Chapter 1: The Foundations: Logic and Proofs

Trần Hòa Phú

Ngày 10 tháng 1 năm 2023

Objectives

- Explain what makes up a correct mathematical arguments
- Introduce tools to construct correct arguments

Propositions (Mệnh đề)

Definition

A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

Example

Sun rises in the east and sets in the west.

Hue is the captial of Vietnam.

$2+3=5$.

$2+3=6$.

Exercise

Which of the following sentences are propositions?

a) $5+7 = 12$.

b) $x+2 = 11$.

c) Answer this question.

d) Today is thursday.

Operations of Propositions

Let p, q are propositions.

- $\neg p$
- $p \wedge q$
- $p \vee q$
- $p \oplus q$
- $p \rightarrow q$
- $p \leftrightarrow q$

Proposition ¬p

Definition

Let p be a proposition.

- $\neg p, (\overline{p})$ is the negation of p.

Example

p : My PC runs Windows.

Proposition ¬p

Definition

Let p be a proposition.

- $\neg p, (\overline{p})$ is the negation of p.

Example

p : My PC runs Windows.

$\neg p$ My PC doesn't run Windows. pause Example

p : This book has at least 300 pages.

Proposition ¬p

Definition

Let p be a proposition.

- $\neg p, (\overline{p})$ is the negation of p.

Example

p : My PC runs Windows.

$\neg p$ My PC doesn't run Windows. pause Example

p : This book has at least 300 pages. $\neg p$ This book doesn't have at least 300 pages. $\neg p$: This book has less than 300 pages.

Exercise

Find the negation of each of these propositions.

a) Tuan has an iphone

b) There is no pollution in Singapore

c) $2+1=3$

Table 1: The truth table for the negation of a Proposition

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

Proposition $p \wedge q$

Definition

Let p and q be propositions.

- $p \wedge q$ is the proposition "p and q".

Example 1

p : Sun rises in the east

q : Sun sets in the west

$p \wedge q$:

Definition

Let p and q be propositions.

- $p \wedge q$ is the proposition "p and q".

Example 1

p : Sun rises in the east

q : Sun sets in the west

$p \wedge q$: Sun rises in the east and sets in the west.

Proposition $p \wedge q$

Definition

Let p and q be propositions.

- $p \wedge q$ is the proposition "p and q".

Example 1

p : Sun rises in the east

q : Sun sets in the west

$p \wedge q$: Sun rises in the east and sets in the west.

Example 2

p : Sun is sunning

q : Sun is raining

$p \wedge q$:

Proposition $p \wedge q$

Definition

Let p and q be propositions.

- $p \wedge q$ is the proposition "p and q".

Example 1

p : Sun rises in the east

q : Sun sets in the west

$p \wedge q$: Sun rises in the east and sets in the west.

Example 2

p : Sun is sunning

q : Sun is raining

$p \wedge q$: "Sun is sunning and it is raining"

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Definition

Let p and q be propositions.

- $p \vee q$ is the proposition "p or q"

Example

p : Sun rises in the east

q : Sun sets in the west

$p \vee q$:

Definition

Let p and q be propositions.

- $p \vee q$ is the proposition "p or q"

Example

p : Sun rises in the east

q : Sun sets in the west

$p \vee q$: Sun rises in the east or it sets in the west.

Example

p : This book has more than 200 pages.

q : This book costs 20 dollars.

$p \vee q$:

Definition

Let p and q be propositions.

- $p \vee q$ is the proposition "p or q"

Example

p : Sun rises in the east

q : Sun sets in the west

$p \vee q$: Sun rises in the east or it sets in the west.

Example

p : This book has more than 200 pages.

q : This book costs 20 dollars.

$p \vee q$: This book has more than 200 pages or it costs 20 dollars.

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Definition

$p \oplus q$: "p or q but not both"

Example

p : It's coffee

q : It's tea

Proposition $p \oplus q$

Definition

$p \oplus q$: "p or q but not both"

Example

p : It's coffee

q : It's tea

$p \oplus q$ "It's coffee or tea but not both"

Definition

$p \oplus q$: "p or q but not both"

Example

p : It's coffee

q : It's tea

$p \oplus q$ "It's coffee or tea but not both"

$p \oplus q$ "It's coffee or tea"

Definition

$p \oplus q$: "p or q but not both"

Example

p : It's coffee

q : It's tea

$p \oplus q$ "It's coffee or tea but not both"

$p \oplus q$ "It's coffee or tea"

Example

p : He plays football well

q : He sings well

$p \oplus q$

Definition

$p \oplus q$: "p or q but not both"

Example

p : It's coffee

q : It's tea

$p \oplus q$ "It's coffee or tea but not both"

$p \oplus q$ "It's coffee or tea"

Example

p : He plays football well

q : He sings well

$p \oplus q$ "He plays football well or sings well but not both."

TABLE 4 **The Truth Table for the Exclusive Or of Two Propositions.**

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Definition

Let p and q be propositions.

- $p \rightarrow q$ is the proposition "if p, then q", "p is sufficient condition for q","q is necessary condition for p"

Definition

Let p and q be propositions.

- $p \to q$ is the proposition "if p, then q", "p is sufficient condition for q","q is necessary condition for p"

Example

p: Maria learns discrete mathematics.

q: Maria will find a good job.

$p \to q$

Definition

Let p and q be propositions.

- $p \to q$ is the proposition "if p, then q", "p is sufficient condition for q","q is necessary condition for p"

Example

p: Maria learns discrete mathematics.

q: Maria will find a good job.

$p \to q$ if Maria learns discrete mathematics, then Maria will find a good job.

Definition

Let p and q be propositions.

- $p \to q$ is the proposition "if p, then q", "p is sufficient condition for q","q is necessary condition for p"

Example

p: Maria learns discrete mathematics.

q: Maria will find a good job.

$p \to q$ if Maria learns discrete mathematics, then Maria will find a good job.

$p \to q$ For Maria to get a good job, it is sufficient for her to learn discrete mathematics.

p->q ch "sai" khi p úng, q sai

TABLE 5 The Truth Table for the Conditional Statement $p \to q$.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Exercise

Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 2$, then $2 + 2 \cancel{=} 5$ $\rightarrow$ F : 0

b) If $1 + 1 = 3$, then $2 + 2 = 4$ $\rightarrow$ T :

c) If $1 + 1 = 3$, then $2 + 2 = 5$ $\rightarrow$ T

d) If monkeys can fly, then $1 + 1 = 3$ $\rightarrow$ T

Definition

Let p and q be propositions.
The $p \leftrightarrow q$ is the proposition "p if and only if q".

Example

p: You can take the flight.

q: You buy a ticket.

$p \leftrightarrow q$:

Definition

Let p and q be propositions.
The $p \leftrightarrow q$ is the proposition "p if and only if q".

Example

p: You can take the flight.
q: You buy a ticket.
$p \leftrightarrow q$: You can take the flight if and only if you buy a ticket.

p <=> q dung khi {p dung, q dung hoac p sai, q sai}

TABLE 6 **The Truth Table for the Biconditional $p \leftrightarrow q$.**

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Exercise

6. Determine whether these biconditionals are true or false.

a) $2 + 2 = 4$ if and only if $1 + 1 = 2$. $\longrightarrow$ T : 1

b) $1 + 1 = 2$ if and only if $2 + 3 \cancel{=} 4$. $\longrightarrow$ F : 0

c) $1 + 1 \cancel{=} 3$ if and only if monkeys ~~can~~ fly. $\longrightarrow$ T : 1

d) $0 \cancel{>} 1$ if and only if $2 > 1$. $\longrightarrow$ F : 0

Truth Table Summary of Connectives

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	T	T	F	T	T
T	F	F	T	T	F	F
F	T	F	T	T	T	F
F	F	F	F	F	T	T

Establishing truth table of proposition given below

$$(p \vee \neg q) \rightarrow (p \wedge q)$$

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators (Thứ tự ưu tiên)

(1) Parentheses from inner to outer

(2) $\neg$

(3) $\wedge$

(4) $\vee$

(5) $\rightarrow$

(6) $\leftrightarrow$

Exercise

Construct a truth table for each of these compound propositions.

a) $p \wedge \neg p$

b) $p \vee \neg p$

c) $(p \vee \neg q) \rightarrow q$

d) $(p \vee q) \rightarrow (p \wedge q)$

e) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

$p \quad q \quad \bar{p} \quad \bar{q} \quad p \wedge \bar{p} \quad p \vee \bar{p} \quad p \vee q \quad p \wedge q$

T T T T

Definition

- Computers represent information using bits.
- A bit is a symbol with two possible values, namely, 0(zero) and 1(one).
- Bit operations: $x \wedge y$(AND), $x \vee y$ (OR), $x \oplus y$(XOR) (x, y are two bits.)

TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1

Definition

A bit string is a sequence of zero or more bits. The length of this string is the number of bits in the string.

Example

101 is a bit string of length 3.

Definition

Operations of bit string: bitwise OR, bitwise AND, bitwise XOR.

Example

Find the bitwise OR, bitwise AND and bitwise XOR of the bit strings 01 1011 0110 and 11 0001 1101.

Solution

01 1011 0110	
11 0001 1101	
11 1011 1111	bitwise *OR*
01 0001 0100	bitwise *AND*
10 1010 1011	bitwise *XOR*

Exercise

Find the bitwise OR, bitwise AND, and bitwise XOR of each of these pairs of bit strings.

a) 101 1110, 010 0001

b) 1111 0000, 1010 1010

Câu sau đây có ý gì?
"Nếu trời mưa, tôi ở nhà và trời không mưa, tôi cũng ở nhà."

- Tautology and Contradiction
- Logical Equivalences
- De Morgan's Laws

Tautology

Definition

A tautology is a compound proposition that is always true

Example

$p \vee \overline{p}$ is always true.

Definition

A tautology is a compound proposition that is always true

Example

$p \vee \overline{p}$ is always true.

p	$\overline{p}$	p $\vee \overline{p}$
T	F	T
F	T	T

Therefore, $p \vee \overline{p}$ is a tautology

Contradiction

Definition

A contradiction is a compound proposition that is always false.

Example

$p \wedge \overline{p}$ is always false.

Definition

A contradiction is a compound proposition that is always false.

Example

$p \wedge \overline{p}$ is always false.

p	$\overline{p}$	p $\wedge \overline{p}$
T	F	F
F	T	F

Therefore, $p \wedge \overline{p}$ is a contradiction.

Definition

The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology (p, q are both True or are both False).
The notation $p \equiv q$ denotes that p and q are logically equivalent.

Example

Show that $\neg(p \land q)$ and $\neg p \lor \neg q$ are logically equivalent.

Example

Show that $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

TABLE 3 **Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.**

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

1.2.2- Logical Equivalences...

Equivalence	Name
$p \wedge T \equiv p$ $\quad$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $\quad$ $p \wedge F \equiv F$	Domination Laws
$p \vee p \equiv p$ $\quad$ $p \wedge p \equiv p$	Idempotent Laws
$\neg(\neg p) \equiv p$	Double Negation Laws
$p \vee q \equiv q \vee p$ $\quad$ $p \wedge q \equiv q \wedge p$	Commutative Laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Laws

1.2.2- Logical Equivalences…

Equivalence	**Name**
$\neg (p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan Laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption Laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation Laws

De Morgan's Laws

Theorem

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

$$\neg(p \lor q) \equiv \neg p \land \neg q$$

Example Find the negation
"He is an actor and he is rich"

De Morgan's Laws

Theorem

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Example Find the negation
"He is an actor and he is rich"
Solution
He is not an actor or he isn't rich".

Theorem

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Example Find the negation

"He is an actor and he is rich"

Solution

He is not an actor or he isn't rich".

Example Find the negation

"John has a cellphone or he has a laptop"

Theorem

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Example Find the negation

"He is an actor and he is rich"

Solution

He is not an actor or he isn't rich".

Example Find the negation

"John has a cellphone or he has a laptop"

Solution

"John doesn't have a cellphone and doesn't have a laptop. "

Problem Find the negation
"If it rains, then I stay at home".

Theorem

$p \to q \equiv \neg p \vee q.$

$\overline{p \to q} \equiv \overline{\overline{p} \vee q} \equiv \overline{\overline{p}} \wedge \overline{q} \equiv p \wedge \overline{q}$

Let p: It rains
q: I stay at home
$p \to q$: "If it rains, then I stay at home".
$\overline{p \to q} \equiv p \wedge \overline{q}$
The answer is: "It rains and I don't stay at home"

Exercise

Prove that

$$(p \to q) \land (\overline{p} \to q) \equiv q$$

Problem

Tìm mệnh đề phủ định các mệnh đề sau:

1) "Mọi bông hồng đều đẹp"

2) "Có một chiếc xe hơi giá rẻ"

Predicates

Example

Let $P(x)$: "$x > 3$".

Predicates

Example

Let $P(x)$: "$x > 3$".

P is the propositional function.

Predicates

Example

Let $P(x)$: "$x > 3$".

P is the propositional function.

$P(x)$ is the value of the propositional function P at x.

Predicates

Example

Let $P(x)$: "$x > 3$".

P is the propositional function.

$P(x)$ is the value of the propositional function P at x.

$P(2)$:

Predicates

Example

Let $P(x)$: "$x > 3$".

P is the propositional function.

$P(x)$ is the value of the propositional function P at x.

$P(2) : 2 > 3$,

Predicates

Example

Let $P(x)$: "$x > 3$".

P is the propositional function.

$P(x)$ is the value of the propositional function P at x.

$P(2) : 2 > 3$,

$P(2)$ is F

Predicates

Example

Let $P(x)$: "$x > 3$".

P is the propositional function.

$P(x)$ is the value of the propositional function P at x.

$P(2) : 2 > 3$,

$P(2)$ is F

$P(4) :$

Predicates

Example

Let $P(x)$: "$x > 3$".

P is the propositional function.

$P(x)$ is the value of the propositional function P at x.

$P(2) : 2 > 3$,

$P(2)$ is F

$P(4) : 4 > 3$

Predicates

Example

Let $P(x)$: "$x > 3$".

P is the propositional function.

$P(x)$ is the value of the propositional function P at x.

$P(2) : 2 > 3$,

$P(2)$ is F

$P(4) : 4 > 3$

$P(4)$ is T

Example

$Q(x, y)$: "$x = y + 3$".

What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$?

Example

$Q(x, y)$: "$x = y + 3$".

What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$?

Solution

$Q(1, 2)$:

Example

$Q(x, y)$: "$x = y + 3$".

What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$?

Solution

$Q(1, 2)$: "$1 = 2 + 3$"

Example

$Q(x, y)$: "$x = y + 3$".

What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$?

Solution

$Q(1, 2)$: "$1 = 2 + 3$"

$Q(1, 2)$ is F

Example

$Q(x, y)$: "$x = y + 3$".

What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$?

Solution

$Q(1, 2)$: "$1 = 2 + 3$"

$Q(1, 2)$ is F

$Q(3, 0)$:

Example

$Q(x, y)$: "$x = y + 3$".

What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$?

Solution

$Q(1, 2)$: "$1 = 2 + 3$"

$Q(1, 2)$ is F

$Q(3, 0)$: "$3 = 0 + 3$"

Example

$Q(x, y)$: "$x = y + 3$".

What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$?

Solution

$Q(1, 2)$: "$1 = 2 + 3$"

$Q(1, 2)$ is F

$Q(3, 0)$: "$3 = 0 + 3$"

$Q(3, 0)$ is T

Exercise

1. Let $P(x)$: "$x \leq 4$".

What are these truth values?

a) $P(0)$

b) $P(4)$

c) $P(6)$

Exercise

Let $P(x)$: "the word x contains the letter a".

What are these truth values?

a) P(orange)

b) P(lemon)

c) P(true)

d) P(false)

Definition

The universal quantification of $P(x)$ is the statement

$\forall x P(x)$: "$P(x)$ for all values of x in the domain"

$\forall$: "for every", "for all" ("với mỗi", "với mọi", "với tất cả")

Example

$P(x)$: "$x + 1 > x$", domain consists of all real numbers.

$\forall x P(x)$

Definition

The universal quantification of $P(x)$ is the statement
$\forall x P(x)$: "$P(x)$ for all values of x in the domain"
$\forall$: "for every", "for all" ("với mỗi", "với mọi", "với tất cả")

Example

$P(x)$: "$x + 1 > x$", domain consists of all real numbers.
$\forall x P(x) \equiv \forall x(x + 1 > x)$:

Definition

The universal quantification of $P(x)$ is the statement
$\forall x P(x)$: "$P(x)$ for all values of x in the domain"
$\forall$: "for every", "for all" ("với mỗi", "với mọi", "với tất cả")

Example

$P(x)$: "$x + 1 > x$", domain consists of all real numbers.
$\forall x P(x) \equiv \forall x (x + 1 > x)$:
"for every real number x, $x + 1 > x$ "
$\forall x P(x)$ is T

Definition

The universal quantification of $P(x)$ is the statement
$\forall x P(x)$: "$P(x)$ for all values of x in the domain"

Example

$P(x)$: "x reads books everyday", domain for x consists of all student.
$\forall x P(x)$

Definition

The universal quantification of $P(x)$ is the statement
$\forall x P(x)$: "$P(x)$ for all values of x in the domain"

Example

$P(x)$: "x reads books everyday", domain for x consists of all student.

$\forall x P(x) \equiv \forall x$("x read books every day"):

Definition

The universal quantification of $P(x)$ is the statement $\forall x P(x)$: "$P(x)$ for all values of x in the domain"

Example

$P(x)$: "x reads books everyday", domain for x consists of all student.

$\forall x P(x) \equiv \forall x$("x read books every day"):

"for every student x,x reads books everyday " "

Definition

The universal quantification of $P(x)$ is the statement
$\forall x P(x)$: "$P(x)$ for all values of x in the domain"

Example

$P(x)$: "x reads books everyday", domain for x consists of all student.
$\forall x P(x) \equiv \forall x$("x read books every day"):
"for every student x,x reads books everyday " "
$\forall x P(x)$: "Every student reads book everyday".

Example

Let $Q(x)$: "$x < 2$".

What is the truth values of the quantificaion $\forall x Q(x)$, where the domain consists of all real numbers?

Example

Let $Q(x)$: "$x < 2$".

What is the truth values of the quantificaion $\forall x Q(x)$, where the domain consists of all real numbers?

Solution

$\forall x Q(x)$:

Example

Let $Q(x)$: "$x < 2$".

What is the truth values of the quantificaion $\forall x Q(x)$, where the domain consists of all real numbers?

Solution

$\forall x Q(x)$: "for every real number $x, x < 2$".

Example

Let $Q(x)$: "$x < 2$".

What is the truth values of the quantificaion $\forall x Q(x)$, where the domain consists of all real numbers?

Solution

$\forall x Q(x)$: "for every real number $x, x < 2$".

So $\forall x Q(x)$ is F since $3 < 2$.

Exercise

Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n + 1 > n)$

b) $\forall n(3n \leq 4n)$

The Existential Quantifier

Definition

The existential quantification of $P(x)$ is the proposition
$\exists x P(x)$ "there exists an element x in the domain such that $P(x)$"
$\exists$: "exists", "some" ("tồn tại", "một vài")

Example

$P(x)$: "$x + 1 > x$", domain consists of all real numbers.
$\exists x P(x)$

Definition

The existential quantification of $P(x)$ is the proposition
$\exists x P(x)$ "there exists an element x in the domain such that $P(x)$"
$\exists$: "exists", "some" ("tồn tại", "một vài")

Example

$P(x)$: "$x + 1 > x$", domain consists of all real numbers.
$\exists x P(x) \equiv \exists x(x + 1 > x)$:

Definition

The existential quantification of $P(x)$ is the proposition
$\exists x P(x)$ "there exists an element x in the domain such that $P(x)$"
$\exists$: "exists", "some" ("tồn tại", "một vài")

Example

$P(x)$: "$x + 1 > x$", domain consists of all real numbers.
$\exists x P(x) \equiv \exists x(x + 1 > x)$: "There exists a real number x such that $x + 1 > x$."
$\exists x P(x)$ is T.

Definition

The existential quantification of $P(x)$ is the proposition $\exists x P(x)$: "there exists an element x in the domain such that $P(x)$"

Example

$P(x)$: "x reads books everyday", domain for x consists of all student.

$\exists x P(x)$:

Definition

The existential quantification of $P(x)$ is the proposition
$\exists x P(x)$: "there exists an element x in the domain such that $P(x)$"

Example

$P(x)$: "x reads books everyday", domain for x consists of all student.

$\exists x P(x)$: "There exists a student x such that x reads books everyday "

Definition

The existential quantification of $P(x)$ is the proposition
$\exists x P(x)$: "there exists an element x in the domain such that $P(x)$"

Example

$P(x)$: "x reads books everyday", domain for x consists of all student.
$\exists x P(x)$: "There exists a student x such that x reads books everyday "
$\exists x P(x)$:"There exists a student who reads book everyday".

Example

$P(x)$: "$x = x + 1$". What is the truth value of the quantification $\exists x P(x)$, where the domain consists of all real numbers?

Solution

$\exists x P(x)$:

Example

$P(x)$: "$x = x + 1$". What is the truth value of the quantification $\exists x P(x)$, where the domain consists of all real numbers?

Solution

$\exists x P(x)$:"there exists a real number x such that $x = x + 1$"

$\exists x P(x)$ is F.

Example

$P(x)$: "$x > 2$".

What is the truth value of $\exists x P(x)$, where the domain consists of the integers?

Solution

$\exists x P(x)$:

Example

$P(x)$: "$x > 2$".

What is the truth value of $\exists x P(x)$, where the domain consists of the integers?

Solution

$\exists x P(x)$:"there exists an integer x such that $x > 2$."

Example

$P(x)$: "$x > 2$".

What is the truth value of $\exists x P(x)$, where the domain consists of the integers?

Solution

$\exists x P(x)$:"there exists an integer x such that $x > 2$."

$\exists x P(x)$ is T since $3 > 2$.

Exercise

Determine the truth value of each of these statements if the domain consists of all integers.

a) $\exists n(2n = 3n)$

b) $\exists n(n = -n)$

Exercise

Let $P(x)$: "$x = x^2$", domain consists of the integers. What are these truth values?
$P(0) \quad P(1) \quad P(2) \quad P(-1) \quad \exists x P(x) \quad \forall x P(x).$

Exercise

Let $P(x)$: "$x = x^2$", domain consists of the integers. What are these truth values?

$P(0) \quad P(1) \quad P(2) \quad P(-1) \quad \exists x P(x) \quad \forall x P(x)$.

Solution

$\exists x P(x)$: " there exists an integer x such that $x = x^2$"

$\exists x P(x)$ is True since $1 = 1^2$

$\forall x P(x)$: "For every integer x, $x = x^2$"

$\forall x P(x)$ is F

Exercise

Let $P(x)$:" x spends more than five hours every weekday in class", where the domain for x consists of all students. Translate these statements into English

a) $\exists x P(x)$

b) $\forall x P(x)$

c) $\exists x \neg P(x)$

d) $\forall x \neg P(x)$

Exercise

Translate these statements into English , where $C(x)$: "x is a comedian", $F(x)$: "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \to F(x))$

b) $\forall x(C(x) \wedge F(x))$

c) $\exists x(C(x) \to F(x))$

d) $\exists x(C(x) \wedge F(x))$

Example

What do the statement $\forall x < 0(x^2 > 0)$ means, where the domain consists of real numbers?

Solution

$\forall x < 0(x^2 > 0)$:"for every real number x with $x < 0$, then $x^2 > 0$"

Example

What do the statement $\forall x < 0(x^2 > 0)$ means, where the domain consists of real numbers?

Solution

$\forall x < 0(x^2 > 0)$:"for every real number x with $x < 0$, then $x^2 > 0$" or "The square of a negative real number is positive".

Example

What do the statement $\forall y \neq 0 (y^3 \neq 0)$ mean where the domain consists of real numbers?

Solution

$\forall y \neq 0 (y^3 \neq 0)$

Example

What do the statement $\forall y \neq 0(y^3 \neq 0)$ mean where the domain consists of real numbers?

Solution

$\forall y \neq 0(y^3 \neq 0)$: "for every real number y with $y \neq 0$, $y^3 \neq 0$"

Example

What do the statement $\forall y \neq 0(y^3 \neq 0)$ mean where the domain consists of real numbers?

Solution

$\forall y \neq 0(y^3 \neq 0)$: "for every real number y with $y \neq 0$, $y^3 \neq 0$".
or: "The cube of every nonzero real number is nonzero".

Negating Quantified Expression

Theorem

$$\overline{\forall x P(x)} \equiv \exists x \overline{P(x)}$$

Example Tìm phủ định của câu "Mọi người Việt Nam đều thích phở".

Solution "Mọi người Việt Nam đều thích phở"

"Mọi người Việt Nam x, x thích phở "

Negating Quantified Expression

Theorem

$$\overline{\forall x P(x)} \equiv \exists x \overline{P(x)}$$

Example Tìm phủ định của câu "Mọi người Việt Nam đều thích phở".

Solution "Mọi người Việt Nam đều thích phở"

"Mọi người Việt Nam x, x thích phở "

Thay "mọi" thành $\forall$ và đặt $P(x)$: "x thích phở".

Negating Quantified Expression

Theorem

$$\overline{\forall x P(x)} \equiv \exists x \overline{P(x)}$$

Example Tìm phủ định của câu "Mọi người Việt Nam đều thích phở".

Solution "Mọi người Việt Nam đều thích phở"
"Mọi người Việt Nam x, x thích phở "
Thay "mọi" thành $\forall$ và đặt $P(x)$: ''x thích phở".
$\equiv \forall x P(x)$
Theo công thức: $\overline{\forall x P(x)} \equiv \exists x \overline{P(x)}$
$\exists x \overline{P(x)}$: "tồn tại người Việt Nam x, x không thích phở".
"Có một người VN không thích phở"

Example

Find negation of the statement $\forall x(x^2 > x)$?

Example

Find negation of the statement $\forall x(x^2 > x)$?

Solution

$$\overline{\forall x(x^2 > x)} \equiv \exists x\overline{(x^2 > x)} \equiv \exists x(x^2 \leq x)$$

Theorem

$$\overline{\exists x P(x)} \equiv \forall x \overline{P(x)}$$

Example Tìm câu phủ định của mệnh đề "Có một con khỉ biết đếm".

Solution

"Có một con khỉ biết đếm"

"Có một con khỉ x, x biết đếm"

Thay "có một" thành $\exists$, và đặt $P(x)$: "x biết đếm"

$\equiv \exists x P(x)$, domain $x \in$ những con khỉ.

Dùng công thức $\overline{\exists x P(x)} \equiv \forall x \overline{P(x)}$

$\forall x \overline{P(x)}$: "Với mọi con khỉ x, x không biết đếm"

"Mọi con khỉ không biết đếm."

Example

What is the negation of the statement $\exists x(x^2 = 2)$?

Example

What is the negation of the statement $\exists x(x^2 = 2)$?

Solution

$$\overline{\exists x(x^2 = 2)} \equiv \forall x\overline{(x^2 = 2)} \equiv \forall x(x^2 \neq 2)$$

Exercise

Tìm mệnh đề phủ định các mệnh đề sau:

1) "Mọi bông hồng đều đẹp"

2) "Có một con mèo có 3 chân"

3) "Mọi xe hơi đều đắt tiền"

4) "Có một chiếc xe máy giá nhiều hơn 200 triệu"

5) "Mọi người đều biết đi xe đạp và thích uống cà phê".

6) $\forall x(x > 1 \vee x < 0)$

Example

Biến đổi câu sau thành mệnh đề logic "tất cả học sinh trong lớp này đã học môn giải tích". Miền xác định là tập hợp tất cả con người

Example

Biến đổi câu sau thành mệnh đề logic "tất cả học sinh trong lớp này đã học môn giải tích". Miền xác định là tập hợp tất cả con người

Solution

"tất cả học sinh trong lớp này đã học môn giải tích"

Example

Biến đổi câu sau thành mệnh đề logic "tất cả học sinh trong lớp này đã học môn giải tích". Miền xác định là tập hợp tất cả con người

Solution

"tất cả học sinh trong lớp này đã học môn giải tích"

$\equiv$ "mỗi học sinh x trong lớp này, x đã học môn giải tích".

Example

Biến đổi câu sau thành mệnh đề logic "tất cả học sinh trong lớp này đã học môn giải tích". Miền xác định là tập hợp tất cả con người

Solution

"tất cả học sinh trong lớp này đã học môn giải tích"
$\equiv$ "mỗi học sinh x trong lớp này, x đã học môn giải tích".
$\equiv$ "Mỗi con người x sao cho nếu x là học sinh trong lớp này thì x đã học môn giải tích"

Example

Biến đổi câu sau thành mệnh đề logic "tất cả học sinh trong lớp này đã học môn giải tích". Miền xác định là tập hợp tất cả con người

Solution

"tất cả học sinh trong lớp này đã học môn giải tích"

$\equiv$ "mỗi học sinh x trong lớp này, x đã học môn giải tích".

$\equiv$ "Mỗi con người x sao cho nếu x là học sinh trong lớp này thì x đã học môn giải tích"

Thay "Mỗi" thành $\forall$, $P(x)$: "x là học sinh trong lớp này".

$Q(x)$: "x đã học môn giải tích"

Example

Biến đổi câu sau thành mệnh đề logic "tất cả học sinh trong lớp này đã học môn giải tích". Miền xác định là tập hợp tất cả con người

Solution

"tất cả học sinh trong lớp này đã học môn giải tích"
$\equiv$ "mỗi học sinh x trong lớp này, x đã học môn giải tích".
$\equiv$ "Mỗi con người x sao cho nếu x là học sinh trong lớp này thì x đã học môn giải tích"
Thay "Mỗi" thành $\forall$, $P(x)$: "x là học sinh trong lớp này".
$Q(x)$: "x đã học môn giải tích"
$\equiv \forall x(P(x) \rightarrow Q(x))$, domain: tập hợp tất cả con người.

Exercise

Biến đổi những câu sau đây thành mệnh đề logic với miền xác định là những sinh vật sống.

"Tất cả sư tử đều hung dữ"

"Một số sư tử không thích săn mồi"

"Một số sinh vật hung dữ không thích ăn thịt".

Solution

Solution

"Tất cả sư tử đều hung dữ"

Solution

"Tất cả sư tử đều hung dữ"

$\equiv$ "Mỗi sinh vật sống x, nếu x là sư tử thì x hung dữ"

Solution

"Tất cả sư tử đều hung dữ"

$\equiv$ "Mỗi sinh vật sống x, nếu x là sư tử thì x hung dữ"

Đặt $A(x)$:"x là sư tử", $B(x)$: "x hung dữ"

Solution

"Tất cả sư tử đều hung dữ"

$\equiv$ "Mỗi sinh vật sống x, nếu x là sư tử thì x hung dữ"

Đặt $A(x)$:"x là sư tử", $B(x)$: "x hung dữ"

$\equiv \forall x(A(x) \to B(x))$, domain: creatures.

"Một số sư tử không thích săn mồi"

"Một số sư tử không thích săn mồi"

$\equiv$ "Một số sinh vật sống x, x là sư tử và x không thích săn mồi"

Đặt $A(x)$: "x là sư tử", $B(x)$: "x thích săn mồi"

"Một số sư tử không thích săn mồi"

$\equiv$ "Một số sinh vật sống x, x là sư tử và x không thích săn mồi"

Đặt $A(x)$: "x là sư tử", $B(x)$: "x thích săn mồi"

$\equiv \exists x(A(x) \wedge \neg B(x))$

"Một số sinh vật hung dữ không thích ăn thịt"
$\equiv$ "Một số sinh vật x, x hung dữ và x không thích ăn thịt".
Đặt $A(x)$: "x hung dữ", $B(x)$: "x thích ăn thịt"
$\equiv \exists x(A(x) \wedge \neg B(x))$

Example

Assume that the domain for the variables x and y consists of all real numbers. What is the meaning of the following statement

$$\forall x \forall y (x + y = y + x)$$

Example

Assume that the domain for the variables x and y consists of all real numbers. What is the meaning of the following statement

$$\forall x \forall y (x + y = y + x)$$

Solution

$\forall x \forall y (x + y = y + x)$: "for every real number x

Example

Assume that the domain for the variables x and y consists of all real numbers. What is the meaning of the following statement

$$\forall x \forall y (x + y = y + x)$$

Solution

$\forall x \forall y (x + y = y + x)$: "for every real number $x$for every real number y,

Example

Assume that the domain for the variables x and y consists of all real numbers. What is the meaning of the following statement

$$\forall x \forall y(x + y = y + x)$$

Solution

$\forall x \forall y(x + y = y + x)$: "for every real number $x$for every real number y,: $x + y = y + x$"
or "$x + y = y + x$ for all real numbers x and y".
$\forall x \forall y(x + y = y + x)$ is T

Example

What is the meaning of the statement $\forall x \exists y (x + y = 0)$, where the domain for x and y consists of all real numbers?

Example

What is the meaning of the statement $\forall x \exists y(x + y = 0)$, where the domain for x and y consists of all real numbers?

Solution

"$\forall x \exists y(x + y = 0)$":for every real number x, there exists a real number y such that $x + y = 0$.

$\forall x \exists y(x + y = 0)$ is T

Example

Translate the statement into logical proposition "The sum of two positive integers is always positive"

Translating Mathematical Statements into Statements Involving Nested Quantifiers

Example

Translate the statement into logical proposition "The sum of two positive integers is always positive"

Solution

"The sum of two positive integers is always positive"

Example

Translate the statement into logical proposition "The sum of two positive integers is always positive"

Solution

"The sum of two positive integers is always positive"
$\equiv$ "for every integer x, for every integer y such that if $x > 0$ and $y > 0$, then $x + y > 0$

Example

Translate the statement into logical proposition "The sum of two positive integers is always positive"

Solution

"The sum of two positive integers is always positive"
$\equiv$ "for every integer x, for every integer y such that if $x > 0$ and $y > 0$, then $x + y > 0$
$\equiv \forall x \forall y((x > 0) \wedge (y > 0) \rightarrow x + y > 0)$

Example

Translate the statement into logical proposition "The sum of two positive integers is always positive"

Solution

"The sum of two positive integers is always positive"
$\equiv$ "for every integer x, for every integer y such that if $x > 0$ and $y > 0$, then $x + y > 0$
$\equiv \forall x \forall y((x > 0) \wedge (y > 0) \rightarrow x + y > 0)$
where the domain for all variables consists of integers.

Exercise

Let

T(x, y) = " the student x takes the class y",

where x represents a student in a university, and y represents a class.

Translate the logical expression into a sentence:

$$\forall y \exists x T(x, y)$$

a.	For each class there is at least a student taking it
b.	There is a student taking all classes
c.	Every student is taking at least one class
d.	There is a class that every student taking it

Exercise

Suppose $L(x,y)$ is the statement

"x buys Christmas gifts for y",

where **x** and **y** are members of a family.

Translate the statement into logical expression

"*Each family member buys Christmas gifts for any other member but not for himself or herself*"

(i) $\exists x(\neg L(x,x))$

(ii) $\forall x(\neg L(x,x))$

(iii) $\forall x(\forall y L(x,y))$

(iv) $\forall x(\forall y((y \neq x) \leftrightarrow L(x,y)))$

Example

Find $\overline{\forall x \exists y(xy = 1)}$ so that no negation precedes a quantifer.

Solution

$$\begin{aligned} &\overline{\forall x \exists y(xy = 1)} \\ \equiv\ &\exists x \overline{\exists y(xy = 1)} \\ \equiv\ &\exists x \forall y \overline{(xy = 1)} \\ \equiv\ &\exists x \forall y (xy \neq 1) \end{aligned}$$

Exercise

1. Find $\overline{\forall x \Big(P(x) \to Q(x) \Big)}$

2. Find $\overline{\exists x \forall y \Big(P(x,y) \to Q(x,y) \Big)}$

3. Find the negation of the statement
"There exists a student who if he graduates, then he will find a good job".

Is the following argument is valid?
"If it rains, then I stay at home"
"It rains"
Therefore, "I stay at home"

Is the following argument is valid?
"If it rains, then I stay at home"
"It doesn't rain"
Therefore, "I don't stay at home"

Argument, Conclusion, Premises, Valid

Definition

- An argument in propositional logic is a sequence of propositions.
- Final proposition is called the conclusion.
- All but the final proposition are called premises.
- An argument is valid if the truth of all its premises implies that the conclusion is true.

Definition

- An argument in propositional logic is a sequence of propositions.
- Final proposition is called the conclusion.
- All but the final proposition are called premises.
- An argument is valid if the truth of all its premises implies that the conclusion is true.

Example

"If it rains, then I stay at home"

"It rains"

Therefore, "I stay at home"

Valid Argument: Modus ponens

Theorem

The following argument is valid

$$\begin{array}{l} p \to q \\ p \\ \therefore q \end{array}$$

Theorem

The following argument is valid

$$p \to q$$
$$p$$
$$\therefore q$$

Example The following argument is valid.

"If you work hard, you pass MAD"

"you work hard"

Therefore, "you pass MAD".

Valid Argument: Modus tollens

Theorem

The following argument is valid

$$\begin{aligned} p \rightarrow q \\ \neg q \\ \therefore \neg p \end{aligned}$$

Valid Argument: Modus tollens

Theorem

The following argument is valid

$$\begin{aligned} p &\to q \\ &\neg q \\ \therefore\ &\neg p \end{aligned}$$

Example The following argument is valid.
"If you are a singer, then you sing well"
"You don't sing well"
Therefore, "you are not a singer".

Valid Argument: Hypothetical syllogism

Theorem

The following argument is valid.

$$p \to q$$
$$q \to r$$
$$\therefore p \to r$$

Valid Argument: Hypothetical syllogism

Theorem

The following argument is valid.

$$\begin{gathered} p \to q \\ q \to r \\ \therefore p \to r \end{gathered}$$

Example

"If it rains today, then we will not have dinner outdoor today"
"If we don't have dinner outdoor today, we will have dinner at home today "
"If it rains today, we will have dinner at home today"

Theorem

The following argument is valid.

$$\begin{array}{c} p \to q \\ q \to r \\ \therefore p \to r \end{array}$$

Example

"If it rains today, then we will not have dinner outdoor today"
"If we don't have dinner outdoor today, we will have dinner at home today "
"If it rains today, we will have dinner at home today"

Theorem

The following argument is valid.

$$\begin{aligned} &p \vee q \\ &\neg p \\ &\therefore q \end{aligned}$$

Valid Argument: Disjunctive syllogism

Theorem

The following argument is valid.

$$
\begin{aligned}
&p \vee q \\
&\neg p \\
&\therefore q
\end{aligned}
$$

Example 1

"Today is Tuesday or Wednesday"
"Today is not Tuesday"
Therefore "Today is Wednesday"

Valid Argument: Addition

Theorem

The following argument is valid.

$$\begin{array}{c} p \\ \therefore p \vee q \end{array}$$

Valid Argument: Addition

Theorem

The following argument is valid.

$$p$$
$$\therefore p \vee q$$

Example

"He is tall"

Therefore, he is tall or he can sing well.

Theorem

The following argument is valid.

$$p \wedge q$$
$$\therefore p$$

Valid Argument: Simplification

Theorem

The following argument is valid.

$$p \wedge q$$
$$\therefore p$$

Example The following argument is valid.
He sings well and can play guitars.
Therefore he sings well.

Valid Argument: Conjunction

Theorem

The following argument is valid.

$$p$$
$$q$$
$$\therefore p \wedge q$$

Valid Argument: Conjunction

Theorem

The following argument is valid.

$$\begin{array}{c} p \\ q \\ \therefore p \wedge q \end{array}$$

Example The following argument is valid

She can speak English

She can speak French

Therefore, she can speak English and French.

Valid Argument: Resolution

Theorem

The following argument is valid.

$$p \vee q$$
$$\neg p \vee r$$
$$\therefore q \vee r$$

Theorem

The following argument is valid.

$$p \vee q$$
$$\neg p \vee r$$
$$\therefore q \vee r$$

Example

"it is not snowing or John is skiing"
"It is snowing or Jack is playing football"
Therefore, "John is skiing or Jacking is playing football"

Theorem

The following argument is NOT VALID

$$\begin{aligned} &p \rightarrow q \\ &\neg p \\ \therefore\ &\neg q \end{aligned}$$

Theorem

The following argument is NOT VALID

$$\begin{array}{l} p \to q \\ \neg p \\ \therefore \neg q \end{array}$$

Example The following argument is NOT VALID.

"Nếu trời mưa thì tôi nghỉ học"

"Trời không mưa"

Vậy tôi không nghỉ học

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism

Summary

p $\therefore\ p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore\ p$	$(p \wedge q) \rightarrow p$	Simplification
p q $\therefore\ p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Exercise

What are conclusions can be drawn if premises are given as below?

"If I win the lottery, then I buy a house"

"If I buy a house, then I have a wife"

"I am single"

Exercise

What are conclusions can be drawn if premises are given as below?

"If I eat spicy foods, then I have strange dreams"

"I have strange dreams if there is thunder while I sleep"

"I did not have strange dream."

Exercise

What are conclusions can be drawn if the premises are given as below?

"I am either clever or lucky"

"I am not lucky"

"If I am lucky, then I will win the lottery"

Exercise

What are conclusions can be drawn if premises are given: "I can speak English or Germany"
"If I can speak Germany, I will study a master degree in German"
"I do not study a master degree in Germany" ?
A. I can speak both English and Germany
B. I can speak Germany but cannot speak English
C. No conclusion is drawn
D. I can speak English but cannot speak Germany.

Theorem

The following argument is valid

$\forall x P(x)$, $x \in$ *domain* D

$\therefore P(c)$, c *is an element of domain* D.

Example 1 The following argument is valid

"All Vietnamese people like Phở "

Therefore, Tuấn like Phở.

Theorem

The following argument is valid

$\forall x(P(x) \to Q(x))$

$P(a)$, *a is a particular element in the domain*

$\therefore Q(a)$

Example The following argument is valid.

"Everyone in this class has taken Calculus"

"Tuan is in this class"

Therefore, Tuan has taken Calculus

Example Show that the following argument is valid.
"Everyone in this class has taken MAD"
"Tuan is in this class"
Therefore, Tuan has taken MAD.

Solution
$P(x)$: "x is in this class"
$Q(x)$: "x has taken MAD"
"Everyone in this class has taken MAD": $\forall x(P(x) \to Q(x))$
"Tuan is in this class":
$\therefore$ "Tuan has taken MAD" $Q(Tuan)$

Theorem

The following argument is valid

$\forall x(P(x) \rightarrow Q(x))$

$\neg Q(a)$, *a is a particular element in the domain*

$\therefore \neg P(a)$

Example The following argument is valid.

"Everyone in this class has taken English"

"Tuan has not taken English"

Therefore, Tuan is not in this class

Example

Show that the following argument is valid.

"Everyone in this class has taken MAD"

"Tuan has not taken MAD"

Therefore, Tuan is not in this class .

Solution

$P(x)$: "x is in this class"

$Q(x)$: "x has taken MAD"

"Everyone in this class has taken MAD":

Example

Show that the following argument is valid.

"Everyone in this class has taken MAD"

"Tuan has not taken MAD"

Therefore, Tuan is not in this class .

Solution

$P(x)$: "x is in this class"

$Q(x)$: "x has taken MAD"

"Everyone in this class has taken MAD":

$\forall x(P(x) \rightarrow Q(x))$

Example

Show that the following argument is valid.

"Everyone in this class has taken MAD"

"Tuan has not taken MAD"

Therefore, Tuan is not in this class .

Solution

$P(x)$: "x is in this class"

$Q(x)$: "x has taken MAD"

"Everyone in this class has taken MAD":

$\forall x(P(x) \to Q(x))$

"Tuan has not taken MAD":

Example

Show that the following argument is valid.

"Everyone in this class has taken MAD"

"Tuan has not taken MAD"

Therefore, Tuan is not in this class .

Solution

$P(x)$: "x is in this class"

$Q(x)$: "x has taken MAD"

"Everyone in this class has taken MAD":

$\forall x(P(x) \rightarrow Q(x))$

"Tuan has not taken MAD":$\neg Q(Tuan)$

$\therefore$ "Tuan is not in this class"

Exercise

What are conclusions can be drawn if the following premises are given as below?

"Every computer science major has a personal computer "

"Ralph does not have a personal computer"

"Ann has a personal computer"

Exercise

What are conclusions can be drawn if the following premises are given as below?

"All insects have six legs"

"Dragonflies are insects"

"Spiders do not have six legs "

Exercise

Determine whether each of the following arguments is valid or not valid.
a) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.

b) Everyone who eats granola everyday is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

c) If Mai knows French, Mai is smart. But Mai doesn't know French. So she is not smart.