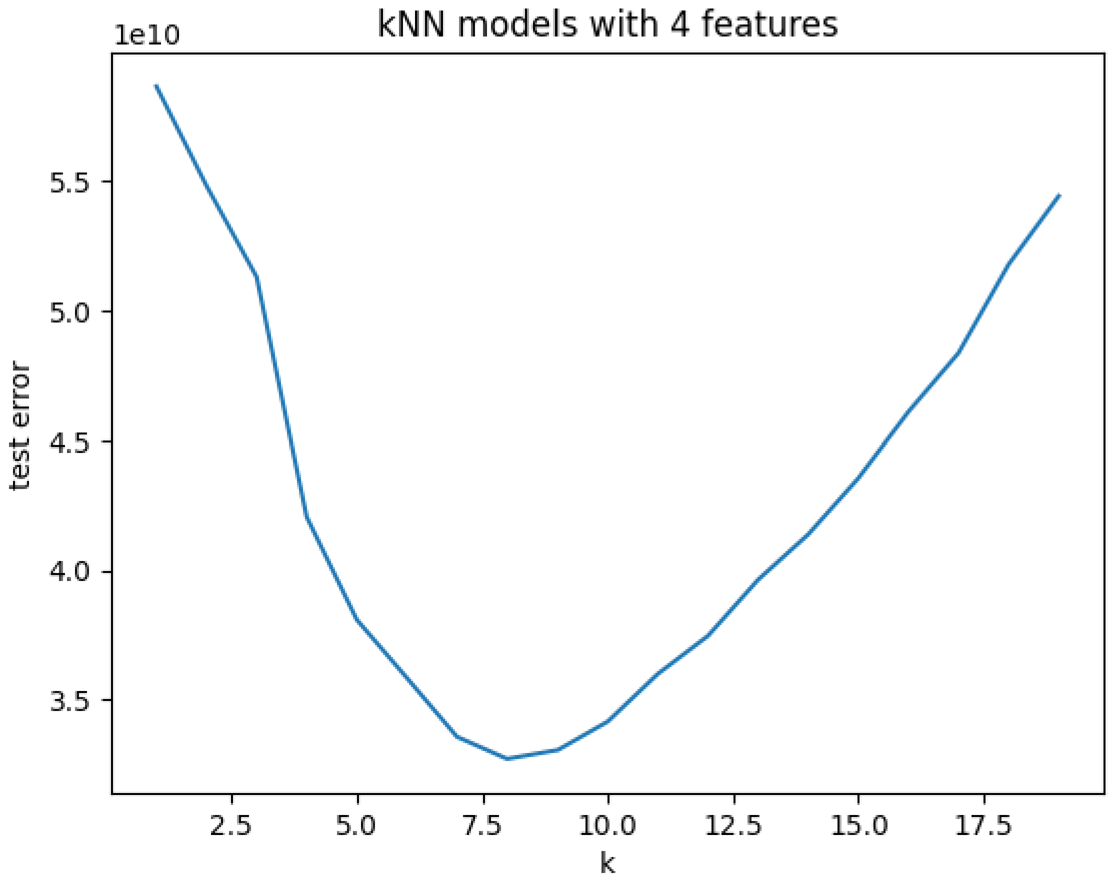
In [26]:
```
from sklearn.model_selection import GridSearchCV

grid_search = GridSearchCV(pipeline,
                           param_grid={
                               "kneighborsregressor__n_neighbors": range(1, 20)
                           },
                           scoring="neg_mean_squared_error",
                           cv=10)
grid_search.fit(X_reg[['geo', 'Year', 'Population', 'Imports']], gdp)
grid_search.best_estimator_
```

Out[26]:

```
▸ Pipeline
  ▸ columntransformer: ColumnTransformer
    ▸ standardscaler        ▸ onehotencoder
    ▸ StandardScaler        ▸ OneHotEncoder
  ▸ KNeighborsRegressor
```

In [27]:
```
df_cv_results_ = pd.DataFrame(grid_search.cv_results_)
df_cv_results_["param_kneighborsregressor__n_neighbors"] = df_cv_results_["param_kneig
df_cv_results_.set_index("param_kneighborsregressor__n_neighbors", inplace = True)
(-df_cv_results_["mean_test_score"]).plot.line(xlabel = "k", ylabel = "test error", ti
```

Out[27]:
```
<Axes: title={'center': 'kNN models with 4 features'}, xlabel='k', ylabel='test erro
r'>
```



The best number of neighbors = 8

Testing Out the Model

K = 8

Features = ['geo', 'Year', 'Population', 'Imports']

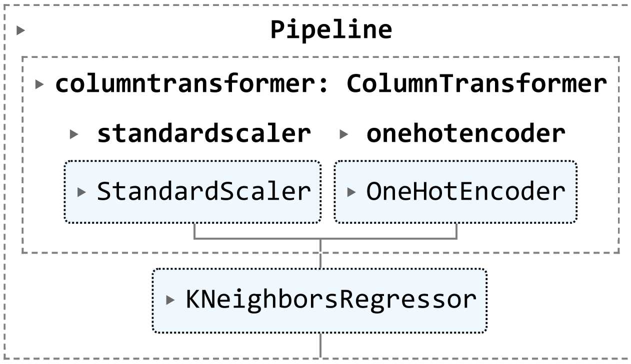
In [33]:
```
from sklearn.model_selection import train_test_split

X_knn1 = X_reg[["geo", "Year", "Population", "Imports"]]
# y = gdp

# split into test and training
X_knn1_train, X_knn1_test, y_knn1_train, y_knn1_test = train_test_split(X_knn1, gdp, t
```

In [34]:
```
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.neighbors import KNeighborsRegressor
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

ct = make_column_transformer(
    (StandardScaler(), ['Year', 'Population', 'Imports']),
    (OneHotEncoder(handle_unknown="ignore"), ['geo']),
    remainder="drop"
)
```

```
pipeline = make_pipeline(
    ct,
    KNeighborsRegressor(n_neighbors=8)
)
```

In [36]:
```
pipeline.fit(X_knn1_train, y_knn1_train)
```

Out[36]:



In [ ]:
```
# cross validation scoring
```

In [42]:
```
scores = -cross_val_score(pipeline,
                          X_knn1_train,
                          y_knn1_train,
                          scoring="neg_root_mean_squared_error",
                          cv=10)
scores.mean()
```

Out[42]:
```
101692.2143834381
```

In [ ]:
```
# test scoring
```

In [38]:
```
print("Train Score : ", pipeline.score(X_knn1_train, y_knn1_train)," Test Score : ", p
```

```
Train Score :  0.9850626807972832  Test Score :  0.9789798238844324
```

In [39]:
```
y_knn1_pred = pipeline.predict(X_knn1_test)
```

RMSE

In [40]:
```
from sklearn.metrics import mean_squared_error

rmse = np.sqrt(mean_squared_error(y_knn1_test, y_knn1_pred))
rmse
```

Out[40]:
```
124842.11379156707
```

Benchmark

In [83]:
```
from sklearn.dummy import DummyRegressor

mean_model = DummyRegressor(strategy="mean")

-cross_val_score(mean_model, X=X_knn1_train, y=y_knn1_train, cv=10,
                 scoring="neg_root_mean_squared_error").mean()
```

Out[83]: 690927.3333981774

In [45]:
```
y_knn1_test.std()
```

Out[45]: 867121.6148310832

Based on these benchmarks, I'd say our model did a much better job!

R^2

In [43]:
```
r_squared = 1 - (((y_knn1_test - y_knn1_pred)**2).mean()) / ((y_knn1_test - y_knn1_pre
r_squared
```

Out[43]: 0.9789898570610891

That's a really good R^2 value. 97.9% of the variability in GDP is explained by this model.

KNN - Predicting for 2021

Removing 2021 from the dataset and seeing how the model performs for an unknown year

In [234...
```
# train
no_2021 = df_eurostat[["geo", "Year", "Population", "Imports", "GDP"]]
no_2021 = no_2021[no_2021["Year"] != 2021]
X_knn_no_2021 = no_2021.drop(columns=["GDP"])
gdp_no_2021 = no_2021["GDP"]


# remove all instances of 2021 (test)
yes_2021 = df_eurostat[["geo", "Year", "Population", "Imports", "GDP"]]
yes_2021 = yes_2021[yes_2021["Year"] == 2021]
X_knn_yes_2021 = yes_2021.drop(columns=["GDP"])
gdp_yes_2021 = yes_2021["GDP"]
```

In [235...
```
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.neighbors import KNeighborsRegressor
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

ct = make_column_transformer(
    (StandardScaler(), ['Year', 'Population', 'Imports']),
    (OneHotEncoder(handle_unknown="ignore"), ['geo']),
    remainder="drop"
)

pipeline = make_pipeline(
    ct,
    KNeighborsRegressor(n_neighbors=8)
)
```

In [236...
```
pipeline.fit(X_knn_no_2021, gdp_no_2021)
```

Out[236]:

Pipeline

columntransformer: ColumnTransformer

standardscaler | onehotencoder

StandardScaler | OneHotEncoder

KNeighborsRegressor

In [237…

```
# cross validation scoring
```

In [238…

```
scores = -cross_val_score(pipeline,
                          X_knn_no_2021,
                          gdp_no_2021,
                          scoring="neg_root_mean_squared_error",
                          cv=10)
scores.mean()
```

Out[238]: 150882.37082297204

In [ ]:

```
# test scoring
```

In [70]:

```
print("Train Score : ", pipeline.score(X_knn_no_2021, gdp_no_2021)," Test Score : ", p
```

```
Train Score :  0.993904443480282  Test Score :  0.9702751426053756
```

In [76]:

```
y_knn_2021_pred = pipeline.predict(X_knn_yes_2021)
```

RMSE

In [239…

```
from sklearn.metrics import mean_squared_error

rmse = np.sqrt(mean_squared_error(gdp_yes_2021, y_knn_2021_pred))
rmse
```

Out[239]: 145482.641500312

Benchmark

In [240…

```
gdp_yes_2021.std()
```

Out[240]: 860533.8484067871

In [222…

```
from sklearn.dummy import DummyRegressor

mean_model = DummyRegressor(strategy="mean")

-cross_val_score(mean_model, X=X_knn_no_2021, y=gdp_no_2021, cv=10,
                 scoring="neg_root_mean_squared_error").mean()
```

Out[222]: 659915.7065210016

Based on this benchmark, I'd say our model did a much better job!

R^2

In [81]:
```
r_squared = 1 - (((gdp_yes_2021 - y_knn_2021_pred)**2).mean()) / ((gdp_yes_2021 - y_kr
r_squared
```

Out[81]:
```
0.970367428062654
```

That's a really good R^2 value. 97.0% of the variability in GDP is explained by this model.

KNN - Comparing to Time Series

Linear Regression - General

Should log transform variables. As seen in the data visualization set, a lot of data is skewed.

In [84]:
```
X_reg
```

Out[84]:

	geo	Year	CPI	Immigrants	Population	Housing Index	emigration	unemployment	total_deatl
0	Austria	2011	93.35	82230.0	8391643.0	81.60	51197.0	3.3	76479
1	Austria	2012	95.75	91557.0	8429991.0	87.57	51812.0	3.5	79436
2	Austria	2013	97.77	101866.0	8479823.0	92.10	54071.0	3.8	79526
3	Austria	2014	99.20	116262.0	8546356.0	95.33	53491.0	4.0	78252
4	Austria	2015	100.00	166323.0	8642699.0	100.00	56689.0	4.1	83073
...	...	...	...	...	...	...	...	...	
280	Slovakia	2017	100.90	7188.0	5439232.0	112.99	3466.0	5.4	53914
281	Slovakia	2018	103.46	7253.0	5446771.0	121.32	3298.0	4.3	54293
282	Slovakia	2019	106.33	7016.0	5454147.0	132.39	3384.0	3.8	53234
283	Slovakia	2020	108.47	6775.0	5458827.0	145.06	2428.0	4.4	59089
284	Slovakia	2021	111.53	5733.0	5447247.0	154.33	3395.0	4.5	73461

285 rows × 11 columns

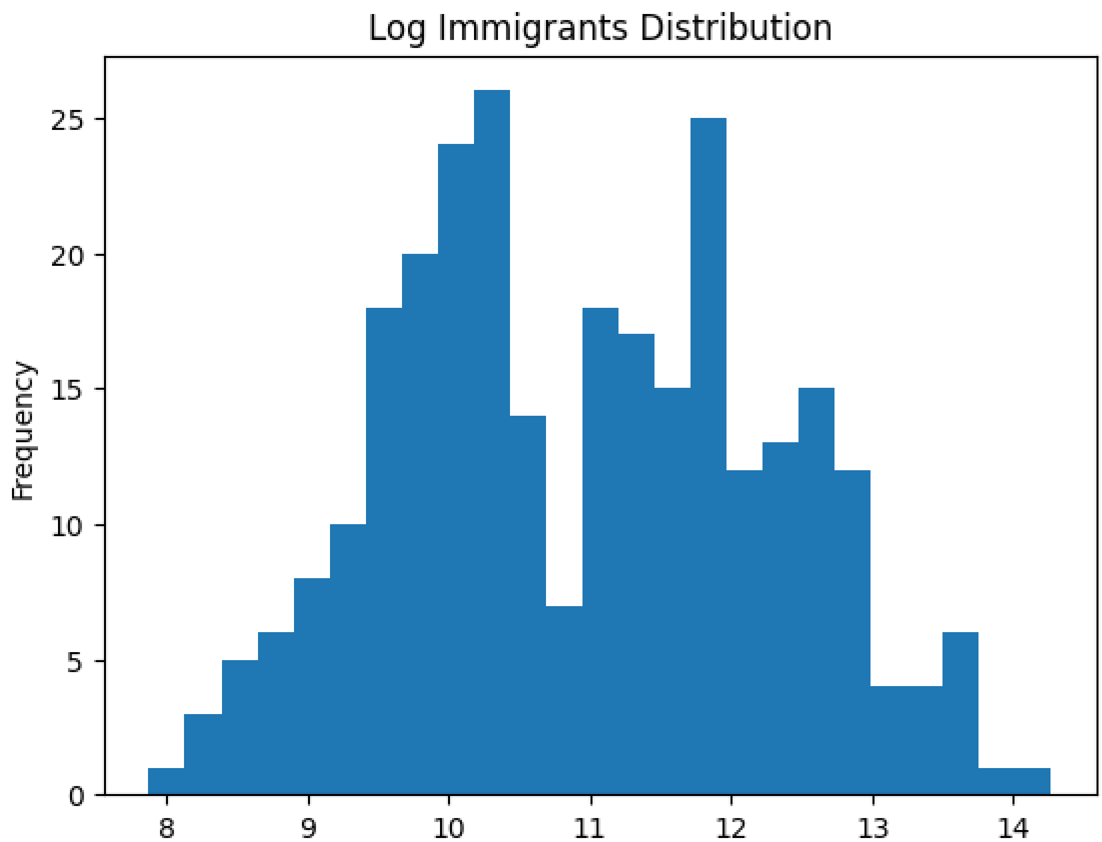
Log Transforming

In [86]:
```
X_reg["log(Immigrants)"] = np.log(X_reg["Immigrants"])
X_reg["log(Immigrants)"].plot.hist(xlabel="Log Number of Immigrants", title="Log Immig
```

```
<ipython-input-86-18e663455fd8>:1: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/us
er_guide/indexing.html#returning-a-view-versus-a-copy
  X_reg["log(Immigrants)"] = np.log(X_reg["Immigrants"])
```

Out[86]:

```
<Axes: title={'center': 'Log Immigrants Distribution'}, ylabel='Frequency'>
```



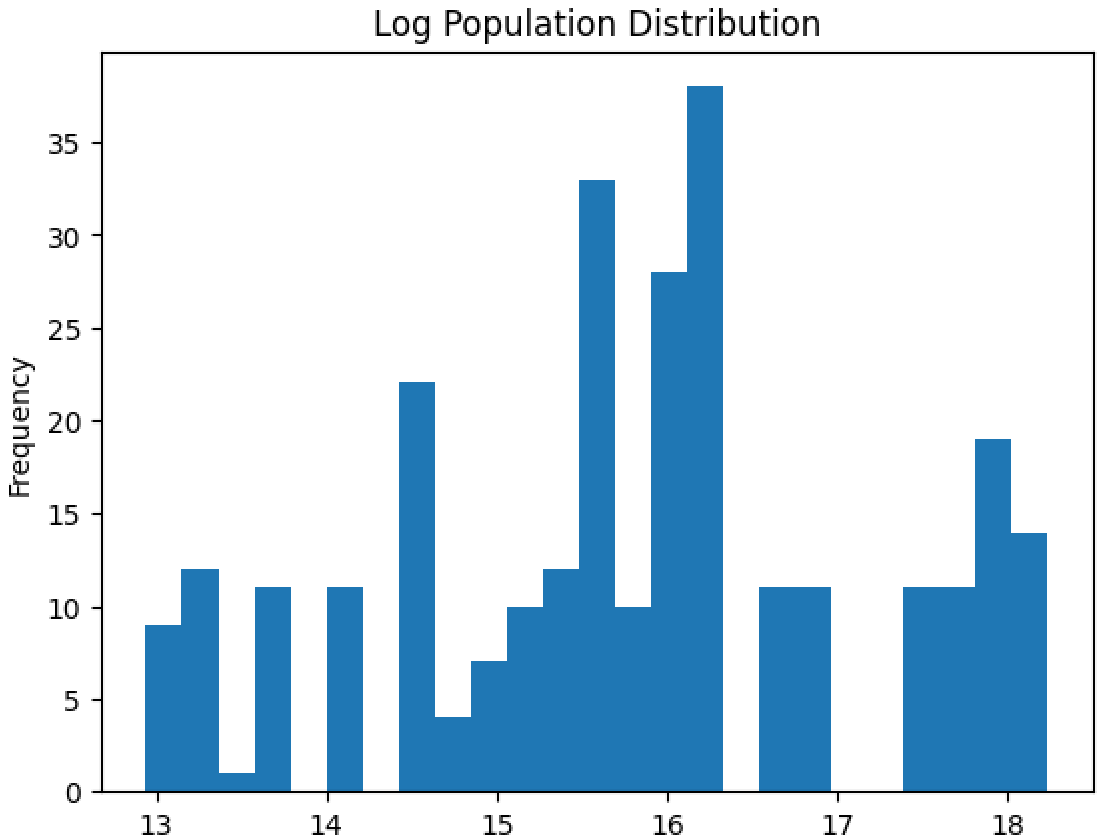
In [107…

```
X_reg["log(Population)"] = np.log(X_reg["Population"])
X_cap["log(Population)"] = np.log(X_cap["Population"])

X_reg["log(Population)"].plot.hist(xlabel="Log Population", title="Log Population Dist
```

```
<ipython-input-107-bcce014eb10e>:2: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/us
er_guide/indexing.html#returning-a-view-versus-a-copy
  X_cap["log(Population)"] = np.log(X_cap["Population"])
```

Out[107]:

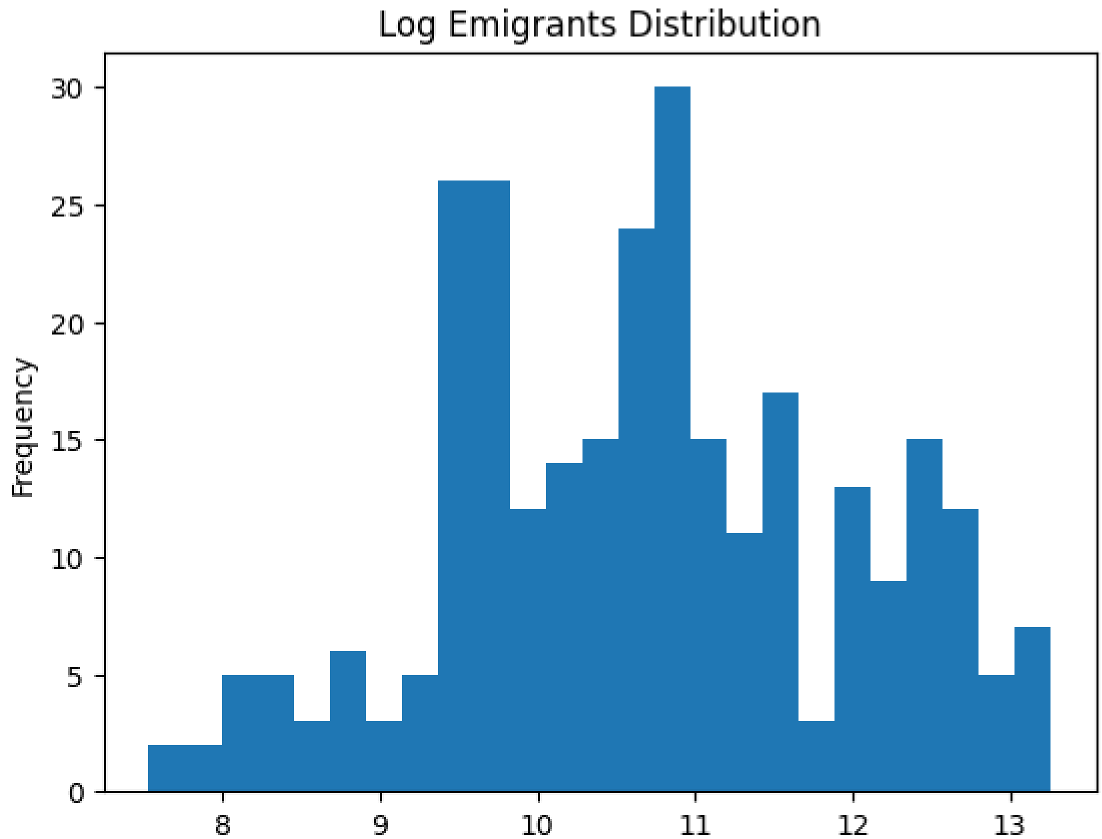
```
<Axes: title={'center': 'Log Population Distribution'}, ylabel='Frequency'>
```



In [88]:
```
X_reg["log(Emigrants)"] = np.log(X_reg["emigration"])
X_reg["log(Emigrants)"].plot.hist(xlabel="Log Number of Emigrants", title="Log Emigrar
```

```
<ipython-input-88-175ad1c3279c>:1: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/us
er_guide/indexing.html#returning-a-view-versus-a-copy
  X_reg["log(Emigrants)"] = np.log(X_reg["emigration"])
```

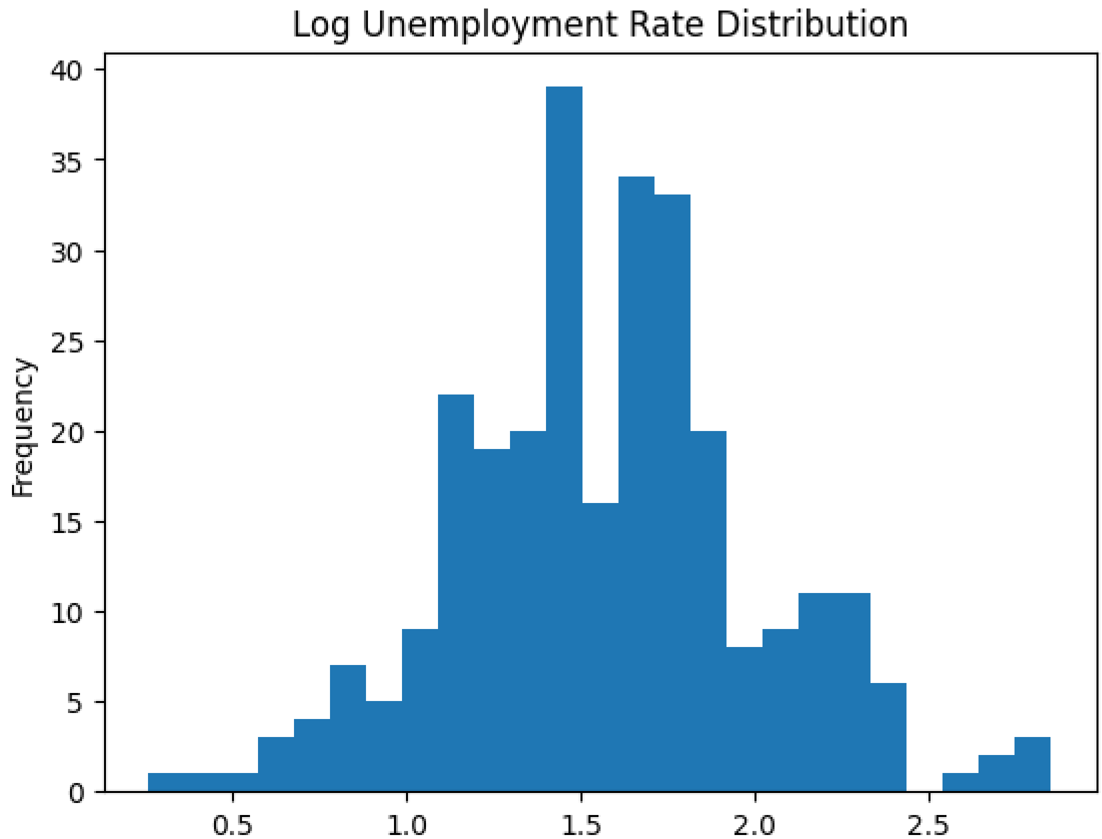
Out[88]:
```
<Axes: title={'center': 'Log Emigrants Distribution'}, ylabel='Frequency'>
```



In [90]:
```
X_reg["log(Unemployment)"] = np.log(X_reg["unemployment"])
X_reg["log(Unemployment)"].plot.hist(xlabel="Log Unemployment Rate", title="Log Unempl
```

```
<ipython-input-90-fa317088e658>:1: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/us
er_guide/indexing.html#returning-a-view-versus-a-copy
  X_reg["log(Unemployment)"] = np.log(X_reg["unemployment"])
```

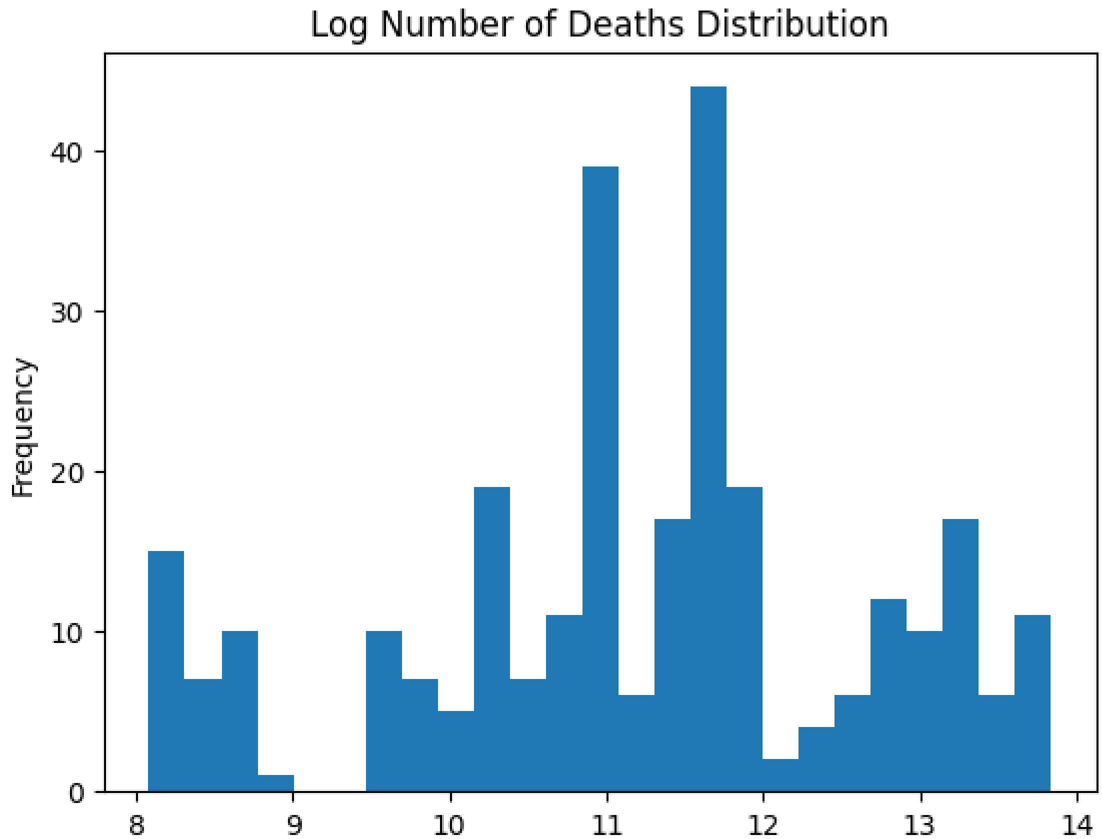
Out[90]:
```
<Axes: title={'center': 'Log Unemployment Rate Distribution'}, ylabel='Frequency'>
```



In [91]:
```
X_reg["log(Deaths)"] = np.log(X_reg["total_deaths"])
X_reg["log(Deaths)"].plot.hist(xlabel="Log Deaths", title="Log Number of Deaths Distri
```

```
<ipython-input-91-95e86eccef03>:1: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/us
er_guide/indexing.html#returning-a-view-versus-a-copy
  X_reg["log(Deaths)"] = np.log(X_reg["total_deaths"])
```

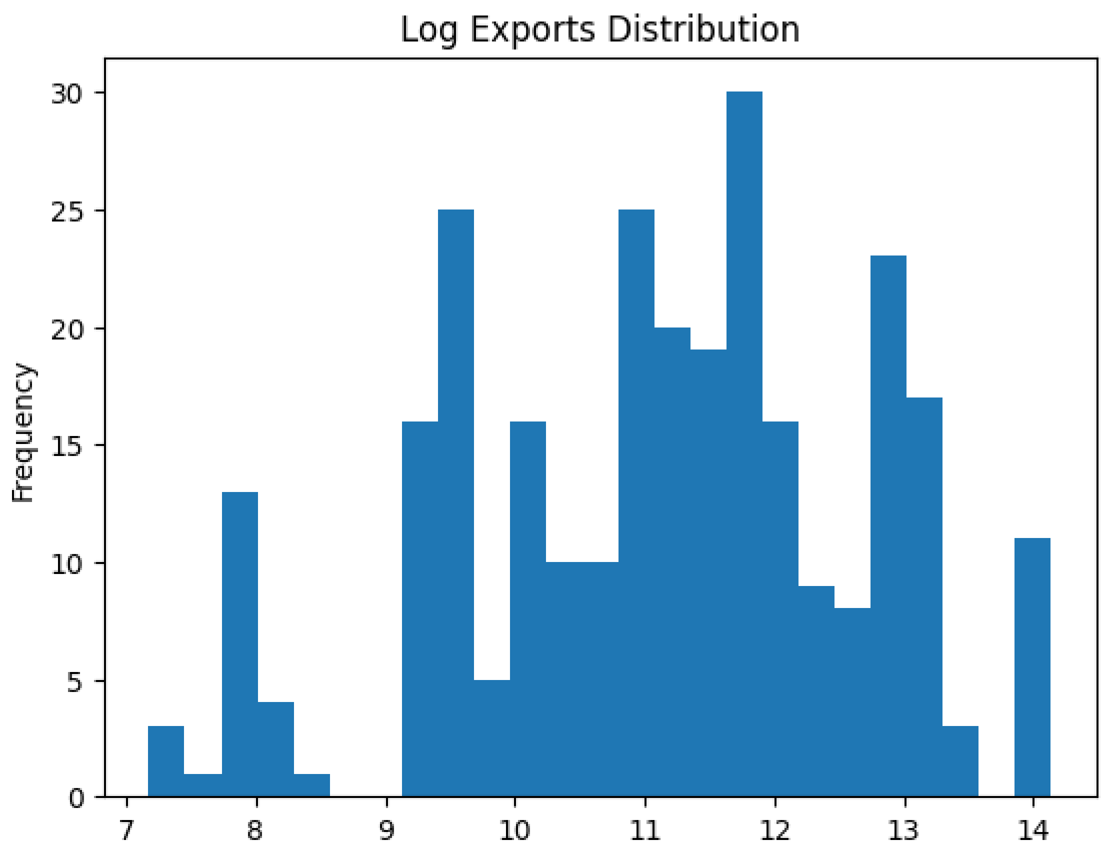
Out[91]:
```
<Axes: title={'center': 'Log Number of Deaths Distribution'}, ylabel='Frequency'>
```



In [92]:
```
X_reg["log(Exports)"] = np.log(X_reg["Exports"])
X_reg["log(Exports)"].plot.hist(xlabel="Log Exports", title="Log Exports Distribution"
```

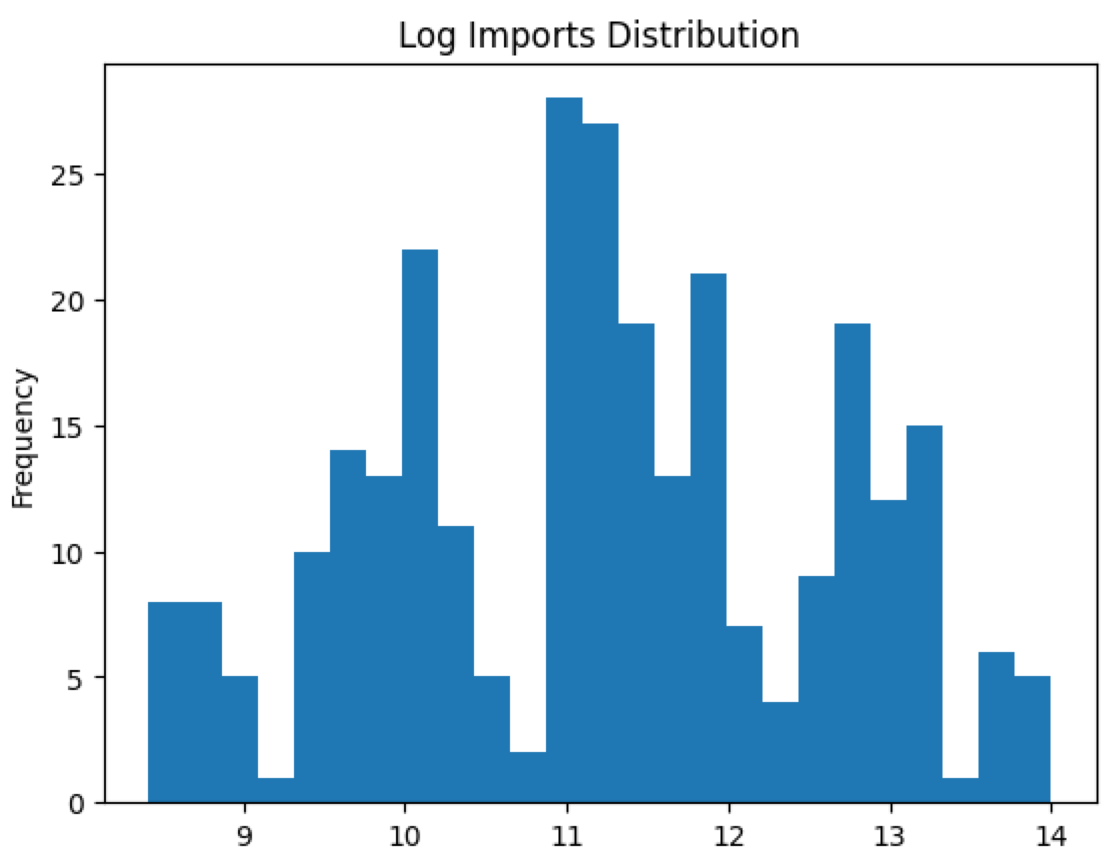
```
<ipython-input-92-884f1a1d6827>:1: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/us
er_guide/indexing.html#returning-a-view-versus-a-copy
  X_reg["log(Exports)"] = np.log(X_reg["Exports"])
```

Out[92]:
```
<Axes: title={'center': 'Log Exports Distribution'}, ylabel='Frequency'>
```



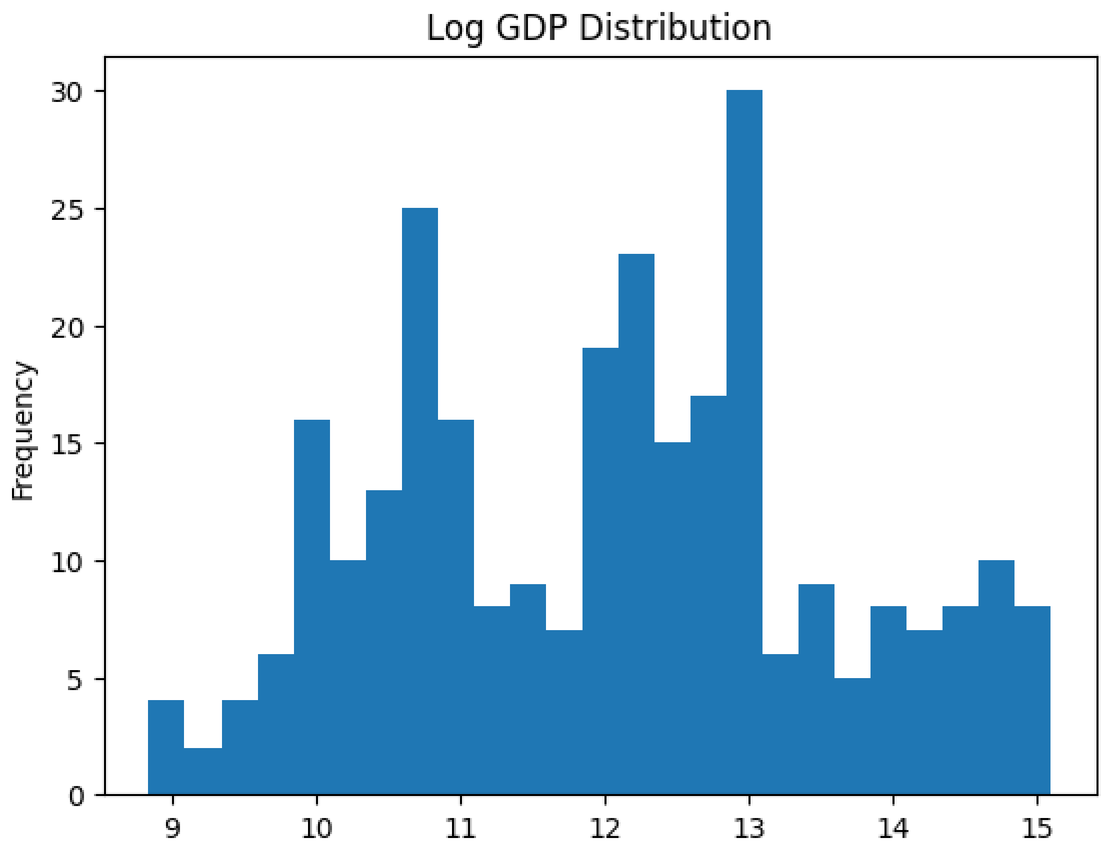
In [93]:
```
X_reg["log(Imports)"] = np.log(X_reg["Imports"])
X_reg["log(Imports)"].plot.hist(xlabel="Log Imports", title="Log Imports Distribution"
```

Out[93]: <Axes: title={'center': 'Log Imports Distribution'}, ylabel='Frequency'>



In [97]:
```python
# also log transform GDP
log_gdp = np.log(gdp)
log_gdp.plot.hist(xlabel="Log GDP", title="Log GDP Distribution", bins=25)
```

Out[97]: <Axes: title={'center': 'Log GDP Distribution'}, ylabel='Frequency'>



Feature Selection

In [101…
```python
from itertools import chain, combinations

def powerset(iterable):
    features = list(iterable)
    return chain.from_iterable(combinations(features, r) for r in range(len(features)+

reg_features = ["geo", "Year", "CPI", "log(Immigrants)", "log(Population)", "Housing I

# Creating a power set of all possible regular features to test for the best model
reg_power_set = []
for subset in powerset(reg_features):
    reg_power_set.append(list(subset))

# First value is empty, so remove
reg_power_set = reg_power_set[1:]
len(reg_power_set)
# That's a lot of combinations
```

Out[101]: 2047

In [102…
```python
# Takes ~5 min to run
```

```python
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

# define function to calculate estimate of test error for a given feature set
def get_cv_error(features):

  quant_vars = []
  cat_vars = []
  for feature in features:
    if X_reg.dtypes[feature] == "int64" or  X_reg.dtypes[feature] == "float64":
      quant_vars.append(feature)
    else:
      cat_vars.append(feature)

  ct = make_column_transformer(
      (StandardScaler(), quant_vars),
      (OneHotEncoder(handle_unknown="ignore"), cat_vars),
      remainder="drop"
  )

  pipeline = make_pipeline(
      ct,
      LinearRegression()
  )
  # errors from cross-validation
  cv_errs = -cross_val_score(pipeline, X=X_reg[features],
                             y=log_gdp,
                             scoring="neg_mean_squared_error", cv=10)
  # calculate average of the cross-validation errors
  return cv_errs.mean()

# calculate and store errors for different feature sets
errs = pd.Series()
for features in reg_power_set:
  errs[str(features)] = get_cv_error(features)

errs
```

```
<ipython-input-102-7e9d91271b2d>:38: FutureWarning: The default dtype for empty Serie
s will be 'object' instead of 'float64' in a future version. Specify a dtype explicit
ly to silence this warning.
  errs = pd.Series()
```

Out[102]:
```
['geo']
1.338080
['Year']
2.444883
['CPI']
2.440426
['log(Immigrants)']
0.517105
['log(Population)']
0.439551

...
['geo', 'Year', 'CPI', 'log(Population)', 'Housing Index', 'log(Emigrants)', 'log(Une
mployment)', 'log(Deaths)', 'log(Exports)', 'log(Imports)']                        0.2
48068
['geo', 'Year', 'log(Immigrants)', 'log(Population)', 'Housing Index', 'log(Emigrant
s)', 'log(Unemployment)', 'log(Deaths)', 'log(Exports)', 'log(Imports)']           0.
250493
['geo', 'CPI', 'log(Immigrants)', 'log(Population)', 'Housing Index', 'log(Emigrant
s)', 'log(Unemployment)', 'log(Deaths)', 'log(Exports)', 'log(Imports)']
0.600841
['Year', 'CPI', 'log(Immigrants)', 'log(Population)', 'Housing Index', 'log(Emigrant
s)', 'log(Unemployment)', 'log(Deaths)', 'log(Exports)', 'log(Imports)']           0.
115330
['geo', 'Year', 'CPI', 'log(Immigrants)', 'log(Population)', 'Housing Index', 'log(Em
igrants)', 'log(Unemployment)', 'log(Deaths)', 'log(Exports)', 'log(Imports)']    0.2
79569
Length: 2047, dtype: float64
```

In [103…
```
errs.sort_values()
```

Out[103]:
```
['Year', 'CPI', 'log(Immigrants)', 'log(Population)', 'log(Unemployment)', 'log(Death
s)', 'log(Exports)']    0.102480
['Year', 'log(Immigrants)', 'log(Population)', 'log(Unemployment)', 'log(Deaths)', 'l
og(Exports)']           0.102568
['log(Immigrants)', 'log(Population)', 'log(Deaths)', 'log(Imports)']
0.102854
['Year', 'CPI', 'log(Immigrants)', 'log(Population)', 'log(Unemployment)', 'log(Death
s)', 'log(Imports)']    0.103289
['Year', 'CPI', 'log(Immigrants)', 'log(Population)', 'log(Deaths)', 'log(Imports)']
0.103661

...
['geo', 'log(Immigrants)', 'log(Population)', 'log(Emigrants)', 'log(Exports)']
3.273926
['geo', 'log(Population)', 'log(Emigrants)', 'log(Deaths)', 'log(Exports)']
3.320444
['geo', 'log(Population)', 'log(Emigrants)', 'log(Exports)']
3.377197
['geo', 'log(Population)', 'log(Exports)']
3.417014
['geo', 'log(Population)', 'log(Deaths)', 'log(Exports)']
3.426497
Length: 2047, dtype: float64
```

Testing Out the Model

Features = ['Year', 'CPI', 'log(Immigrants)', 'log(Population)', 'log(Unemployment)', 'log(Deaths)', 'log(Exports)']

In [241…

```
from sklearn.model_selection import train_test_split

X_lin1 = X_reg[['Year', 'CPI', 'log(Immigrants)', 'log(Population)', 'log(Unemployment
# y = gdp

# split into test and training
X_lin1_train, X_lin1_test, y_lin1_train, y_lin1_test = train_test_split(X_lin1, log_gc
```

In [242…

```
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

# ct = make_column_transformer(
#     (StandardScaler(), ['Year', 'Population', 'Imports']),
#     (OneHotEncoder(handle_unknown="ignore"), ['geo']),
#     remainder="drop"
# )

pipeline = make_pipeline(
    StandardScaler(),
    LinearRegression()
)
```

In [243…

```
pipeline.fit(X_lin1_train, y_lin1_train)
```

Out[243]:

```
▸ Pipeline
  ▸ StandardScaler
  ▸ LinearRegression
```

In [ ]:

```
# cross validation scoring
```

In [244…

```
scores = -cross_val_score(pipeline,
                          X_lin1_train,
                          y_lin1_train,
                          scoring="neg_root_mean_squared_error",
                          cv=10)
scores.mean()
```

Out[244]:

```
0.2752669104287434
```

In [ ]:

```
# test scoring
```

In [245…

```
print("Train Score : ", pipeline.score(X_lin1_train, y_lin1_train)," Test Score : ", p
```

```
Train Score :  0.966234579395702  Test Score :  0.971134488846023
```

In [246…

```
y_lin1_pred = pipeline.predict(X_lin1_test)
```

RMSE

In [247…
```
from sklearn.metrics import mean_squared_error

rmse = np.sqrt(mean_squared_error(y_lin1_test, y_lin1_pred))
rmse
```

Out[247]:
```
0.2739277626249072
```

Benchmark

In [248…
```
from sklearn.dummy import DummyRegressor

mean_model = DummyRegressor(strategy="mean")

-cross_val_score(mean_model, X=X_lin1_train, y=y_lin1_train, cv=10,
                           scoring="neg_root_mean_squared_error").mean()
```

Out[248]:
```
1.4660836855778345
```

In [249…
```
y_lin1_test.std()
```

Out[249]:
```
1.6236166560977188
```

Based on these benchmarks, I'd say our model did a much better job!

R^2

In [125…
```
r_squared = 1 - (((y_lin1_test - y_lin1_pred)**2).mean()) / ((y_lin1_test - y_lin1_pre
r_squared
```

Out[125]:
```
0.9711439844190322
```

That's a really good R^2 value. 97.1% of the variability in GDP is explained by this model.

Linear Regression General - Predicting for 2021

Removing 2021 from the dataset and seeing how the model performs for an unknown year

In [250…
```
# train
X_lin1 = X_reg[['Year', 'CPI', 'log(Immigrants)', 'log(Population)', 'log(Unemployment

# concatenate X and y
df_combined = pd.concat([X_lin1, pd.DataFrame(log_gdp)], axis=1)

no_2021 = df_combined[df_combined["Year"] != 2021]
X_lin1_no_2021 = no_2021.drop(columns=["GDP"])
log_gdp_no_2021 = no_2021["GDP"]

yes_2021 = df_combined[df_combined["Year"] == 2021]
```

```python
X_lin1_yes_2021 = yes_2021.drop(columns=["GDP"])
log_gdp_yes_2021 = yes_2021["GDP"]
```

In [251…

```python
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

# ct = make_column_transformer(
#     (StandardScaler(), ['Year', 'Population', 'Imports']),
#     (OneHotEncoder(handle_unknown="ignore"), ['geo']),
#     remainder="drop"
# )

pipeline = make_pipeline(
    StandardScaler(),
    LinearRegression()
)
```

In [252…

```python
pipeline.fit(X_lin1_no_2021, log_gdp_no_2021)
```

Out[252]:

```
Pipeline
  StandardScaler
  LinearRegression
```

In [ ]:

```python
# cross validation scoring
```

In [253…

```python
scores = -cross_val_score(pipeline,
                          X_lin1_no_2021,
                          log_gdp_no_2021,
                          scoring="neg_root_mean_squared_error",
                          cv=10)
scores.mean()
```

Out[253]:

```
0.32017677586391696
```

In [ ]:

```python
# test scoring
```

In [156…

```python
print("Train Score : ", pipeline.score(X_lin1_no_2021, log_gdp_no_2021)," Test Score :
```

```
Train Score :  0.9676111102787208  Test Score :  0.9669075610328213
```

In [157…

```python
y_lin1_2021_pred = pipeline.predict(X_lin1_yes_2021)
```

RMSE

In [254…

```python
from sklearn.metrics import mean_squared_error

rmse = np.sqrt(mean_squared_error(log_gdp_yes_2021, y_lin1_2021_pred))
rmse
```

Out[254]: 0.26376247487392557

Benchmark

In [223…
```python
from sklearn.dummy import DummyRegressor

mean_model = DummyRegressor(strategy="mean")

-cross_val_score(mean_model, X=X_lin1_no_2021, y=log_gdp_no_2021, cv=10,
                           scoring="neg_root_mean_squared_error").mean()
```

Out[223]: 1.491461500108686

In [159…
```python
log_gdp_yes_2021.std()
```

Out[159]: 1.4786496693288973

Based on this benchmark, I'd say our model did a much better job!

R^2

In [161…
```python
r_squared = 1 - (((log_gdp_yes_2021 - y_lin1_2021_pred)**2).mean()) / ((log_gdp_yes_20
r_squared
```

Out[161]: 0.9672226359601809

That's a really good R^2 value. 96.7% of the variability in GDP is explained by this model.

Linear Regression - Per Capita Model

In [108…
```python
X_cap
```

Out[108]:

	geo	Year	CPI	Immigrants_pcap	Population	Housing Index	Emigrants_pcap	unemployment
0	Austria	2011	93.35	0.009799	8391643.0	81.60	0.006101	3.3
1	Austria	2012	95.75	0.010861	8429991.0	87.57	0.006146	3.5
2	Austria	2013	97.77	0.012013	8479823.0	92.10	0.006376	3.8
3	Austria	2014	99.20	0.013604	8546356.0	95.33	0.006259	4.0
4	Austria	2015	100.00	0.019244	8642699.0	100.00	0.006559	4.1
...	...	...	...	...	...	...	...	...
280	Slovakia	2017	100.90	0.001322	5439232.0	112.99	0.000637	5.4
281	Slovakia	2018	103.46	0.001332	5446771.0	121.32	0.000605	4.3
282	Slovakia	2019	106.33	0.001286	5454147.0	132.39	0.000620	3.8
283	Slovakia	2020	108.47	0.001241	5458827.0	145.06	0.000445	4.4
284	Slovakia	2021	111.53	0.001052	5447247.0	154.33	0.000623	4.5

285 rows × 12 columns

In [109…

```python
from itertools import chain, combinations

def powerset(iterable):
    features = list(iterable)
    return chain.from_iterable(combinations(features, r) for r in range(len(features)+

reg_features = ["geo", "Year", "CPI", "Immigrants_pcap", "log(Population)", "Housing I

# Creating a power set of all possible regular features to test for the best model
reg_power_set = []
for subset in powerset(reg_features):
    reg_power_set.append(list(subset))

# First value is empty, so remove
reg_power_set = reg_power_set[1:]
len(reg_power_set)
# That's a lot of combinations
```

Out[109]: 2047

In [110…

```python
# Takes ~5 min to run

from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

# define function to calculate estimate of test error for a given feature set
def get_cv_error(features):

  quant_vars = []
```

```python
  cat_vars = []
  for feature in features:
    if X_cap.dtypes[feature] == "int64" or  X_cap.dtypes[feature] == "float64":
      quant_vars.append(feature)
    else:
      cat_vars.append(feature)

  ct = make_column_transformer(
      (StandardScaler(), quant_vars),
      (OneHotEncoder(handle_unknown="ignore"), cat_vars),
      remainder="drop"
  )

  pipeline = make_pipeline(
      ct,
      LinearRegression()
  )
  # errors from cross-validation
  cv_errs = -cross_val_score(pipeline, X=X_cap[features],
                             y=gdp_cap,
                             scoring="neg_mean_squared_error", cv=10)
  # calculate average of the cross-validation errors
  return cv_errs.mean()

# calculate and store errors for different feature sets
errs = pd.Series()
for features in reg_power_set:
  errs[str(features)] = get_cv_error(features)

errs
```

```
<ipython-input-110-a430b3d49c61>:38: FutureWarning: The default dtype for empty Serie
s will be 'object' instead of 'float64' in a future version. Specify a dtype explicit
ly to silence this warning.
  errs = pd.Series()
```

Out[110]:
```
['geo']
3.560759e+08
['Year']
4.037711e+08
['CPI']
4.095859e+08
['Immigrants_pcap']
3.698054e+08
['log(Population)']
4.448045e+08

...
['geo', 'Year', 'CPI', 'log(Population)', 'Housing Index', 'Emigrants_pcap', 'unemplo
yment', 'Deaths_pcap', 'Exports_pcap', 'Imports_pcap']                        4.282401
e+09
['geo', 'Year', 'Immigrants_pcap', 'log(Population)', 'Housing Index', 'Emigrants_pca
p', 'unemployment', 'Deaths_pcap', 'Exports_pcap', 'Imports_pcap']            4.400176
e+09
['geo', 'CPI', 'Immigrants_pcap', 'log(Population)', 'Housing Index', 'Emigrants_pca
p', 'unemployment', 'Deaths_pcap', 'Exports_pcap', 'Imports_pcap']             5.18634
6e+09
['Year', 'CPI', 'Immigrants_pcap', 'log(Population)', 'Housing Index', 'Emigrants_pca
p', 'unemployment', 'Deaths_pcap', 'Exports_pcap', 'Imports_pcap']            3.816170
e+08
['geo', 'Year', 'CPI', 'Immigrants_pcap', 'log(Population)', 'Housing Index', 'Emigra
nts_pcap', 'unemployment', 'Deaths_pcap', 'Exports_pcap', 'Imports_pcap']    3.919832
e+09
Length: 2047, dtype: float64
```

In [111…
```
errs.sort_values()
```

Out[111]:
```
['geo', 'Immigrants_pcap', 'Imports_pcap']
1.342052e+08
['geo', 'Immigrants_pcap', 'Deaths_pcap', 'Imports_pcap']
1.383472e+08
['geo', 'CPI', 'Immigrants_pcap', 'Deaths_pcap', 'Imports_pcap']
1.414693e+08
['geo', 'Imports_pcap']
1.450625e+08
['geo', 'Immigrants_pcap', 'Emigrants_pcap', 'Imports_pcap']
1.453131e+08

...
['geo', 'log(Population)', 'Deaths_pcap']
1.221979e+10
['geo', 'log(Population)', 'Emigrants_pcap', 'unemployment', 'Deaths_pcap']
1.223113e+10
['geo', 'Immigrants_pcap', 'log(Population)', 'Emigrants_pcap', 'unemployment', 'Deat
hs_pcap']    1.235351e+10
['geo', 'Immigrants_pcap', 'log(Population)', 'Emigrants_pcap', 'Deaths_pcap']
1.299331e+10
['geo', 'log(Population)', 'Emigrants_pcap', 'Deaths_pcap']
1.441981e+10
Length: 2047, dtype: float64
```

Testing Out the Model

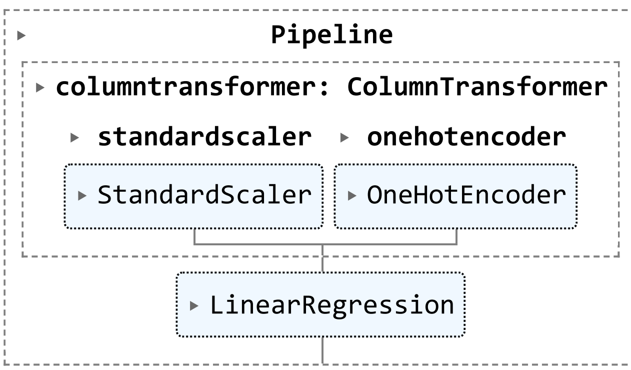
Features = ['geo', 'Immigrants_pcap', 'Imports_pcap']

In [163…
```
from sklearn.model_selection import train_test_split

X_lin2 = X_cap[['geo', 'Immigrants_pcap', 'Imports_pcap']]
# y = gdp

# split into test and training
X_lin2_train, X_lin2_test, y_lin2_train, y_lin2_test = train_test_split(X_lin2, gdp_ca
```

In [171…
```
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

ct = make_column_transformer(
    (StandardScaler(), ['Immigrants_pcap', 'Imports_pcap']),
    (OneHotEncoder(handle_unknown="ignore"), ['geo']),
    remainder="drop"
)

pipeline = make_pipeline(
    ct,
    LinearRegression()
)
```

In [172…
```
pipeline.fit(X_lin2_train, y_lin2_train)
```

Out[172]:



In [ ]:
```
# cross validation scoring
```

In [173…
```
scores = -cross_val_score(pipeline,
                          X_lin2_train,
                          y_lin2_train,
                          scoring="neg_root_mean_squared_error",
                          cv=10)
scores.mean()
```

Out[173]:
```
3700.35721021237
```

In [ ]:
```
# test scoring
```

In [174…
```
print("Train Score : ", pipeline.score(X_lin2_train, y_lin2_train)," Test Score : ", p
```

```
Train Score :  0.9652896984504861  Test Score :  0.9842289358292329
```

In [176...
```
y_lin2_pred = pipeline.predict(X_lin2_test)
```

RMSE

In [178...
```
from sklearn.metrics import mean_squared_error

rmse = np.sqrt(mean_squared_error(y_lin2_test, y_lin2_pred))
rmse
```

Out[178]:
```
2256.045641959205
```

Benchmarks

In [179...
```
from sklearn.dummy import DummyRegressor

mean_model = DummyRegressor(strategy="mean")

-cross_val_score(mean_model, X=X_lin2_train, y=y_lin2_train, cv=10,
                           scoring="neg_root_mean_squared_error").mean()
```

Out[179]:
```
19659.644333209537
```

In [180...
```
y_lin2_test.std()
```

Out[180]:
```
18090.661855816146
```

Based on these benchmarks, I'd say our model did a much better job!

R^2

In [181...
```
r_squared = 1 - (((y_lin2_test - y_lin2_pred)**2).mean()) / ((y_lin2_test - y_lin2_pre
r_squared
```

Out[181]:
```
0.9842338631197548
```

That's a really good R^2 value. 98.42% of the variability in GDP is explained by this model.

Linear Regression - No Transformation

Creating to see if we can create an ensemble method

In [112...
```
from itertools import chain, combinations

def powerset(iterable):
    features = list(iterable)
    return chain.from_iterable(combinations(features, r) for r in range(len(features)+

reg_features = ["geo", "Year", "CPI", "Immigrants", "Population", "Housing Index", "en

# Creating a power set of all possible regular features to test for the best model
reg_power_set = []
for subset in powerset(reg_features):
```

```
    reg_power_set.append(list(subset))

# First value is empty, so remove
reg_power_set = reg_power_set[1:]
len(reg_power_set)
# That's a lot of combinations
```

Out[112]:
```
2047
```

In [113…
```
# Takes ~5 min to run

from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

# define function to calculate estimate of test error for a given feature set
def get_cv_error(features):

  quant_vars = []
  cat_vars = []
  for feature in features:
    if X_reg.dtypes[feature] == "int64" or  X_reg.dtypes[feature] == "float64":
      quant_vars.append(feature)
    else:
      cat_vars.append(feature)

  ct = make_column_transformer(
      (StandardScaler(), quant_vars),
      (OneHotEncoder(handle_unknown="ignore"), cat_vars),
      remainder="drop"
  )

  pipeline = make_pipeline(
      ct,
      LinearRegression()
  )
  # errors from cross-validation
  cv_errs = -cross_val_score(pipeline, X=X_reg[features],
                             y=gdp,
                             scoring="neg_mean_squared_error", cv=10)
  # calculate average of the cross-validation errors
  return cv_errs.mean()

  ["geo", "Year", "CPI", "Immigrants", "Population", "Housing Index", "emigration", "u

# calculate and store errors for different feature sets
errs = pd.Series()
for features in reg_power_set:
  errs[str(features)] = get_cv_error(features)

errs
```

```
<ipython-input-113-8f9f8e8e9b42>:40: FutureWarning: The default dtype for empty Serie
s will be 'object' instead of 'float64' in a future version. Specify a dtype explicit
ly to silence this warning.
  errs = pd.Series()
```

Out[113]:
```
['geo']
1.810583e+11
['Year']
5.979601e+11
['CPI']
5.937839e+11
['Immigrants']
1.308372e+11
['Population']
7.291611e+10

...
['geo', 'Year', 'CPI', 'Population', 'Housing Index', 'emigration', 'unemployment',
'total_deaths', 'Exports', 'Imports']                    1.011950e+12
['geo', 'Year', 'Immigrants', 'Population', 'Housing Index', 'emigration', 'unemploym
ent', 'total_deaths', 'Exports', 'Imports']           1.013701e+12
['geo', 'CPI', 'Immigrants', 'Population', 'Housing Index', 'emigration', 'unemployme
nt', 'total_deaths', 'Exports', 'Imports']            1.067768e+12
['Year', 'CPI', 'Immigrants', 'Population', 'Housing Index', 'emigration', 'unemploym
ent', 'total_deaths', 'Exports', 'Imports']           5.580780e+10
['geo', 'Year', 'CPI', 'Immigrants', 'Population', 'Housing Index', 'emigration', 'un
employment', 'total_deaths', 'Exports', 'Imports']    9.581895e+11
Length: 2047, dtype: float64
```

In [115…:
```
errs.sort_values()
```

Out[115]:
```
['geo', 'Immigrants', 'emigration', 'total_deaths', 'Imports']
2.573979e+10
['geo', 'CPI', 'Immigrants', 'Housing Index', 'emigration', 'total_deaths', 'Import
s']    2.597665e+10
['geo', 'Immigrants', 'Housing Index', 'emigration', 'total_deaths', 'Imports']
2.602533e+10
['geo', 'Year', 'CPI', 'emigration', 'total_deaths', 'Imports']
2.616357e+10
['geo', 'Year', 'CPI', 'Immigrants', 'emigration', 'total_deaths', 'Imports']
2.624142e+10

...
['geo', 'Population', 'Housing Index', 'unemployment']
3.723337e+12
['geo', 'Immigrants', 'Population', 'unemployment']
3.932125e+12
['geo', 'Immigrants', 'Population']
3.964237e+12
['geo', 'Population', 'unemployment']
4.119505e+12
['geo', 'Population']
4.331985e+12
Length: 2047, dtype: float64
```

Testing Out the Model

Features = ['geo', 'Immigrants', 'emigration', 'total_deaths', 'Imports']

In [197…:
```
from sklearn.model_selection import train_test_split

X_lin3 = X_reg[['geo', 'Immigrants', 'emigration', 'total_deaths', 'Imports']]
# y = gdp
```

```python
# split into test and training
X_lin3_train, X_lin3_test, y_lin3_train, y_lin3_test = train_test_split(X_lin3, gdp, t
```

In [198…

```python
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

ct = make_column_transformer(
    (StandardScaler(), ['Immigrants', 'emigration', 'total_deaths', 'Imports']),
    (OneHotEncoder(handle_unknown="ignore"), ['geo']),
    remainder="drop"
)

pipeline = make_pipeline(
    ct,
    LinearRegression()
)
```

In [199…

```python
pipeline.fit(X_lin3_train, y_lin3_train)
```

Out[199]:

```
Pipeline
columntransformer: ColumnTransformer
standardscaler    onehotencoder
StandardScaler    OneHotEncoder
LinearRegression
```

In [ ]:

```python
# cross validation scoring
```

In [201…

```python
scores = -cross_val_score(pipeline,
                         X_lin3_train,
                         y_lin3_train,
                         scoring="neg_root_mean_squared_error",
                         cv=10)
scores.mean()
```

Out[201]:

```
38021.17430563266
```

In [ ]:

```python
# test scoring
```

In [202…

```python
print("Train Score : ", pipeline.score(X_lin3_train, y_lin3_train)," Test Score : ", p
```

```
Train Score :  0.9983561233574343  Test Score :  0.9975992775706017
```

In [205…

```python
y_lin3_pred = pipeline.predict(X_lin3_test)
```

RMSE

In [206…
```
from sklearn.metrics import mean_squared_error

rmse = np.sqrt(mean_squared_error(y_lin3_test, y_lin3_pred))
rmse
```
Out[206]:
```
42190.426261967914
```

Benchmark

In [207…
```
from sklearn.dummy import DummyRegressor

mean_model = DummyRegressor(strategy="mean")

-cross_val_score(mean_model, X=X_lin3_train, y=y_lin3_train, cv=10,
                            scoring="neg_root_mean_squared_error").mean()
```
Out[207]:
```
690927.3333981774
```

In [224…
```
y_lin3_test.std()
```
Out[224]:
```
867121.6148310832
```

Based on these benchmarks, I'd say our model did a much better job!

R^2

In [209…
```
r_squared = 1 - (((y_lin3_test - y_lin3_pred)**2).mean()) / ((y_lin3_test - y_lin3_pre
r_squared
```
Out[209]:
```
0.9975993796502242
```

That's a really good R^2 value. 99.75% of the variability in GDP is explained by this model.

Linear Regression No Transformation - Predicting for 2021

Removing 2021 from the dataset and seeing how the model performs for an unknown year

In [225…
```
# train
X_lin3 = X_reg[['geo', 'Immigrants', 'emigration', 'total_deaths', 'Imports', 'Year']]

# concatenate X and y
df_combined = pd.concat([X_lin3, pd.DataFrame(gdp)], axis=1)

no_2021 = df_combined[df_combined["Year"] != 2021]
X_lin3_no_2021 = no_2021.drop(columns=["GDP", "Year"])
gdp_no_2021 = no_2021["GDP"]

yes_2021 = df_combined[df_combined["Year"] == 2021]
X_lin3_yes_2021 = yes_2021.drop(columns=["GDP", "Year"])
gdp_yes_2021 = yes_2021["GDP"]
```

In [226…
```python
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

ct = make_column_transformer(
    (StandardScaler(), ['Immigrants', 'emigration', 'total_deaths', 'Imports']),
    (OneHotEncoder(handle_unknown="ignore"), ['geo']),
    remainder="drop"
)

pipeline = make_pipeline(
    ct,
    LinearRegression()
)
```

In [227…
```python
pipeline.fit(X_lin3_no_2021, gdp_no_2021)
```

Out[227]:
```
Pipeline
columntransformer: ColumnTransformer
standardscaler        onehotencoder
StandardScaler        OneHotEncoder
        LinearRegression
```

In [ ]:
```python
# cross validation scoring
```

In [228…
```python
scores = -cross_val_score(pipeline,
                         X_lin3_no_2021,
                         gdp_no_2021,
                         scoring="neg_root_mean_squared_error",
                         cv=10)
scores.mean()
```

Out[228]:
```
129483.09053418187
```

In [ ]:
```python
# test scoring
```

In [217…
```python
print("Train Score : ", pipeline.score(X_lin3_no_2021, gdp_no_2021)," Test Score : ",
```

```
Train Score :  0.9988080413836591  Test Score :  0.9918942551268459
```

In [218…
```python
y_lin3_2021_pred = pipeline.predict(X_lin3_yes_2021)
```

RMSE

In [229…
```python
from sklearn.metrics import mean_squared_error

rmse = np.sqrt(mean_squared_error(gdp_yes_2021, y_lin3_2021_pred))
rmse
```

Out[229]: 75970.98427215328

Benchmark

In [230…
```
gdp_yes_2021.std()
```

Out[230]: 860533.8484067871

In [232…
```
from sklearn.dummy import DummyRegressor

mean_model = DummyRegressor(strategy="mean")

-cross_val_score(mean_model, X=X_lin3_no_2021, y=gdp_no_2021, cv=10,
                           scoring="neg_root_mean_squared_error").mean()
```

Out[232]: 659915.7065210016

Based on this benchmark, I'd say our model did a much better job!

R^2

In [231…
```
r_squared = 1 - (((gdp_yes_2021 - y_lin3_2021_pred)**2).mean()) / ((gdp_yes_2021 - y_l
r_squared
```

Out[231]: 0.9919075657037448

That's a really good R^2 value. 99.2% of the variability in GDP is explained by this model.

Ensemble

Combine KNN and Linear Regression w/o Transformation

In [255…
```
# KNN pipeline
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.neighbors import KNeighborsRegressor
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

k_ct = make_column_transformer(
    (StandardScaler(), ['Year', 'Population', 'Imports']),
    (OneHotEncoder(handle_unknown="ignore"), ['geo']),
    remainder="drop"
)

k_pipeline = make_pipeline(
    k_ct,
    KNeighborsRegressor(n_neighbors=8)
)
```

In [256…
```
# Linear Regression pipeline
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.linear_model import LinearRegression
```

```python
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

l_ct = make_column_transformer(
    (StandardScaler(), ['Immigrants', 'emigration', 'total_deaths', 'Imports']),
    (OneHotEncoder(handle_unknown="ignore"), ['geo']),
    remainder="drop"
)

l_pipeline = make_pipeline(
    l_ct,
    LinearRegression()
)
```

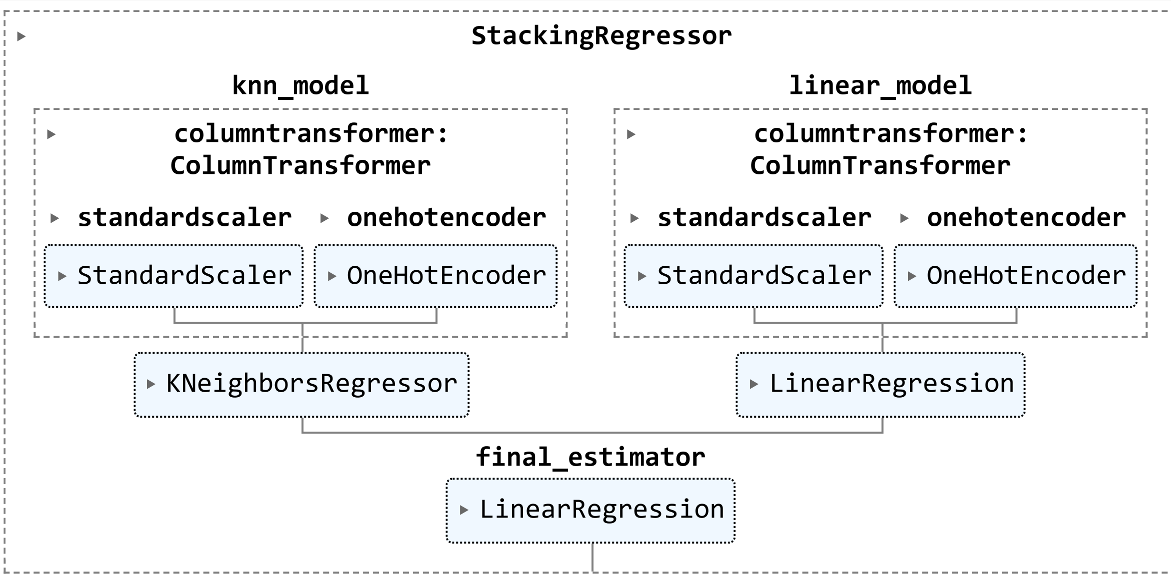
In [260…

```python
X_train = X_reg[['Immigrants', 'emigration', 'total_deaths', 'Imports', 'geo', 'Year',
y_train = gdp
```

In [261…

```python
from sklearn.ensemble import StackingRegressor

stack_model = StackingRegressor([
    ("knn_model", k_pipeline),
    ("linear_model", l_pipeline)],
    final_estimator=LinearRegression()
)
stack_model.fit(X_train, y_train)
```

Out[261]:



Testing Out the Model

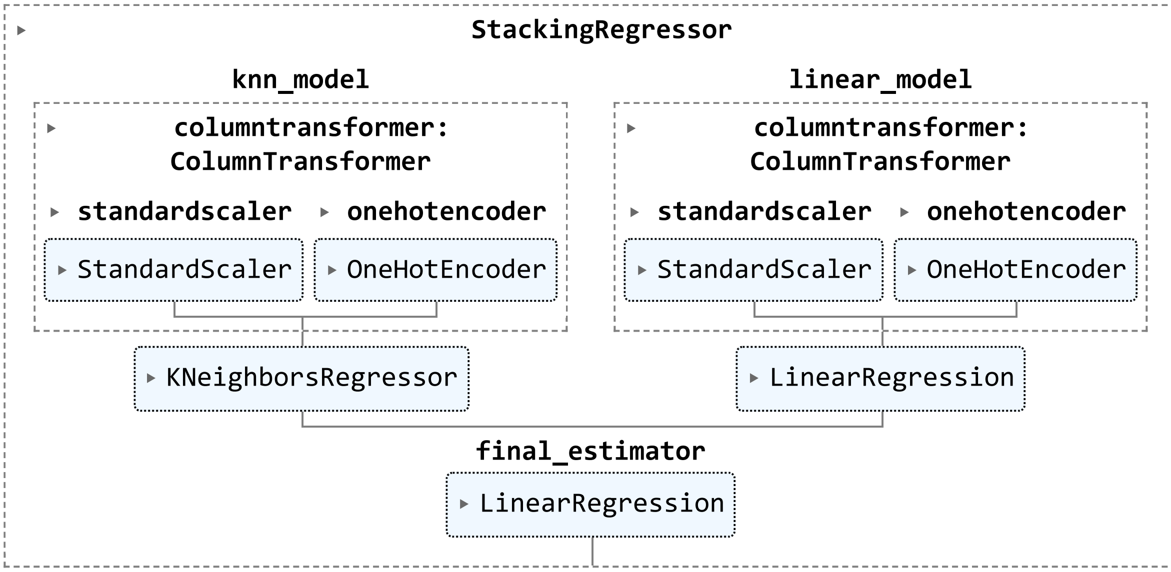
In [262…

```python
from sklearn.model_selection import train_test_split

# split into test and training
X_ens_train, X_ens_test, y_ens_train, y_ens_test = train_test_split(X_train, y_train,
```

In [263…

```python
stack_model.fit(X_ens_train, y_ens_train)
```

Out[263]:



In [ ]:

```
# cross validation scoring
```

In [264…

```
scores = -cross_val_score(stack_model,
                          X_ens_train,
                          y_ens_train,
                          scoring="neg_root_mean_squared_error",
                          cv=10)
scores.mean()
```

Out[264]:

```
39914.05547766272
```

In [ ]:

```
# test scoring
```

In [266…

```
print("Train Score : ", stack_model.score(X_ens_train, y_ens_train)," Test Score : ",
```

```
Train Score :  0.9982782899929161  Test Score :  0.9975861888851629
```

In [267…

```
y_ens_pred = stack_model.predict(X_ens_test)
```

RMSE

In [268…

```
from sklearn.metrics import mean_squared_error

rmse = np.sqrt(mean_squared_error(y_ens_test, y_ens_pred))
rmse
```

Out[268]:

```
42305.280563096756
```

Benchmark

In [269…

```
from sklearn.dummy import DummyRegressor

mean_model = DummyRegressor(strategy="mean")

-cross_val_score(mean_model, X=X_ens_train, y=y_ens_train, cv=10,
                           scoring="neg_root_mean_squared_error").mean()
```

Out[269]: 690927.3333981774

In [271…
```
y_ens_test.std()
```

Out[271]: 867121.6148310832

Based on these benchmarks, I'd say our model did a much better job!

R^2

In [273…
```
r_squared = 1 - (((y_ens_test - y_ens_pred)**2).mean()) / ((y_ens_test - y_ens_pred.me
r_squared
```

Out[273]: 0.997586400245855

That's a really good R^2 value. 99.8% of the variability in GDP is explained by this model.

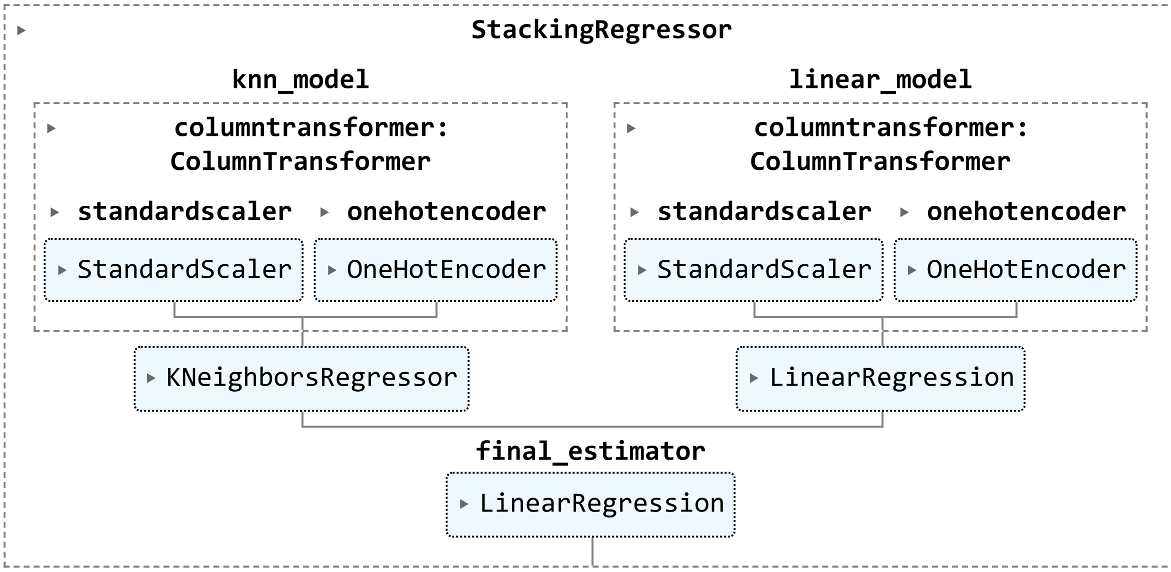
Ensemble - Predicting for 2021

Removing 2021 from the dataset and seeing how the model performs for an unknown year

In [275…
```
X_ens = X_reg[['Immigrants', 'emigration', 'total_deaths', 'Imports', 'geo', 'Year', '
y_train = gdp

# concatenate X and y
df_combined = pd.concat([X_ens, pd.DataFrame(gdp)], axis=1)

no_2021 = df_combined[df_combined["Year"] != 2021]
X_ens_no_2021 = no_2021.drop(columns=["GDP"])
gdp_no_2021 = no_2021["GDP"]

yes_2021 = df_combined[df_combined["Year"] == 2021]
X_ens_yes_2021 = yes_2021.drop(columns=["GDP"])
gdp_yes_2021 = yes_2021["GDP"]
```

In [276…
```
stack_model.fit(X_ens_no_2021, gdp_no_2021)
```

Out[276]:



In [ ]:
```
# cross validation scoring
```

In [277…
```
scores = -cross_val_score(stack_model,
                          X_ens_no_2021,
                          gdp_no_2021,
                          scoring="neg_root_mean_squared_error",
                          cv=10)
scores.mean()
```

Out[277]:
```
183450.02876797522
```

In [ ]:
```
# test scoring
```

In [280…
```
print("Train Score : ", stack_model.score(X_ens_no_2021, gdp_no_2021)," Test Score : "
```

```
Train Score :  0.989778742246415  Test Score :  0.9896728805773415
```

In [281…
```
y_ens_2021_pred = stack_model.predict(X_ens_yes_2021)
```

RMSE

In [282…
```
from sklearn.metrics import mean_squared_error

rmse = np.sqrt(mean_squared_error(gdp_yes_2021, y_ens_2021_pred))
rmse
```

Out[282]:
```
85751.33452716541
```

Benchmark

In [283…
```
gdp_yes_2021.std()
```

Out[283]:
```
860533.8484067871
```

In [284…
```
from sklearn.dummy import DummyRegressor

mean_model = DummyRegressor(strategy="mean")
```

```
-cross_val_score(mean_model, X=X_ens_no_2021, y=gdp_no_2021, cv=10,
                           scoring="neg_root_mean_squared_error").mean()
```

Out[284]:

```
659915.7065210016
```

Based on this benchmark, I'd say our model did a much better job!

R^2

In [286…

```
r_squared = 1 - (((gdp_yes_2021 - y_ens_2021_pred)**2).mean()) / ((gdp_yes_2021 - y_er
r_squared
```

Out[286]:

```
0.9896796766731947
```

That's a really good R^2 value. 98.968% of the variability in GDP is explained by this model.

Time Series

In [301…

```
df_eurostat_future = pd.read_csv("https://raw.githubusercontent.com/SDuncan5/Eurostat-
df_eurostat_future = df_eurostat_future.drop(columns=["Unnamed: 0"])
```

In [314…

```
df_eurostat_future["log(Immigrants)"] = np.log(df_eurostat_future["Immigrants"])
df_eurostat_future["log(Population)"] = np.log(df_eurostat_future["Population"])
df_eurostat_future["log(Emigrants)"] = np.log(df_eurostat_future["emigration"])
df_eurostat_future["log(Unemployment)"] = np.log(df_eurostat_future["unemployment"]).f
df_eurostat_future["log(Deaths)"] = np.log(df_eurostat_future["total_deaths"])
df_eurostat_future["log(Exports)"] = np.log(df_eurostat_future["Exports"])
df_eurostat_future["log(Imports)"] = np.log(df_eurostat_future["Imports"])
df_eurostat_future["log(GDP)"] = np.log(df_eurostat_future["GDP"])

log_gdp = np.log(gdp)
```

Comparing Transformed Linear Model to Time Series Data

In [315…

```
# train
X_time_train = X_reg[['Year', 'CPI', 'log(Immigrants)', 'log(Population)', 'log(Unempl
y_time_train = log_gdp
X_time_test = df_eurostat_future[['Year', 'CPI', 'log(Immigrants)', 'log(Population)',
y_time_test = df_eurostat_future["log(GDP)"]
# concatenate X and y
#X_train_combined = pd.concat([X_time_train, pd.DataFrame(log_gdp)], axis=1)

# no_2021 = df_combined[df_combined["Year"] != 2021]
# X_lin1_no_2021 = no_2021.drop(columns=["GDP"])
# log_gdp_no_2021 = no_2021["GDP"]

# yes_2021 = df_combined[df_combined["Year"] == 2021]
# X_lin1_yes_2021 = yes_2021.drop(columns=["GDP"])
# log_gdp_yes_2021 = yes_2021["GDP"]
```

In [316…

```
from sklearn.preprocessing import StandardScaler, OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
from sklearn.compose import make_column_transformer
from sklearn.model_selection import cross_val_score

pipeline = make_pipeline(
    StandardScaler(),
    LinearRegression()
)
```

In [317…

```
pipeline.fit(X_time_train, y_time_train)
```

Out[317]:

```
Pipeline
  StandardScaler
  LinearRegression
```

In [ ]:

```
# cross validation scoring
```

In [318…

```
scores = -cross_val_score(pipeline,
                         X_time_train,
                         y_time_train,
                         scoring="neg_root_mean_squared_error",
                         cv=10)
scores.mean()
```

Out[318]:

```
0.3132950858973453
```

In [ ]:

```
# test scoring
```

In [319…

```
print("Train Score : ", pipeline.score(X_time_train, y_time_train)," Test Score : ", p
```

```
Train Score :  0.9679465698710387  Test Score :  0.9695005058467016
```

In [320…

```
y_time_pred = pipeline.predict(X_time_test)
```

RMSE

In [321…

```
from sklearn.metrics import mean_squared_error

rmse = np.sqrt(mean_squared_error(y_time_test, y_time_pred))
rmse
```

Out[321]:

```
0.251391018311425
```

Benchmark

In [323…

```
from sklearn.dummy import DummyRegressor

mean_model = DummyRegressor(strategy="mean")
```

```
-cross_val_score(mean_model, X=X_time_train, y=y_time_train, cv=10,
                           scoring="neg_root_mean_squared_error").mean()
```

Out[323]:

```
1.485843915942742
```

In [324…

```
y_time_test.std()
```

Out[324]:

```
1.444108715440894
```

Based on this benchmark, I'd say our model did a much better job!

R^2

In [325…

```
r_squared = 1 - (((y_time_test - y_time_pred)**2).mean()) / ((y_time_test - y_time_pre
r_squared
```

Out[325]:

```
0.9695592364290889
```

That's a really good R^2 value. 96.96% of the variability in GDP is explained by this model.